QCD Topology, axions and electromagnetic fields

Lattice 2024, Liverpool, 29th July 2024

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What is topology in QCD?

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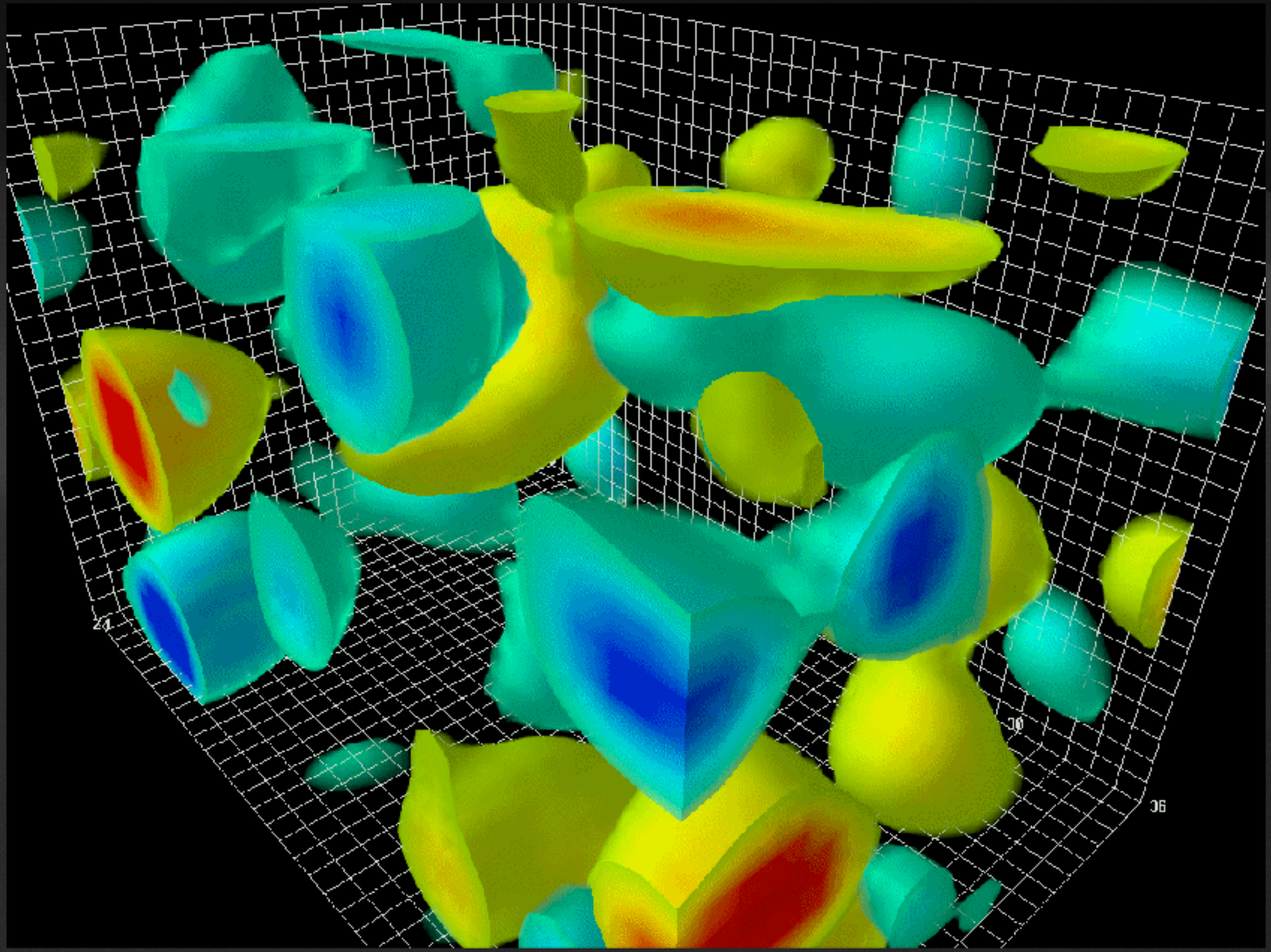
Non-trivial vacuum !



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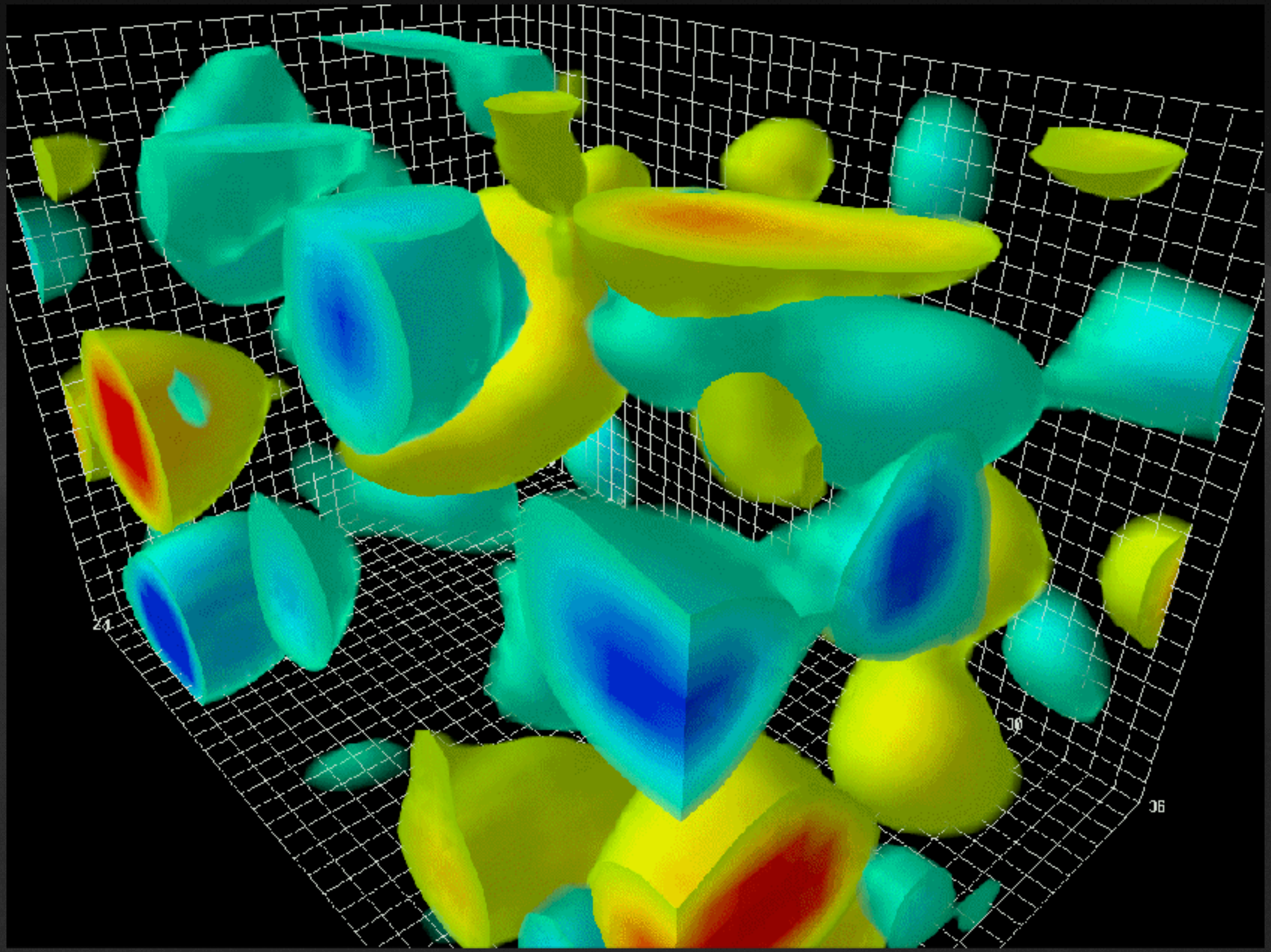
Classification of gluon field configurations

$Q_{\rm top}$



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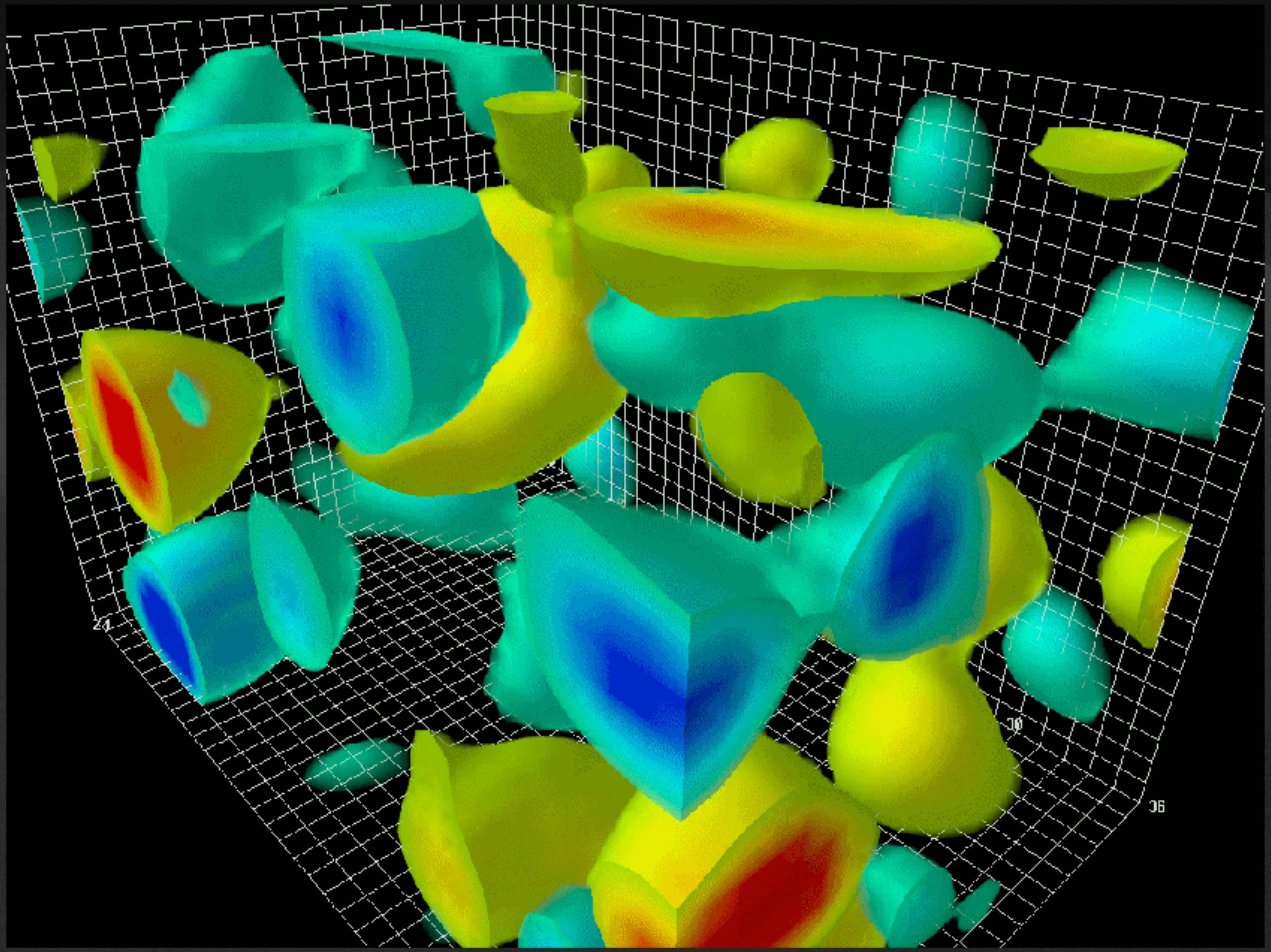
Chirality + magnetic fields



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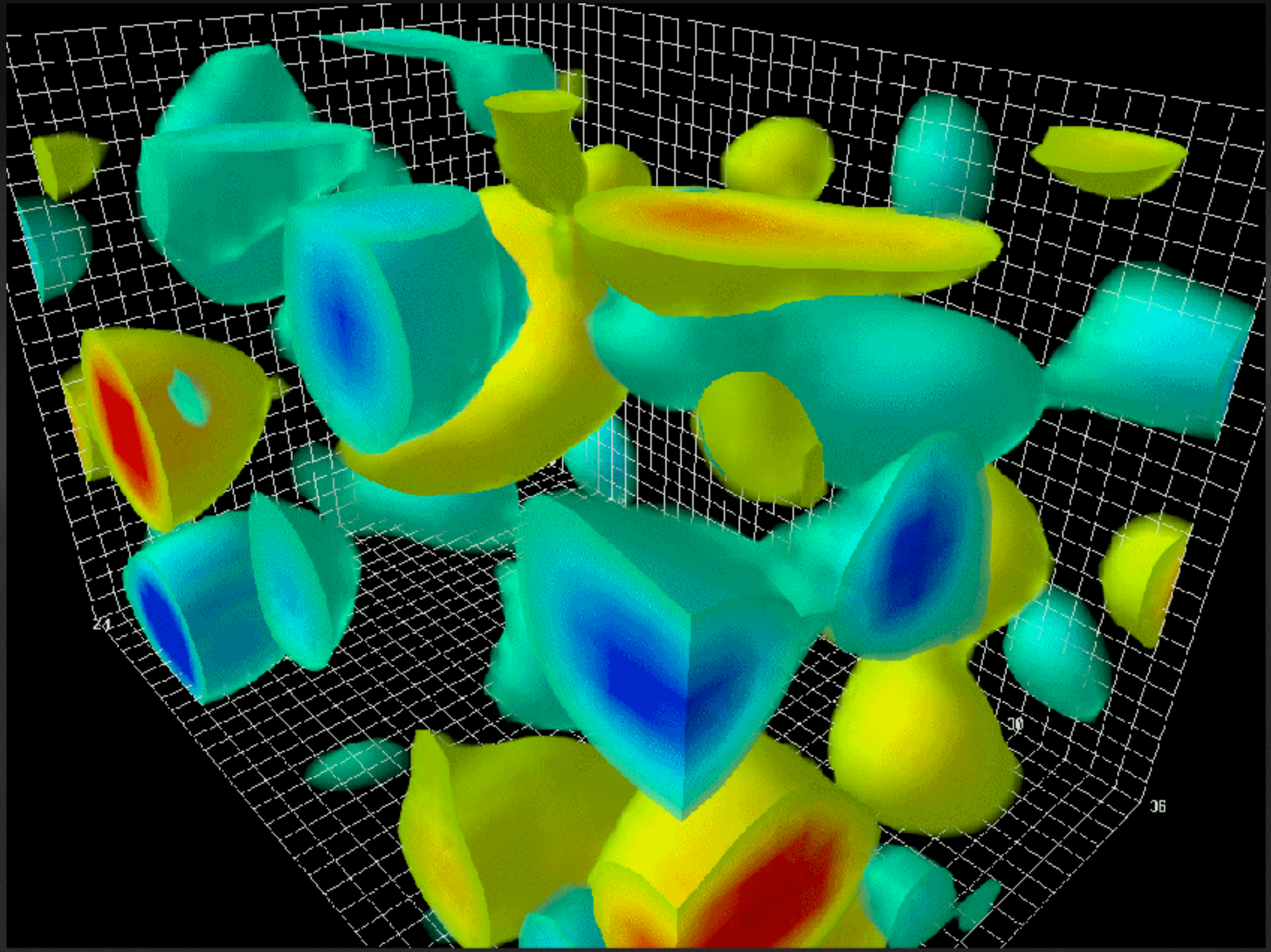
Chiral Magnetic Effect (CME)



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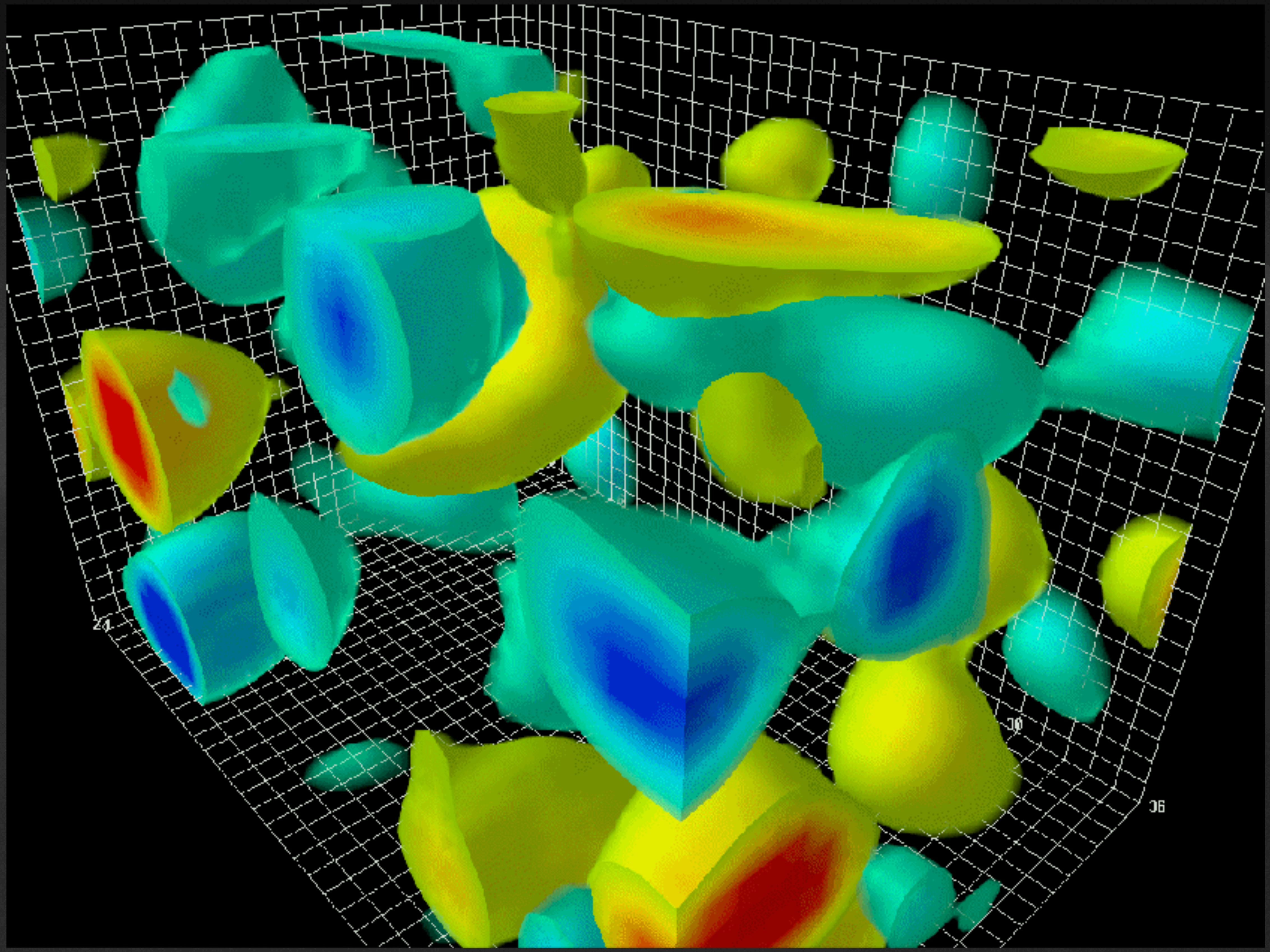
Intimately related to axions

$$\theta(x) \equiv \frac{a(x)}{f_a}$$



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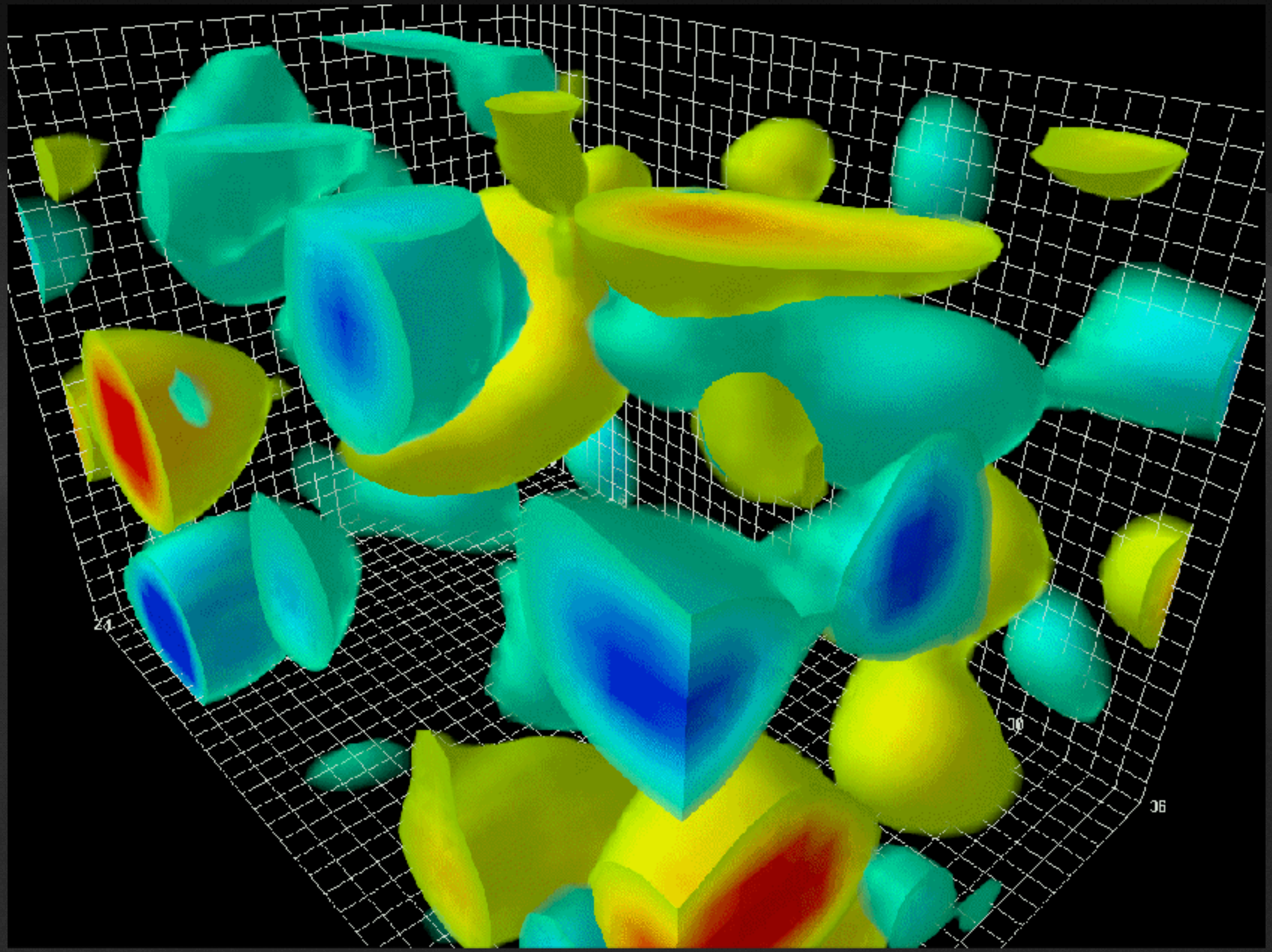

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$$Q_{\text{top}} = N_R - N_L$$

Index theorem

Outline

- ▶ Topology with magnetic fields
  - Topological susceptibility
- ▶ Topology with electromagnetic fields
  - Axion-photon coupling

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Non-perturbatively + finite T

Lattice QCD

That's our goal!

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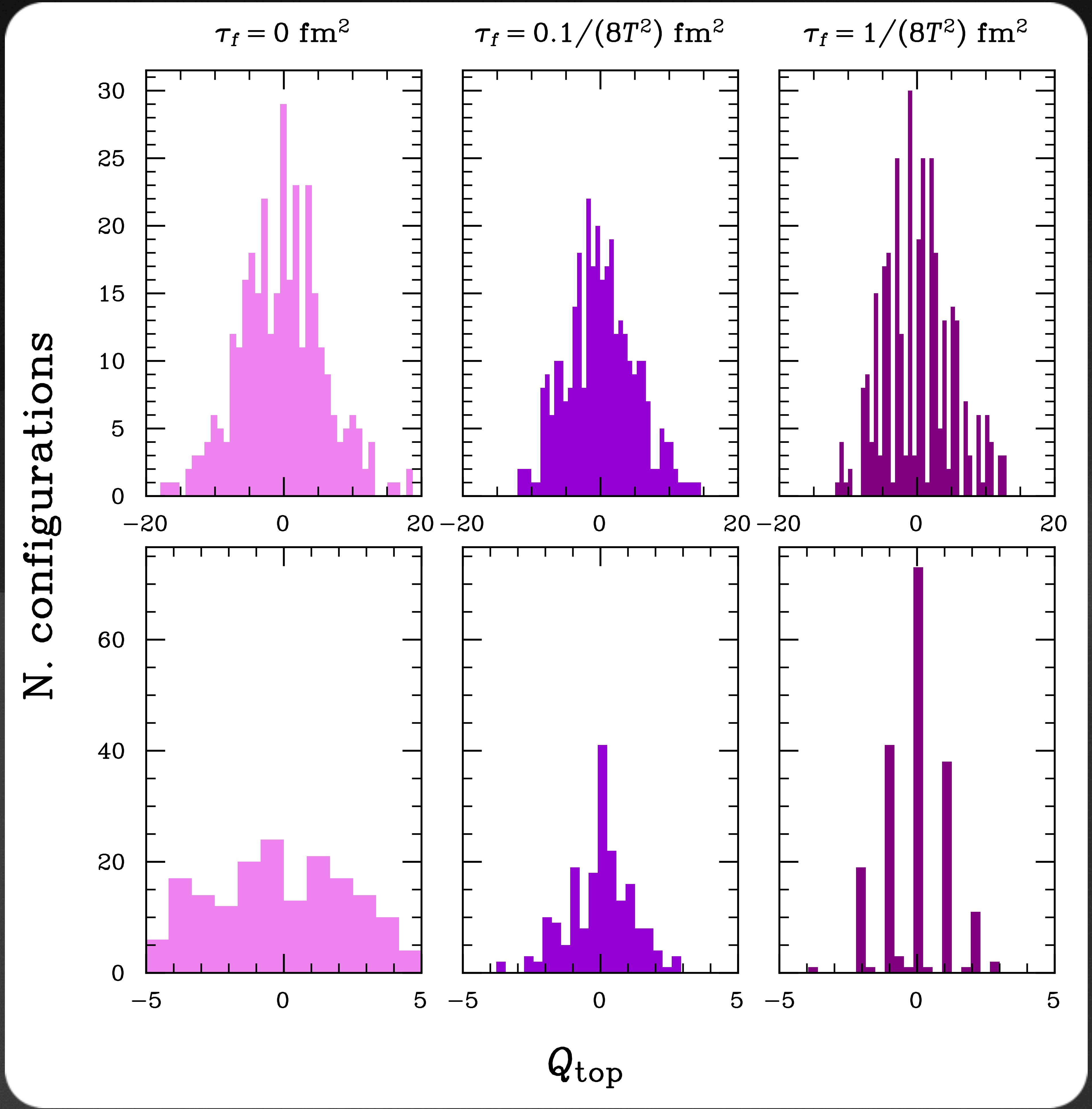
2+1 improved staggered quarks at the physical point

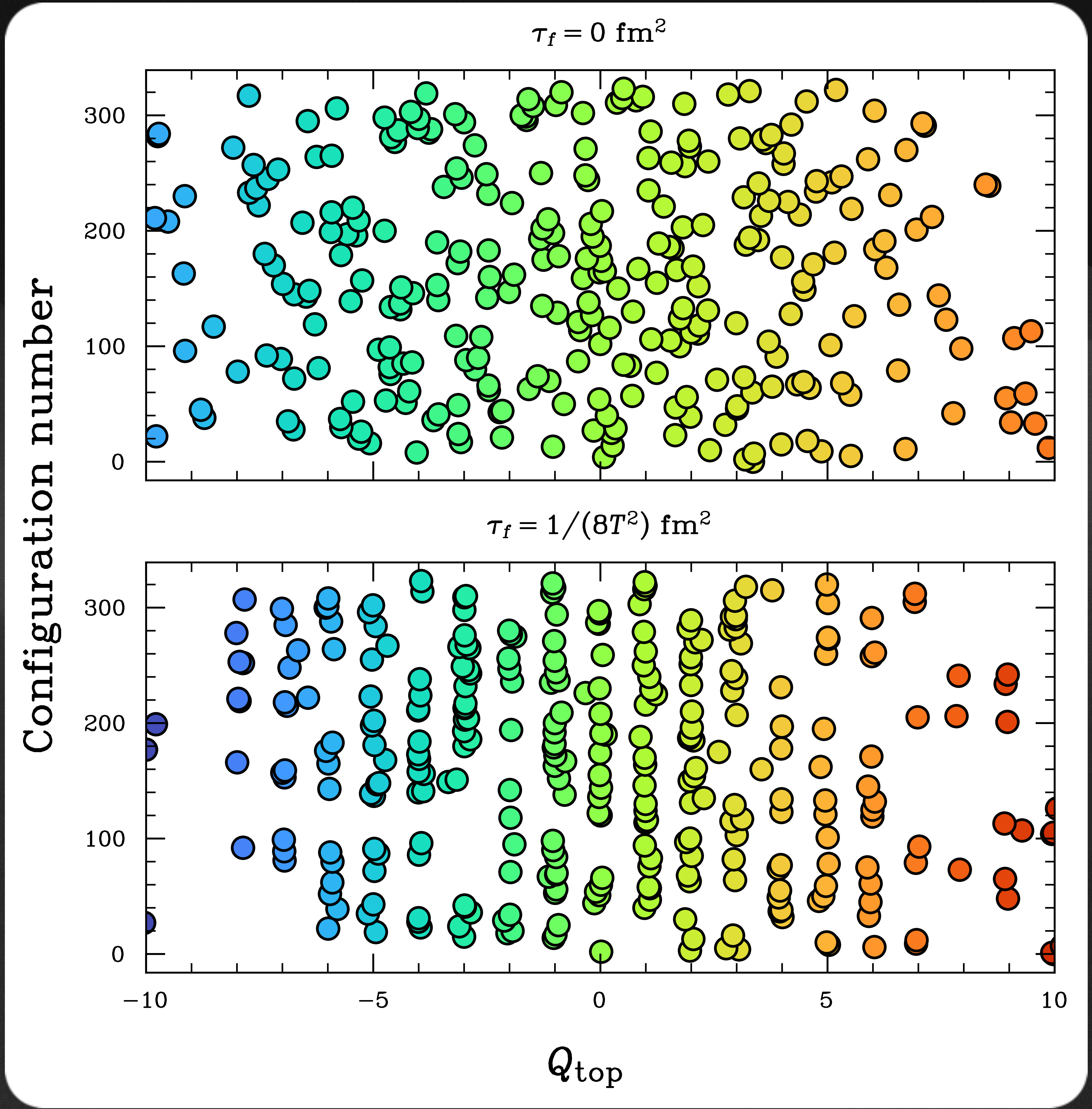
$eB = 0.5\,\mathrm{GeV}^2$

Topology on the lattice

$T = 150\,\mathrm{MeV}$

$T = 212\,\mathrm{MeV}$





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χ_{top} at finite magnetic field: low T

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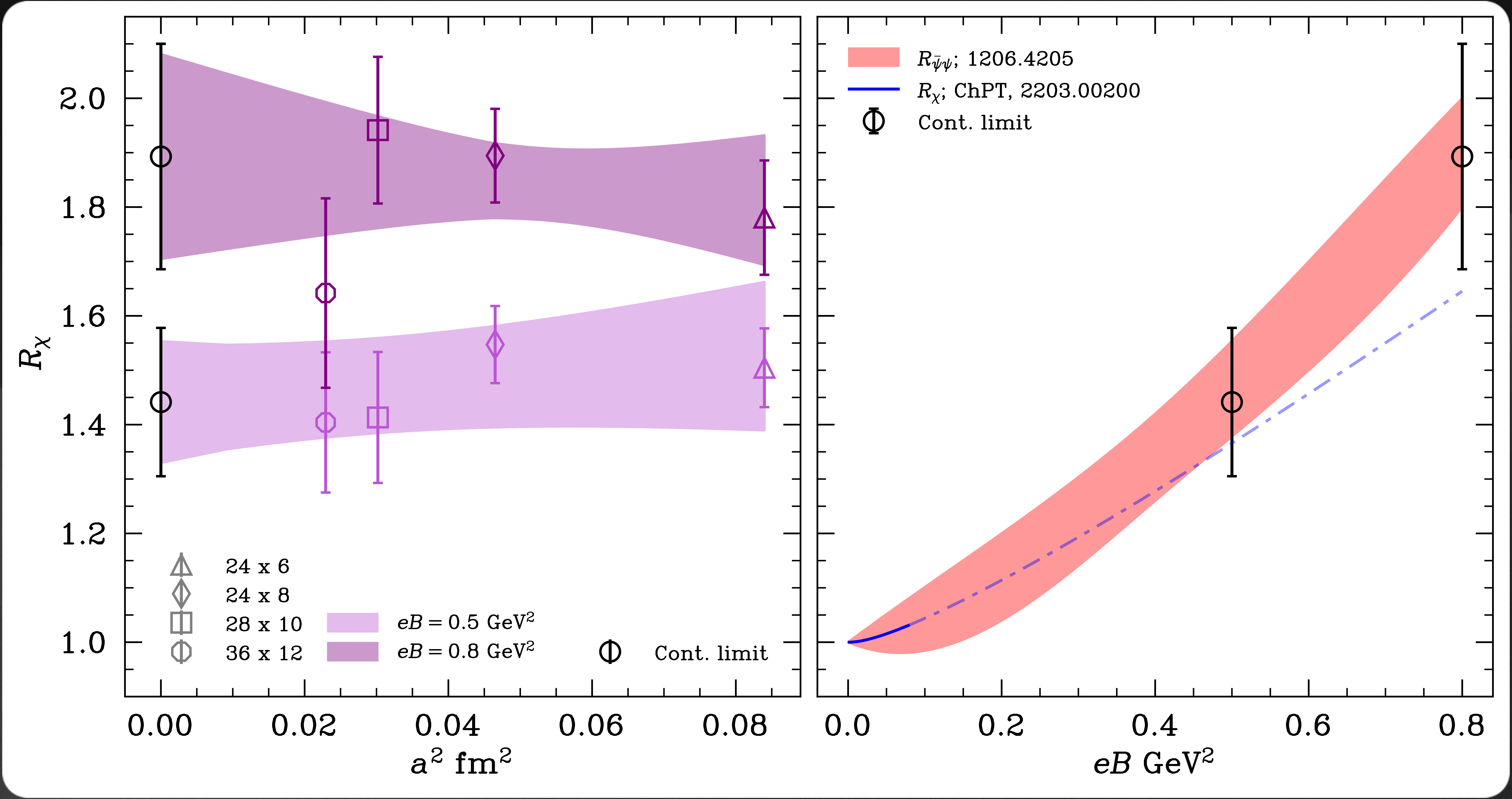
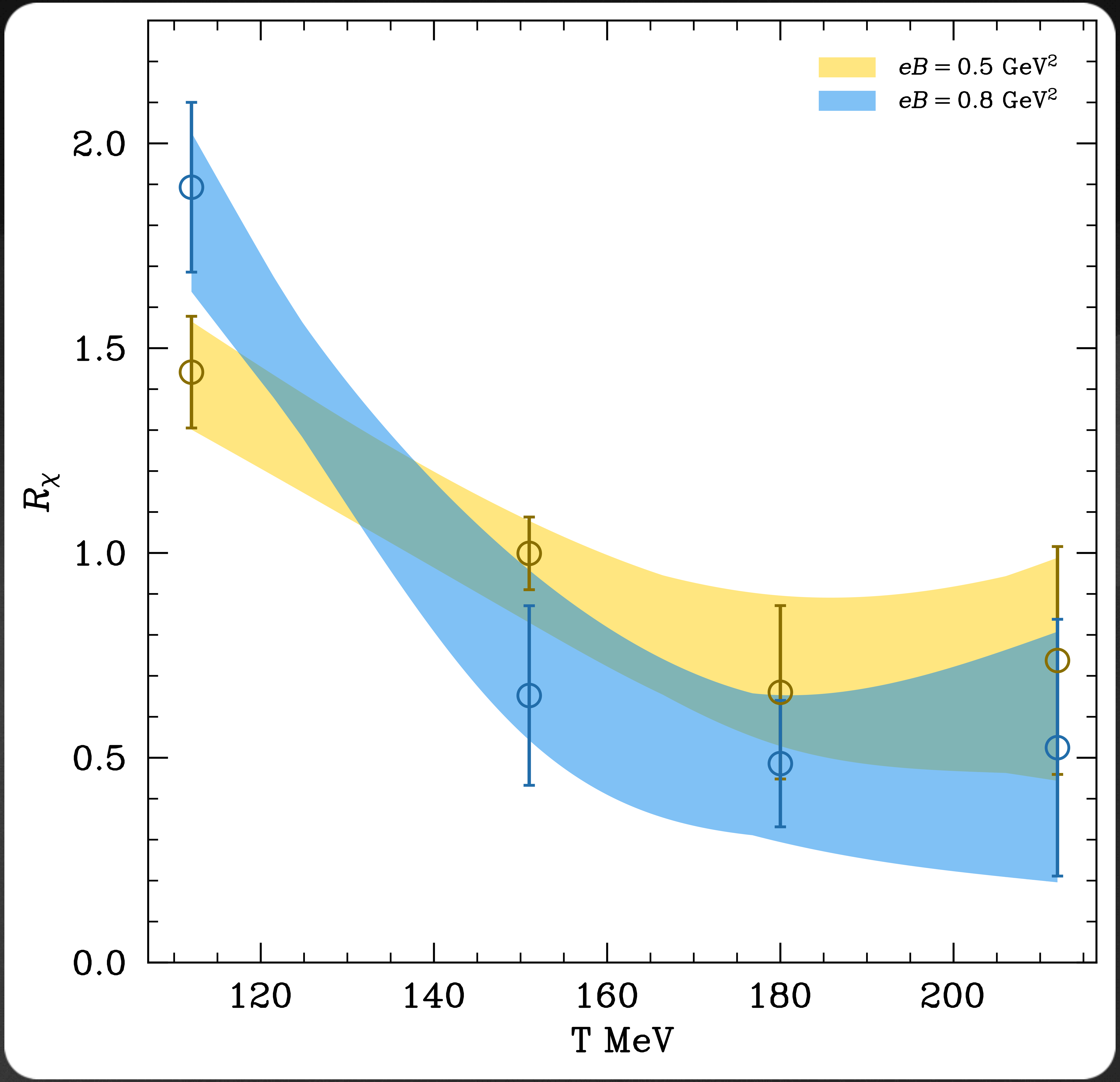


$T = 112$ MeV

χ_{top} at finite magnetic field



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$Q_{\text{top}} + Q_{EM} = N_R - N_L$ → $g < 0$!

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$g_{a\gamma\gamma}^{\text{model}} + g_{a\gamma\gamma}^{\text{QCD}}$

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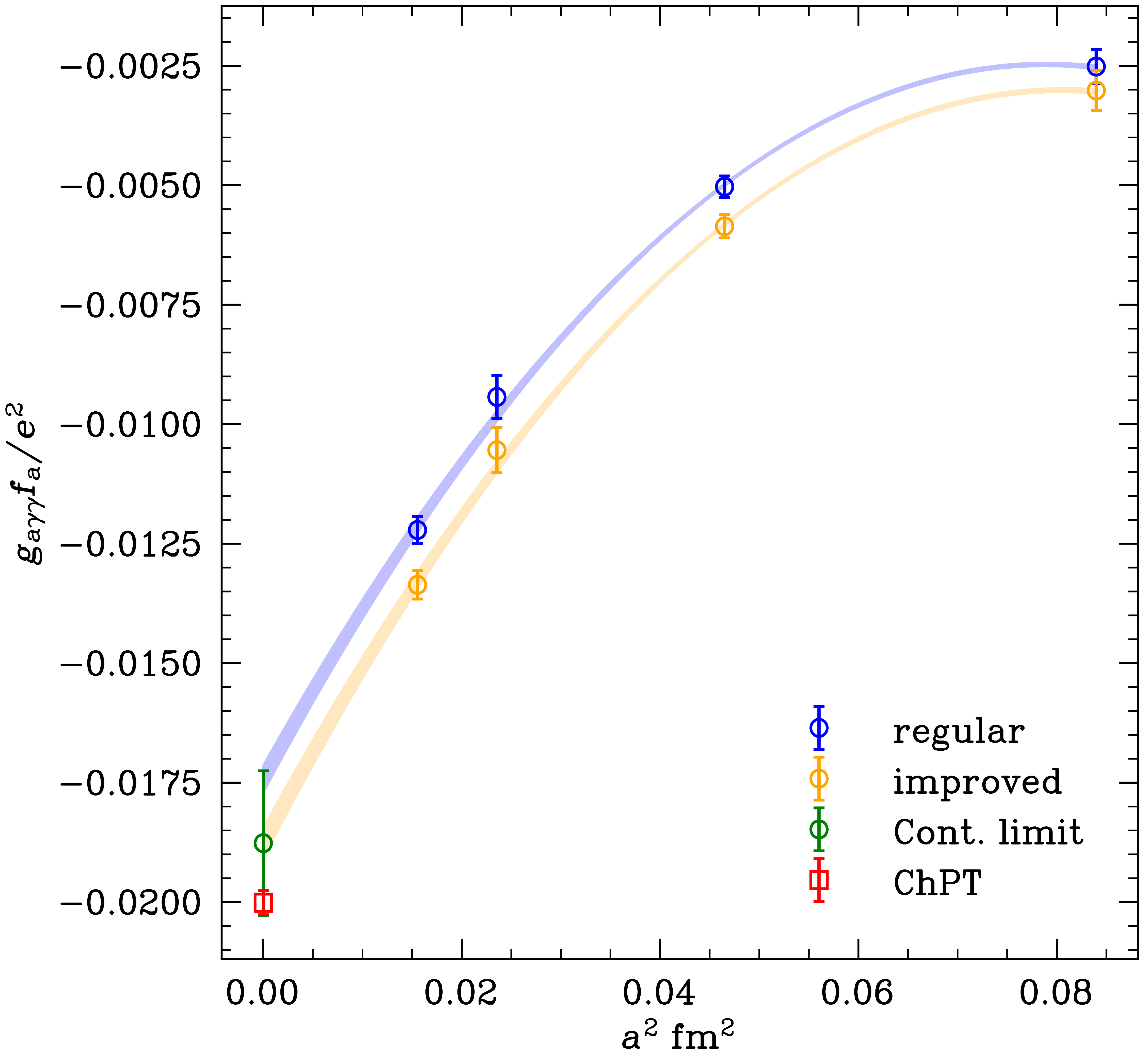
ChPT (NLO) [6]: $g_{a\gamma\gamma}^{\text{QCD}} f_a = -0.0243(5)\, e^2$

Axion-photon coupling



$T = 0$

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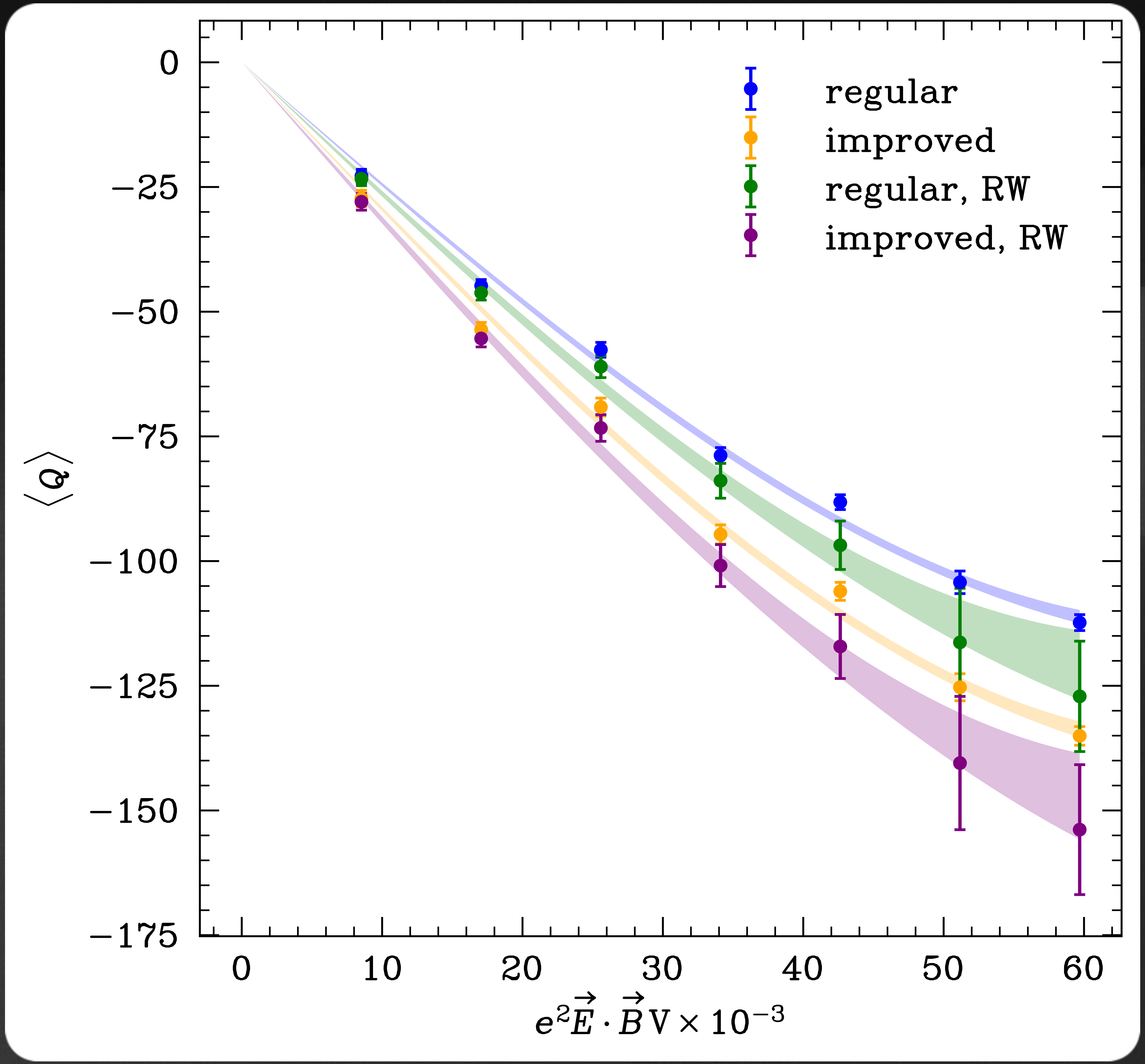
Caveat: sometimes too many eigenvalues!

Aproximate the reweighting factor: Lanczos quadrature

In progress!

Effect of the partial reweighting



$24^3 \times 32, T = 0$

AWI with EM fields

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$$\partial_\mu J_5^\mu = 2m\bar{\psi}\gamma_5\psi + 2q_{\rm top} + 2q_{em}$$

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$$\int d^4x \partial_\mu J_5^\mu = \int d^4x 2m\bar{\psi}\gamma_5\psi + \int d^4x 2q_{\text{top}} + \int d^4x 2q_{em}$$

AWI with EM fields

$$0 = mV_4 \bar{\psi}\gamma_5 \psi + Q_{\text{top}} + Q_{em}$$

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$$0 = mV_4\langle\bar{\psi}\gamma_5\psi\rangle_{EB} + \langle Q_{\text{top}}\rangle_{EB} + N_c\langle Q_{em}\rangle_{EB}$$

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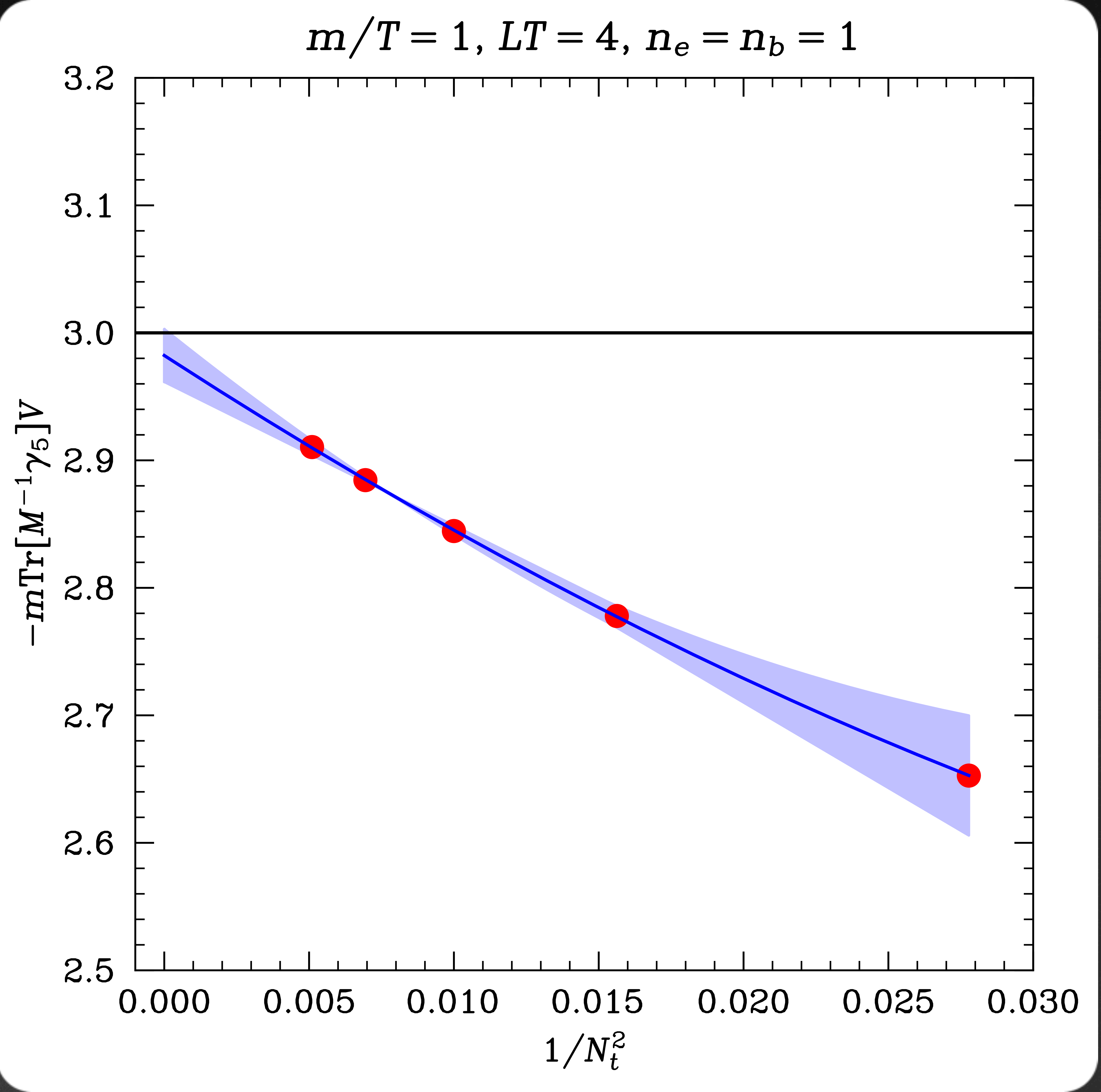
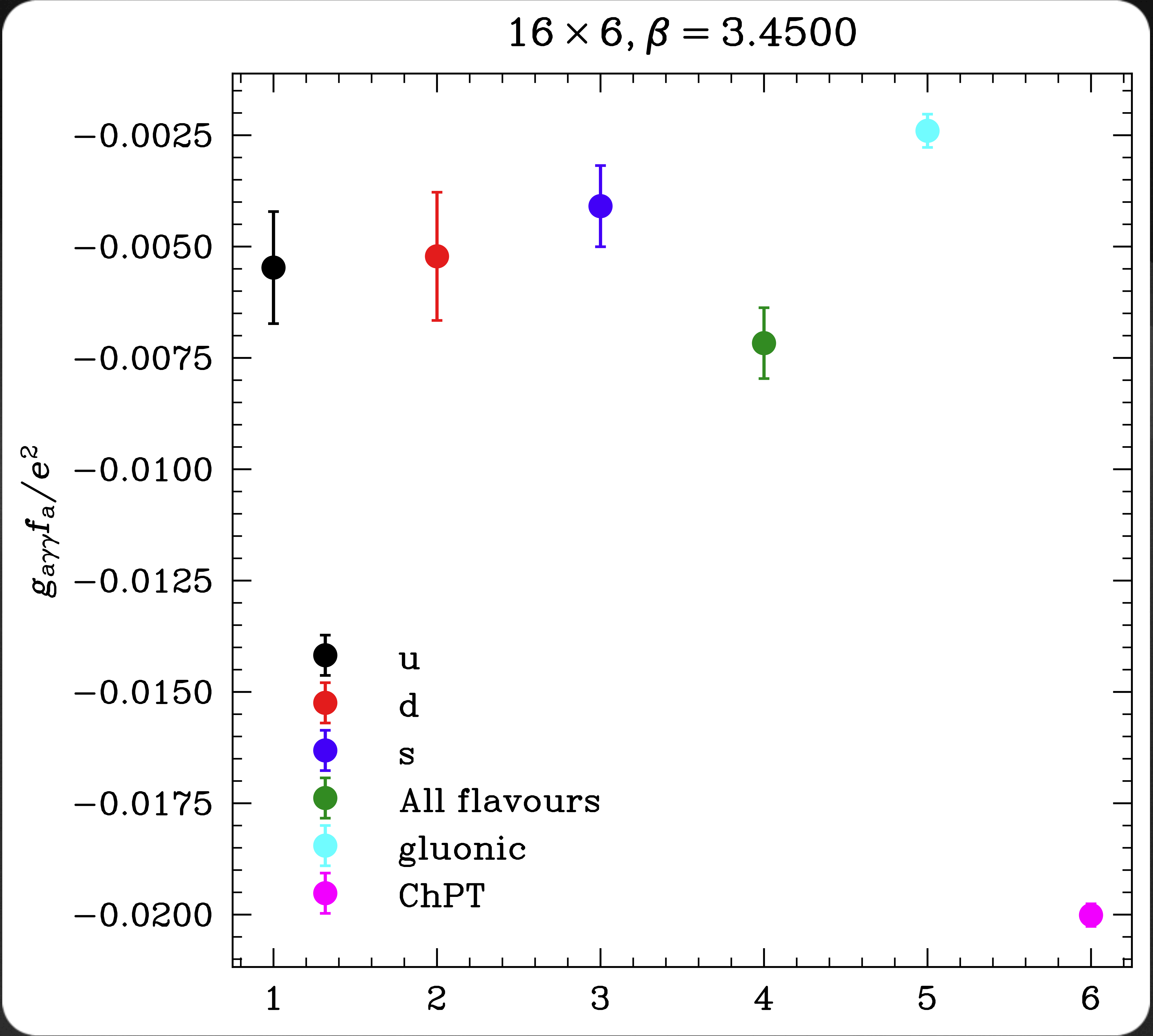
$$g_{a\gamma\gamma} f_a / e^2 \propto \frac{\langle \bar{\psi} \gamma_5 \psi \rangle_{EB}}{\langle \bar{\psi} \gamma_5 \psi \rangle_0} - 1$$

Gluonic vs AWI

Free case





QCD

Summary

- How EM fields affect topological observables
- First non-perturbative calculation of the dependence of χ_{top} with the magnetic field at finite temperatures (publication coming soon!)
- First non-perturbative calculation of the axion-photon coupling

Outlook

- Investigate the reweighting (exact and approximate) and the AWI method for the axion-photon coupling

Thank you for your attention!

References

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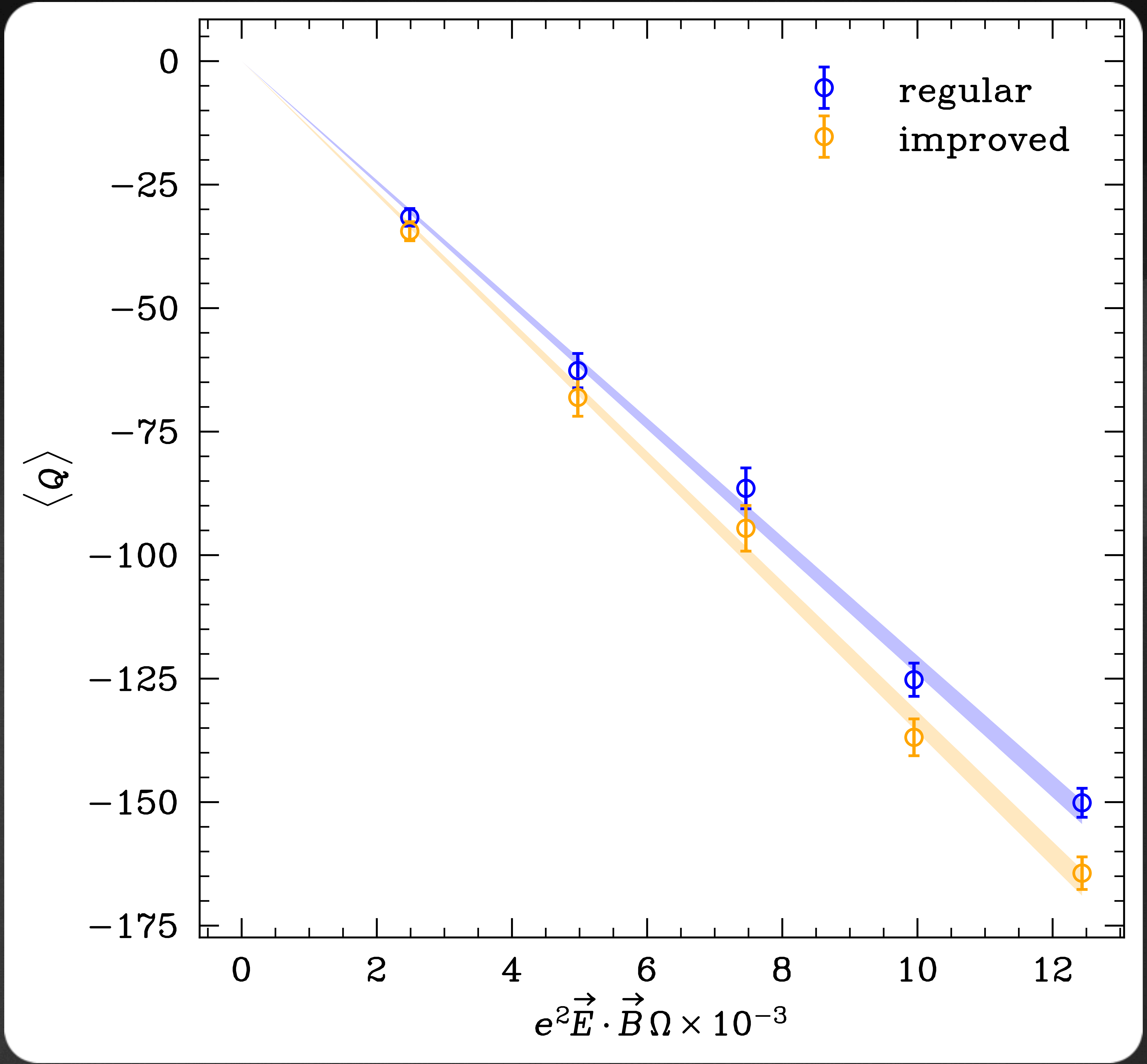
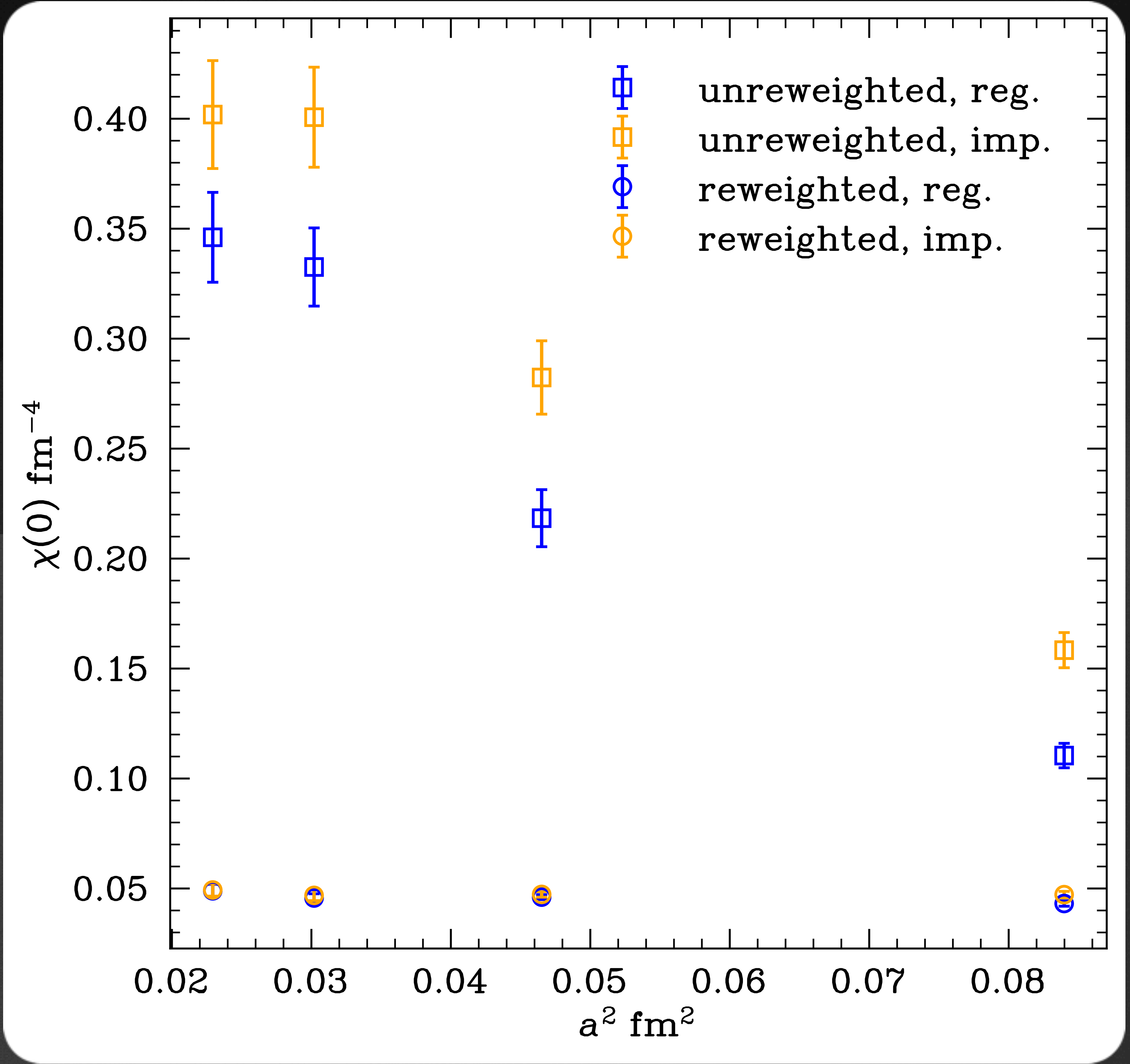
EM response of Q_{top}

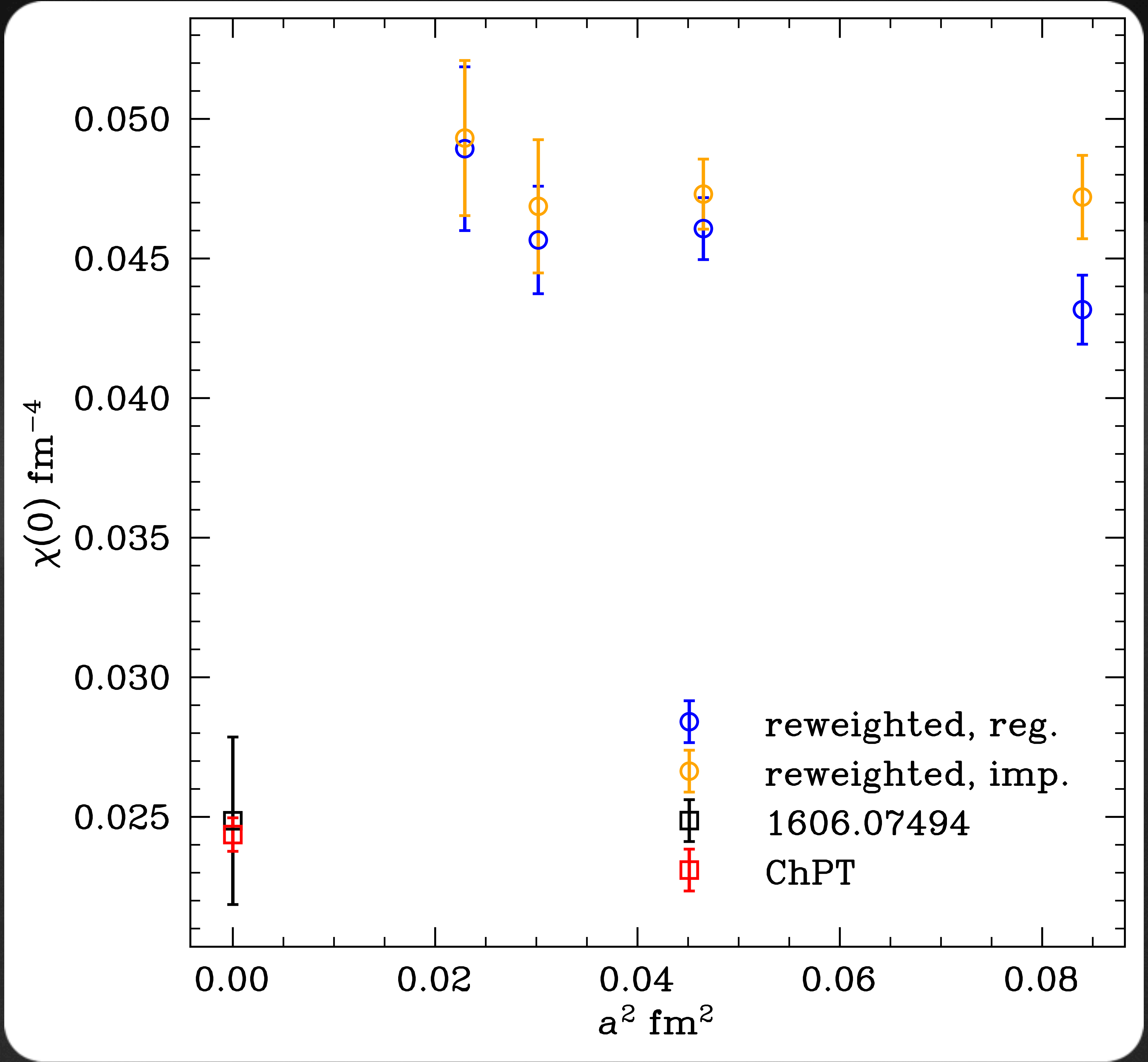


$40^3 \times 48, T = 0$

Effect of the reweighing

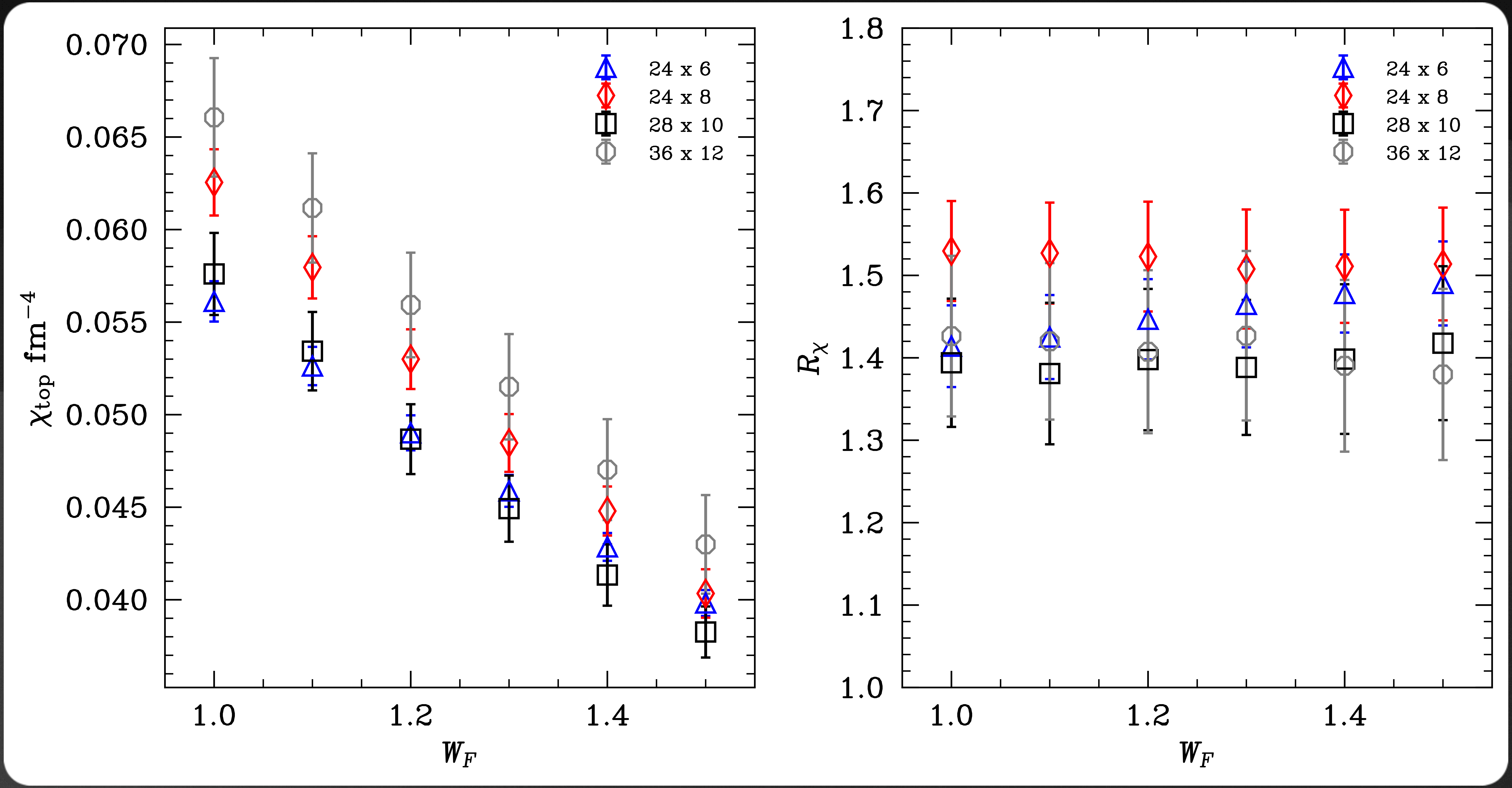
$T = 112\,\text{MeV}, eB = 0\,\text{GeV}^2$





Window reweighting

$T = 112\,\text{MeV}, eB = 0.5\,\text{GeV}^2$



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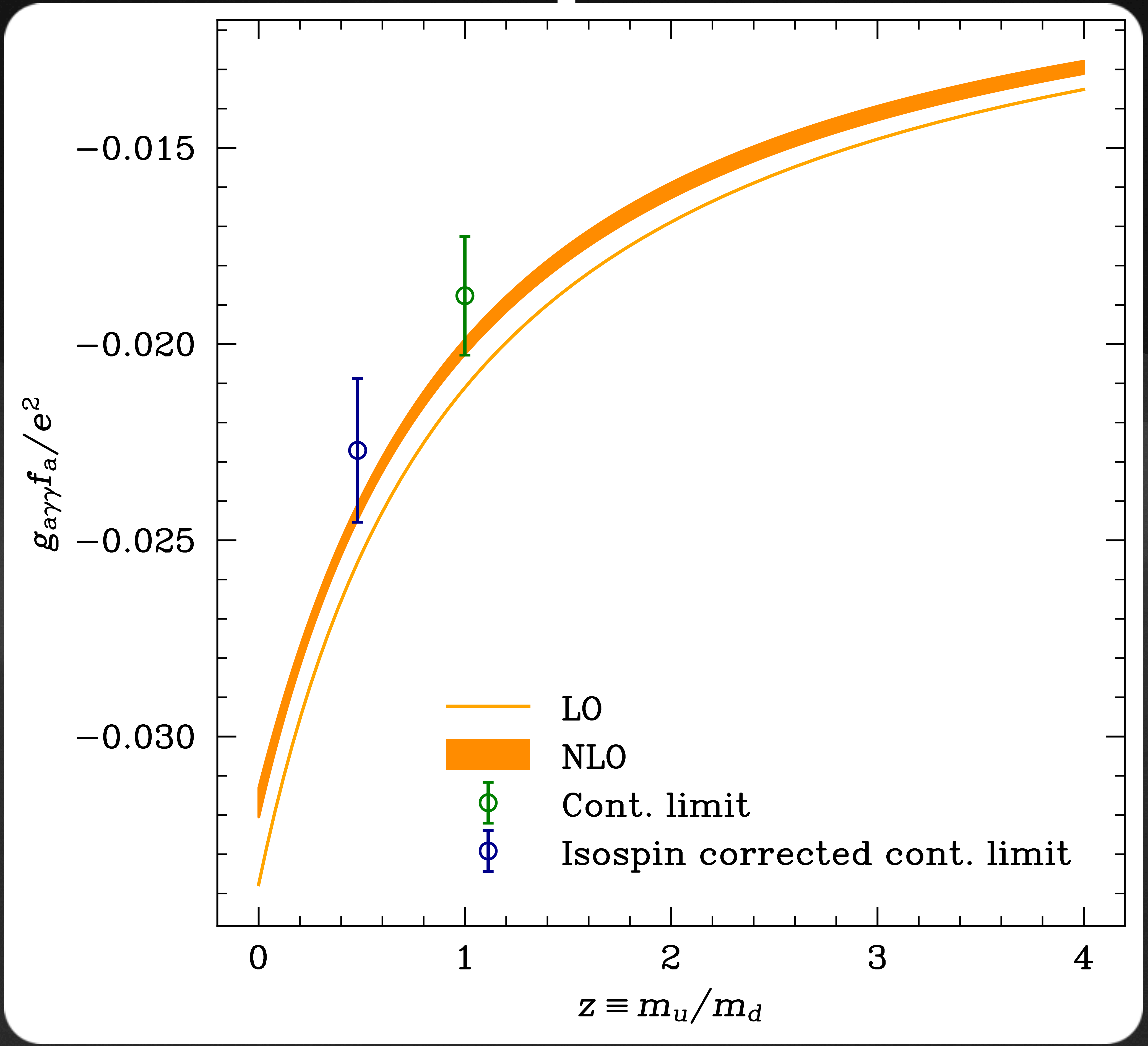

Our result: $g_{a\gamma\gamma}^{\text{phys}} f_a = -\,0.023(2)\,e^2$