Localization of Dirac modes in a finite temperature SU(2)-Higgs model

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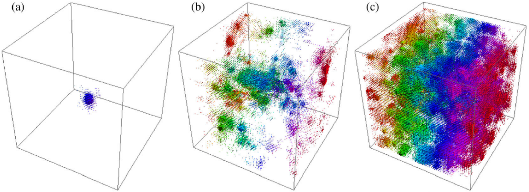
Introduction I

- The connection between deconfinement and chiral symmetry restoration at the finite temperature QCD transition is still not fully understood.
- Low Dirac modes could be key in understanding this connection.
  - Chiral symmetry breaking is controlled by the density $\rho(\lambda)$ of low modes according to the Banks-Casher relation

$$\left|\langle\bar{\psi}(x)\psi(x)\rangle\right| \overset{m\to 0}{=} \pi\rho(0).$$

  - Localization of low modes (up to a mobility edge λ_c) of the Dirac operator was observed in QCD and other gauge theories above the deconfinement transition [Giordano and Kovács, 2021]



From Ref. [Ujfalusi et al., 2015]

- "Sea/islands" picture of localization: in the deconfined phase modes get "trapped" on "islands" of
  1. Polyakov-loop fluctuations [Bruckmann et al., 2011]
  2. gauge field fluctuations that decrease correlations in the temporal direction [Baranka and Giordano, 2022]

  within the "sea" of ordered Polyakov loops.
- Only ordering of the Polyakov-loop is needed → localization of low modes expected in a generic gauge theory

- To test the "sea/islands" picture: check thermal transitions in which the Polyakov-loop gets ordered other than the deconfinement transition
- Another test of the "sea/islands" picture: changing the type of dynamic matter
- Fixed length ($\lambda \to \infty$) SU(2)-Higgs model carries out both tests [Baranka and Giordano, 2023]

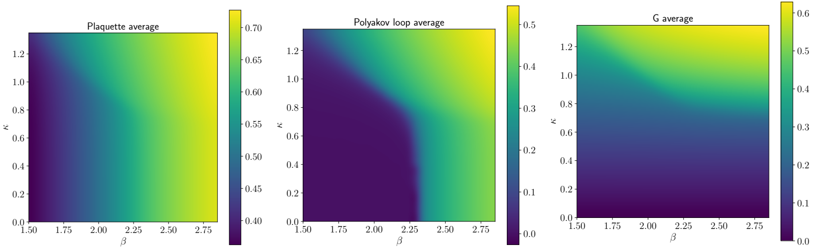
$$S = -\frac{1}{2}\sum_{n}\left[\beta\sum_{\mu<\nu}\mathrm{tr}\,U_{\mu\nu}(n) + \kappa\sum_{\mu}\mathrm{tr}\,G_{\mu}(n)\right]$$

$$G_{\mu}(n) = \phi(n)^{\dagger}U_{\mu}(n)\phi(n+\mu)$$

$U_{\mu}(n)$: link variables (gauge field)
$\phi(n)$: site variables (scalar field represented by a SU(2) matrix)

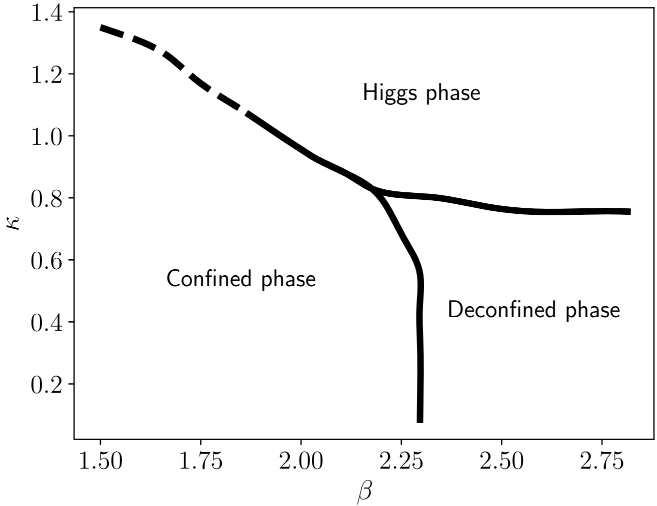
Mapping the phase diagram

Phase diagram was studied at zero temperature [Bonati et al., 2010] $\rightarrow$ do it at finite temperature $\rightarrow$ N_t fixed, $T = \frac{1}{N_t a}$



The structure of the phase diagram is the one expected from $T = 0$ studies; we checked that the transitions are crossovers

Sketch of phase diagram



Spectral statistics I

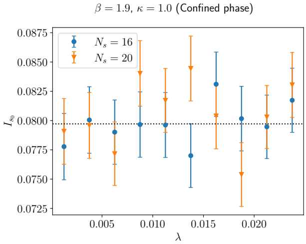
- Localization and spectral statistics are connected
  - Delocalized modes $\rightarrow$ strongly correlated under small changes of the gauge field $\rightarrow$ RMT (Random Matrix theory) statistics
  - Localized modes $\rightarrow$ uncorrelated under small changes of the gauge field for large volumes $\rightarrow$ Poisson statistics
- Concentrate on the universal spectral statistical properties $\rightarrow$ unfold the spectrum

$$s_i = (\lambda_{i+1} - \lambda_i)\rho(\lambda_i)$$

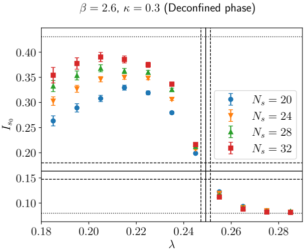
- Distribution $p(s)$ of unfolded spacings s_i:
  - $p_{\mathrm{RMT}} \simeq a_\beta s^\beta e^{-b_\beta s^2}$, where β depends on the symmetry class ($\beta = 4$ in our case)
  - $p_{\mathrm{Poisson}} = e^{-s}$

Spectral statistics II

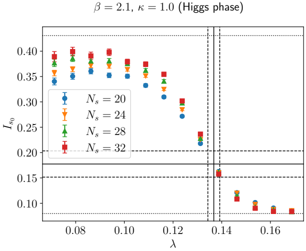
- $I_{s_0} = \int_0^{s_0} p(s)ds$ measured locally in the spectrum is a convenient observable to detect changes in the localization properties
- The mobility edge λ_c is the transition point in the spectrum between different statistical behaviors $\rightarrow$ the point where the spectral statistics are volume-independent $\rightarrow$ determine the crossing point for the various pairs of system sizes
- The critical value for I_{s_0} is expected to be universal $\rightarrow$ once the critical I_{s_0} is determined, the mobility edge can be found using just one volume



$\beta = 1.9,\ \kappa = 1.0$ (Confined phase)
$N_s = 16$
$N_s = 20$
I_{s_0}
λ



$\beta = 2.6,\ \kappa = 0.3$ (Deconfined phase)
$N_s = 20$
$N_s = 24$
$N_s = 28$
$N_s = 32$
I_{s_0}
λ



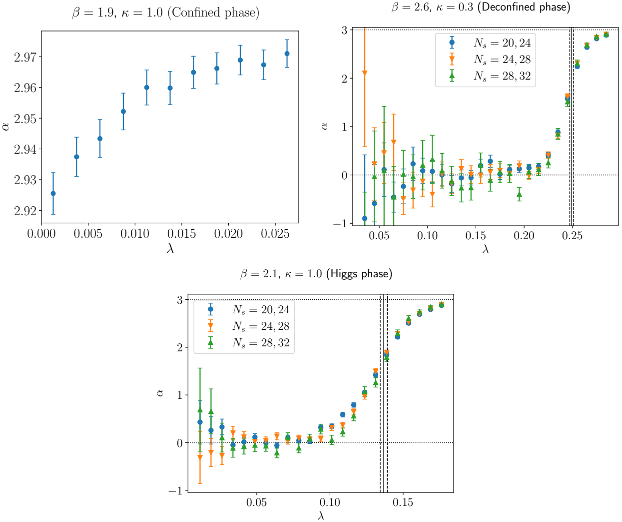
$\beta = 2.1,\ \kappa = 1.0$ (Higgs phase)
$N_s = 20$
$N_s = 24$
$N_s = 28$
$N_s = 32$
I_{s_0}
λ

Participation ratio and fractal dimension

$$\mathrm{PR}_l = \frac{1}{N_t N_s^3} \mathrm{IPR}_l^{-1}\,, \quad \mathrm{IPR}_l = \sum_n \|\psi_l(n)\|^4\,,$$

- $\mathrm{PR} = \frac{\text{occupied volume}}{\text{lattice volume}}$
- $\mathrm{PR} \cdot V \sim N_s^\alpha$ at large N_s, α the fractal dimension of modes
  - $\alpha = 0 \rightarrow$ Localized mode
  - $\alpha = 3 \rightarrow$ Delocalized mode

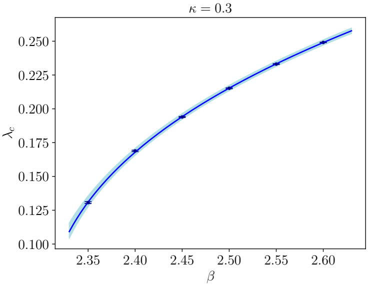
Fractal dimension

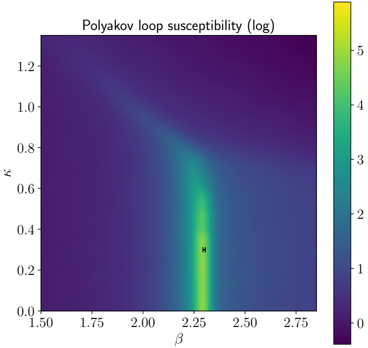


Fitted function:

$$\lambda_c = a(\beta - \beta_c)^b$$

The mobility edge disappears in the transition region.





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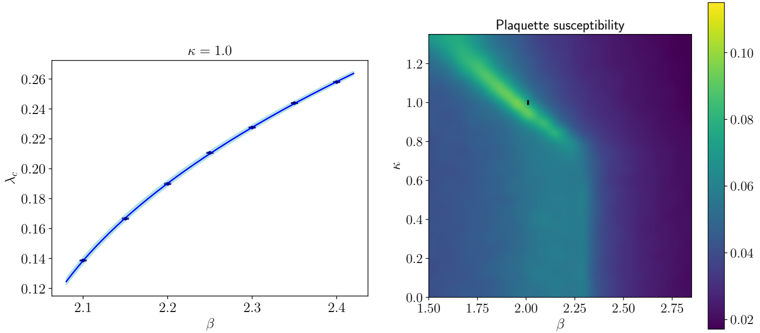
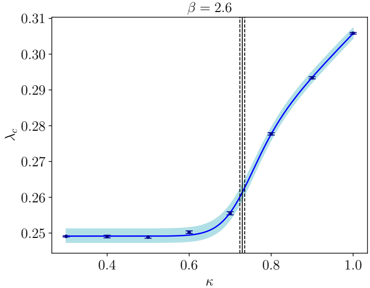
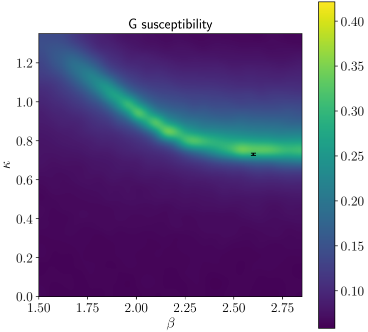


Fitted function:

$$\lambda_c = a \cdot (1 - \sigma(d \cdot (\kappa - \kappa_c))) + (b\kappa + c)\sigma(d \cdot (\kappa - \kappa_c))$$

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$





Conclusions

- Study of the relationship between localization and Polyakov-loop ordering extended to a system with a dynamical scalar field (SU(2)-Higgs theory)
- Phase diagram mapped at finite temperature
- "Sea/islands" picture is confirmed: works without regard to
  1. type of dynamical matter
  2. type of transition (the ordering of the Polyakov loop is enough)
- Possible extension:
  1. study the localization properties of the covariant Laplacian (was only studied at zero temperature [Greensite et al., 2005])
  2. detailed study of low β, large κ region of the phase diagram [Greensite and Matsuyama, 2022]

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