Flavour singlet mixing in Sp(4) gauge theory with fermions in multiple representations

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`based on 2405.05765` with

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slides at fzierler.github.io/talks/, data + analysis code on Zenodo see e.g. Ferretti,Karateev [1312.5330], Ferretti [1604.06467] Ayyar et.al. [1710.00806] [1801.05809] [1802.09644] Bergner,Piemonte [2008.02855][2111.15335] Cossu et. al. [1904.08885] Bennett et.al. [2202.05516] [2311.14663]

Theories With Multiple Fermion Representations

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} + \bar{\mathrm{Q}}^{\mathrm{i}}\left(\mathrm{i}\not{\mathrm{D}} - \mathrm{m}_{\mathrm{i}}^{\mathrm{f}}\right)\mathrm{Q}^{\mathrm{i}} + \bar{\Psi}^{\mathrm{j}}\left(\mathrm{i}\not{\mathrm{D}} - \mathrm{m}_{\mathrm{j}}^{\mathrm{as}}\right)\Psi^{\mathrm{j}}$$

- Gauge theory of group G with field strength tensor $F_{\mu\nu}$
- Two species of fermions Q and Ψ under different irreps of G

`Applications:`

- **Composite Higgs+top**(fundamental and antisymmetric fermions)
- Models of supersymmetric physics (fundamental + adjoint)

[1] Kosower (Phys.Lett.B.144, 1984) Witten (Nucl.Phys.B.223, 1983) Peskin (Nucl.Phys.B.175, 1980)

[2] Witten (Nucl.Phys.B.149, 1979) (Nucl.Phys.B.156, 1979) Veneziano (Nucl.Phys.B.159, 1979)

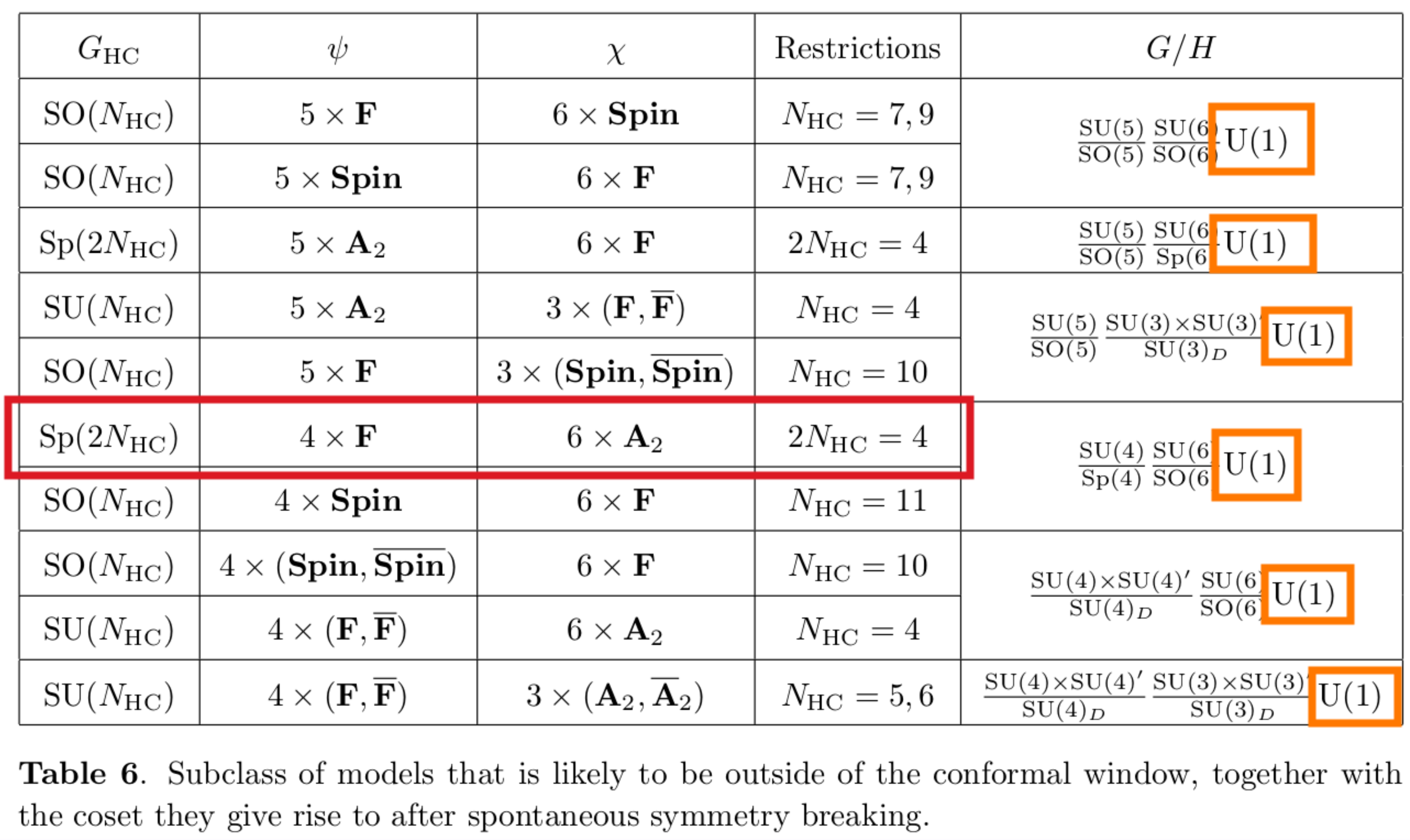
[3] Belyaev et.al. [1512.07242]

Chiral Symmetry and Extra Goldstone Bosons

- One breaking pattern for every fermion representation [1]
  - complex: $SU(N_f) \times SU(N_f) \rightarrow SU(N_f)$
  - pseudoreal: $SU(2N_f) \rightarrow Sp(2N_f)$
  - real: $SU(2N_f) \rightarrow SO(2N_f)$
- And one axial $U(1)$ for each representation
  - one (combination of) $U(1)$ broken by axial anomaly! [2]
  - Additional $U(1)$ Goldstone for multiple representations! [3]
  - mixed state with contributions from different reps

table taken from Feretti [1604.06467], see also Ferretti,Karateev [1312.5330]

Composite Higgs + Top Realisations



G_{HC}	ψ	χ	Restrictions	G/H
$\mathrm{SO}(N_{\mathrm{HC}})$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$N_{\mathrm{HC}} = 7, 9$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)} \frac{\mathrm{SU}(6)}{\mathrm{SO}(6)} \boxed{\mathrm{U}(1)}$
$\mathrm{SO}(N_{\mathrm{HC}})$	$5 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\mathrm{HC}} = 7, 9$	
$\mathrm{Sp}(2N_{\mathrm{HC}})$	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	$2N_{\mathrm{HC}} = 4$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)} \frac{\mathrm{SU}(6}{\mathrm{Sp}(6} \boxed{\mathrm{U}(1)}$
$\mathrm{SU}(N_{\mathrm{HC}})$	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	$N_{\mathrm{HC}} = 4$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)} \frac{\mathrm{SU}(3) \times \mathrm{SU}(3)}{\mathrm{SU}(3)_D} \boxed{\mathrm{U}(1)}$
$\mathrm{SO}(N_{\mathrm{HC}})$	$5 \times \mathbf{F}$	$3 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$N_{\mathrm{HC}} = 10$	
$\mathrm{Sp}(2N_{\mathrm{HC}})$	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	$2N_{\mathrm{HC}} = 4$	$\frac{\mathrm{SU}(4)}{\mathrm{Sp}(4)} \frac{\mathrm{SU}(6}{\mathrm{SO}(6} \boxed{\mathrm{U}(1)}$
$\mathrm{SO}(N_{\mathrm{HC}})$	$4 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\mathrm{HC}} = 11$	
$\mathrm{SO}(N_{\mathrm{HC}})$	$4 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$6 \times \mathbf{F}$	$N_{\mathrm{HC}} = 10$	$\frac{\mathrm{SU}(4) \times \mathrm{SU}(4)'}{\mathrm{SU}(4)_D} \frac{\mathrm{SU}(6}{\mathrm{SO}(6} \boxed{\mathrm{U}(1)}$
$\mathrm{SU}(N_{\mathrm{HC}})$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	$N_{\mathrm{HC}} = 4$	
$\mathrm{SU}(N_{\mathrm{HC}})$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}_2})$	$N_{\mathrm{HC}} = 5, 6$	$\frac{\mathrm{SU}(4) \times \mathrm{SU}(4)'}{\mathrm{SU}(4)_D} \frac{\mathrm{SU}(3) \times \mathrm{SU}(3)}{\mathrm{SU}(3)_D} \boxed{\mathrm{U}(1)}$

Table 6. Subclass of models that is likely to be outside of the conformal window, together with the coset they give rise to after spontaneous symmetry breaking.

Our model: $Sp(4)$ with 2 fundamental + 3 antisymmetric

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} + \bar{\mathrm{Q}}^{\mathrm{i}}\left(\mathrm{i}\not{D} - \mathrm{m}_{\mathrm{i}}^{\mathrm{f}}\right)\mathrm{Q}^{\mathrm{i}} + \bar{\Psi}^{\mathrm{j}}\left(\mathrm{i}\not{D} - \mathrm{m}_{\mathrm{j}}^{\mathrm{as}}\right)\Psi^{\mathrm{j}}$$

- **Non-perturbative input needed for pheno $\Rightarrow$ Lattice**
- $5 + 20 + 1$ pseudo-Goldstones + 1 $U(1)_A$ state
- The two $U(1)$ states will mix: both are 0^- iso-singlets
- First explorations with heavy dynamical fermions
- **Goal: Determine mass spectrum and mixing angle**
- This is the first study of singlet mesons in this theory!

coset: $\mathbf{U(1)} \times \frac{SU(4)}{Sp(4)} \times \frac{SU(6)}{SO(6)}$

Other aspects of Sp(4) gauge theories at Lattice2024:

- Spectroscopy & spectral densities (**N.Forzano, Wednesday 11:35**)
- Finite-T phase transitions (**D.Mason, Wednesday 12:15**)
- $\pi\pi$ scattering and Dark Matter (**Y.Dengler, Thursday 10:00**)
- Chimera baryon ($QQ\psi$) spectroscopy (**H.Hsiao, Friday 11:35**)

[1] e.g. Belyaev et.al. [1610.06591], Cacciapaglia et.al. [1902.06890] Franzosi et.al. [2106.12615]

[2] DeGrand et.al. [1605.07738]

The axial $U(1)$ states

- pseudoscalar flavour-singlets: similar to η and η' of QCD
- **Potentially light singlet can have large pheno implications!**
- probed by the following operators:

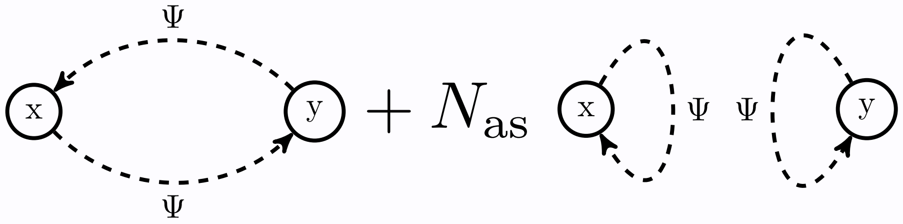
$$O_{\eta^{\mathrm{f}}} = \left(\bar{Q}^1\gamma_5 Q^1 + \bar{Q}^2\gamma_5 Q^2\right)/\sqrt{2}$$
$$O_{\eta^{\mathrm{as}}} = \left(\bar{\Psi}^1\gamma_5 \Psi^1 + \bar{\Psi}^2\gamma_5 \Psi^2 + \bar{\Psi}^3\gamma_5 \Psi^3\right)/\sqrt{3}$$

- These two states will mix: Light PNGB state η'_l + heavier state η'_h
  - mixing angle in general $\phi \neq 0$
  - Effective field theory in chiral limit has been developed [2]

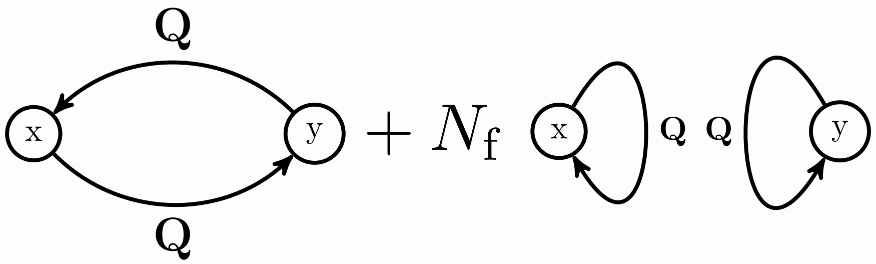
Lattice Investigation: Masses

- Variational Analysis with $O_{\eta^{\mathrm{f}}}$ and $O_{\eta^{\mathrm{as}}}$ operators
- Several levels of Wuppertal smearing

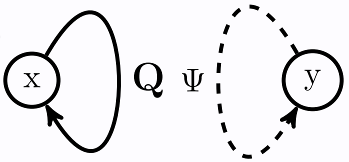
$$\langle \bar{O}_{\eta^{\mathrm{as}}}(x) O_{\eta^{\mathrm{as}}}(y) \rangle = -$$



$$\langle \bar{O}_{\eta^{\mathrm{f}}}(x) O_{\eta^{\mathrm{f}}}(y) \rangle = -$$



$$\langle \bar{O}_{\eta^{\mathrm{f}}}(x) O_{\eta^{\mathrm{as}}}(y) \rangle = +\sqrt{N_{\mathrm{as}} N_{\mathrm{f}}}$$



- N_{f} and N_{as} enhance singlet contributions
- non-vanishing fermion masses $(m_{\mathrm{f}}, m_{\mathrm{as}})$ suppress them

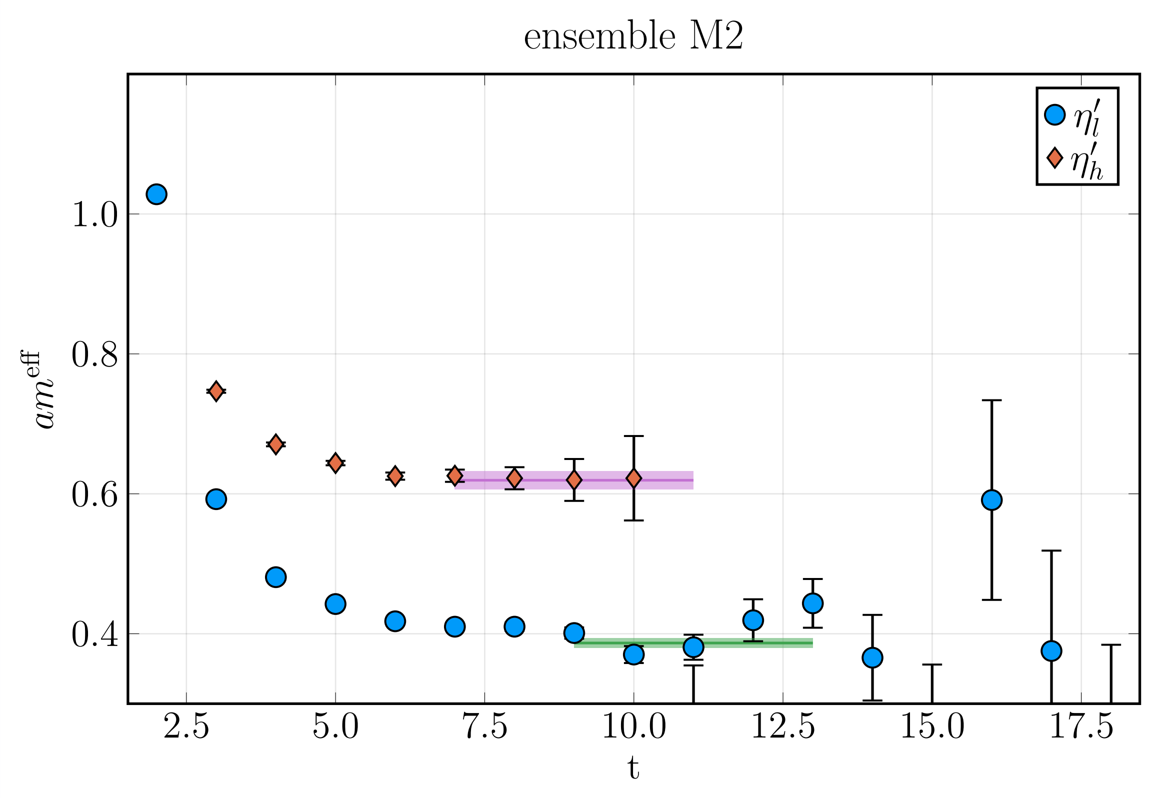
Available Dynamical Ensembles

Label	β	am_0^{as}	am_0^{f}	N_t	N_s	N_{conf}
M1	6.5	-1.01	-0.71	48	20	479
M2	6.5	-1.01	-0.71	64	20	698
M3	6.5	-1.01	-0.71	96	20	436
M4	6.5	-1.01	-0.70	64	20	709
M5	6.5	-1.01	-0.72	64	32	295

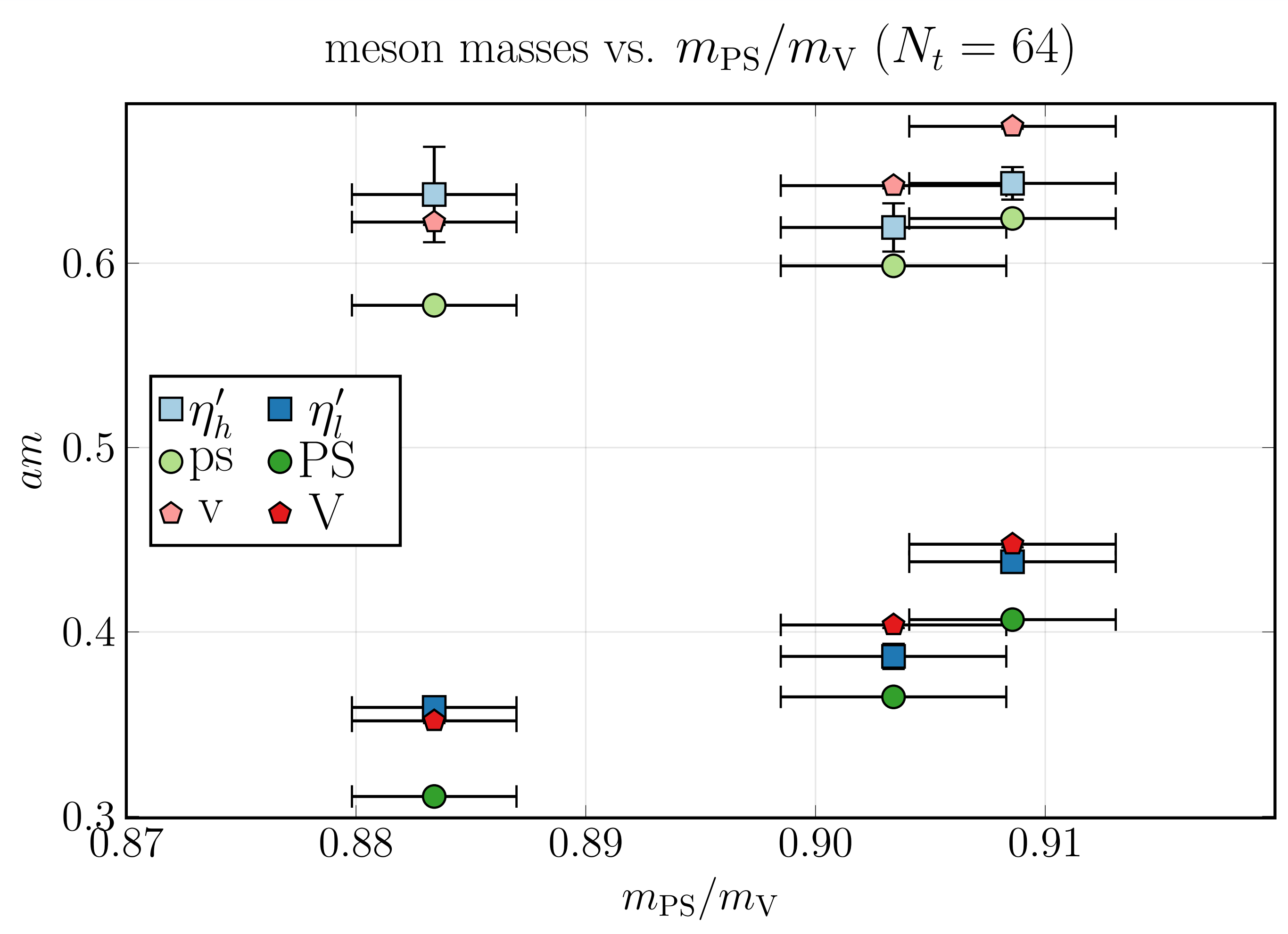
- Ensembles generated using GPUs with GRID
- Measurements performed with HiRep on CPUs

Results: Effective masses for η_l' and η_h'

- Variational analysis for correlation matrix $C_{ij}(t)$
- n^{th} eigenvalue falls off exponentially with energy E_n
- Masses from fits to correlator at large t



Results: Pseudoscalar Singlet Masses



- Spectrum likely dominated by heavy fermion masses

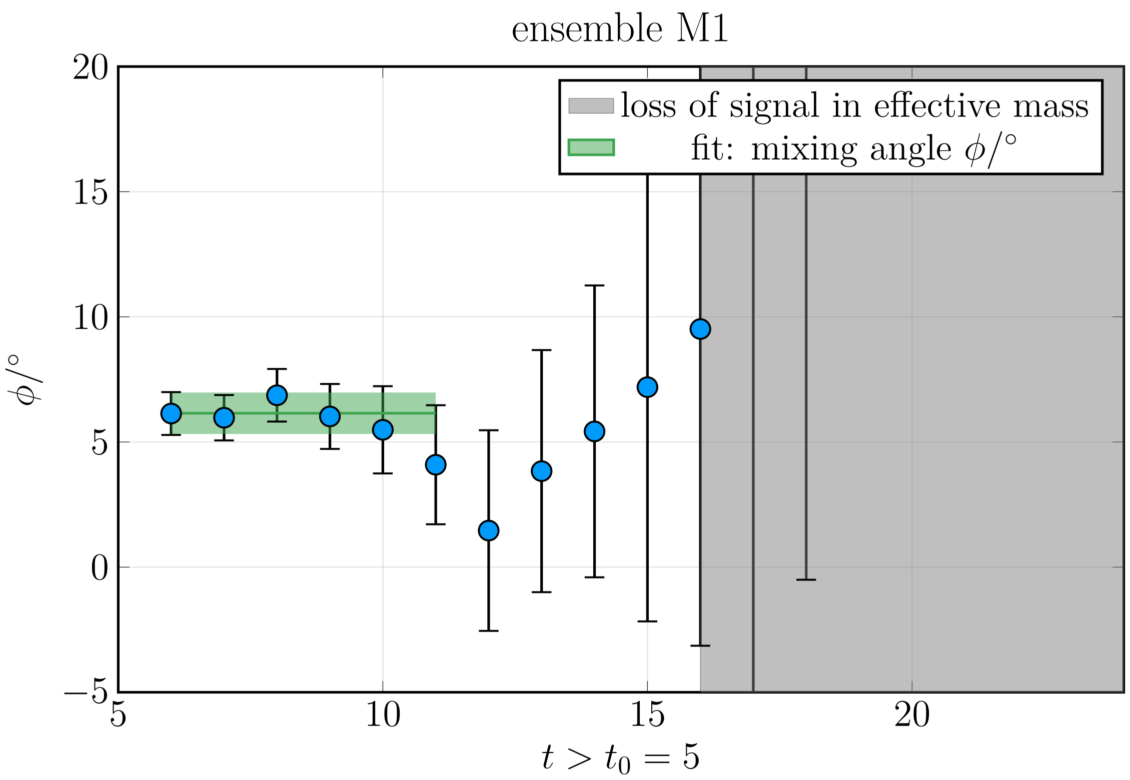
Lattice Investigation: Mixing Angle

- Obtained from operator mixing (no signal for decay constants)
- Use of flavour basis justifies use of single mixing angle [1]

$$\begin{pmatrix} \langle 0|O_{\eta^{\mathrm{f}}}|\eta_l'\rangle & \langle 0|O_{\eta^{\mathrm{as}}}|\eta_l'\rangle \\ \langle 0|O_{\eta^{\mathrm{f}}}|\eta_h'\rangle & \langle 0|O_{\eta^{\mathrm{as}}}|\eta_h'\rangle \end{pmatrix} = \begin{pmatrix} A_{\mathrm{f}}^{\eta_l'} & A_{\mathrm{as}}^{\eta_l'} \\ A_{\mathrm{f}}^{\eta_h'} & A_{\mathrm{as}}^{\eta_h'} \end{pmatrix} \equiv \begin{pmatrix} A_{\eta_l'} \cos\phi & A_{\eta_l'} \sin\phi \\ -A_{\eta_h'} \sin\phi & A_{\eta_h'} \cos\phi \end{pmatrix}$$

- Matrix elements are obtained from the eigenvectors of the GEVP
- Expected to be constant for all timeslices t
- Test for dominance of fermion masses:
  - $m_{\mathrm{fermions}} \to \infty$ implies that $\phi \to 0$

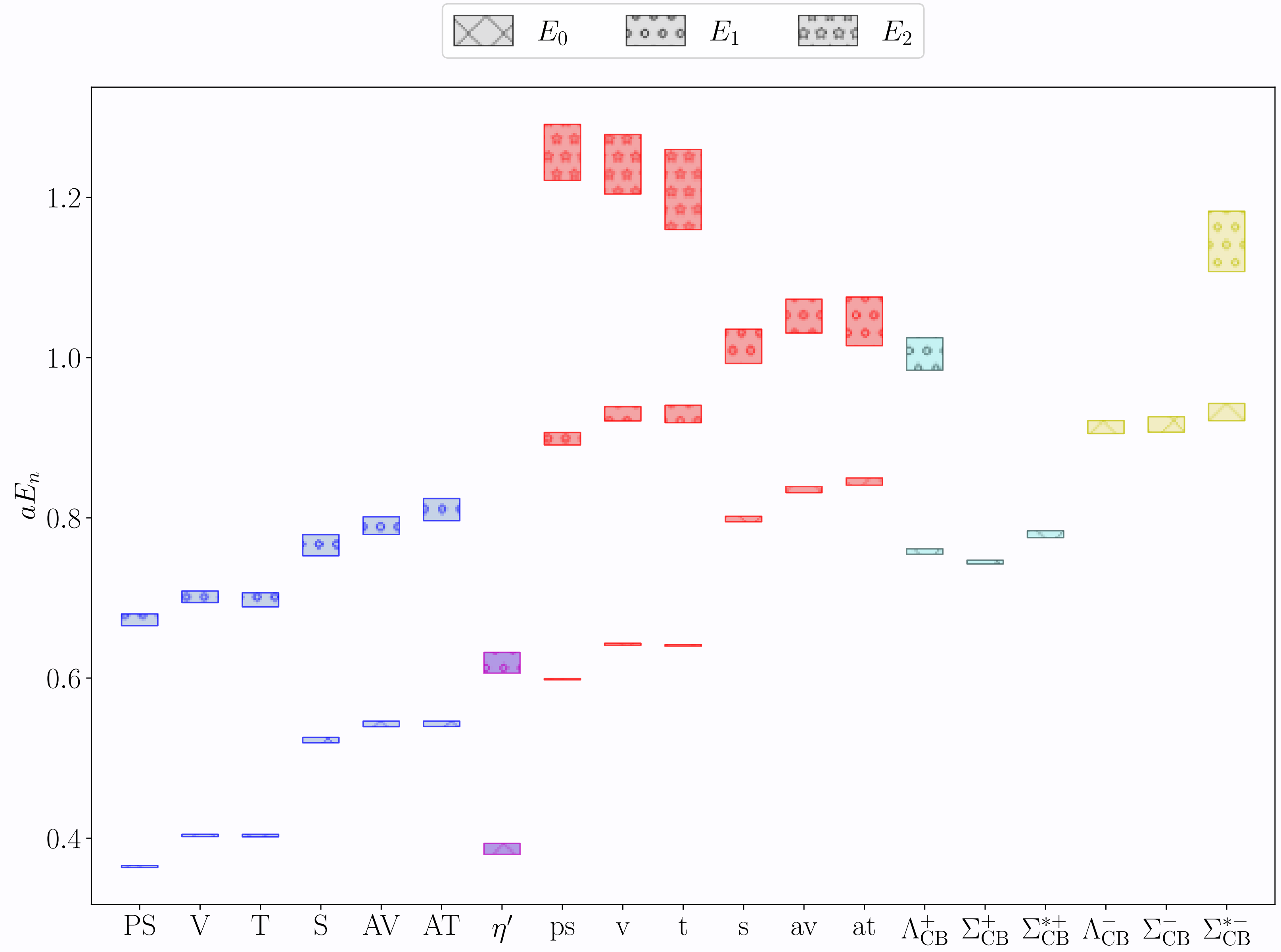
Results: Mixing Anlge ϕ small



- Consistently small mixing angles

Label	β	N_t	N_s	$\phi/°$
M1	6.5	48	20	**6.15(83)**
M2	6.5	64	20	**6.07(63)**
M3	6.5	96	20	**6.16(66)**
M4	6.5	64	20	**7.44(58)**
M5	6.5	64	32	**6.61(54)**

Results: Meson and baryon spectrum of single ensemble



Summary

- **First direct measurement of singlet mesons in multirep theory**
- Extraction of masses and mixing angles possible!
- Currently restricted to heavy dynamical fermions
- So far only one lattice spacing

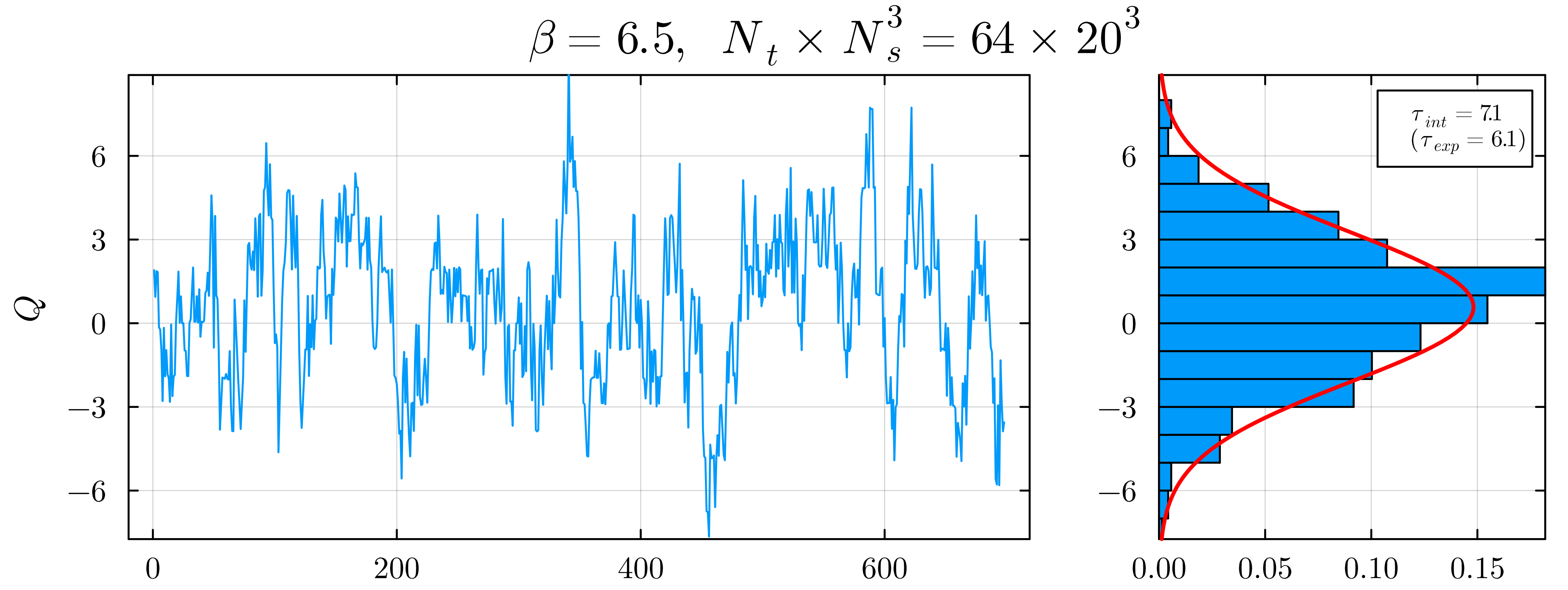
Outlook

- Different lattice spacings (i.e. different values of β)
- Scalar singlet 0^+ f_0/σ channel, and lighter fermions
- Mixing with 0^- glueball states

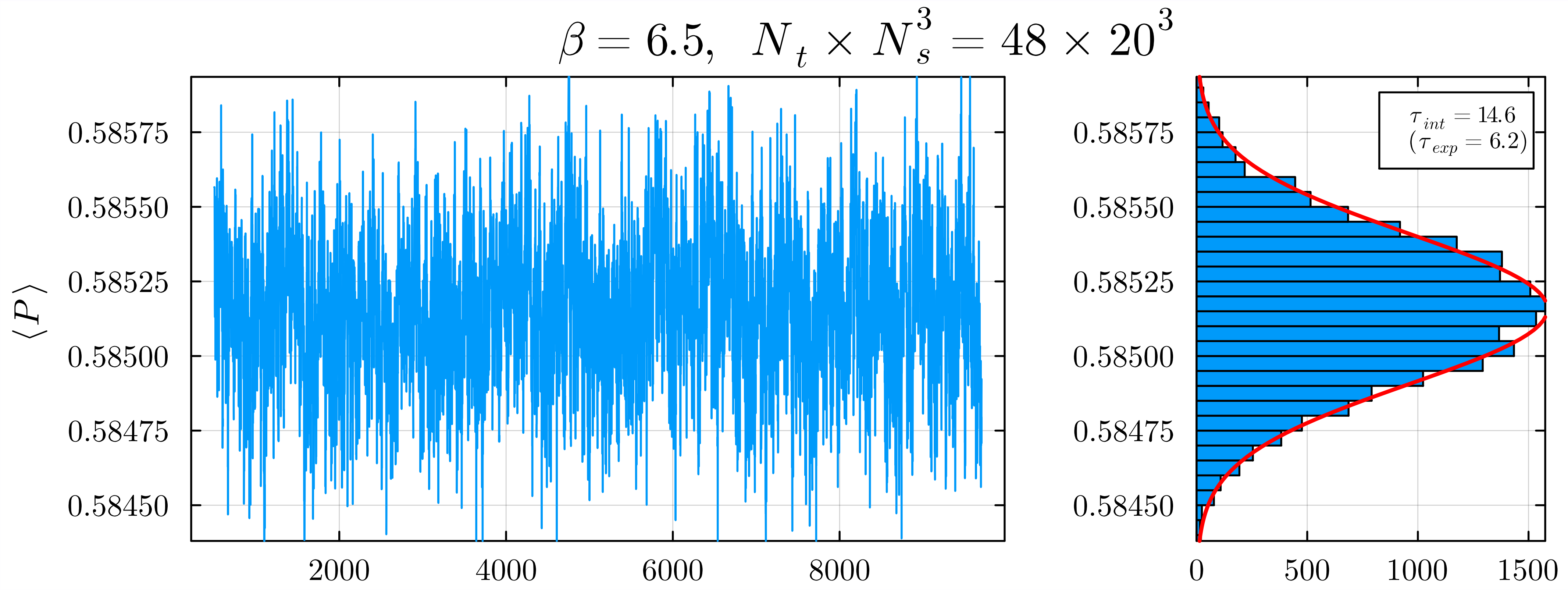
Back-up slides

Topology of ensembles

- not frozen, but correlations in topological charge Q

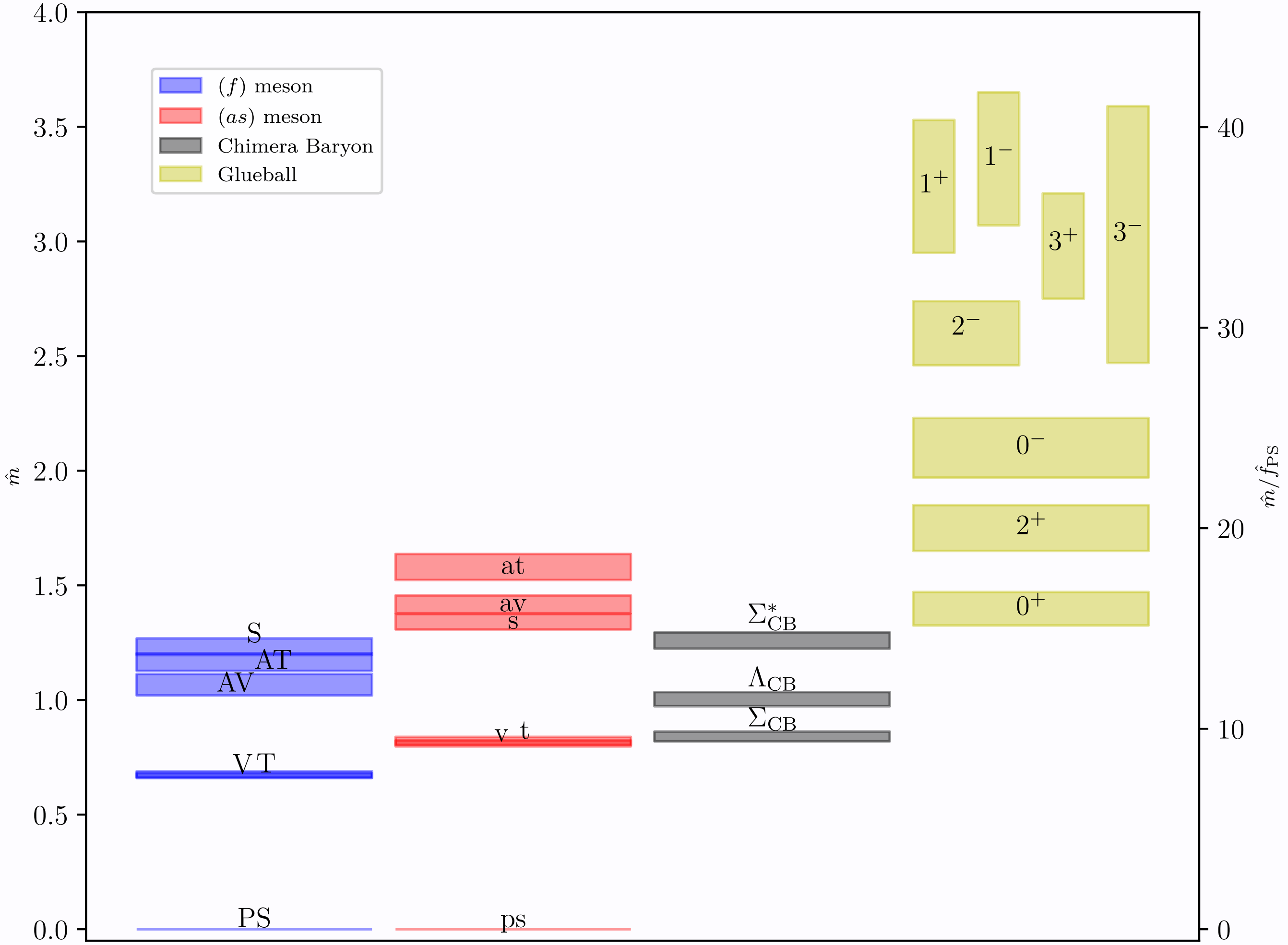


Configurations chosen such that plaquette is uncorrelated



Glueballs potentially relevant

- Suggested by quenched spectrum



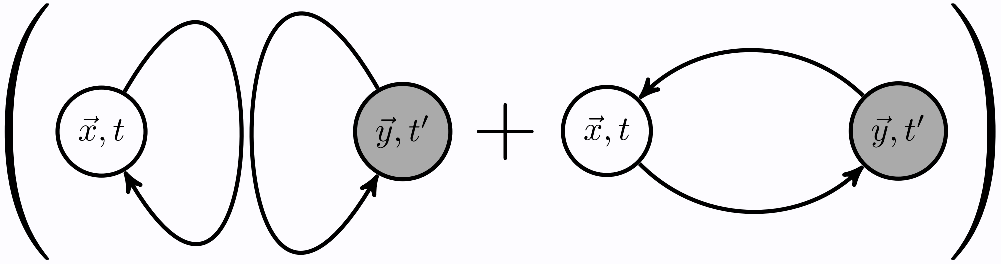
Lattice spectroscopy: Getting meson masses

- Construct operator with same quantum numbers
- Energy levels from Euclidean correlator

$$C_{\mathcal{O}}(t) = \sum_n \frac{1}{2E_n} \langle 0|\mathcal{O}|n\rangle^* \langle n|\mathcal{O}|0\rangle e^{-E_n t}.$$

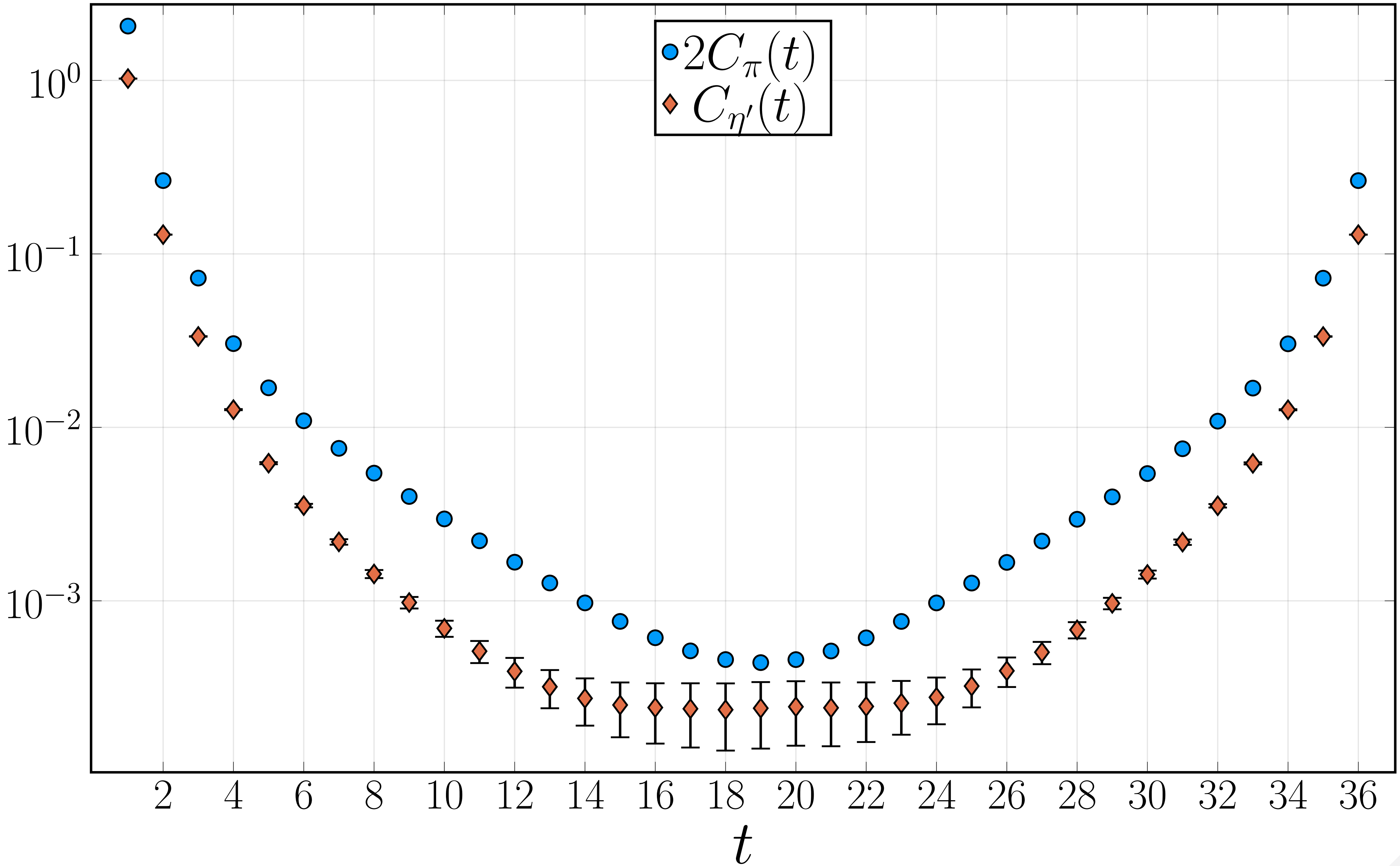
- For mesons a generic correlator

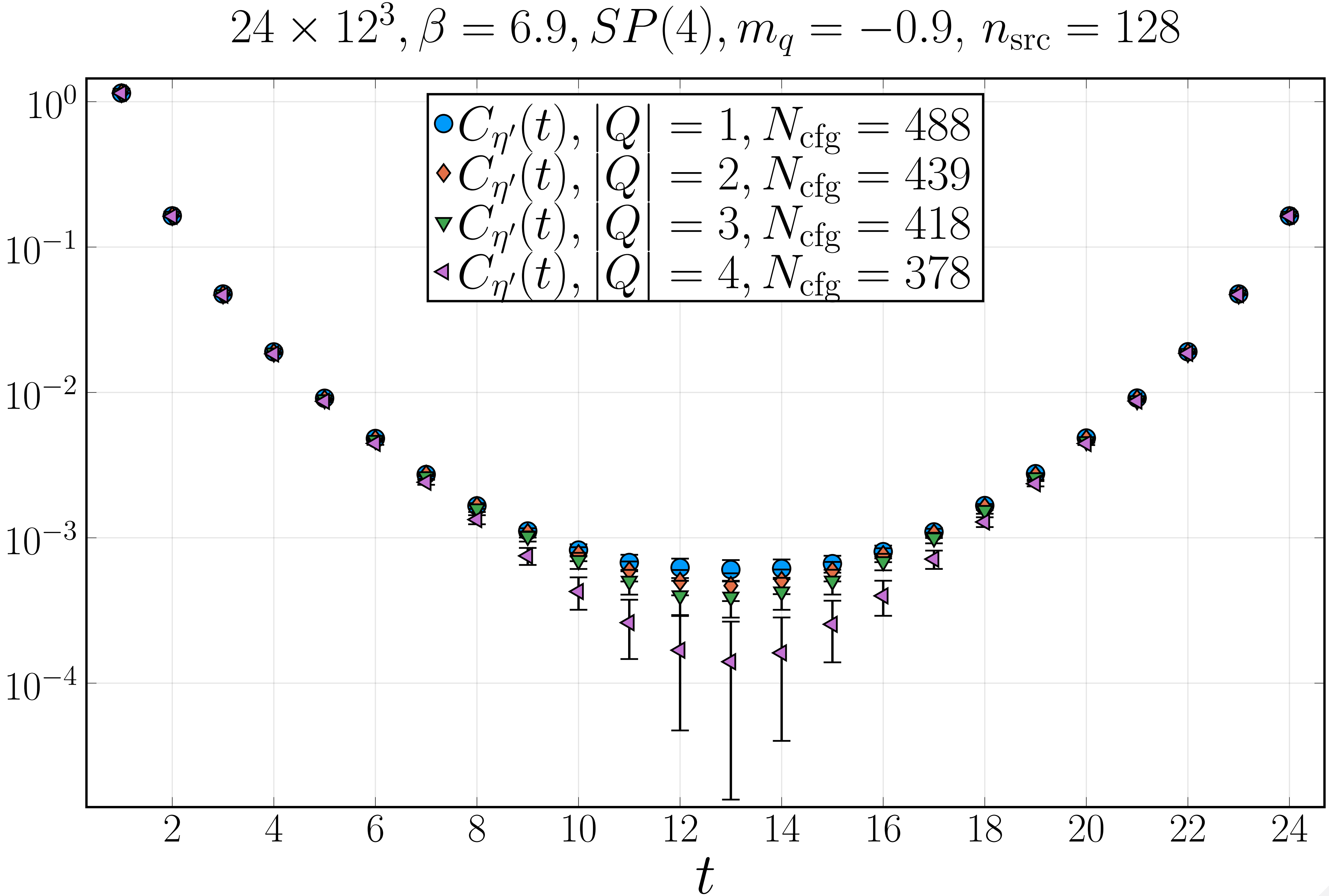
$$C(t-t') = \sum_{\vec{x},\vec{y}}$$



$$+ \underbrace{\text{const.}}_{=|\langle 0|O|0\rangle|^2}$$

$36 \times 28^3, \beta = 7.2, SP(4), m_q = -0.794,\ n_{\text{src}} = 288$





$24 \times 12^3, \beta = 6.9, SP(4), m_q = -0.9,\ n_{\rm src} = 128$

Flavour symmetry: Pseudo-real representation

- Higher symmetry than QCD-like (complex rep) theories
- Mixing of left- and right-handed Weyl components

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ -SCu_R^* \\ -SCd_R^* \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix} \qquad \begin{matrix} C \ldots \text{charge conj.} \\ S \ldots \text{colour matrix} \end{matrix}$$

$$\mathcal{L}_{\mathrm{DM}} = i\bar{\Psi}\not{D}\Psi - \frac{1}{2}\left(\Psi^T SCM\Psi + h.c.\right)$$

- Mass matrix M proportional to symplectic invariant tensor
- generators $\tau_a : S\tau_a S = -\tau_a^T$
- **Very similar pattern for real representation**