Supersymmetric QCD on the Lattice: Fine-tuning and Counterterms for the Yukawa and Quartic Couplings

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Contents

Supersymmetric QCD on the Lattice (1)

- Study the strong interactions between the particles and their superpartners [1] [2]
- Extend Wilson's formulation of the QCD action $\rightarrow$ superpartner fields [3] [4]
- Standard discretization $\rightarrow$ quarks (ψ), squarks ($A_\pm$) and gluinos (λ) $\rightarrow$ on the lattice points whereas gluons (u_μ) $\rightarrow$ on the links between adjacent points:

$$U_\mu(x) = \exp[igaT^\alpha u_\mu^\alpha(x + a\hat{\mu}/2)]$$

[1] J. Giedt, Int. J. Mod. Phys. A24 (2009) 4045-4095
[2] D. Schaich, PoS (LATTICE2018) 005
[3] D. Schaich, Eur. Phys. J. ST 232 (2023) no.3, 305-320
[4] G. Bergner and S. Catterall, Int. J. Mod. Phys. A 31 (2016) no.22, 1643005

Supersymmetric QCD on the Lattice (2)

- For Wilson-type quarks and gluinos, the Euclidean action $\mathcal{S}^L_{\rm SQCD}$ on the lattice becomes [5]:

$$\mathcal{S}^L_{\rm SQCD} = a^4 \sum_x \Big[\frac{N_c}{g^2} \sum_{\mu,\,\nu} \left(1 - \frac{1}{N_c}\,{\rm Tr}\, U_{\mu\,\nu}\right) + \sum_\mu {\rm Tr}\left(\bar\lambda \gamma_\mu \mathcal{D}_\mu \lambda\right) - a\frac{r}{2}{\rm Tr}\left(\bar\lambda \mathcal{D}^2 \lambda\right)$$
$$+ \sum_\mu \left(\mathcal{D}_\mu A_+^\dagger \mathcal{D}_\mu A_+ + \mathcal{D}_\mu A_- \mathcal{D}_\mu A_-^\dagger + \bar\psi \gamma_\mu \mathcal{D}_\mu \psi\right) - a\frac{r}{2}\bar\psi \mathcal{D}^2 \psi$$
$$+ i\sqrt{2} g \left(A_+^\dagger \bar\lambda^\alpha T^\alpha P_+ \psi - \bar\psi P_- \lambda^\alpha T^\alpha A_+ + A_- \bar\lambda^\alpha T^\alpha P_- \psi - \bar\psi P_+ \lambda^\alpha T^\alpha A_-^\dagger\right)$$
$$+ \frac{1}{2} g^2 (A_+^\dagger T^\alpha A_+ - A_- T^\alpha A_-^\dagger)^2 - m(\bar\psi\psi - m A_+^\dagger A_+ - m A_- A_-^\dagger)\Big],$$

- $P_\pm = (1 \pm \gamma_5)/2$ and $U_{\mu\nu}(x) = U_\mu(x) U_\nu(x + a\hat\mu) U^\dagger_\mu(x + a\hat\nu) U^\dagger_\nu(x)$
- m $\rightarrow$ the mass of the matter fields (which may be flavor-dependent)
- $\mathcal{D} \rightarrow$ the standard covariant derivative in the fundamental/adjoint representation [5]
- $a \rightarrow$ lattice spacing, $r \rightarrow$ Wilson parameter, $N_c \rightarrow$ number of colors
- $T^\alpha \rightarrow$ generators of SU(N_c), $g \rightarrow$ coupling constant

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507

Symmetries of the Supersymmetric QCD Action (1)

Parity ($\mathcal{P}$):

$$\mathcal{P}: \begin{cases} U_0(x) \to U_0(x_{\mathcal{P}})\,, \qquad U_k(x) \to U_k^{\dagger}(x_{\mathcal{P}} - a\hat{k})\,, \qquad k = 1,2,3 \\ \psi(x) \to \gamma_0 \psi(x_{\mathcal{P}}) \\ \bar{\psi}(x) \to \bar{\psi}(x_{\mathcal{P}})\gamma_0 \\ \lambda^{\alpha}(x) \to \gamma_0 \lambda^{\alpha}(x_{\mathcal{P}}) \\ \bar{\lambda}^{\alpha}(x) \to \bar{\lambda}^{\alpha}(x_{\mathcal{P}})\gamma_0 \\ A_{\pm}(x) \to A_{\mp}^{\dagger}(x_{\mathcal{P}}) \\ A_{\pm}^{\dagger}(x) \to A_{\mp}(x_{\mathcal{P}}) \end{cases}$$

where $x_P = (-\mathrm{x}, x_0)$

Symmetries of the Supersymmetric QCD Action (2)

- Charge conjugation ($\mathcal{C}$):

$$\mathcal{C}: \begin{cases} U_\mu(x) \to U_\mu^\star(x)\,, \quad \mu = 0,1,2,3 \\ \psi(x) \to -C\bar{\psi}(x)^T \\ \bar{\psi}(x) \to \psi(x)^T C^\dagger \\ \lambda(x) \to C\bar{\lambda}(x)^T \\ \bar{\lambda}(x) \to -\lambda(x)^T C^\dagger \\ A_\pm(x) \to A_\mp(x) \\ A_\pm^\dagger(x) \to A_\mp^\dagger(x) \end{cases}$$

- The matrix C satisfies: $(C\gamma_\mu)^T = C\gamma_\mu$, $C^T = -C$ and $C^\dagger C = 1$

Symmetries of the Supersymmetric QCD Action (3)

$U(1)_R$ which rotates the quark and gluino fields in opposite direction:

$$\mathcal{R}: \begin{cases} \psi_f(x) \to e^{i\theta\gamma_5}\psi_f(x) \\ \bar{\psi}_f(x) \to \bar{\psi}_f(x) e^{i\theta\gamma_5} \\ \lambda(x) \to e^{-i\theta\gamma_5}\lambda(x) \\ \bar{\lambda}(x) \to \bar{\lambda}(x) e^{-i\theta\gamma_5} \end{cases}$$

$U(1)_A$ which rotates the squark and the quark fields in the same direction as follows:

$$\chi: \begin{cases} \psi_f(x) \to e^{i\theta'\gamma_5}\psi_f(x) \\ \bar{\psi}_f(x) \to \bar{\psi}_f(x) e^{i\theta'\gamma_5} \\ A_\pm(x) \to e^{i\theta'} A_\pm(x) \\ A^\dagger_\pm(x) \to e^{-i\theta'} A^\dagger_\pm(x) \end{cases}$$

What do we calculate and why? (1)

- We calculate the renormalization factors of the Yukawa and quartic couplings of the $\mathcal{N} = 1$ Supersymmetric QCD
- Introduce the appropriate counterterms to the regularised Lagrangian $\rightarrow$ the bare parameters [6]
- Calculate perturbatively the relevant three-point and four-point Green's functions using both dimensional regularization (DR) in $D = 4 - 2\epsilon$ dimensions and lattice regularization (LR)

[6] P. Athron and D. J. Miller, Phys. Rev. D 76 (2007), 075010

What do we calculate and why? (2)

- Lagrangian parameters of the classical action do not include quantum fluctuations
- Restore Supersymmetry in the continuum limit [7] [8] [9]
- Important ingredients in extracting nonperturbative information for supersymmetric theories through lattice simulations
- This work is a sequel to earlier investigations on SCQD and completes the one-loop fine-tuning of the SQCD action on the lattice $\rightarrow$ paving the way for numerical simulations of SQCD [5] [10]

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[7] G. Curci and G. Veneziano, Nucl. Phys. B292 (1987) 555

[8] F. Farchioni et al., Eur. Phys. J. D 76 (2002), 719

[9] S. Ali et al., Eur. Phys. J. C 78 (2018) 404

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507

[10] M. Costa and H. Panagopoulos, Phys. Rev. D 99 (2019) no.7, 074512

Coupling Constants of the Action of SQCD

- Bare coupling constants appearing in the lattice action are not typically all identical
- Gauge coupling $g \rightarrow$ gluons couple with quarks, squarks, gluinos and other gluons with the same gauge coupling constant
- Yukawa interactions and four-squark interactions contain a potentially different coupling constant $\rightarrow$ must be fine-tuned on the lattice
- A similar situation holds for quark and squark masses [5]

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507

Computational Setup (1)

- Examine the behavior under $\mathcal{P}$ and $\mathcal{C}$ of all dimension-4 operators having one gluino, one quark and one squark field

Operators	$\mathcal{C}$	$\mathcal{P}$
$A_+^\dagger \bar\lambda P_+ \psi$	$-\bar\psi P_+ \lambda A_-^\dagger$	$A_- \bar\lambda P_- \psi$
$\bar\psi P_- \lambda A_+$	$-A_- \bar\lambda P_- \psi$	$\bar\psi P_+ \lambda A_-^\dagger$
$A_- \bar\lambda P_- \psi$	$-\bar\psi P_- \lambda A_+$	$A_+^\dagger \bar\lambda P_+ \psi$
$\bar\psi P_+ \lambda A_-^\dagger$	$-A_+^\dagger \bar\lambda P_+ \psi$	$\bar\psi P_- \lambda A_+$
$A_+^\dagger \bar\lambda P_- \psi$	$-\bar\psi P_- \lambda A_-^\dagger$	$A_- \bar\lambda P_+ \psi$
$\bar\psi P_+ \lambda A_+$	$-A_- \bar\lambda P_+ \psi$	$\bar\psi P_- \lambda A_-^\dagger$
$A_- \bar\lambda P_+ \psi$	$-\bar\psi P_+ \lambda A_+$	$A_+^\dagger \bar\lambda P_- \psi$
$\bar\psi P_- \lambda A_-^\dagger$	$-A_+^\dagger \bar\lambda P_- \psi$	$\bar\psi P_+ \lambda A_+$

Computational Setup (2)

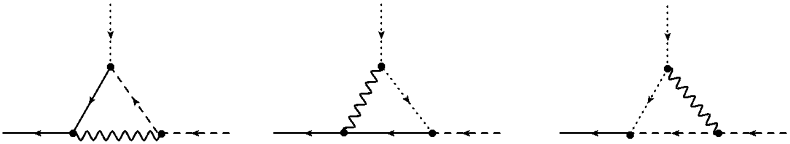
- Two linear combinations of Yukawa-type operators which are invariant under $\mathcal{P}$ and $\mathcal{C}$:

$$Y_1 \equiv A_+^\dagger \bar{\lambda} P_+ \psi - \bar{\psi} P_- \lambda A_+ + A_- \bar{\lambda} P_- \psi - \bar{\psi} P_+ \lambda A_-^\dagger \quad (1)$$
$$Y_2 \equiv A_+^\dagger \bar{\lambda} P_- \psi - \bar{\psi} P_+ \lambda A_+ + A_- \bar{\lambda} P_+ \psi - \bar{\psi} P_- \lambda A_-^\dagger \quad (2)$$

- All terms within each of the combinations in Eqs. (1) and (2) are multiplied by the same Yukawa coupling, g_{Y_1} and g_{Y_2}, respectively
- In the classical continuum limit: $g_{Y_1} \to g$ and $g_{Y_2} \to 0$

Computational Setup (3)

- Compute, perturbatively, the relevant three-point Green's functions with external gluino-quark-squark fields
- Three one-loop Feynman diagrams that enter the computation of the three-point amputated Green's functions for the Yukawa couplings:



- Wavy (solid) line $\rightarrow$ gluons (quarks)
- Dotted (dashed) line $\rightarrow$ squarks (gluinos)
- An arrow entering (exiting) a vertex $\rightarrow$ $\lambda, \psi, A_+, A_-^\dagger$ ($\bar{\lambda}, \bar{\psi}, A_+^\dagger, A_-$) field

Computational Setup (4)

- We impose renormalization conditions $\rightarrow$ in the cancellation of divergences in the corresponding bare three-point amputated Green's functions with external gluino-quark-squark fields
- The renormalization factors $\rightarrow$ to remove all divergences
- The application of the renormalization factors on the bare Green's functions $\rightarrow$ the renormalized Green's functions $\rightarrow$ independent of the regulator (ϵ in DR, a in LR) $\rightarrow$ renormalized Green's functions at a given scheme, but derived via different regularizations, should coincide

Renormalization Factors of the Fields and the Coupling Constants

- The renormalization factors of the fields and the gauge coupling constant:

$$\psi \equiv \psi^B = Z_\psi^{-1/2}\, \psi^R, \tag{3}$$

$$u_\mu \equiv u_\mu^B = Z_u^{-1/2}\, u_\mu^R, \tag{4}$$

$$\lambda \equiv \lambda^B = Z_\lambda^{-1/2}\, \lambda^R, \tag{5}$$

$$c \equiv c^B = Z_c^{-1/2}\, c^R, \tag{6}$$

$$g \equiv g^B = Z_g^{-1}\, \mu^\epsilon\, g^R \tag{7}$$

- The Yukawa coupling is renormalized as follows:

$$g_{Y_1} \equiv g_{Y_1}^B = Z_{Y_1}^{-1} Z_g^{-1} \mu^\epsilon g^R \tag{8}$$

- The definition of the renormalization mixing matrix for the squark fields:

$$\begin{pmatrix} A_+^R \\ A_-^{R\,\dagger} \end{pmatrix} = \left(Z_A^{1/2}\right) \begin{pmatrix} A_+^B \\ A_-^{B\,\dagger} \end{pmatrix} \tag{9}$$

Renormalization Condition in DR

- In the *DR* and $\overline{\rm MS}$ scheme this 2×2 mixing matrix is diagonal[5]
- Green's function in *DR* with external squark field $A_+ \rightarrow$ the renormalization condition:

$$\langle\lambda(q_1)A_+(q_3)\bar\psi(q_2)\rangle\Big|^{\overline{\rm MS}} = Z_\psi^{-1/2} Z_\lambda^{-1/2} (Z_A^{-1/2})_{++} \langle\lambda(q_1)A_+(q_3)\bar\psi(q_2)\rangle\Big|^{\rm bare} \tag{10}$$

- In the right-hand side $\rightarrow$ all coupling constants $\rightarrow$ expressed in terms of their renormalized values
- The left-hand side $\rightarrow$ the $\overline{\rm MS}$ renormalized Green's function
- The other renormalization conditions which involve the external squark fields $A_+^\dagger, A_-, A_-^\dagger$ are similar

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507

Renormalization Factor of the Yukawa Coupling in DR (1)

- Have checked that no IR divergences will be generated → calculation of the corresponding diagrams by setting to zero one of the external momenta
- One-loop Green's function for the Yukawa coupling for zero gluino momentum in DR with external squark field A_+:

$$\langle \lambda^{\alpha_1}(0)\bar{\psi}(q_2)A_+(q_3)\rangle^{DR,1\text{loop}} = -i\,(2\pi)^4\delta(q_2-q_3)\frac{g_Y g^2}{16\pi^2}\frac{1}{4\sqrt{2}N_c}T^{\alpha_1}\times$$
$$\Bigg[-3(1+\gamma_5)+((1+\alpha)(1+\gamma_5)+8\gamma_5 c_{\rm hv})N_c^2$$
$$+(1+\gamma_5)(-\alpha+(3+2\alpha)N_c^2)\left(\frac{1}{\epsilon}+\log\left(\frac{\bar{\mu}^2}{q_2^2}\right)\right)\Bigg] \qquad (11)$$

- $c_{\rm hv}=0,1$ for the naïve and 't Hooft-Veltman (HV) prescription of γ_5, respectively [11] [12] and α → gauge parameter

[11] M. S. Chanowitz et al., Nucl. Phys. B 159 (1979), 225-243
[12] G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B 44 (1972), 189-213

Renormalization Factor of the Yukawa Coupling in DR (2)

- Results of the renormalization factors in *DR* [5]:

$$Z_{\psi}^{DR,\overline{\text{MS}}} = 1 + \frac{g^2\, C_F}{16\,\pi^2}\frac{1}{\epsilon}\,(1+\alpha)\,, \qquad Z_{A}^{DR,\overline{\text{MS}}} = \left(1 + \frac{g^2\, C_F}{16\,\pi^2}\frac{1}{\epsilon}\,(-1+\alpha)\right)\mathbb{1}$$

$$Z_{\lambda}^{DR,\overline{\text{MS}}} = 1 + \frac{g^2}{16\,\pi^2}\frac{1}{\epsilon}\,(\alpha\, N_c + N_f)\,, \qquad Z_{g}^{DR,\overline{\text{MS}}} = 1 + \frac{g^2}{16\,\pi^2}\frac{1}{\epsilon}\left(\frac{3}{2}N_c - \frac{1}{2}N_f\right)$$

- $C_F = (N_c^2 - 1)/(2\,N_c)$ → Casimir operator and N_f → number of flavors
- For all Green's functions and all choices of the external momenta which we consider, we obtain the same value of $Z_{Y_1}^{DR,\overline{\text{MS}}}$ [13]:

$$\boldsymbol{Z_{Y_1}^{DR,\overline{\text{MS}}} = 1 + \mathcal{O}(g^4)} \tag{12}$$

- $Z_{Y_1}^{DR,\overline{\text{MS}}}$ → **at the quantum-level, the renormalization process involving the Yukawa interaction is not affected by one-loop corrections**

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507
[13] M. Costa, H. Herodotou and H. Panagopoulos, Phys. Rev. D 109 (2024) no.3, 035015

Renormalization Condition on the Lattice

- On the lattice the renormalization mixing matrix for the squark fields $\rightarrow$ non diagonal $\rightarrow$ the component $A_+(A_-)$ mixes with $A_-^\dagger(A_+^\dagger)$ [5]
- The $\chi \times \mathcal{R}$ symmetry is broken
- Introducing mirror Yukawa counterterms in the action
- The renormalization condition is the following:

$$\langle \lambda(q_1) A_+(q_3) \bar{\psi}(q_2) \rangle \Big|^{\overline{\rm MS}} = Z_\psi^{-1/2} Z_\lambda^{-1/2} \langle \lambda(q_1) \big((Z_A^{-1/2})_{++} A_+(q_3) + (Z_A^{-1/2})_{+-} A_-^\dagger(q_3) \big) \bar{\psi}(q_2) \rangle \Big|^{\rm bare} \quad (13)$$

- The bare coupling g_{Y_2} arises in the right hand side of the renormalization condition

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507

Renormalization Factor of the Yukawa Coupling on the Lattice (1)

- The difference between the one-loop $\overline{\rm MS}$-renormalized and bare lattice Green's functions for zero gluino momentum and with external squark field A_+:

$$\langle\lambda^{\alpha_1}(0)A_+(q_3)\bar\psi(q_2)\rangle^{\overline{\rm MS},1{\rm loop}} - \langle\lambda^{\alpha_1}(0)A_+(q_3)\bar\psi(q_2)\rangle^{LR,1{\rm loop}}$$
$$= -i\,(2\pi)^4\delta(q_2-q_3)\frac{g_Y g^2}{16\pi^2}\frac{1}{4\sqrt{2}N_c}T^{\alpha_1}\times$$
$$\Big[-3.7920\alpha(1+\gamma_5)+(-3.6920+5.9510\gamma_5+7.5840\alpha(1+\gamma_5)-8\gamma_5 c_{\rm hv})N_c^2$$
$$+(1+\gamma_5)(\alpha-(3+2\alpha)N_c^2)\log\left(a^2\bar\mu^2\right)\Big] \qquad (14)$$

- Alternative choices of the external momenta give the same results for these differences

Renormalization Factor of the Yukawa Coupling on the Lattice (2)

- By combining this difference and by recalling renormalization factors of fields and gauge coupling on the lattice → the renormalization factors [13]:

$$Z_{Y_1}{}^{LR,\overline{\rm MS}} = 1 + \frac{g^2}{16\,\pi^2}\left(\frac{1.45833}{N_c} + 2.40768 N_c + 0.520616 N_f\right) \tag{15}$$

$$g_{Y_2}{}^{LR,\overline{\rm MS}} = \frac{g^3}{16\,\pi^2}\left(\frac{-0.040580}{N_c} + 0.45134 N_c\right) \tag{16}$$

- **Gauge independent** → the $\overline{\rm MS}$ renormalization factors for gauge invariant objects are gauge-independent
- **Finite**

[13] M. Costa, H. Herodotou and H. Panagopoulos, Phys. Rev. D 109 (2024) no.3, 035015

Computational Setup (1)

- Identify the four external squark fields → gauge symmetry constraints → two squarks to lie in the fundamental representation and the other two in the antifundamental
- Ten possibilities for choosing the 4 external squarks:

$$(A_+^\dagger A_+)(A_+^\dagger A_+), \quad (A_- A_-^\dagger)(A_- A_-^\dagger),$$
$$(A_+^\dagger A_+)(A_- A_-^\dagger), \quad (A_+^\dagger A_-^\dagger)(A_+^\dagger A_-^\dagger), \quad (A_- A_+)(A_- A_+), \quad (A_- A_+)(A_+^\dagger A_-^\dagger),$$
$$(A_+^\dagger A_+)(A_+^\dagger A_-^\dagger), \quad (A_+^\dagger A_+)(A_- A_+), \quad (A_- A_-^\dagger)(A_+^\dagger A_-^\dagger), \quad (A_- A_-^\dagger)(A_- A_+)$$

- Pairs of squark fields in parenthesis → color-singlet combinations

Computational Setup (2)

- Take into account $\mathcal{C}$ and $\mathcal{P}$ to construct combinations which are invariant under these symmetries
- There are five combinations [14]:

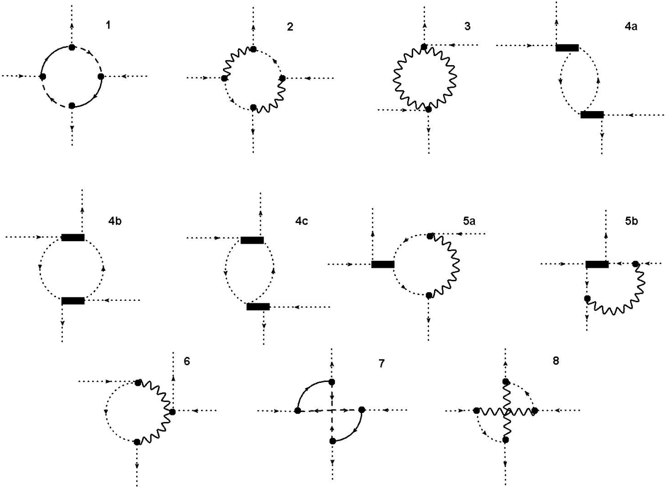
Operators	$\mathcal{C}$	$\mathcal{P}$
$\lambda_1(A_+^\dagger\, T^\alpha\, A_+ + A_-\, T^\alpha\, A_-^\dagger)^2/2$	+	+
$\lambda_2[(A_+^\dagger A_-^\dagger)^2 + (A_- A_+)^2]$	+	+
$\lambda_3(A_+^\dagger A_+)(A_- A_-^\dagger)$	+	+
$\lambda_4(A_+^\dagger A_-^\dagger)(A_- A_+)$	+	+
$\lambda_5(A_+^\dagger A_-^\dagger + A_- A_+)(A_+^\dagger A_+ + A_- A_-^\dagger)$	+	+

- The tree-level values of λ_i which satisfy Supersymmetry are:

$$\lambda_1 = g^2, \;\; \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0 \tag{17}$$

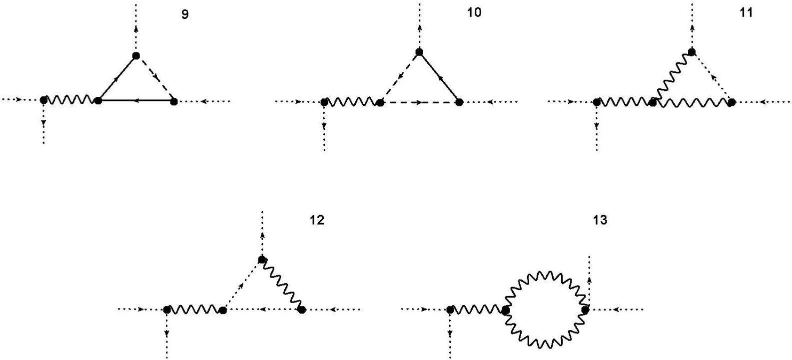
[14] B. Wellegehausen and A. Wipf, PoS LATTICE2018 (2018), 210

Computational Setup (3)



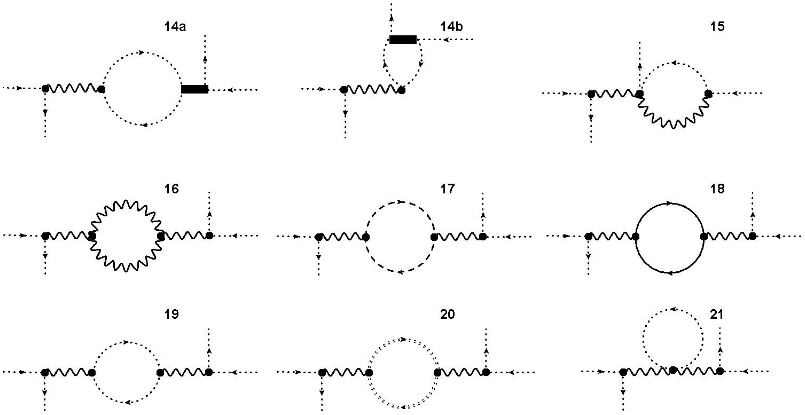
- A wavy (solid) line → gluons (quarks)
- A dotted (dashed) line → squarks (gluinos)
- A solid rectangle → the 4-squark vertex

Computational Setup (4)



- A wavy (solid) line → gluons (quarks)
- A dotted (dashed) line → squarks (gluinos)

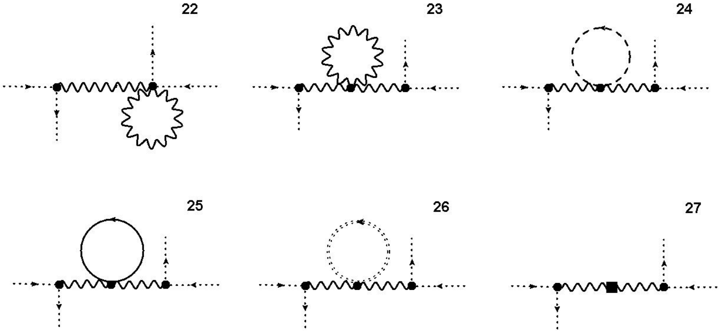
Computational Setup (5)



- A wavy (solid) line → gluons (quarks)
- A dotted (dashed) line → squarks (gluinos)
- A solid rectangle → the 4-squark vertex

Computational Setup (6)

- Additional one-loop Feynman diagrams leading to the fine-tuning of the quartic couplings on the lattice:



- A wavy (solid) line → gluons (quarks)
- A dotted (dashed) line → squarks (gluinos)
- A "double dashed" line → ghost field
- The solid box in diagram 27 → measure part of the lattice action

Renormalization Condition in DR

- Compute the diagrams by setting 2 external squark momenta of fields in the fundamental representation to zero
- The renormalization factor of the gauge parameter Z_α [5]:

$$\alpha^R = Z_\alpha^{-1}\, Z_u\, \alpha^B \tag{18}$$

- The quartic coupling is renormalized as follows:

$$\lambda_1 = Z_{\lambda_1}^{-1} Z_g^{-2} \mu^{2\epsilon}\, (g^R)^2 \tag{19}$$

- Green's function in *DR* with external squark fields A_+ and $A_+^\dagger$ → the renormalization condition:

$$\langle A_+(q_1)A_+^\dagger(q_2)A_+(q_3)A_+^\dagger(q_4)\rangle\Big|^{\overline{\mathrm{MS}}} = (Z_A^{-2})_{++}\langle A_+(q_1)A_+^\dagger(q_2)A_+(q_3)A_+^\dagger(q_4)\rangle\Big|^{\mathrm{bare}} \tag{20}$$

- In the right-hand side all coupling constants and the gauge parameter → expressed in terms of their renormalized values

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507

Renormalization Factor of the Quartic Coupling in DR (1)

- The difference between the $\overline{\mathrm{MS}}$-renormalized and the corresponding bare Green's function:

$$\langle A^{\dagger\alpha_1}_{+\,f_1}(q_1)A^{\dagger\alpha_2}_{+\,f_2}(q_2)\,A^{\alpha_3}_{+\,f_3}(q_3)A^{\alpha_4}_{+\,f_4}(q_4)\rangle^{\overline{\mathrm{MS}},1\mathrm{loop}} - \langle A^{\dagger\alpha_1}_{+\,f_1}(q_1)A^{\dagger\alpha_2}_{+\,f_2}(q_2)\,A^{\alpha_3}_{+\,f_3}(q_3)A^{\alpha_4}_{+\,f_4}(q_4)\rangle^{DR,1\mathrm{loop}}$$
$$= \frac{g^4}{64\,\pi^2\,N_c^2}\,\frac{1}{\epsilon}\Bigg[\Big(2+4\,N_c^2+\alpha\,(-2\alpha+3\,(1+\alpha)\,N_c^2)-2\,N_c\,N_f\Big)\times$$
$$\Big(\delta_{f_1 f_3}\delta_{f_2 f_4}(-\delta^{\alpha_1\alpha_3}\delta^{\alpha_2\alpha_4}+N_c\,\delta^{\alpha_1\alpha_4}\delta^{\alpha_2\alpha_3})$$
$$+\,\delta_{f_1 f_4}\delta_{f_2 f_3}(-\delta^{\alpha_1\alpha_4}\delta^{\alpha_2\alpha_3}+N_c\,\delta^{\alpha_1\alpha_3}\delta^{\alpha_2\alpha_4})\Big)\Bigg] \tag{21}$$

Renormalization Factor of the Quartic Coupling in DR (2)

- By utilizing the renormalization condition, the renormalization factors of the fields, the gauge coupling and the gauge parameter in DR, we obtain:

$$Z_{\lambda_1}{}^{DR,\overline{\mathrm{MS}}} = 1 + \mathcal{O}(g^4) \tag{22}$$

- At the quantum level, the renormalization of the quartic coupling in DR remains unaffected by one-loop corrections
- Terms proportional to λ_2 - λ_5 do not manifest in the $\overline{\mathrm{MS}}$ renormalization using DR

Summary - Conclusions

- The multiplicative renormalization of the Yukawa coupling and the coefficient of the mirror Yukawa counterterm on the lattice $\rightarrow$ **finite and gauge independent**
- At the quantum level, the renormalization of the quartic coupling in DR **remains unaffected by one-loop corrections**
- The renormalization of the quartic coupling on the lattice is **underway** $\rightarrow$ a finite, but nontrivial renormalization for λ_1, as well as nonzero finite couplings λ_2 - λ_5 for the corresponding counterterms

Thank you!

Thank you for your attention!

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