Dilaton Effective Theory and Soft Theorems

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mostly based on

Del Debbio, RZ	JHEP'22 2112.1364	Dilaton new phase?
RZ	PRD, 2306.06752	broken χ-sym.@IRFP - pions
RZ	2306.12914	Dilaton improves Goldstones
Shifman RZ	PRD, 2310.16449	β_*' in N=1 confomal window
RZ	PRD 2312.13761	broken χ-sym.@IRFP - pions & dilaton

Extensive list of Refs in papers

Lattice 2024 - Liverpool - 30 July 2024

Overview

- **Dilaton soft theorem** & **improvement term**

  ⇒ *model-independent constraint, operator $\mathcal{O}$ generating dilaton mass*

$$\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}} = d - 2$$

- **Interpretation** assuming **QCD=IR-CFT$_{\text{SSB}}$** is consistent

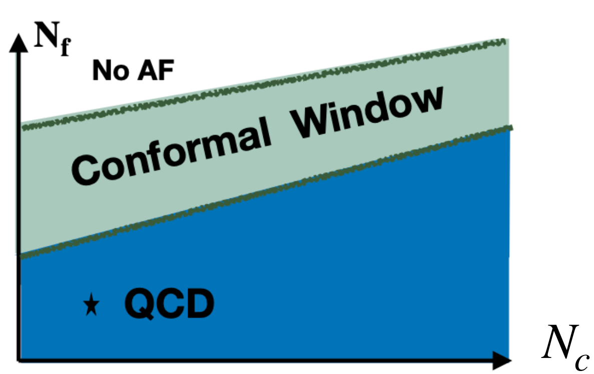
- Does it **make sense** to consider **chirally broken** phase as **IRFP**?

  Yes, in $\mathcal{N} = 1$ SUSY gauge theories (Seiberg dualities)

- **Conclusions & Outlook**

Theories

	no SSB $\langle \bar{q}q \rangle = 0$	SSB $\langle \bar{q}q \rangle \neq 0$	
no flow $\beta = 0$	**CFT** powerful tools	**CFT**$_{\text{SSB}}$ less known	$m_D = 0$
flow $\beta \neq 0$	**IR-CFT** good understanding e.g. hyperscaling "conformal window"	**IR-CFT**$_{\text{SSB}}$ in progress is QCD of this type?	$m_D = 0?$



Dilaton D

Dilaton (formal basics)

- **What is a dilaton?**
  0^{++}-Goldstone due to spontaneous breaking of scale symmetry (1970)

- **SSB?** Goldstone current (eg. chiral) $\langle \pi^b | J^a_{\mu 5} | 0 \rangle = i q_\mu F_\pi$
  couples to Goldstone (eg. pion) s.t. $Q^a_5 | 0 \rangle \neq 0$ vacuum non-invariant

- **Dilatation current** defined EMT: $J^\mu_D = x_\nu T^{\mu\nu}$, analogy **dilaton decay constant**

$$\langle D | T_{\mu\nu} | 0 \rangle = \frac{F_D}{d-1} (m_D^2 \eta_{\mu\nu} - q_\mu q_\nu) \qquad (1)$$

- **Dilaton mass?** Could be due to explicit symmetry breaking (quark mass)*

$$\langle D | T^\rho_\rho | D \rangle = 2 m_D^2 \qquad (2)$$

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* generally valid, unless dilaton massless as then $\langle N | T^\rho_\rho | N \rangle = 0$ and $m_N \neq 0$ Del Debbio, RZ '21 JHEP

Dilation EFT basics

Isham, Salam, Strathdee, Mack, Zumino ca '70

- Non-linear representation: $\hat{\chi} = \exp(-D/F_D)$ $(\chi = F_D \hat{\chi})$

$$\mathscr{L}_{LO} = \frac{1}{2}\hat{\chi}^{d-4}(\partial\chi)^2 + \frac{\kappa_d}{4} R\chi^{d-2} + V(\hat{\chi}) + \text{ other sectors (e.g. pions)}$$

kinetic

=0 if dilaton massless

(locally) Weyl invariant: $g_{\mu\nu} \to e^{-2\alpha(x)} g_{\mu\nu}\,, \quad \hat{D} \to \hat{D} - \alpha(x)$

- Improvement term* $\kappa_d = \frac{2}{(d-1)(d-2)}$, reproduces eq (1) (RZ, 2306.12914)

$$\langle D | T_{\mu\nu} | 0 \rangle = \frac{F_D}{d-1}(m_D^2 \eta_{\mu\nu} - q_\mu q_\nu)$$

$\Rightarrow$ it is a **must** and will play a further role very soon …..

- Other sectors: *compensator mechanism*: $\delta\mathscr{L} = -\, m_\phi^2/2\phi^2\hat{\chi}^{d-2}$ (restores Weyl-inv.)

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* improvement term also solves pion improvement problem & helps for flow thms (e.g. a-thm) RZ, 2306.12914

Dilaton mass and soft theorems

RZ 2312.13761

- Assume operator $\mathcal{O} \subset T^\rho_\rho$ responsible for dilation mass

$$\langle D | T^\rho_\rho | 0 \rangle = F_D m_D^2 \qquad (1')$$

$$\langle D | T^\rho_\rho | D \rangle = 2 m_D^2 \qquad (2)$$

recall our first principles equations

- Idea: using soft-dilation thm on (2) $\Rightarrow$ learn sthg about $\mathcal{O}$

$$\lim_{q \to 0} \langle D(q) \beta | \mathcal{O}(0) | \alpha \rangle = -\frac{1}{F_D} \langle \beta | i [Q_D, \mathcal{O}(0)] | \alpha \rangle + \lim_{q \to 0} iq \cdot R$$

$$i[Q_D, \mathcal{O}(x)] = (\Delta_{\mathcal{O}} + x \cdot \partial) \mathcal{O}(x)$$

$$R_\mu = -\frac{i}{F_D} \int d^d x e^{iq \cdot x} \langle \beta | T J^D_\mu(x) \mathcal{O}(0) | \alpha \rangle$$

Dilaton soft theorem applied to equation (2)

$$2m_D^2 = \langle D|\mathcal{O}(x)|D\rangle = -\frac{1}{F_D}\langle 0|i[Q_D,\mathcal{O}(x)]|D\rangle = -(\Delta_{\mathcal{O}} + x\cdot\partial)\langle 0|\mathcal{O}(x)|D\rangle$$

- There is **x-dependence** in matrix element: $\langle 0|\mathcal{O}(x)|D(p)\rangle = F_{\mathcal{O}}e^{-ipx}$

- Interpret as distribution to be smeared out

$$\mathbb{1}_V[x\cdot\partial\langle 0|\mathcal{O}(x)|D\rangle] = -d\frac{1}{V}\int_V d^dx\langle 0|\mathcal{O}(x)|D\rangle$$

Physics: form **wave packet**
(validates integration by parts as boundary-terms automatically vanish (finite wave packet))

$$\mathbb{1}_V = \frac{1}{V}\int_V d^dx$$

… concluding

$$2m_D^2 = \frac{1}{F_D}(d - \Delta_{\mathcal{O}})\langle 0|T^\rho{}_\rho|D(0)\rangle = (d - \Delta_{\mathcal{O}})m_D^2$$

↗ $F_D m_D^2$ by (1')

⇒ Operator giving mass to dilation ought to be of scaling dimension

$$\Delta_{\mathcal{O}} = d - 2$$

important result

EFT interpretation of $\Delta_{\mathcal{O}} = d-2$

- What does $\mathcal{O} \subset T^{\rho}_{\rho}$ mean in EFT? $V \supset a\hat{\chi}^{\Delta_{\mathcal{O}}} + \dots$

$$V_{\Delta_{\mathcal{O}}} = \frac{F_D^2 m_D^2}{\Delta_{\mathcal{O}} - d}\left(\frac{1}{\Delta_{\mathcal{O}}}\hat{\chi}^{\Delta_{\mathcal{O}}} - \frac{1}{d}\hat{\chi}^{d}\right) = c + \frac{1}{2}m_D^2 D^2 + f(\Delta_{\mathcal{O}})D^3$$

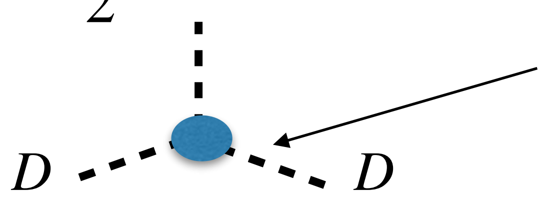
Zumino-term 70'

(In soft-thm mimicks $x\cdot\partial$-term)

- So, how come $\Delta_{\mathcal{O}} = d-2$ is constrained?

The **improvement-term** is **not innocent** $\langle D | T^{\rho}_{\rho}|_{imp} | D\rangle = 0$

With $T^{\rho}_{\rho}|_{imp} = -\frac{F_D}{2}\partial^2 D$ as otherwise $\langle D | T^{\rho}_{\rho} | D\rangle = 2m_D^2$ does not hold



kinetic $D(\partial D)^2$

potential $f(\Delta_{\mathcal{O}})D^3$

$\Rightarrow$ tadpole of improvement term leads to $\Delta_{\mathcal{O}} = d-2$ constraint

End of part 1 - bonus run 1

QCD is IR-CFT$_{SSB}$

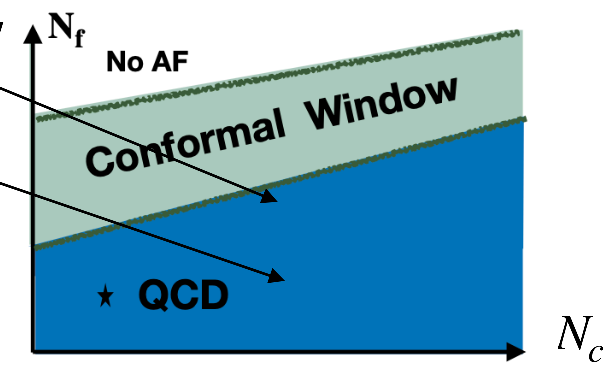
Switch gears .... assume QCD is IR-CFT$_{\text{SSB}}$

Really another talk (here .. nutshell-version)

- Under this assumptions shown (many ways - backup) RZ, 2306.06752, 2312.13761

$$\gamma_* = -\gamma_{\bar{q}q}|_{\mu=0} = 1^* \qquad \beta'_* = 0$$

- Whereas often assumed IR-CFT$_{\text{SSB}}$ interpretation below sill of CW, here we **explore** in **all of χ-broken phase**



- Dilaton? In QCD $\sigma = f_0(500)$ natural candidate
  Q: what is the m_σ in chiral limit? A: nobody knows
  However, reasoning works equally for $m_D = 0$ and $m_D \neq 0$

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* consistent with N=1 SUSY and lattice fits Appelquist, Ingoldby, Pia'17i , and Golterman,Neil,Shamir'19

QCD@IR-CFT$_{SSB}$ interpretation of $\Delta_{\mathcal{O}} = d-2$

$d = 4$

$$T^\rho_\rho|_{phys} = \frac{\beta}{2g} G^2 + N_f m_q (1+\gamma_m)\bar{q}q$$

- $m_q = 0$: only $\mathcal{O} = cG^2$ with $\Delta_{G^2} = 4 + \beta'_* = 4 \neq 2$

  $\Rightarrow$ how G^2 can give mass to dilation is **unclear** (to me)

- $m_q \neq 0$: then $\mathcal{O} = c\bar{q}q$ with $\Delta_{\bar{q}q} = 3 - \gamma_* = 2 \quad \Leftrightarrow \quad \gamma_* = 1$

  $\Rightarrow$ if $m_D = 0$, deforming $m_q \neq 0$ **dilation-GMOR** $\quad F_D^2 m_D^2 = -4 N_f m_q \langle \bar{q}q \rangle$

(previous works 70' and 80' difference $\gamma_* = 1$)

In literature:

a) tradition $\langle G^2 \rangle \neq 0 \;\Leftrightarrow\; m_D \neq 0$ and $\Delta_{\mathcal{O}} = 4$, e.g Golterman & Shamir

b) or no constraint at all $\Delta_{\mathcal{O}}$ + quark mass Appelquist, Ingoldby, Piai & LSD

End of part II - bonus run II

Does it **make sense** to consider **chirally broken** phase **IR-CFT$_{SSB}$?**

$\mathcal{N} = 1$ SUSY gauge theories (Seiberg duality)

electric theory | **magnetic theory**

$SU(N)$ & $2N_f$ chiral matter fields

$SU(N_D)$ & $N_D = N_f - N$ .. matter &

$M_i^{\bar{j}}$ colour neutral **meson field** (adjoint)

Dual IR? a) **global symmetries match** IR b) some operators known to match

a) e.g. $$\langle T^{\rho}{}_{\rho}(x) T^{\alpha}{}_{\alpha}(0) \rangle_{\rm el} \overset{\rm IR}{\longleftrightarrow} \langle T^{\rho}{}_{\rho}(x) T^{\alpha}{}_{\alpha}(0) \rangle_{\rm mag}$$

b) e.g. $$\tilde{Q}^{\bar{j}} Q_i \quad \leftrightarrow \quad M_i^{\bar{j}}$$

below CW
(chiral sym. broken)

$$N + 1 < N_f < \frac{3}{2} N$$

IR-free
magnetic phase

$$2 - \gamma_* = \Delta_{\tilde{Q}Q} = \Delta_M = 1 \quad \Leftrightarrow \quad \gamma_* = 1$$

- Q: *Does it make sense to extend below CW-boundary?*
  A: **At least in $\mathcal{N}=1$ SUSY** gauge theory

- We can get **further inspiration** from $\mathcal{N}=1$....

$$\Delta_{G^2} = 4 + \beta'_* = \Delta_{T^\rho_\rho} \quad \Rightarrow \quad \langle T^\rho{}_\rho(x) T^\rho{}_\rho(0)\rangle_{CW} \propto \frac{1}{(x^2)^{4+\beta'_*}}$$

$$\langle T^\rho{}_\rho(x) T^\alpha{}_\alpha(0)\rangle_{\rm el} \stackrel{\rm IR}{\longleftrightarrow} \langle T^\rho{}_\rho(x) T^\alpha{}_\alpha(0)\rangle_{\rm mag}$$

$$\Rightarrow \quad \left(\beta'_*|_{\rm el} = \beta'_*|_{\rm mag} \right)_{\rm CW}$$

Anselmi, Grisaru, Johanson 97'
Shifman RZ '23

- ***Below CW?*** Magnetic IR-free, thus $\beta'_*|_{\rm mag} = 0 \;\Rightarrow\; \beta'_*|_{\rm el} = 0$ **by continuity**

Summary

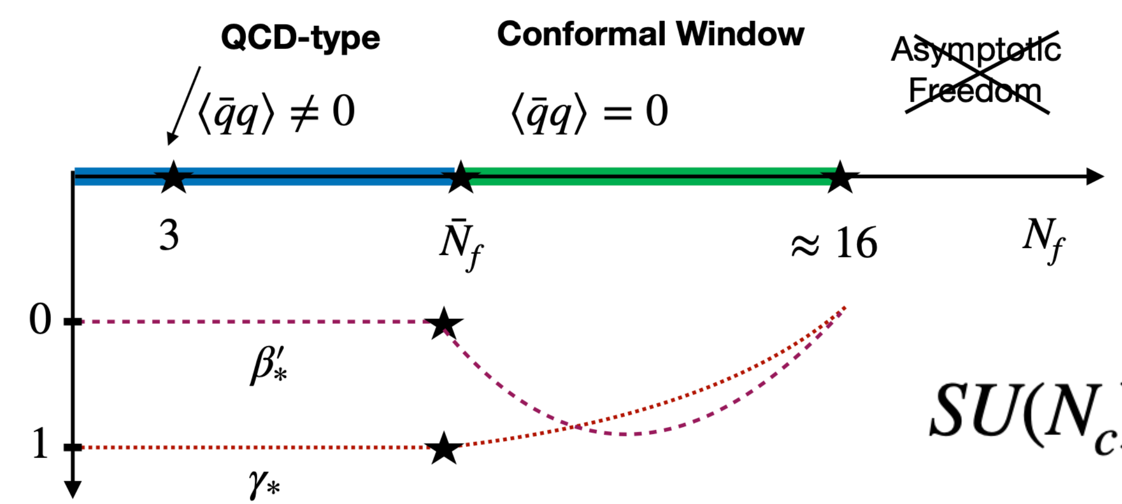
- **Dilaton soft-thms & improvement-term** go hand in hand

  ⇒ *model-independent constraint, operator $\mathcal{O}$ generating dilaton mass*

$$\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}} = d - 2$$

  not clear to me, how to formulate dilaton-EFT with massive dilaton $(m_q = 0)$
  possible to find a solvable near conformal model and work it out in full?

- **QCD = IR-CFT$_{\text{SSB}}$?**
  a) looks consistent (not covered in any detail .. time)
  b) $\mathcal{N} = 1$ SUSY, looks like a dilaton phase can be extended
  c) its **dilaton-EFT** prefers (implies?) **integer scaling dimensions**



$SU(N_c)\ , N_c = 3$

Outlook

- *Q: Can the dilaton remain massless when there is a flow into IRFP?*

  A: yes it cab d=3 model Cresswell-Hogg Litim'23 and Cresswell-Hogg Litim, RZ '24

  Methods presented seem to work - consistency in the dilaton-GMOR relation

- *Q: Can $\sigma = f_0(500)$ **meson** be a **dilaton?***

  A1: likely more special than many people think (e.g. light in chiral limit)

  A2: dilaton-EFT. - **width** works qualitatively ..
  - **mass** issues with a) strange quark & b) convergence.

  A3: efforts needed: lattice, FRG, Dyson-Schwinger, Roy equations & Bootstrap?

The End - Thank You

Backup

- *Q: Can* ***Higgs*** *be a* ***dilaton?***

A: probably yes, **if** $F_\pi/F_D \approx 1$ for $N_f = 2$ (weak force)

- gauge theory G' with one doublet (narrow dilaton)
- one does not need massless dilaton
- coupled to SM via Yukawa-sector as EFT
- is approximately satisfied in nucleon potential models puzzling as there is no symmetry reason known (yet)

Interesting open problems …
Hope to learn more during workshop - thank you!

Matching scalar adjoint correlator

$$m_q = 0$$

$$S^a = \bar{q} T^a q$$

$$\langle S^a(x) S^a(0) \rangle_{\text{CD-QCD}} = \langle S^a(x) S^a(0) \rangle_{\chi\text{PT}} \,, \quad \text{for } x^2 \to \infty$$

deep-IR

$$\langle \mathcal{O}(x) \mathcal{O}^\dagger(0) \rangle_{\text{CFT}} \propto (x^2)^{-\Delta_{\mathcal{O}}}$$

$$\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}}$$

$$\langle S^a(x) S^a(0) \rangle_{\text{CD-QCD}} \propto (x^2)^{-(3-\gamma_*)}$$

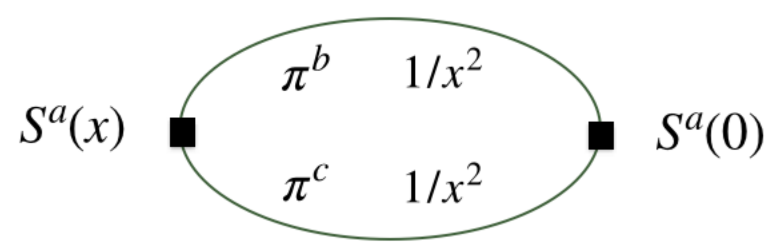
$$\Delta_{S^a} = d_{S^a} - \gamma_*$$

source theory

Gasser & Leutwyler'84

$$S^a|_{\text{LO}} = -\frac{F_\pi^2 B_0}{2} \text{Tr}[T^a U^\dagger + U T^a] \propto B_0 d^{abc} \pi^b \pi^c + \ldots$$

$$\langle S^a(x) S^a(0) \rangle_{\chi\text{PT}} \propto B_0^2 d^{abc} d^{abc} \langle \pi^a(x) \pi^a(0) \rangle^2 \propto \frac{1}{x^4}$$



CFT-scaling matches Goldstone EFT

$$\gamma_* = 1$$

2nd main result

Trace anomaly & Feynman-Hellmann thm

$m_q \neq 0$

$$2m_\pi^2 = \langle \pi^a | T^\rho{}_\rho | \pi^a \rangle$$

$$T^\rho_\rho|_{phys} = \beta/(2g)\, G^2 + N_f m_q (1+\gamma_m)\bar{q}q$$

Ellis, Chanowist, Crewther, Minkowski
Adler, Duncan, Nielsen, Collins, Jogelekar '72-75 '

$$\partial_{\ln m_q} E_\pi = N_f m_q \langle \hat{\pi} | \bar{q}q | \hat{\pi} \rangle + \mathcal{O}(m_q^2)$$

main ingredient: $\partial_{m_q} \langle \hat{\pi}(p') | \hat{\pi}(p) \rangle = 0$

rewrite using GMOR $m_\pi^2 \propto m_q$ (QCD):

$$2m_\pi^2 = \langle \pi | \beta/(2g) G^2 + N_f m_q (1+\gamma_m) \bar{q}q | \pi \rangle$$

$$2m_\pi^2|_{m_q} = 2N_f m_q \langle \pi | \bar{q}q | \pi \rangle$$

reduces to GMOR double soft-pion thm

1. Note that these two **must equate at** $\mathcal{O}(m_q)$, also in standard QCD

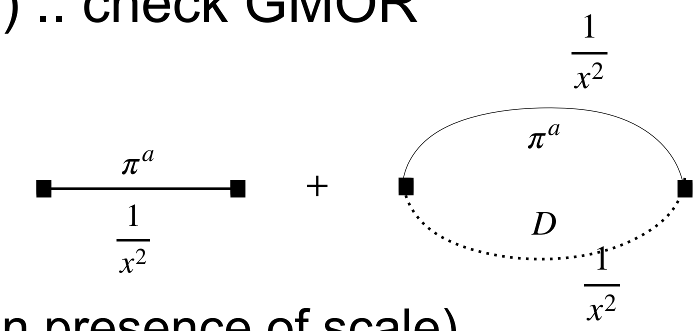
2. Note that $\beta \to \beta_* = 0\,, \gamma_m \to \gamma_* = 1$ seems a simple $\mathcal{O}(m_q)$-solution

$\Rightarrow \gamma_* = 1$ follows once more

*residue $\mathcal{O}(q^2, m_\pi^2) \Rightarrow$ pole no "dramatic" effect

Interpretation & comments

- Works with and without dilation ($m_D = 0$, $m_D \neq 0$) .. check GMOR



- 1) *Accidental?* can be **derived** in **other ways**

  i) $P^a = \bar{q}\gamma_5 T^a q$-correlator
  (breakdown of state-operator correspondence or RG in presence of scale)

  ii) hyperscaling $m_\pi^2 \propto m_q^{\frac{2}{1+\gamma_*}} \propto m_q$ (need to argue)

  iii) low energy thm for pion gravitational form factor RZ, 2306.12914v2

- 2) *Accidental?* **consistent** with **end** of **conformal window** in

  a) $\mathcal{N} = 1$ SUSY gauge theories b) other approaches & lattice

  Suggests: not accidental at boundary

*However, does it **make sense** to **extend below CW**-boundary?*
$\Rightarrow$ look at $\mathcal{N} = 1$

non-standard

$\beta'_* = 0$ important since ..

- Power-running $\delta g \propto \mu^{\beta'_*}$ $\Rightarrow$ **log-running**

$$\delta g \propto \frac{1}{|\beta''_*| \ln(\mu/\lambda_{IR})}$$

$\Rightarrow$ seems can **drop** $\mathscr{L}_{\textbf{anom}}(\beta'_*)$ from LO Lagrangian

as anomaly reproduced in extending "EMT in ✗PT" Donoghue & Leutwyler 90'

$\Rightarrow$ **log-running**, sign of **mass-gap**. QCD asymptotes into Goldstone-EFT

- Makes light (or massless) dilaton more probable since: $m_D = \mathcal{O}(\beta'_*) \to \mathcal{O}(\beta''_*)$

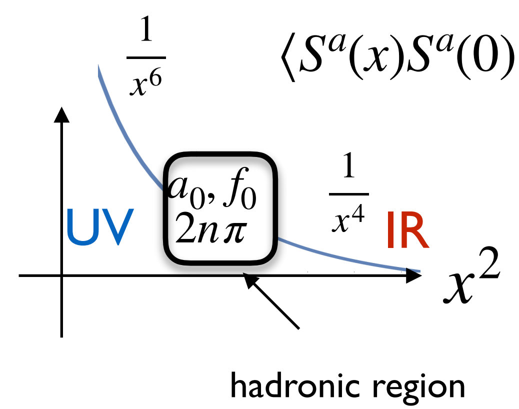
- Argument in favour of Seiberg dual for QCD (possibly hidden local symmetry)

An emerging picture

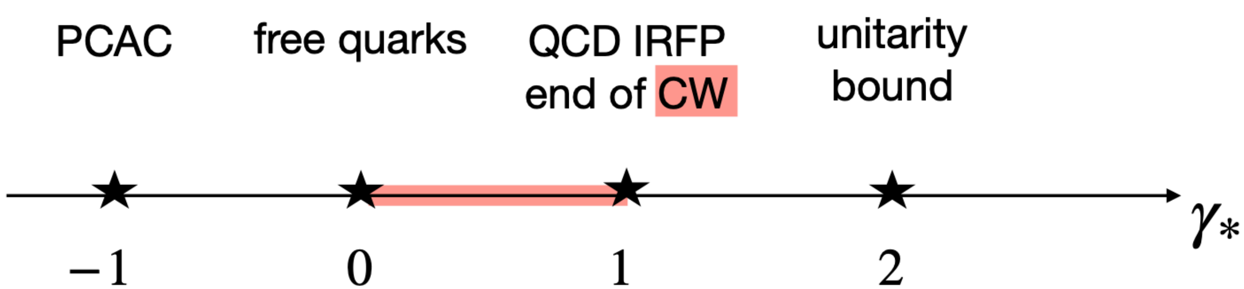
- Message seems to be: integer γ_* is special

$\gamma_* = 2$ unitarity bound (Mack'77) = 1 free scalar
$\gamma_* = 1$ lower end of CW = 2 free scalars $\Delta_{S^a}^{UV} = 2$ } degenerate $\mathcal{N} = 1$ SUSY
$\gamma_* = 0$ upper end of CW = 2 free quarks $\Delta_{S^a}^{UV} = 3$

$\gamma_* = -1$ PCAC bound (Wilson'69)



QCD-like theories (no scalars) $\quad \gamma_m = -\gamma_{\bar{q}q}|_{\mu=0} = \gamma_*$



- Conformal window only uses 1/3 of allowed γ_*-range

RG derivation of $\beta'_* = 0$

RG-consideration*: $\langle \pi | G^2 | \pi \rangle \propto m_q^{\frac{2+\beta'_*}{y_m}}$

pion-GMOR $\langle \pi | G^2 | \pi \rangle = \mathcal{O}(m_q)$ $y_m = 1 + \gamma_* = 2$

$\Leftrightarrow \quad \beta_* = 0$

* $\langle \pi | G^2 | \pi \rangle \propto F_\pi^2$ since $\langle \pi | \bar{q} q | \pi \rangle \propto F_\pi^2$ by GMOR

The higgs boson as a dilaton

universal part

- If **v = 0, SM conformal** (up to log-running), Higgs like a dilaton

$$(1+\frac{h}{v}) \to \chi = e^{-\frac{D}{F_D}} \to (1+\frac{h}{F_D})$$

If number of **doublets = 1** $\Rightarrow v = F_\pi$

- $r = \frac{F_\pi}{F_D} = 1$ is the Standard Model limit

- One can deduce indirectly: $r_{QCD} = 1.0(2) \pm$ syst, **intriguing!**

  a) **no symmetry reason** for this to happen (however, systematics…)

  b) closeness to unity, **LO-invisible @ LHC**

Why does the dilaton couple like the Higgs?

non-universal part

1. popular just before LHC $\quad G_{CFT} = G_{SM} \times G' \quad + \quad \delta\mathscr{L}_{\not{CFT}} = c\mathcal{O}$

Golfberger et al, Terning et al etc

new-sector

in trouble: $\delta_{SM}(gg \to h) \propto \delta_{SM}(h \to \gamma\gamma) \propto \Delta\beta_{decoupled}$ = too large

when it is said that *"the dilaton as a Higgs has been excluded by the LHC"*. then that's what people mean.

2. another idea (Cata, Crewther'Tunstall, 18') $\quad G_{SM}^{\text{no Higgs}} \xleftrightarrow{\text{Yukawa}} G'$

$$\mathscr{L} \supset \frac{1}{4} v^2 tr[D^\mu U D_\mu U^\dagger] - v\bar{q}_L Y_d U \mathscr{D}_R + \dots$$

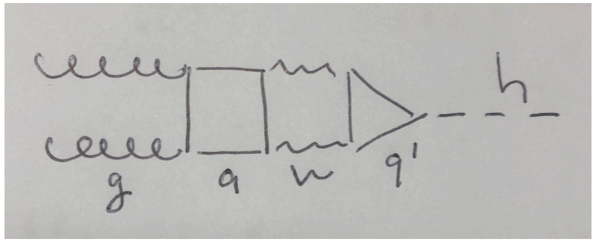
$$U = \exp(i2T^a\pi^a/F_\pi) \qquad U \to V_L U V_Y\,, \quad V_Y = e^{iyT_3}$$

how to bring back the higgs/dilaton is not addressed in that paper, one cannot use the compensator argument as only G' IR-CFT.

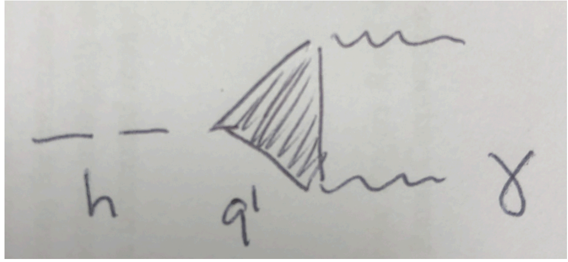
In 2312.13761 it is argued that if there is a symmetry reason for $r_2 \approx 1$, then same reason might enforce the right coupling aka

$$\mathcal{L} \supset \frac{1}{4} v^2 e^{-2D/F_D} \mathrm{Tr}[D^\mu U D_\mu U^\dagger] - v e^{-D/F_D} \bar{q}_L Y_d U \mathcal{D}_R + \ldots$$

- **Constraints?**



$\delta_{SM}(gg \to h) =$ NNLO



$\delta_{SM}(h \to \gamma\gamma) =$ non-perturatbive

EWPO: e.g. S-parameter $\delta S = \mathcal{O}(2\%)$ if $r_2 = 1$

most “dangerous one” looks like $h \to \gamma\gamma$
… to be continued & discussed or other idea

- **Higgs-dilaton potential?**

radiatively induced aka composite Higgs with $\Lambda_{G'} = \mathcal{O}(1)\, \Lambda_{EW}$

What is a **dilaton**?

- Always: particle vacuum quantum numbers $J^{PC} = 0^{++}$
  Otherwise: few different meanings

1. **Goldstone boson*** of spontaneously **broken scale invariance** of strong interactions 1968-1970 then largely forgotten
   (resurrected as Higgs as dilaton pre-LHC)

this talk

2. **Scalar component of gravity (gravi-scalar)**
   Brans-Dicke, supergravity (string theory)

3. A **name** for a **light** $J^P = 0^+$ **scalar** in context of approximate scale inv.

   However, it is not a Goldstone (no limit when it's massless…)

Types of Renormalisation Group (RG)-flow

- assume UV fixed point (e.g. asymptotic freedom) g^*_{UV} , IR flow?

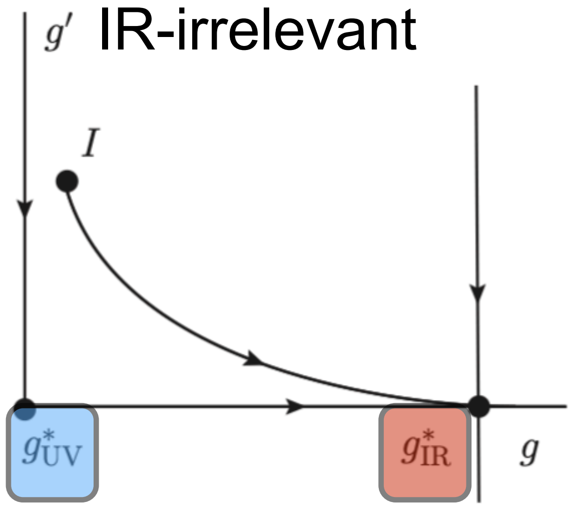
IR fixed point g^*_{IR}

conformal window

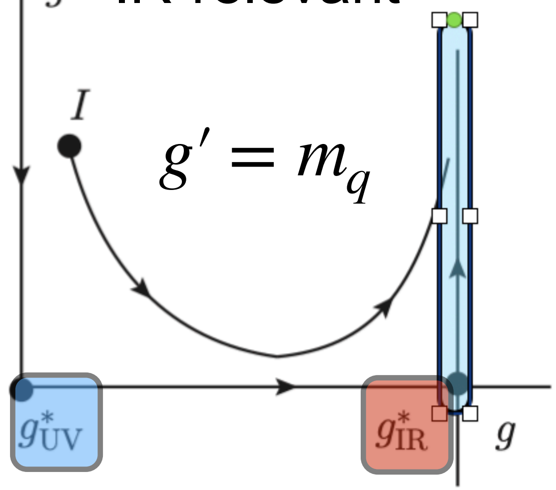
no IR fixed point

QCD-picture

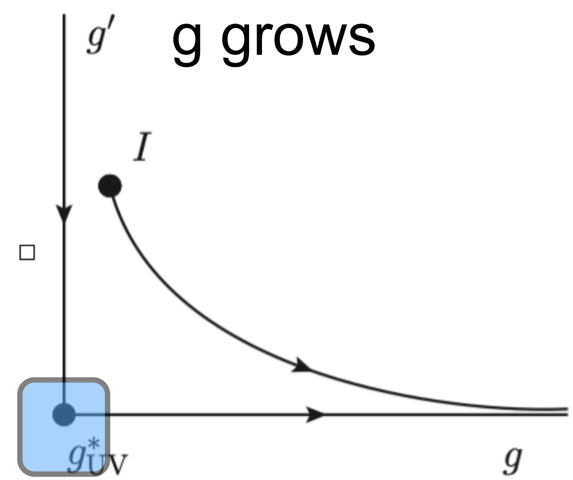
IR-irrelevant



IR-relevant

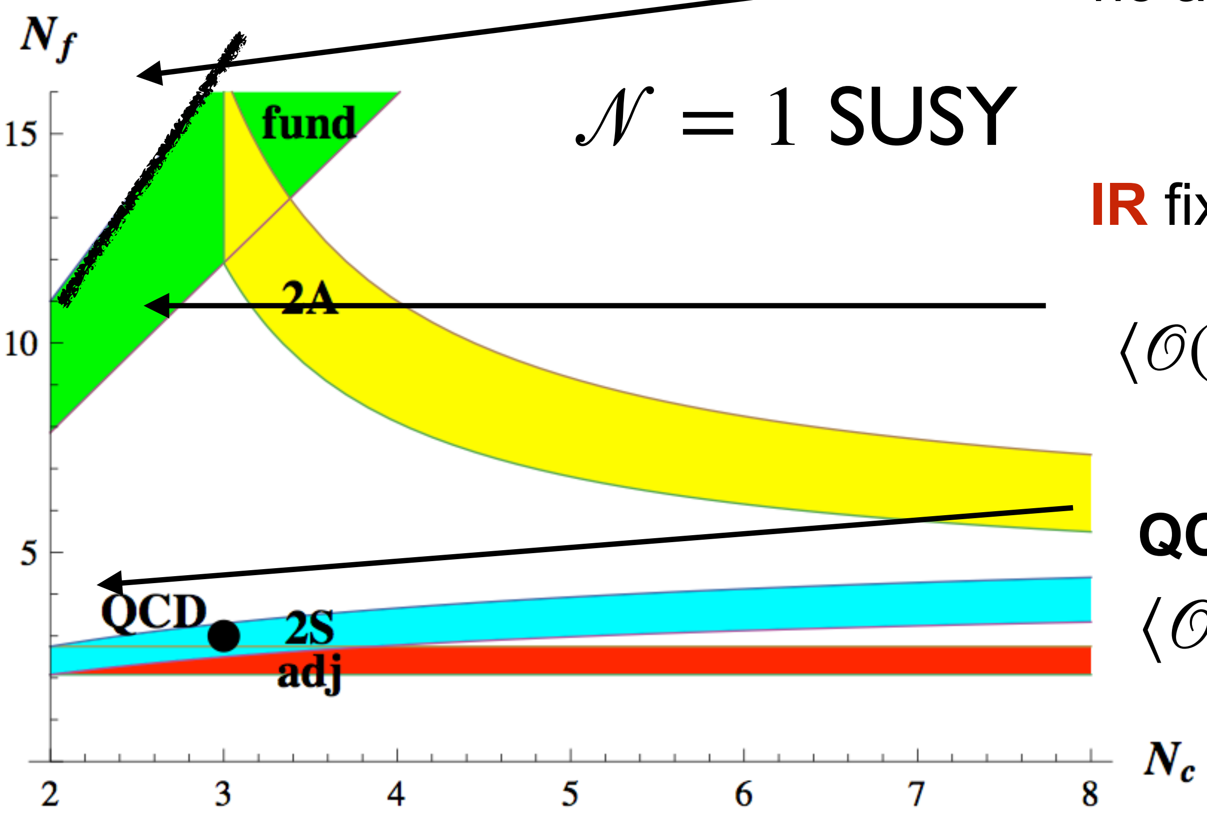


IR-irrrelevant &
g grows



Phases of gauge theories - **Conformal Window**

- gauge theory **massless quarks** in some **irrep** (e.g. fund. of say $SU(N_c)$)
- Focus on green = fund irrep



$\mathcal{N} = 1$ SUSY

no asymptotic freedom (ignore)

IR fixed point = **conformal window**

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle_{CFT} \propto \frac{1}{(x^2)^{\Delta_{\mathcal{O}}}} \quad x^2 \to \infty$$

QCD: *chiral SSB* & *confinement*

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle_{QCD} \propto \text{complicated}$$

QCD@low energy: pion EFT = χPT

- QCD $\langle \bar{q}q \rangle \neq 0$ **breaks** chiral $SU_L(N_f) \times SU_L(N_f) \to SU_V(N_f)$ (*isospin*) **spontaneously,** $N_f^2 - 1$ **Goldstones = pions** $[\, m_\pi^2 = \mathcal{O}(m_q) \,]$

- CCWZ construction $U = e^{i\pi^a T^a / F_\pi}$

$$\mathcal{M} \equiv \text{diag}(m_{q_1}, \ldots, m_{q_{N_f}})$$

$$\mathcal{L}^{\chi PT}_{LO} = \frac{F_\pi^2}{4} \text{Tr}[\partial^\mu U \partial_\mu U^\dagger] + \frac{B_0 F_\pi^2}{2} \text{Tr}[\mathcal{M} U^\dagger + U \mathcal{M}^\dagger]$$

kinetic — *m_q-term (spurion technique)* — GMOR $m_\pi^2 F_\pi^2 = -2 m_q \langle \bar{q}q \rangle$

PCAC GMOR, Goldberger-Treiman
LO: Weinberg '67
NLO: Weinberg '79
Gasser Leutweyler '84,'85
NNLO: Bijnes, Colangelo, Gasser …

- QCD $\langle \bar{q}q \rangle \neq 0$ also **breaks scale symmetry,** possibly spontaneously?
If yes, **1** (pseudo) **Goldstones = dilaton**

$$\mathcal{L}^{d\chi PT}_{LO} = later$$

$$m_D^2 = \mathcal{O}(m_q, \beta'_*)$$

does Goldstone mass remember the flow?
(Not settled - If CFT SSB then massless)

IRFP-interpretation - assumptions

- scaling @IRFP with SSB: $\langle \bar{q}q \rangle \neq 0$

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \propto \frac{1}{(x^2)^{\Delta_{\mathcal{O}}}} + \text{ GB-corrections}$$

$x^2 \to \infty$

$\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}}$

- **assume** exists a scheme: $\beta_* = \beta|_{\mu=0} = 0$

$$\beta = \beta'_* \delta g + \beta''_* \frac{(\delta g)^2}{2} + \mathcal{O}((\delta g)^3) \ , \quad \delta g \equiv g - g_* \ .$$

$$T^\rho{}_\rho|_{\text{phys}} = \frac{\beta}{2g} G^2 + \sum_q m_q (1+\gamma_m) \bar{q}q$$

- idea: QCD@IRFP $\leftrightarrow$ EFT (**dilaton)-χPT** for $x^2 \to \infty$

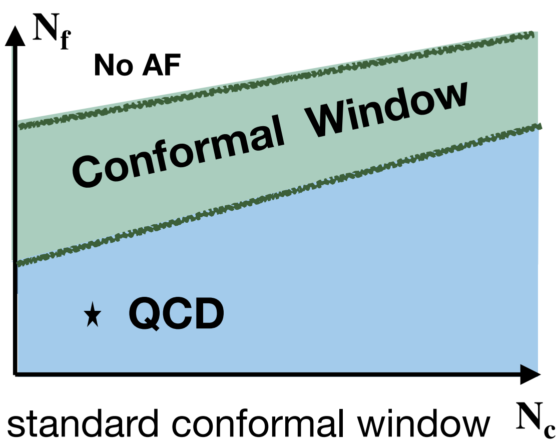
determine anomalous dimension: e.g* $\gamma_{m_q} = -\gamma_{\bar{q}q}|_{\mu=0} \equiv \gamma_*$

*** main quantity in CW-hunt. and Walking technicolor** $-1 \le \gamma_* \le 2$ **allowed range**

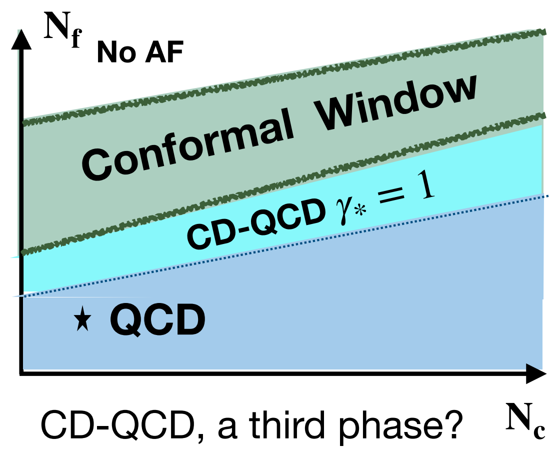
irrelevant(PCAC) unitarity

End of main part and …

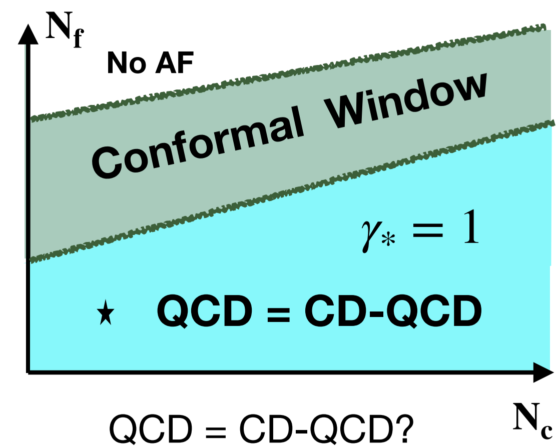
- At least any of these three possibilities is logically possible.
  Option 1 is what is what is taken for granted in standard view.



standard conformal window



CD-QCD, a third phase?



QCD = CD-QCD?

- Hope, convinced you that option 2 & 3 are not as absurd as .. I thought as well.

- Important: under assumptions got back consistent results.

Before going to T^ρ_ρ-correlator …

…. pause and introduce EFT: **dilaton-χPT**

chiral	dilatation	
$J^a_{5\mu} = \bar{q} T^a \gamma_\mu \gamma_5 q$	$J^D_\mu(x) = x^\nu T_{\mu\nu}(x)$	sym. currents
$\langle \pi^b(q) \vert J^a_{5\mu} \vert 0 \rangle = i F_\pi q_\mu \delta^{ab}$	$\langle D(q) \vert J^D_\mu \vert 0 \rangle = i F_D q_\mu$	decay constants= order parameters
$U = e^{i\pi^a T^a / F_\pi}$	$\chi \equiv F_D e^{-D/F_D}$	coset rep.
$U \to L U R^\dagger$	$\chi \to \chi e^{\alpha(x)}$	transformation
$(L, R) \in SU(N_f)_L \otimes SU(N_f)_R$	$\alpha(x) \in \mathbb{R}$	

Isham, Salam, Strathdee,
Mack, Zumino ca '70

Leading order dilaton-χPT

- Building principle: enforce Weyl invariance

$$g_{\mu\nu} \to e^{-2\alpha} g_{\mu\nu} \qquad \chi \to \chi e^{\alpha} \qquad U \to U$$

$$\Delta_{\bar{q}q} = 3 - \gamma_* = 2$$

quark mass = expl. sym-breaking

$$\mathcal{L}_{\rm LO}^{\rm d\chi PT} = \frac{F_\pi^2}{4} \hat{\chi}^2 {\rm Tr}[\partial^\mu U \partial_\mu U^\dagger] + \frac{B_0 F_\pi^2}{2} \hat{\chi}^{\Delta_{\bar{q}q}} ({\rm Tr}[\mathcal{M} U^\dagger + U \mathcal{M}^\dagger] + \frac{1}{2} (\partial \chi)^2$$

standard-extend χPT + dilaton **global Weyl inv.**

$$- \frac{\Delta_{\bar{q}q}}{4} {\rm Tr}[\mathcal{M} + \mathcal{M}^\dagger] \hat{\chi}^4) + \frac{1}{12} \chi^2 R + \mathcal{L}_{\rm anom}(\beta'_*) + V(\chi) \, ,$$

removes tadpole (**Zumino-term**)

local Weyl inv. - solves Goldstone improvement problem (another talk RZ 2306.12914)

matches trace anomaly (talk later)

potential (big unknown) last part …

Ready for T^{ρ}_{ρ}-correlator …

- Trace of EMT: $T^{\rho}{}_{\rho}|_{\text{phys}} = \frac{\beta}{2g} G^2$

$$(\gamma_{G^2})_* = \beta'_* \quad \Rightarrow \Delta_{T^{\rho}{}_{\rho}} = \Delta_{G^2} = 4 + \beta'_*$$

$$\beta = \beta'_* \delta g + \beta''_* \frac{(\delta g)^2}{2} + \mathcal{O}((\delta g)^3) \ , \quad \delta g \equiv g - g_*$$

- Formally (& RG)

$$\langle T^{\rho}{}_{\rho}(x) T^{\rho}{}_{\rho}(0) \rangle \propto (\beta'_* \delta g + \beta''_* \frac{(\delta g)^2}{2})^2 \frac{1}{(x^2)^{4+\beta'_*}}$$

- EFT **difference between ✗PT and dilaton-✗PT** (with improvement RZ 2306.12914)

$$T^{\rho}{}_{\rho}|^{\text{LO}}_{\chi\text{PT}} = -\frac{1}{2} \partial^2 \pi^a \pi^a \ , \quad T^{\rho}{}_{\rho}|^{\text{LO}}_{d\chi\text{PT}} = 0$$

$$\langle T^{\rho}{}_{\rho}(x) T^{\rho}{}_{\rho}(0) \rangle^{\text{LO}}_{\chi\text{PT}} \propto \frac{1}{x^8} \ , \quad \langle T^{\rho}{}_{\rho}(x) T^{\rho}{}_{\rho}(0) \rangle^{\text{LO}}_{\text{d}\chi\text{PT}} \propto 0$$

- χPT implies $\beta'_* = 0$ for dχPT not obvious (need RG-tools)

2nd main result

$\beta'_* = 0$ seems important for consistency

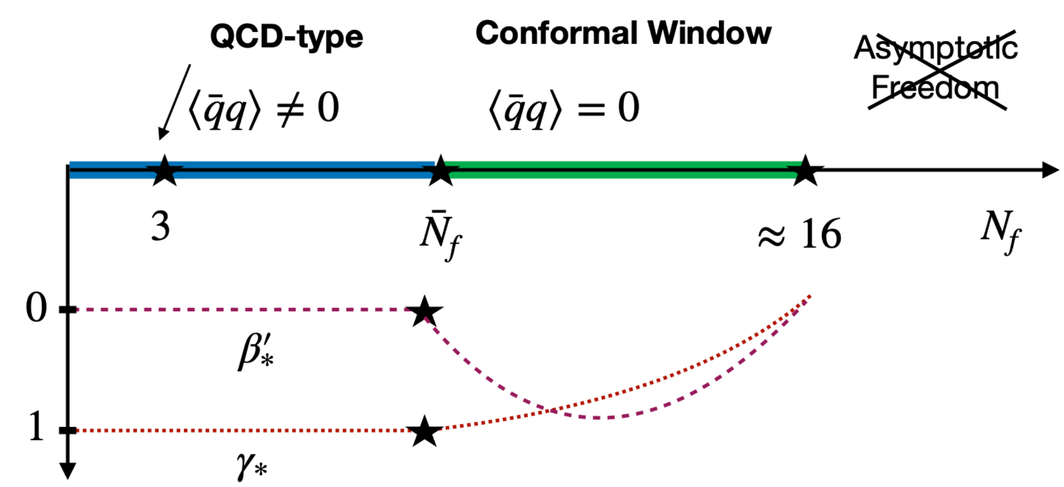
$$\delta g \propto \frac{1}{|\beta''_*| \ln(\mu/\lambda_{IR})}$$

- Power-running $\delta g \propto \mu^{\beta'_*}$ $\Rightarrow$ **log-running**
  - $\Rightarrow$ seems can **drop** $\mathscr{L}_{\textbf{anom}}(\beta'_*)$ from LO Lagrangian
    as anomaly reproduced in extending "EMT in ✗PT" Donoghue & Leutwyler 90'
  - $\Rightarrow$ **log-running**, sign of **mass-gap**. QCD asymptotes into Goldstone-EFT
- Makes light (or massless) dilaton more probable since: $m_D = \mathcal{O}(\beta'_*) \to \mathcal{O}(\beta''_*)$
- Continuous **matching** to **N=1 SUSY** conformal window $\beta'_* \to 0$ @boundary
  Anselmi, Grisaru, Johanson 97' Shifman RZ '23

$$\beta'_*|_{\rm el} = \beta'_*|_{\rm mag} \quad \Leftrightarrow \quad \langle T^\rho{}_\rho(x) T^\alpha{}_\alpha(0)\rangle_{\rm mag} \overset{\rm IR}{\longleftrightarrow} \langle T^\rho{}_\rho(x) T^\alpha{}_\alpha(0)\rangle_{\rm el}$$

- Summary figure:

$SU(N_c)$, $N_c = 3$



The essence of QCD and the dilaton

- **A dilaton in QCD?** Who? Consensus it would be the $\sigma \equiv f_0(500)$-meson

$$\sqrt{s_\sigma} = m_\sigma - \frac{i}{2}\Gamma_\sigma = (441^{+16}_{-8} - i272^{+9}_{-12.5})\,\mathrm{MeV}\;,$$

Caprini, Colangelo, Leutwyler'06
Roy-equations+input

- **Question**: does m_σ become massless or nearly massless in chiral limit?
  Fact: *nobody knows*, some indication it becomes lighter.

- using **dilaton-χPT:**
  1) can reproduce width ($SU(3)_F$-analysis): $\Gamma_\sigma = 616\,{}^{-108}_{+146} \pm \text{syst}^*$ MeV
  2) soft-mass even too large (EFT-convergence broken)

- **Concluding**: 1) success (already 1970's) 2) inconclusive
  Hence, not bad but there could be more to it …

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* notion of σ decay constant F_σ not well-defined, took model-value needs further thought

The higgs boson as a dilaton

Attention: different ways to implement ...
some universal and some not.

- If **v = 0, SM conformal** (up to log-running), Higgs like a dilaton

$$(1+\frac{h}{v}) \to \chi = e^{-\frac{D}{F_D}} \to (1+\frac{h}{F_D})$$

universal

If number of **doublets = 1** $\Rightarrow$ $v = F_\pi$ and $r = \frac{F_\pi}{F_D}$ determines diff. to SM

- One can deduce indirectly: $r_{QCD} = 1.0(2) \pm$ syst, **intriguing!**
  a) **no symmetry reason** for this to happen (however, systematics…)
  b) closeness to unity, **LO-invisible @ LHC**

- An idea for model: **new gauge sector IRFP**,
  EWSB as in technicolor and **dilaton** as **naturally light Higgs**

$$\mathcal{L} \supset \frac{1}{4} v^2 e^{-2D/F_D} \mathrm{Tr}[D^\mu U D_\mu U^\dagger] - v e^{-D/F_D} \bar{q}_L Y_d U \mathcal{D}_R + \ldots$$

non-universal

Like SM@LO but **why** coupled in this way?
Suspect, if there is a symmetry reason for $r \approx 1$,
then same reason enforces Lagrangian as above.

to be continued …

Massive Hadrons in Conformal Phase

Chiral limit $m_q \to 0$ resolve the contradiction below

$$\langle \phi(p) \,|\, T^{\mu}_{\mu} \,|\, \phi(p) \rangle \begin{cases} = 2m_\phi^2 & \text{standard formula} \\ = 0 & \text{with (massless) dilaton} \end{cases}$$

"The dilaton can hide the nucleon mass"

Del Debbio, RZ JHEP'22 2112.1364

Gravitational Form Factors

focus scalar instead of nucleon

- parameterise using Lorentz & translation invariance ($\partial^\mu T_{\mu\nu} = 0$)

$$\langle \varphi(p') | T_{\mu\nu} | \varphi(p) \rangle = 2\mathscr{P}_\mu \mathscr{P}_\nu G_1(q^2) + (q_\mu q_\nu - q^2 \eta_{\mu\nu}) G_2(q^2)$$

$\mathscr{P} = \frac{1}{2}(p + p')\,, \quad q = p - p'$ momentum transfer

- consider soft limit $q \to 0$ then G_2 drops and using $P_\mu = \int d^3x T^0_\mu$

$$\langle \phi(p) | T^\mu_\mu | \phi(p) \rangle = 2m_\phi^2$$

$$G_1(0) = 1$$

… seems the end of the road (for massive hadrons and conformality)

- Let's have another look at*

$$\langle \varphi(p') \,|\, T_{\mu\nu} \,|\, \varphi(p) \rangle = 2P_\mu P_\nu G_1(q^2) + (q_\mu q_\nu - q^2 \eta_{\mu\nu}) G_2(q^2)$$

$\langle \phi(p) \,|\, T^\mu_\mu \,|\, \phi(p) \rangle = 2m_\phi^2$ does **not** need to **hold if**

$$G_2(q^2) = \frac{r}{q^2} + \ldots$$ Goldstone pole (the **dilaton)**

- That is already a bit of a shock - can we make this quantitative?

  Yes in soft limit, as then can use $G_1(0) = 1$ and vanishing trace imposes

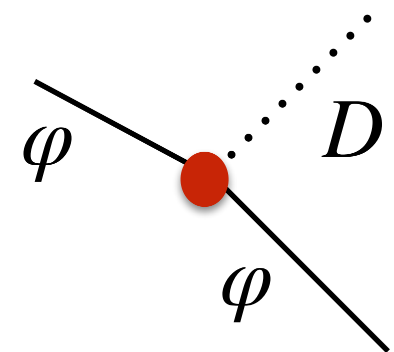
$$r = \frac{2m_\phi^2}{(d-1)}$$

*e.g lecture notes Gell-Mann '69 (pre-QCD), no details worked out

Computation of Residue (new)

$$r = \frac{2m_\phi^2}{(d-1)}$$

- need to know $\langle D\varphi\,|\,\varphi\rangle = i(2\pi)^d\delta\left(\sum p_i\right)\ g_{\varphi\varphi D}$
- can get it via **compensator trick (Weyl scaling)**



$$g_{\mu\nu} \to e^{-2\alpha} g_{\mu\nu}\,, \quad \varphi \to e^{\alpha}\varphi \quad \Rightarrow \quad D \to D - \alpha F_D$$

compensates m_φ^2 by dilaton, regain``conformal inv'': $\delta_\alpha\sqrt{-g}\,\mathscr{L}^{eff} = 0$

$$\mathscr{L}^{eff} \supset -e^{-2D/F_D}\frac{1}{2}m_\varphi^2\varphi^2 \quad \Rightarrow \quad g_{D\varphi\varphi} = \frac{2m_\varphi^2}{F_D}$$

$$r = \frac{2m_\phi^2}{(d-1)}$$

- now apply the LSZ formula (or dispersion theory)

$$\langle D\varphi|\varphi\rangle = \lim_{q^2\to 0}(-i)\frac{q^2}{Z_D}\int d^dx e^{iq\cdot x} P_2^{\mu\nu} T^{(\varphi)}_{\mu\nu}(p,p',x)$$

$$= \lim_{q^2\to 0}(-i)\frac{q^2}{Z_D} G_2(q^2)(2\pi)^d \delta\left(\sum p_i\right)$$

use EMT as dilaton interpolator

$Z_D = -F_D/(d-1)$

- from where we get exactly the right residue

$$r = \lim_{q^2\to 0} q^2 G_2(q^2) = -g_{\varphi\varphi D} Z_D = \frac{2m_\varphi^2}{d-1}$$

- Rather encouraging. The **approach** is **self-consistent**!

The dilaton improves Goldstones

based on
2306.12914 RZ

The standard improved scalar field

- Two terms curved space, no dim. couplings* $\mathcal{L} = \frac{1}{2}\left((\partial\varphi)^2 - \xi R \varphi^2\right)$

$$T^{\rho}{}_{\rho} = -d_{\varphi}(\partial\varphi)^2 + \xi(d-1)\partial^2\varphi^2 = (d-1)(\xi-\xi_d)\partial^2\varphi^2$$

↑ eom

- Conformal $T^{\rho}_{\rho} = 0$, only for $\xi = \xi_d \equiv \frac{(d-2)}{4(d-1)} \to \frac{1}{6}$ (d=4)

  - improved EMT Callan, Coleman, Jackiw'70, finite EMT (necessary as observable)
  - earlier in GR: Penrose'64 required by weak equivalence principle Chernikov&Tagirov'68
  - finite integrated Casimir-effect deWitt'75

- Heuristically, $\mathscr{L} \propto R\phi^2$, not possible to write with coset field $U = e^{i\frac{\pi^a T^a}{F_\pi}}$

Dolgov & Voloshin'82 Leutwyler-Shifman '89, Donoghue-Leutwyler' 91

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* may also work in flat space from start, but less elegant

Intermezzo on relevance for flow theorems

- Focus d=2 for simplicity, Weyl anomaly $T^{\rho}_{\rho} = cR$ reveals central charge of CFT.

  c-theorem (Zamalodchikov'86).: $\Delta c = c_{UV} - c_{IR} \geq 0$

  Cardy'88.: $\Delta c \propto \int d^2x\, x^2 \langle T^{\rho}_{\rho}(x) T^{\rho}_{\rho}(0) \rangle \quad \Rightarrow \quad T^{\rho}_{\rho} \to 0$ in UV and IR fast enough

  d=2 ok, Goldstone special anyway

- d=4, if **Goldstones not improvable** $T^{\rho}_{\rho} = -\frac{1}{2}\partial^2\pi^2$, then **log-IR divergence**

  a-thm* & $\Box R$-flow analogue formula IR-divergent

$\Rightarrow$ Goldstone improvement desirable

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*for a-thm, Luty, Polchiniski, Rattazzi'12 ' provide argument formula is IR-onvergent as inclusive enough

The Goldstone improvement proposal

- dilaton-pion system improvement

$$\mathcal{L}_{\rm LO} = \mathcal{L}_{\rm kin,4} + \mathcal{L}_4^R - V_4(\chi)$$

$$\mathcal{L}_{\rm kin,d} = \frac{F_\pi^2}{4}\hat{\chi}^{d-2}{\rm Tr}[\partial^\mu U \partial_\mu U^\dagger] + \frac{1}{2}\chi^{d-4}(\partial\chi)^2$$

standard Lag.

$$\mathcal{L}_d^R = \frac{\kappa}{4} R\, \chi^{d-2}$$

0, no mass (later..)

improvement term, κ to be **determined**

- **locally Weyl invariant** $\Rightarrow$ conformal invariance.

$$\kappa = \kappa_d \equiv \frac{2}{(d-1)(d-2)} \stackrel{d\to 4}{\to} \frac{1}{3}$$

Compared to $\xi_4 = 1/6$ like a ``double improvement" (more to say)

- realises decay constant in EFT

$$\langle 0|T_{\mu\nu}|D(q)\rangle \stackrel{\rm def}{=} \frac{F_D}{d-1}(m_D^2\eta_{\mu\nu} - q_\mu q_\nu) \quad = \quad \langle 0|T^R_{\mu\nu}|D(q)\rangle = \langle 0|\frac{1}{6}(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\chi^2|D(q)\rangle$$

3a. **Improvement** $T^{\rho}_{\rho} = 0$ **use of equation of motion**

- dilaton eom: $\chi \partial^2 \chi = 2\mathcal{L}^{\pi}_{\text{kin},4} - \partial_{\ln \chi} V_4$

$$T_{\mu\nu} = \frac{F_\pi^2}{2} \hat{\chi}^2 \text{Tr}[\partial_\mu U \partial_\nu U^\dagger] + \partial_\mu \chi \partial_\nu \chi - \eta_{\mu\nu}(\mathcal{L}_{\text{kin},4} - V_4) + T^R_{\mu\nu}$$

$$T^R_{\mu\nu} = \frac{\kappa}{2}(g_{\mu\nu}\partial^2 - \partial_\mu \partial_\nu)\chi^2$$

$$\begin{aligned} T^{\rho}{}_{\rho}|_{V=0} &= \frac{3}{2}\kappa \partial^2 \chi^2 - 2\mathcal{L}^{\pi}_{\text{kin},4} - 2\mathcal{L}^{D}_{\text{kin},4} \\ &\overset{eom}{=} \frac{3}{2}\kappa \partial^2 \chi^2 - (\partial\chi)^2 - \chi \partial^2 \chi \\ &= (3\kappa - 1)\{\chi \partial^2 \chi + (\partial \chi)^2\} = 0 \end{aligned}$$

$$\kappa = \kappa_4 = \frac{1}{3}$$

- **works** as expected from **local Weyl invariance**, also works d-dim curved space