The observable spectrum for GUT-like theories

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Grand Unified Theories

importance of systematic control

nontrivial “broken-group” observables

Gauge invariance

BRST breaks down for nonabelian theories

elementary fields are unphysical

the Fröhlich–Morchio–Strocchi mechanism

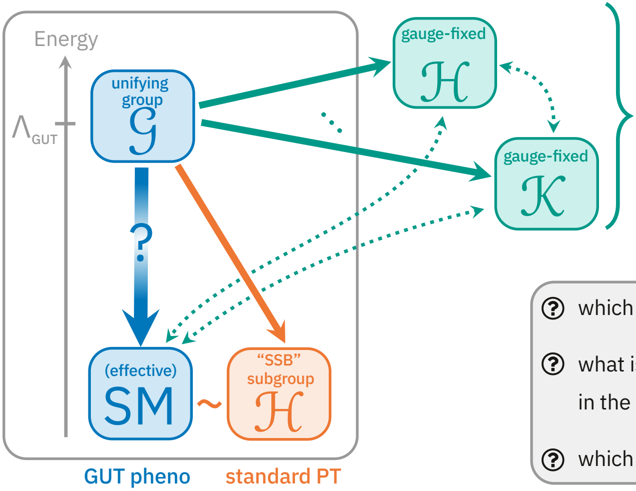
Spectroscopy

toy SU(3) model to test FMS mechanism

discrepancies with naive perturbation theory

relevance of nonperturbative physics

Gauge-invariant approach to grand unified theories



our approach

- subgroup depends on gauge choice
- different spectra possible
- nontrivial mapping to SM spectrum
- no assumption that BEH $\Longrightarrow$ low-energy

(?) which assumptions carry over from SM pheno?

(?) what is the correspondence between bound states in the unbroken theory and SM observables?

(?) which GUT groups are plausible?

Maas, arXiv: 1712.04721v4 — review article (see “FAQs” at end for a good overview)

Elementary fields form an unphysical state space

nonabelian gauge group + local gauge-fixing condition:

no unique solutions beyond PT

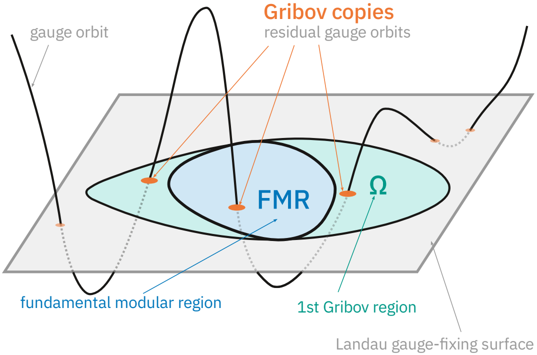
BRST insufficient to fix gauge

ξ-invariance $\not\Rightarrow$ gauge invariance

perturbative state space is gauge-dependent

elementary fields (and e.g. Higgs vev)

are not reliable order parameters

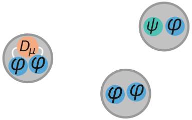


can we work directly with composite states in perturbation theory instead?

Gribov, Nucl. Phys. B (1978) | Singer, Comm. Math. Phys. (1978) | Fujikawa, Nucl. Phys. B (1983)

Fröhlich–Morchio–Strocchi approach: composite states

elementary:	ψ	$W_\mu^{(a)}$	φ
composite:	$\varphi^\dagger\psi$	$i\varphi^\dagger D_\mu\varphi$	$\varphi^\dagger\varphi$
	fermion	vector boson	“Higgs”



fix to **choice** of gauge with $\langle\varphi\rangle \neq 0$

Higgs mechanism: $\boxed{\mathcal{G}} \xrightarrow{\text{breaks to}} \boxed{\mathcal{H}}$

physical observable: $\mathcal{G}$-singlet $\Longleftrightarrow$ sum of $\mathcal{H}$-singlets

exact, nonperturbative relation

boundstate-boundstate correspondence after gauge-fixing is nontrivial in general:

important for BSM model building!

Sondenheimer, arXiv:1912.08680v2 | Fröhlich, Morchio and Strocchi, Phys. Lett B (1980) & Nucl. Phys. B (1981)

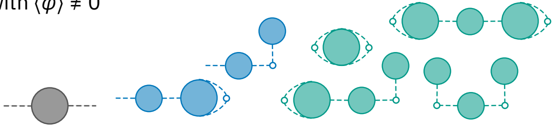
Perturbation theory: “bound-state Higgs” vs 1^- vector singlet

[here: SU(N) Yang–Mills with single fundamental scalar]

expand in choice of gauge with $\langle\varphi\rangle \neq 0$

e.g. $\varphi(x) = v\hat{n} + \eta(x)$

$h(x) = 2\,\mathrm{Re}[\hat{n}^\dagger\eta(x)]$



$$\underbrace{\langle(\varphi^\dagger\varphi)(x)(\varphi^\dagger\varphi)(y)\rangle_c}_{\text{bound-state mass}} = v^2\langle h(x)h(y)\rangle_c + \underbrace{2v\langle h(x)(\eta^\dagger\eta)(y)\rangle_c + \langle(\eta^\dagger\eta)(x)(\eta^\dagger\eta)(y)\rangle_c}_{\text{extra terms neglected in standard picture}}$$

coincides with standard PT

$$\underbrace{\langle(\varphi^\dagger D_\mu\varphi)^\dagger(x)(\varphi^\dagger D_\nu\varphi)(y)\rangle_c}_{\text{singlet} \implies \text{only one pole}} = v^2 c^{ab}\langle W_\mu^{(a)}(x)W_\nu^{(b)}(y)\rangle_c + \underbrace{O(\eta/v) + \ldots}_{\text{don't affect pole structure}}$$

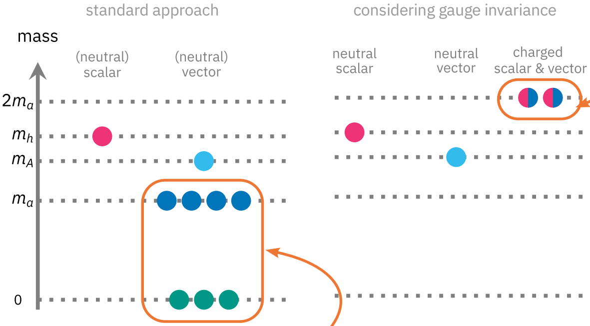
conflicts with standard PT
for SU($N > 2$)

poles coincide to all orders in perturbation theory!

Maas and Sondenheimer, arXiv: 2009.06671v2 | Fröhlich, Morchio and Strocchi, Phys. Lett. B (1980) & Nucl. Phys. B (1981)

Gauge invariance qualitatively changes the PT spectrum

[Example: SU(N) Yang–Mills with single fundamental scalar]



composite operators with custodial charge

1^- singlet $\implies$ loss of degeneracy due to structure of gauge group

$$\underbrace{\langle(\varphi^\dagger D_\mu\varphi)^\dagger(x)(\varphi^\dagger D_\nu\varphi)(y)\rangle}_{\text{singlet} \implies \text{only one pole}}$$

Standard PT still works well when:

gauge group is fundamental SU(2)

$\implies$ enhanced charge symmetry

or

operators are explicitly gauge invariant

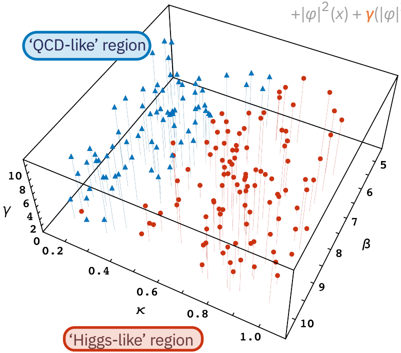
Toy model to test FMS approach: SU(3) + YM

“GUT-like”*

$$\mathcal{L} = \frac{1}{2}\,\mathrm{tr}(W_{\mu\nu}W^{\mu\nu}) + |D\varphi|^2 - \lambda(|\varphi|^2 - f^2)^2$$

$$\beta\,\mathrm{Re\,tr}\sum_{\mu<\nu}[\,\mathbb{1} - U_{\mu\nu}(x)] - \kappa\sum_{\pm\mu}\varphi^\dagger(x)U^R_\mu(x)\varphi(x+\hat{\mu})$$

$$+|\varphi|^2(x) + \gamma(|\varphi|^2(x) - 1)^2$$



$$\lambda \sim \frac{\gamma}{\kappa^2}$$

$$g \sim \frac{1}{\sqrt{\beta}}$$

Generalisation of SM gauge-weak sector

single scalar $\varphi \in \mathrm{SU}(3)$ or $\varphi \in \mathrm{su}(3)$

Breaks to nontrivial gauge group

$\mathrm{SU}(3) \to \mathrm{SU}(2)$ or $\mathrm{SU}(2)\times\mathrm{U}(1),\ \mathrm{U}(1)\times\mathrm{U}(1)$

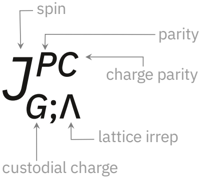
Nontrivial custodial group

global U(1) or Z_2

- ? what is the stable spectrum?
- ? are the lighter states charged?
- ? do lattice results support FMS?

Lattice results: Maas and Törek, arXiv:1804.04453

Constructing an operator basis in different channels



$J^{PC}_{G;\Lambda}$

spin

parity

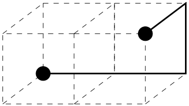
charge parity

lattice irrep

custodial charge

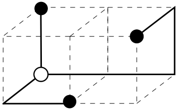
$\varphi^\dagger \cdot (D_{\mu_1} \ldots D_{\mu_n} \varphi)$

U(1)-neutral, gauge-scalar



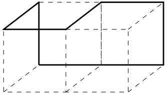
$(D_{\mu_1} \ldots D_{\mu_{n_1}} \varphi) \cdot \left[(D_{\nu_1} \ldots D_{\nu_{n_2}} \varphi) \times (D_{\rho_1} \ldots D_{\rho_{n_3}} \varphi) \right]$

U(1)-charged, gauge-scalar



$\mathrm{tr}\left[(D_{\mu_1} \ldots D_{\mu_{n_1}} F_{\nu_1 \rho_1}) \ldots (D_{\sigma_1} \ldots D_{\sigma_{n_R}} F_{\nu_R \rho_R}) \right]$

gaugeball (+ scalar insertions for adjoint)



States for any (J, M) via ‘ladder operators’:

$\tilde{D}_\pm = \mp i(D_1 \pm i D_2)/\sqrt{2}\,, \quad \tilde{D}_0 = i D_3$

Continuum → lattice: project onto O_h irreps via Clebsch–Gordan coefficients

Project to required parity/charge parity

Smear $\underbrace{\text{links}}_{\text{stout}}$ and $\underbrace{\text{scalars}}_{\text{APE}}$ to enlarge basis

Dudek et al., arXiv:1004.4930 | Wurtz and Lewis, arXiv:1307.1492

Implementation details

Setup

SU(3) + YM + single scalar

3D coupling space (β, κ, γ)

isotropic lattice: $L = 10, 12, \dots, 32$

Gauge fixing

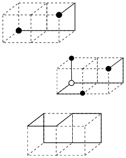
Landau ’t Hooft or Unitary

Stochastic OR

Operator basis

on fixed timeslice

momentum boosts



Heatbath + OR updates

- Cabbibo–Marinari method
- Scalar OR: rotate $\varphi(x)$ around vector $\propto \frac{\partial S}{\partial \varphi(x)}$
- Adjoint case: approx. HB/OR
  + accept/reject step

Smearing

Stout (links), APE (scalars)

Spectroscopy

variational analysis

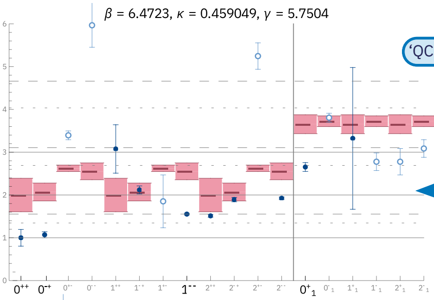
fitting to plateaus of $C(t)$

scattering from stable states

$V \to \infty$ extrapolation

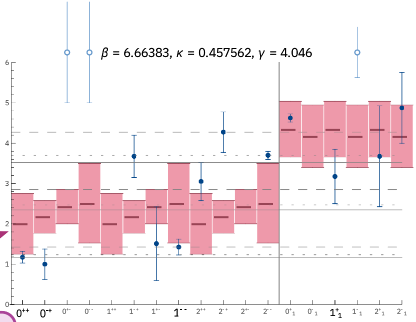
Jenny, Maas and Riederer, arXiv:2204.02756 — fitting/variational analysis details

[NB: everything normalised to lightest mass]



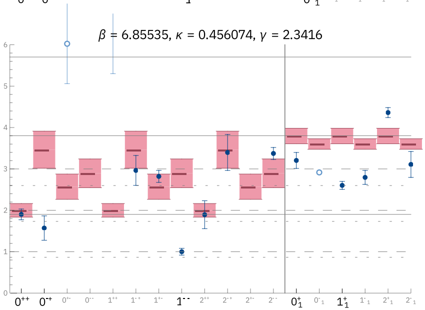
$\beta = 6.4723,\ \kappa = 0.459049,\ \gamma = 5.7504$

‘QCD-like’ region



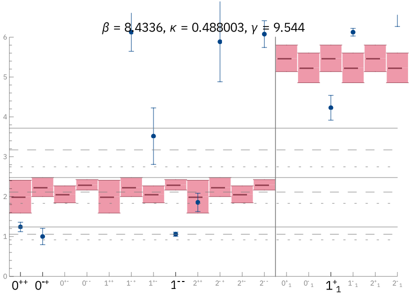
$\beta = 6.66383,\ \kappa = 0.457562,\ \gamma = 4.046$

close to phase boundary



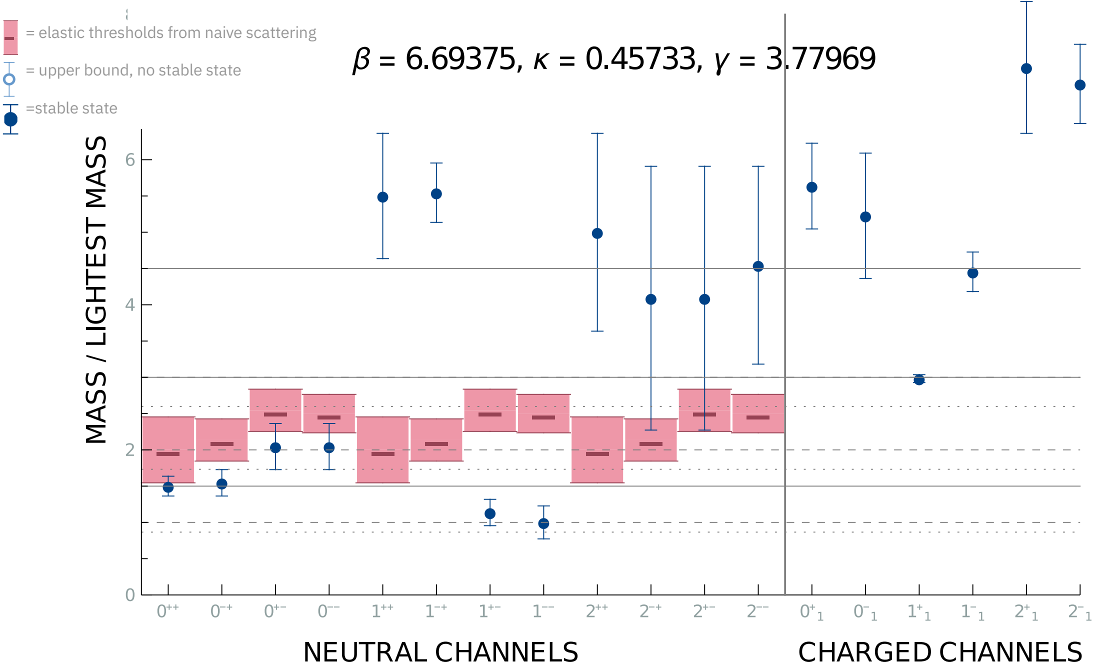
$\beta = 6.85535,\ \kappa = 0.456074,\ \gamma = 2.3416$

with Higgs effect



$\beta = 8.4336,\ \kappa = 0.488003,\ \gamma = 9.544$

$\beta = 6.69375, \kappa = 0.45733, \gamma = 3.77969$



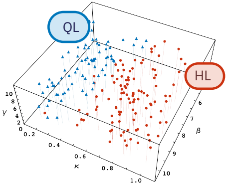
Features of the fundamental spectrum

Phase structure

clear indication of 2 phases:

4 apparent LCPs (2 for QL, 2 for HL)

order of transition/crossover unclear



Generic features

presence of massive charged bound-states

two distinct phases with different orderings of 0^{++} and 1^{--}

degeneracies across different channels

light pseudoscalar 0^{+-}

so far seems (?) consistent with FMS

appears not to be any SSB of custodial U(1)

QCD-like

lightest state is 0^{++}; $m_S < m_V$

strongly coupled scalars

uncharged part ≠ pure YM

heavier U(1)-charged states

Phase boundary

massless modes?

Higgs-like

lightest state is 1^{--}; $m_V < m_S$

degeneracies across channels

The adjoint-scalar case

More interesting

multiple breaking patterns — $SU(2) \times U(1)$ / $U(1)^\star \times U(1)$

more applications to BSM physics

More difficult

(presumably) massless modes

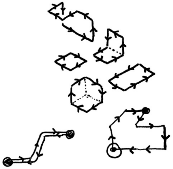
challenges in taking continuum limit

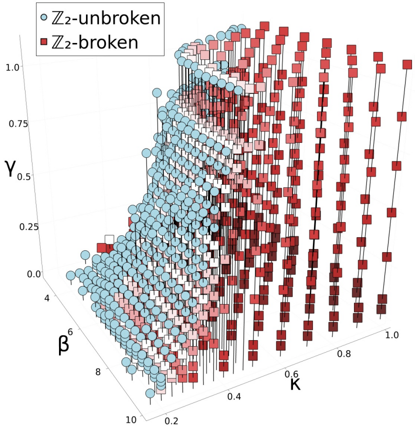
(even!) noisier

Spectroscopy (work in progress)

automatising larger operator basis

stable and scattering states





Dobson, Maas and Riederer, arXiv:2211.05812 — plot for $\mathbb{Z}_2$ phase diagram

Summary and outlook

Systematic control matters

gauge invariance has a qualitative effect on nonperturbative spectra

qualitative differences, including at small coupling

Results

qualitative differences from pure Yang–Mills, and from SU(2)

FMS: nontrivial field theory effects can still be treated perturbatively

Work in progress

understanding fundamental spectrum (analytically?)
preliminary adjoint spectrum
automatising large operator basis
full scattering analysis
check consistency with FMS

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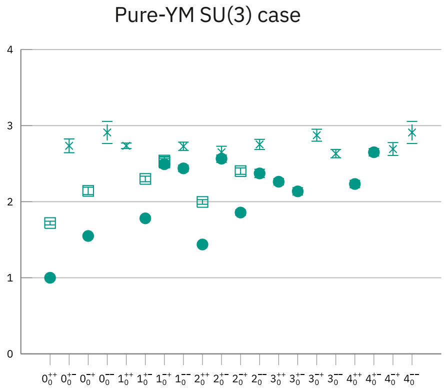
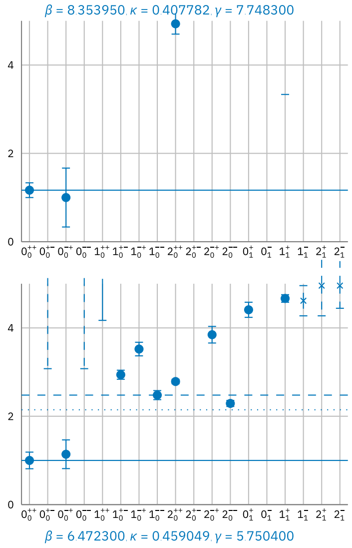
Comparison to pure Yang–Mills

[normalised to lightest mass]



SU(3) YM + scalar
(deep QCD-like region)



data (left) from Athenodorou and Teper, arXiv:2106.00364