Updates on the density of states method in finite temperature Symplectic gauge theories

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Overview

Sp(4) and Beyond the Standard Model Physics

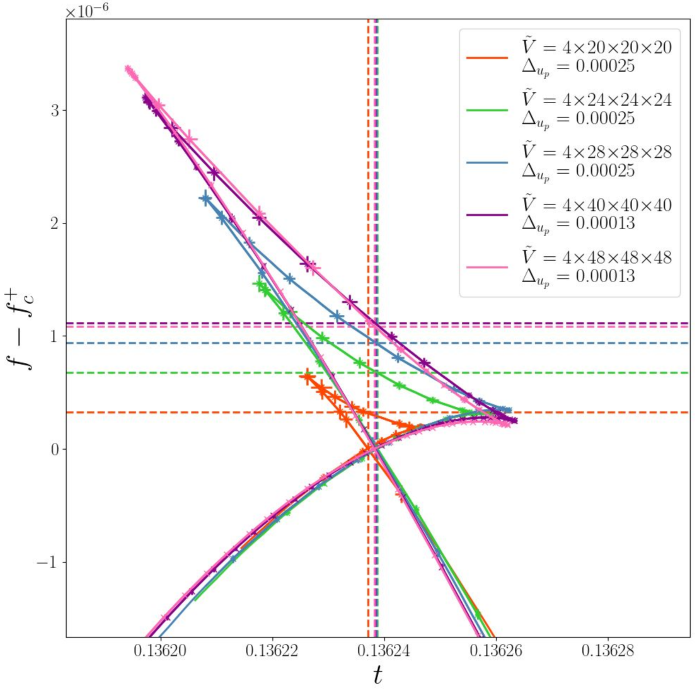
The Deconfinement Phase Transition on the lattice

The Linear Logarithmic Relaxation Method

Results

This work follows of the methodology developed in: arxiv:2305.07463

Pure Sp(4) with results for $N_t = 4$ in the thermodynamic limit



Beyond the standard model physics

Standard models of particle physics contains two smooth cross-overs:

- Chiral symmetry breaking
- Electroweak symmetry breaking

Early universe first order phase transitions can generate:

- Matter-Antimatter asymmetry
- Long wavelength gravitational wave background

Phase co-existence of solutions can lead to nucleation of bubbles

Lattice simulations can inform us on the dynamics of the bubbles

Beyond the standard model physics

Add new strongly interacting sectors:

- Baryogenesis
- Dark matter
- Composite Higgs

Sp(4) gauge theories have been proposed as a possible solution to each of these problems

Lattice set-up

$$S[U] = \frac{6\tilde{V}}{a^4}(1 - u_p[U]) \qquad Z(\beta) = \int [DU] e^{-\beta S[U]}$$

Isotropic hypercubic lattice with spacing a and volume $\tilde{V} = a^4 N_t \times N_s^3$

Temperature $T = 1/aN_t$ by changing $\beta(a)$ for $N_t < N_s$

$U_\mu \in Sp(4) \in SU(4)$ satisfying the condition:

$$U_\mu \Omega (U_\mu)^T = \Omega \qquad \mathbf{\Omega} = \begin{pmatrix} 0 & \mathbb{1}_{2\times 2} \\ -\mathbb{1}_{2\times 2} & 0 \end{pmatrix}$$

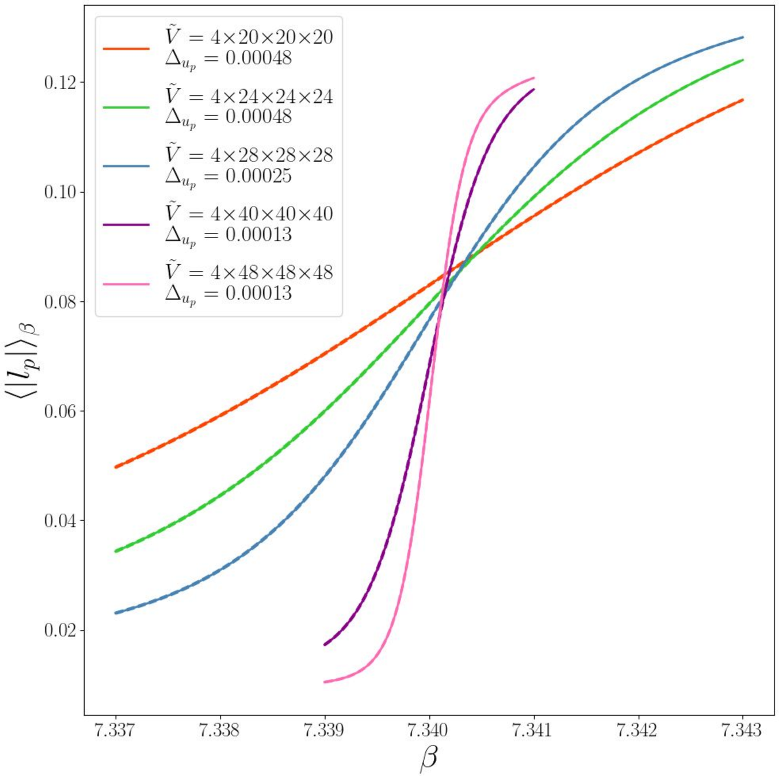
Deconfinement

$$\langle l_p \rangle_\beta \equiv \left\langle \frac{1}{4N_s^3} \sum_{\vec{n}_s} \mathrm{Tr} \left(\prod_{n_t=0}^{N_t-1} U_0(n_t, \vec{n}_s) \right) \right\rangle_\beta$$

$$\begin{cases} = 0 \text{ confined phase} \\ \neq 0 \text{ deconfined phase} \end{cases}$$

The Polyakov loop is the order parameter associated with the breaking of the centre symmetry in the transition

Centre symmetry for Sp(2N) is $\mathbb{Z}_2$

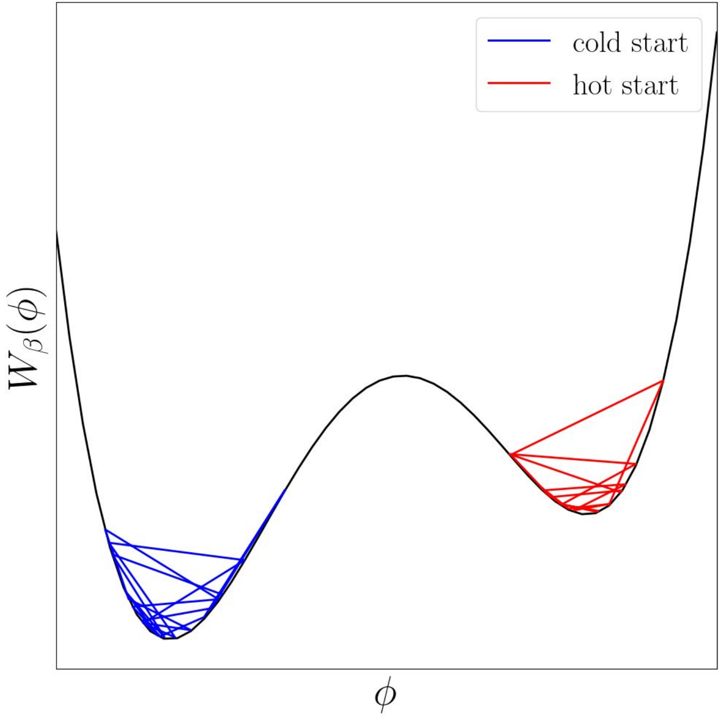


Simulating first order phase transitions

For accurate results configurations must tunnel between phases several times

In the large volume limit the potential barrier grows

More configurations must be generated to ensure the system explores both phases appropriately



Density of States

We want to calculate some observable

$$\langle \mathcal{O} \rangle = \frac{1}{Z_\beta} \int [DU] \mathcal{O}[U] e^{-\beta S[U]} \qquad Z_\beta = \int [DU] e^{-\beta S[U]}$$

The density of states

$$\rho(E) = \int [DU] \delta(S[U] - E)$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z_\beta} \int dE \rho(E) \mathcal{O}[E] e^{-\beta E} \qquad Z_\beta = \int dE \rho(E) e^{-\beta E}$$

$$P_\beta(E) = \frac{1}{Z_\beta} \rho(E) e^{-\beta E}$$

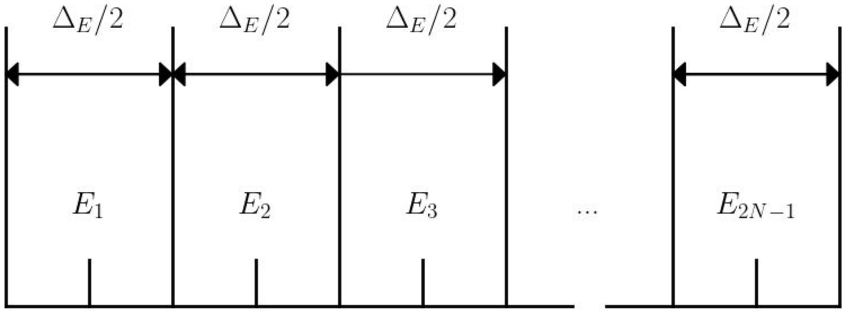
The Linear Logarithmic Relaxation method

Make the ansatz for ϱ(E) for a small region $E_n - \Delta_E/4 \leq E \leq E_n + \Delta_E/4$

$$\ln \tilde{\rho}(E) = a_n(E - E_n) + c_n$$

From continuity of $\tilde{\rho}(E)$

$$c_n = c_1 + \frac{\Delta_E}{4}a_1 + \frac{\Delta_E}{2}\sum_{k=2}^{n-1} a_k + \frac{\Delta_E}{4}a_n$$



Can find a_n by iteratively solving:

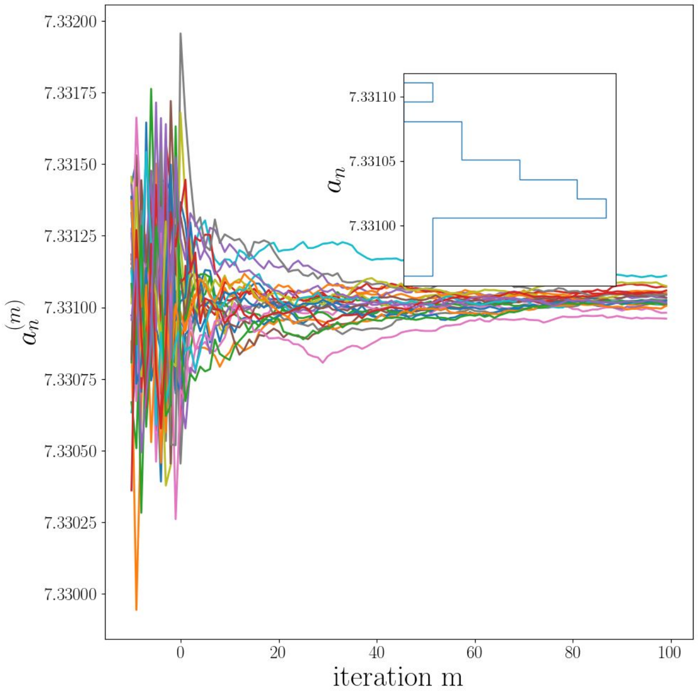
$$\langle\langle E - E_n \rangle\rangle_n(a_n) = \langle\langle u_p - (u_p)_n \rangle\rangle_n(a_n) = 0$$

$$(u_p)_n = 1 - a^4 E_n/6\tilde{V}$$

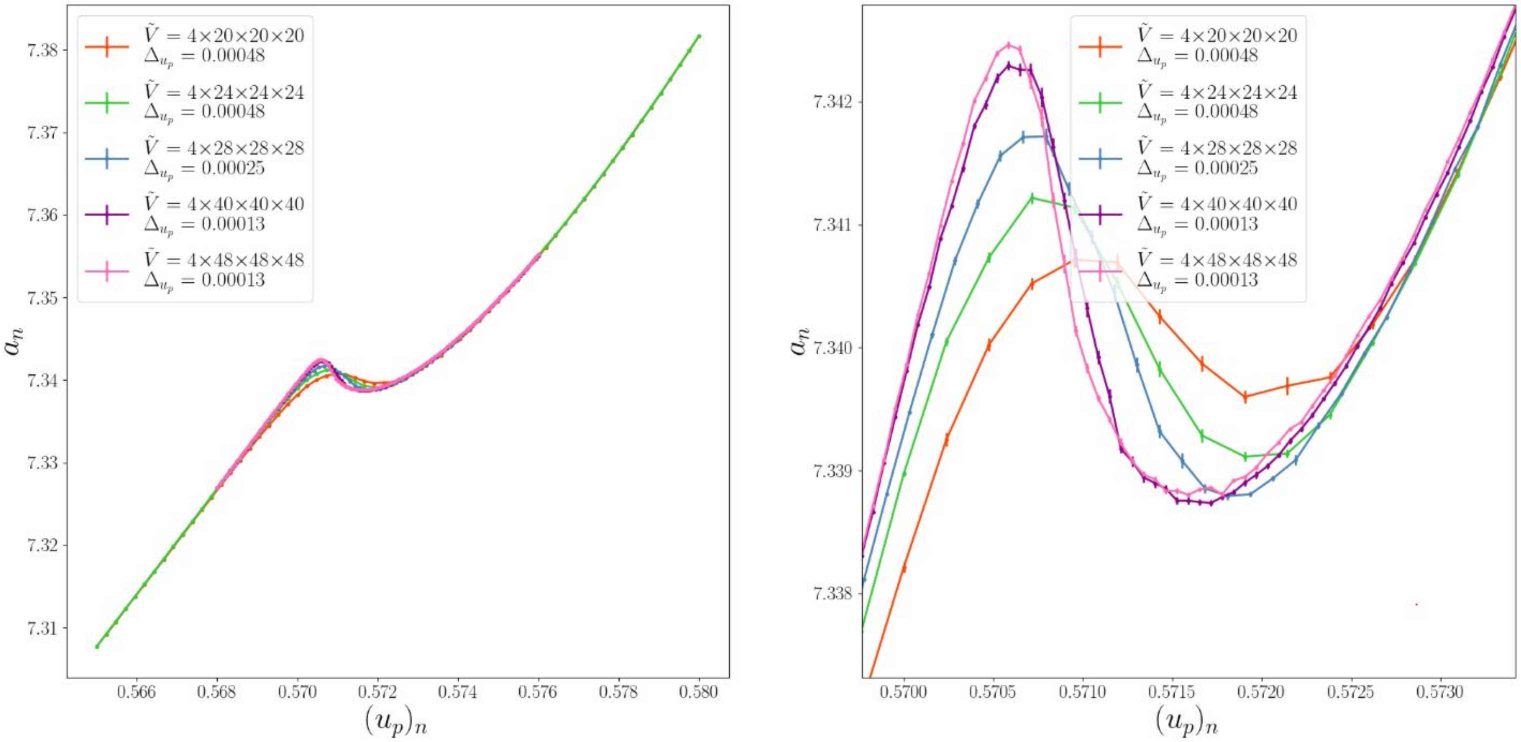
Errors on a_n

Repeat the determination of $\{a_n\}_{n=1}^{2N-1}$
use statistical error on repeats to estimate truncation error

Calculate observables using each set $\{a_n\}_{n=1}^{2N-1}$
and bootstrap to find errors

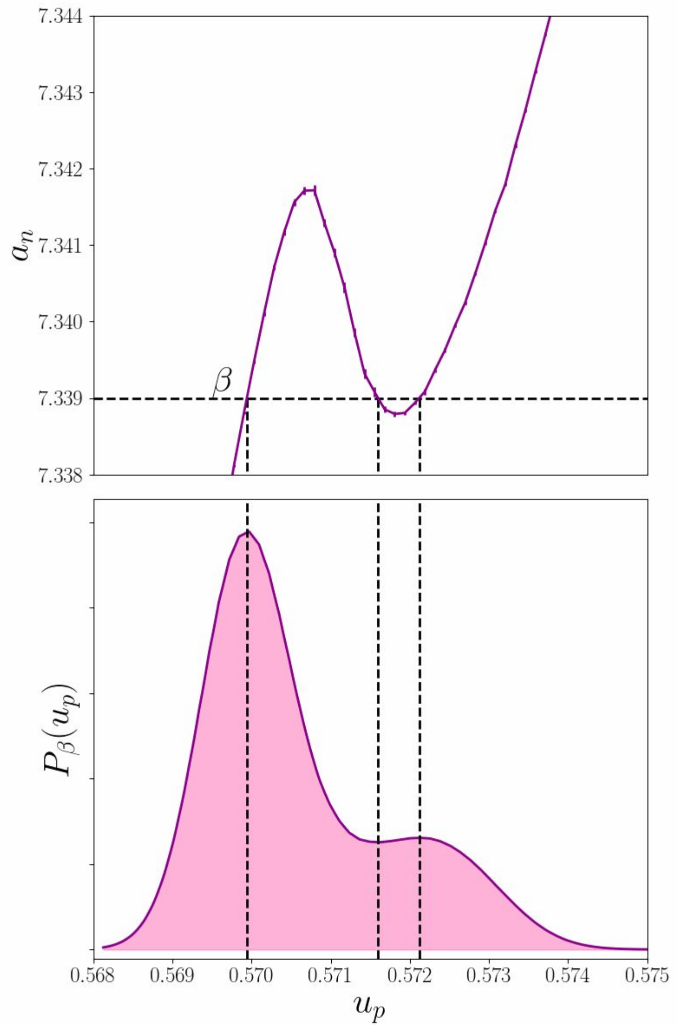


Density of states



Plaquette distribution

$$P_\beta(u_p) = \frac{1}{Z(\beta)}\rho(E)e^{-\beta E}\Big|_{E=\frac{6\tilde{V}}{a^4}(1-u_p)}$$



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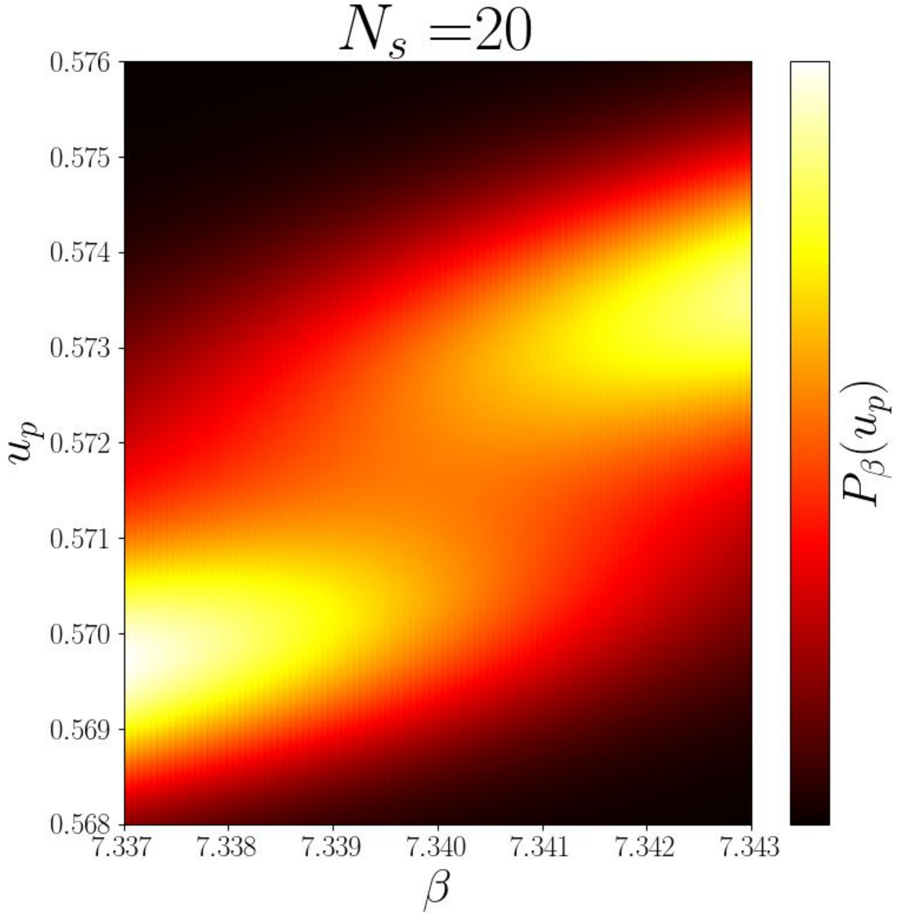

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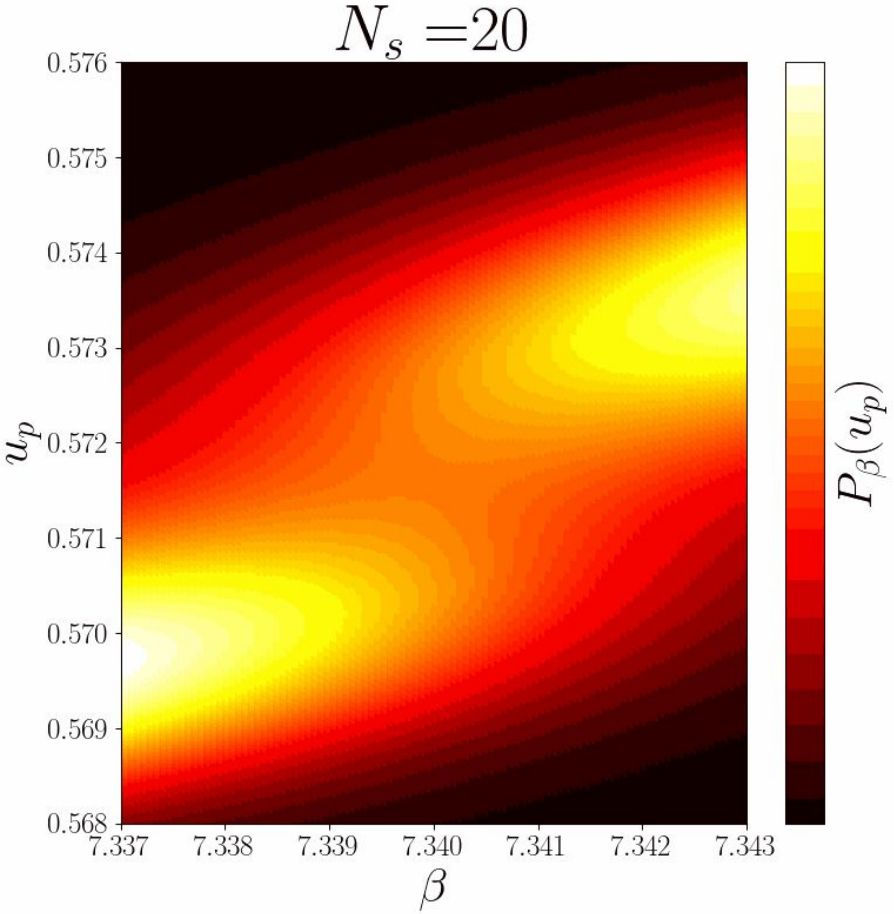

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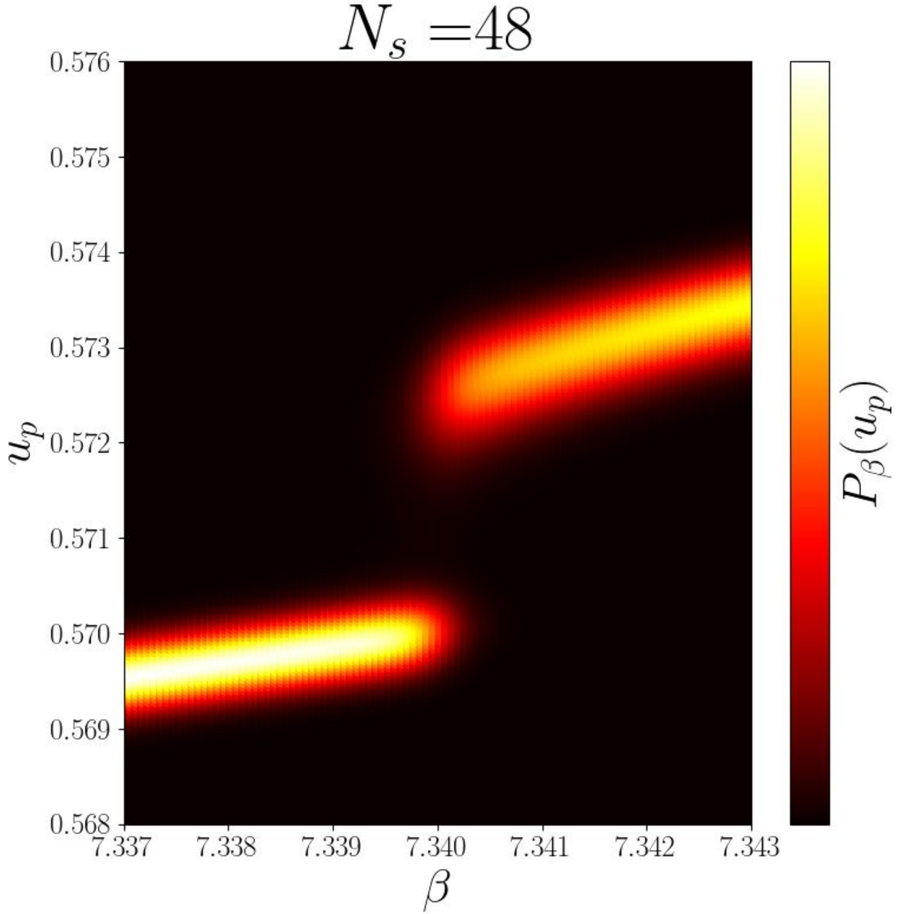

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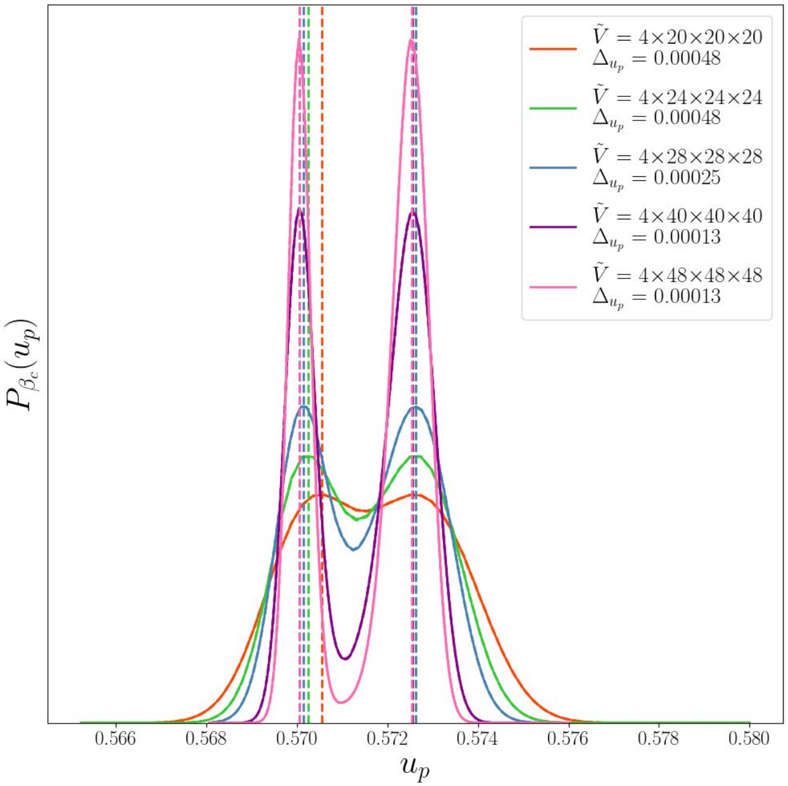

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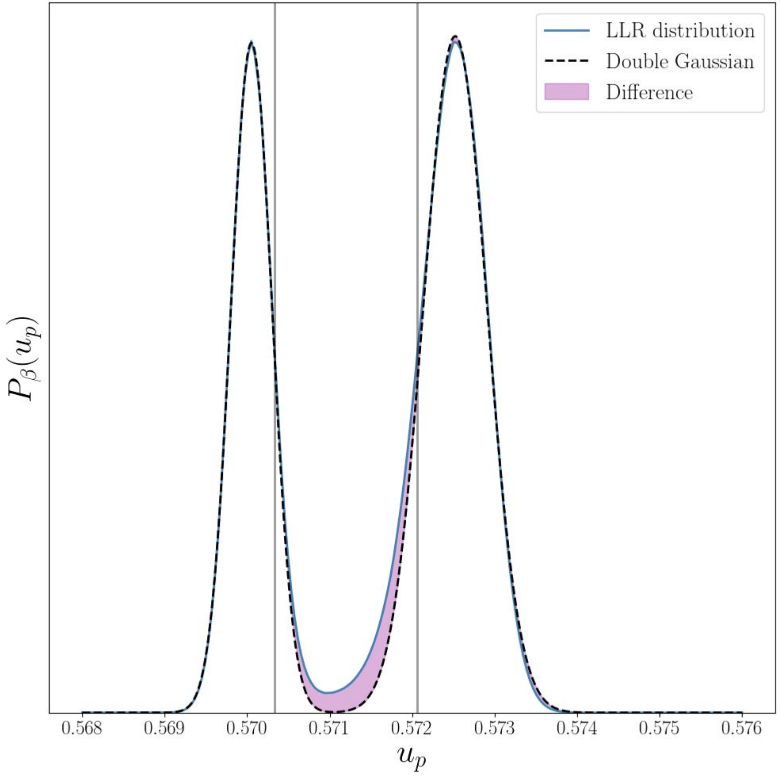
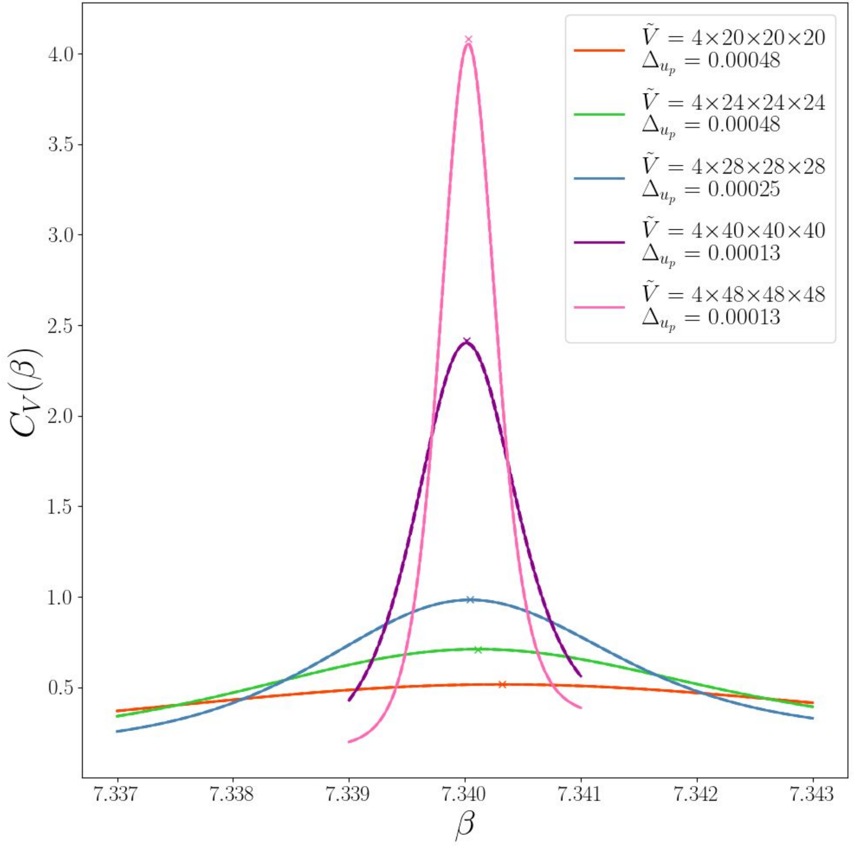
Double Gaussian approximation:
The energy distribution is made up of two Gaussian distributions each in pure phase states.



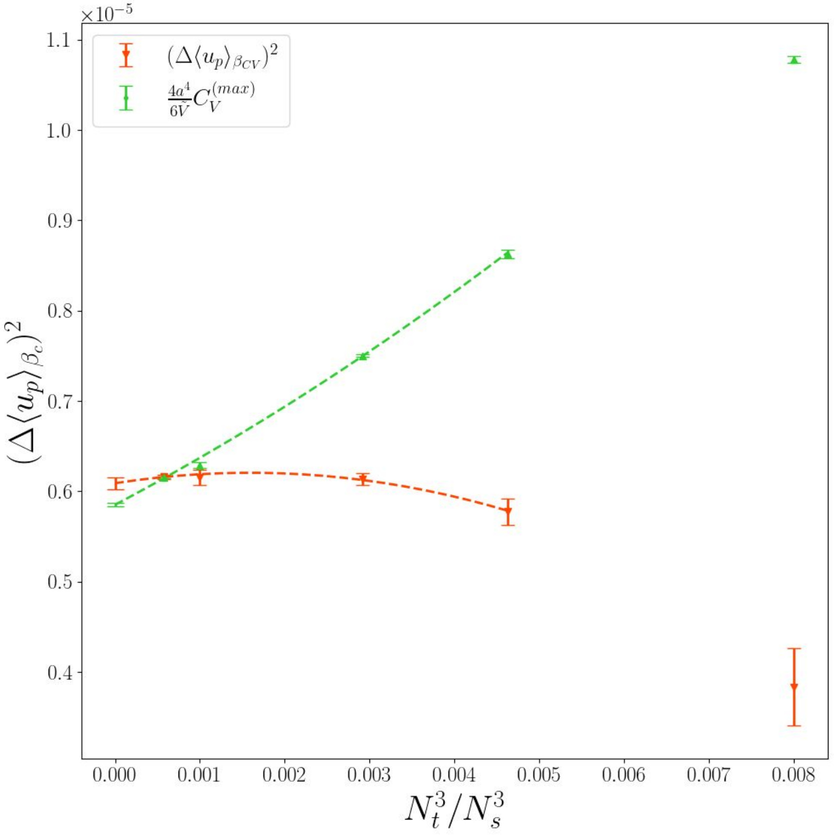
Specific heat

$$C_V \equiv \frac{6\tilde{V}}{a^4}\left(\langle u_p^2\rangle_\beta - \langle u_p\rangle_\beta^2\right)$$



Specific heat

$$C_V^{(max)} \xrightarrow{\frac{N_t}{N_s}\to 0} \frac{6\tilde{V}}{4}\left(\Delta\langle u_p\rangle_{\beta_c}\right)^2$$



Conclusion

Models of physics beyond the standard model with first order phase transitions are phenomenologically interesting for baryogenesis and gravitational waves

Sp(4) pure gauge theory contains a first order deconfinement transition

LLR method can be used to study first order transitions

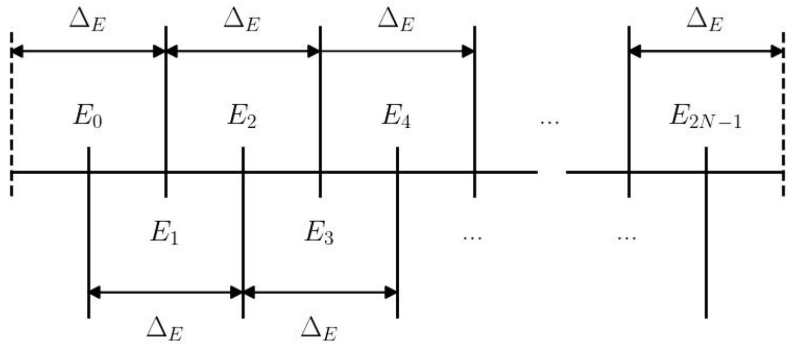
We are seeing indications of the effects of mixed phase configurations

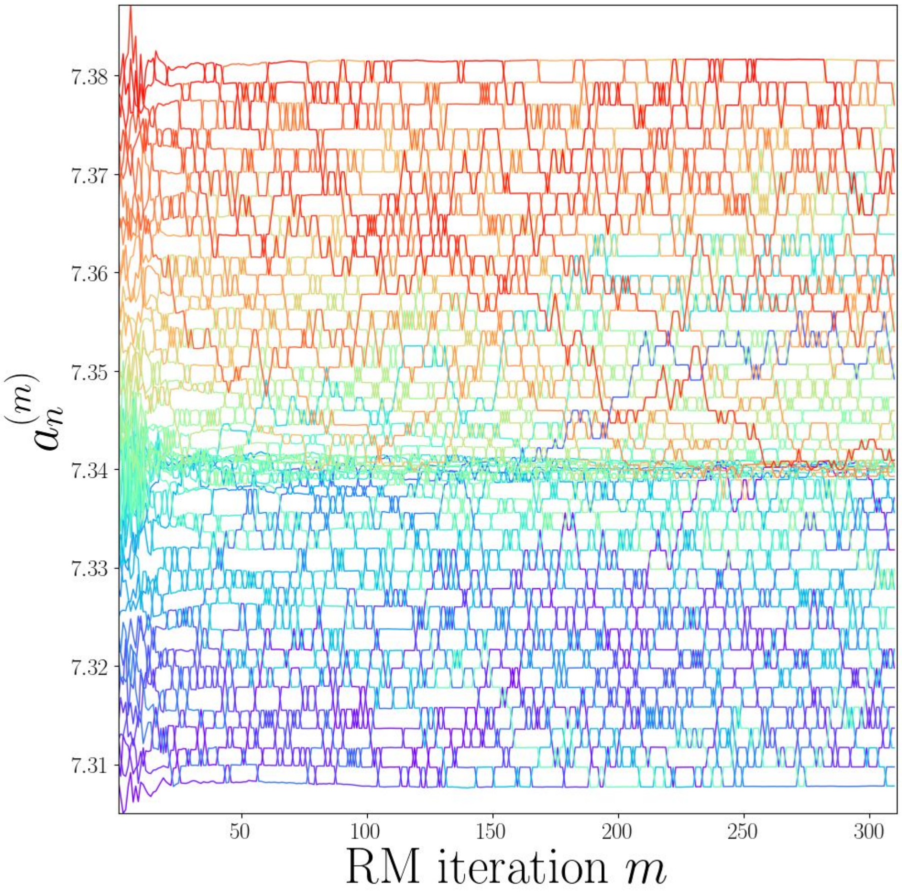
Thanks for listening

Backup: Ergodicity

To solve ergodicity problem we use replica exchange
After each RM iteration if replicas are in crossover region, consider swap with probability:

$$P_{\text{swap}} = \min\left(1, e^{(a_n^{(m)} - a_{n-1}^{(m)})(E_n^{(m)} - E_{n-1}^{(m)})}\right)$$





Backup: Bulk phase transition

Fortunately, in contrast to $SU(N)$ with $N \geq 4$, both in $Sp(2)$ and in $Sp(3)$ no bulk phase transition (which could obscure the finite temperature transition) has been found.

Holland, K., M. Pepe, and U-J. Wiese. "The deconfinement phase transition of Sp (2) and Sp (3) Yang–Mills theories in 2+ 1 and 3+ 1 dimensions." *Nuclear physics B* 694.1-2 (2004): 35-58.

Similar results found in : Bennett, Ed, et al. "Glueballs and strings in S p (2 N) Yang-Mills theories." *Physical Review D* 103.5 (2021): 054509.

The Linear Logarithmic Relaxation method

Finding a_n :

Choose initial guess $a_n^{(0)}$

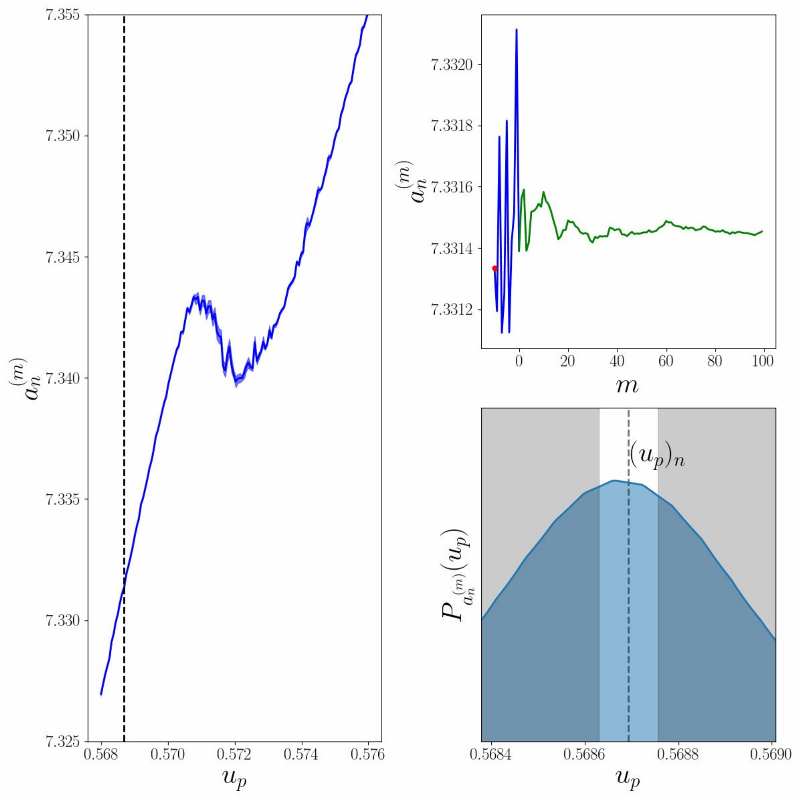
Improve guess iteratively using

Newton-Raphson Method:

$$a_n^{(m+1)} \simeq a_n^{(m)} - \left(\frac{12}{\Delta_E^2}\right) \langle\langle E - E_n \rangle\rangle_n (a_n^{(m)})$$

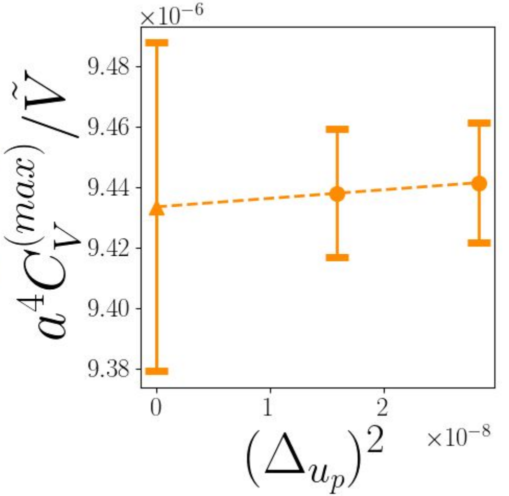
Then, Robbins-Monro method:

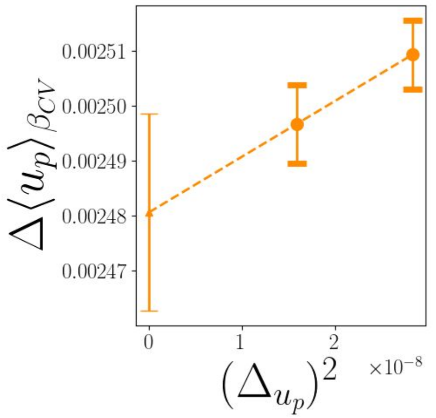
$$a_n^{(m+1)} \simeq a_n^{(m)} - \frac{1}{m+1} \left(\frac{12}{\Delta_E^2}\right) \langle\langle E - E_n \rangle\rangle_n (a_n^{(m)})$$



Vanishing interval size limit

At each volume we use several interval sizes in order to carefully analyse the limit of vanishing interval size.





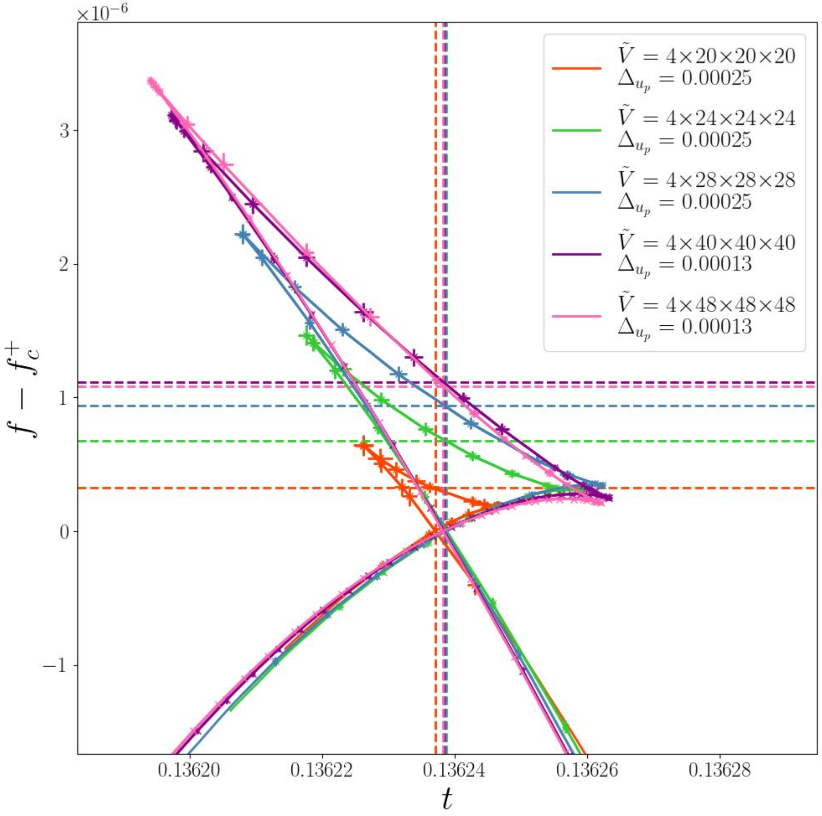
Free energy

$$F(t) = E - ts$$

$$t = 1/a_n$$

$$s = \ln \rho$$

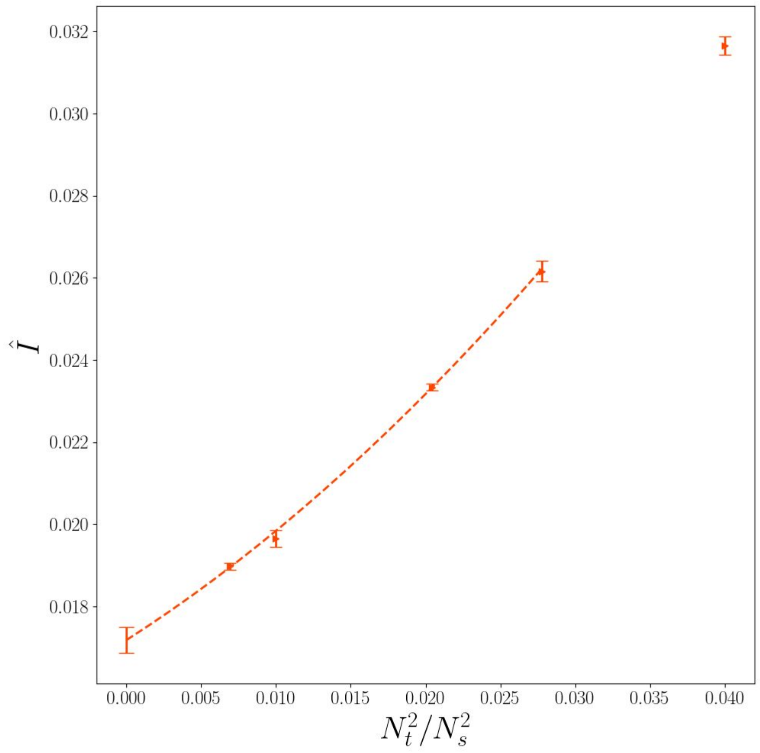
$$f(t) = a^4(F(t) + \Sigma t)/(\tilde{V})$$



Free energy

$$\frac{\Delta F}{t_c} = -\ln\left(\frac{P_{min}}{P_{max}}\right)$$

$$\hat{\sigma}_{cd} = \frac{N_t^2}{2N_s^2}\ln\left(\frac{P_{min}}{P_{max}}\right) + \frac{N_t^2}{2N_s^2}\left(\frac{1}{2}\ln(N_s) + c\right) = \hat{I} + \frac{N_t^2}{2N_s^2}c$$ [1]



[1]Lucini, Biagio, Michael Teper, and Urs Wenger. "Properties of the deconfining phase transition in SU (N) gauge theories." *Journal of High Energy Physics* 2005.02 (2005): 033.