Precision determination of the Wilson-flow scale w_0

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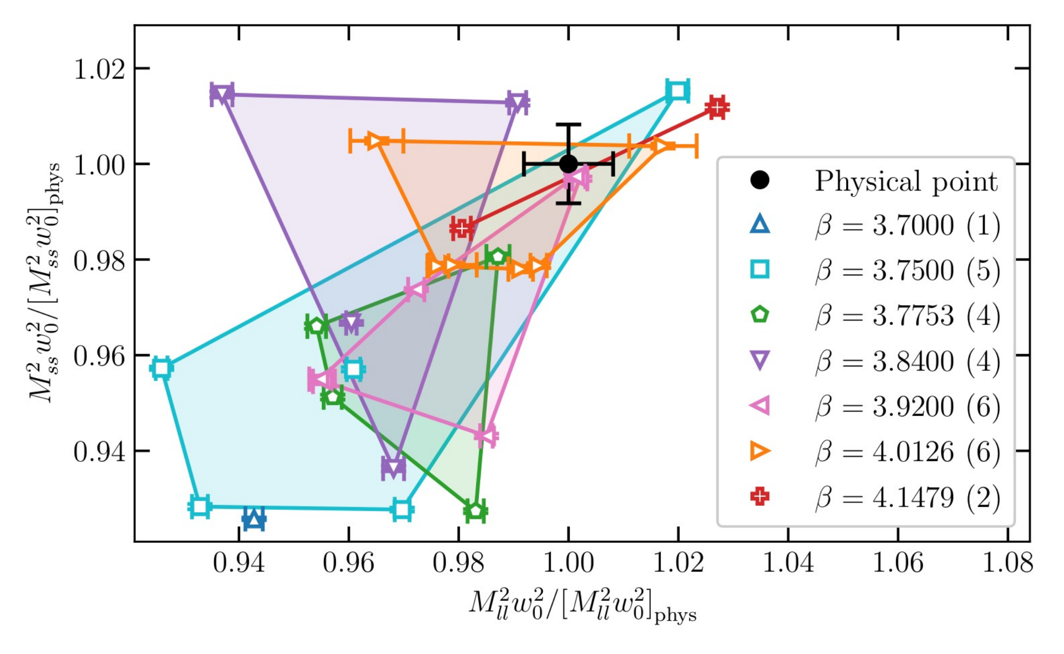
LATTICE 2024
LIVERPOOL

Physical point



- $N_f = 2 + 1 + 1$ staggered fermions
- Stout smearing $n = 4$, $\rho = 0.125$

➢ M_π and M_{ss} around physical point with the finest lattice spacing 0.048 fm

➢ The lattice scale is set by Ω baryon mass

➢ Precise determination of w_0 from M_Ω

Ω correlation function

- For the Ω baryon, two staggered baryon operators in [1] and an operator which only couples to a single taste [2]

$$\Omega_{\mathrm{VI}}(t) = \sum_{x_k even} \epsilon_{abc} \left[S_1\chi_a S_{12}\chi_b S_{13}\chi_c - S_2\chi_a S_{21}\chi_b S_{23}\chi_c + S_3\chi_a S_{31}\chi_b S_{32}\chi_c\right](x)$$

$$\Omega_{\mathrm{XI}}(t) = \sum_{x_k even} \epsilon_{abc} \left[S_1\chi_a S_2\chi_b S_3\chi_c\right](x)$$

$$\Omega_{\mathrm{Ba}}(t) = \left[2\delta_{\alpha 1}\delta_{\beta 2}\delta_{\gamma 3} - \delta_{\alpha 3}\delta_{\beta 1}\delta_{\gamma 2} - \delta_{\alpha 2}\delta_{\beta 3}\delta_{\gamma 1} + (\cdots \beta \leftrightarrow \gamma \cdots)\right]$$

$$\sum_{x_k even} \epsilon_{abc} \left[S_1\chi_{a\alpha} S_{12}\chi_{b\beta} S_{13}\chi_{c\gamma} - S_2\chi_{a\alpha} S_{21}\chi_{b\beta} S_{23}\chi_{c\gamma} + S_3\chi_{a\alpha} S_{31}\chi_{b\beta} S_{32}\chi_{c\gamma}\right](x)$$

- Wuppertal smearing connects 2a lattice spacing

$$\left[\hat{W}v\right]_x = (1-\sigma)v_x + \frac{\sigma}{6}\sum_{\mu=1,2,3}\left(U^{3d}_{\mu,x}U^{3d,\dagger}_{\mu,x+\mu}v_{x+2\mu} + U^{3d}_{u,x-\mu}U^{3d,\dagger}_{\mu,x-2\mu}v_{x-2\mu}\right)$$

[1] M. F. L. Golterman and J. Smit, Nucl. Phys. B 255 (1985), 328-340
[2] J. A. Bailey, Phys. Rev. D 75 (2007), 114505 [arXiv:hep-lat/0611023

GEVP method

- Staggered Ω correlators with positive and oscillating negative parity states:

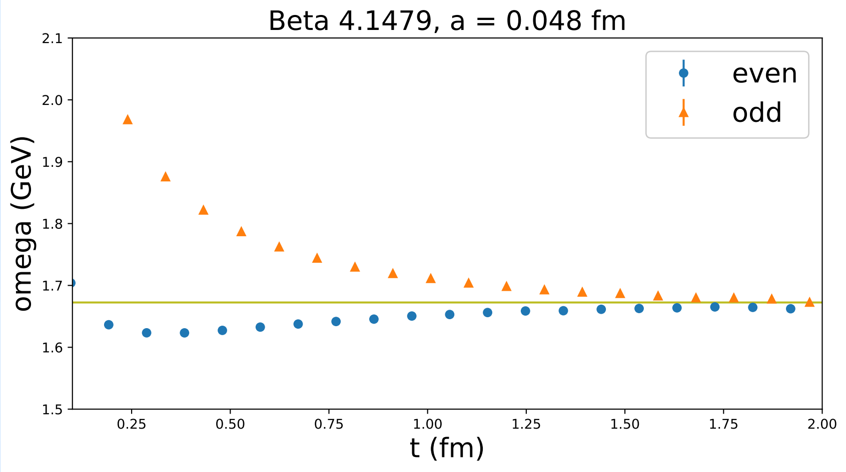
$$H(t, A, M) = A_0 e^{-M_0 t} + (-1)^{t+1} A_1(M_1, t) e^{-M_1 t} + A_2 e^{-M_2 t} + (-1)^{t+1} A_3 e^{-M_3 t} + \cdots$$

- Use time shift to create an `new` operator [1]

$$H(t + 2t_s) = \sum_i \left[A'_i e^{+2M_i t_s}\right] e^{-M_i t}$$

- Combine smear-point, smear-smear, point-point correlators into one Matrix for GEVP

- Presence of oscillating makes the time shifted GEVP procedure more efficient



$$C(t \in \text{even/odd}) = \frac{1}{2} log(\frac{C(t+2)}{C(t)})$$

[1] C. Aubin, and K. Orginos, AIP Conf.Proc. 1374 (2011) 1, 621-624

GEVP method

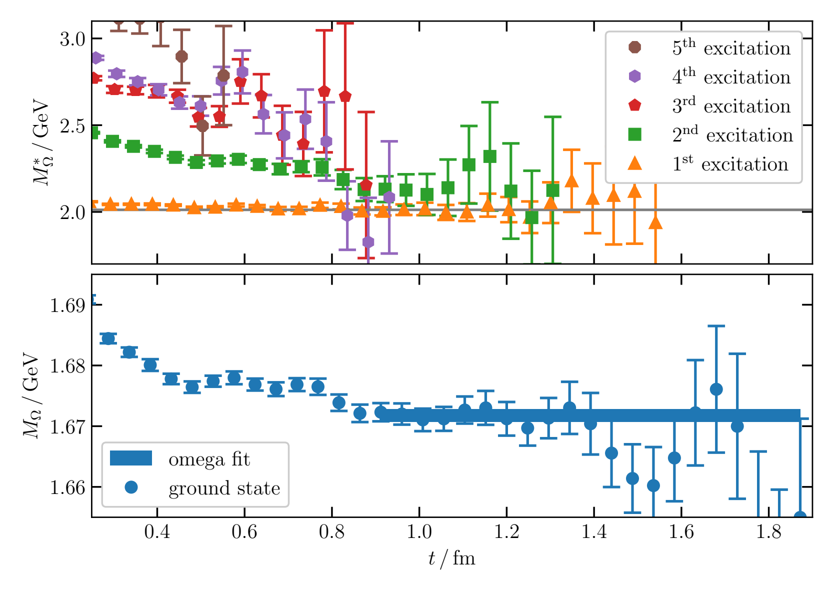
$$
\mathbf{H}(t) = \left(\begin{array}{cc|cccc}
H^{pp}_{t+2t_p+0} & H^{pp}_{t+2t_p+1} & H^{ps}_{t+t_p+0} & H^{ps}_{t+t_p+1} & H^{ps}_{t+t_p+2} & H^{ps}_{t+t_p+3} \\
H^{pp}_{t+2t_p+1} & H^{pp}_{t+2t_p+2} & H^{ps}_{t+t_p+1} & H^{ps}_{t+t_p+2} & H^{ps}_{t+t_p+3} & H^{ps}_{t+t_p+4} \\
\hline
H^{sp}_{t+t_p+0} & H^{sp}_{t+t_p+1} & H^{ss}_{t+0} & H^{ss}_{t+1} & H^{ss}_{t+2} & H^{ss}_{t+3} \\
H^{sp}_{t+t_p+1} & H^{sp}_{t+t_p+2} & H^{ss}_{t+1} & H^{ss}_{t+2} & H^{ss}_{t+3} & H^{ss}_{t+4} \\
H^{sp}_{t+t_p+2} & H^{sp}_{t+t_p+3} & H^{ss}_{t+2} & H^{ss}_{t+3} & H^{ss}_{t+4} & H^{ss}_{t+5} \\
H^{sp}_{t+t_p+3} & H^{sp}_{t+t_p+4} & H^{ss}_{t+3} & H^{ss}_{t+4} & H^{ss}_{t+5} & H^{ss}_{t+6}
\end{array}\right)
$$

- Point source correlators get additional time shift t_p to suppress its large excited state effects

- Solve the Generalized Eigenvalue Problem (GEVP)

$$\mathbf{H}(t_a)v_i(t_a,t_b) = \lambda_i(t_a,t_b)\mathbf{H}(t_b)v_i(t_a,t_b)$$

➢ Ground state is extracted wth 0.1% precision



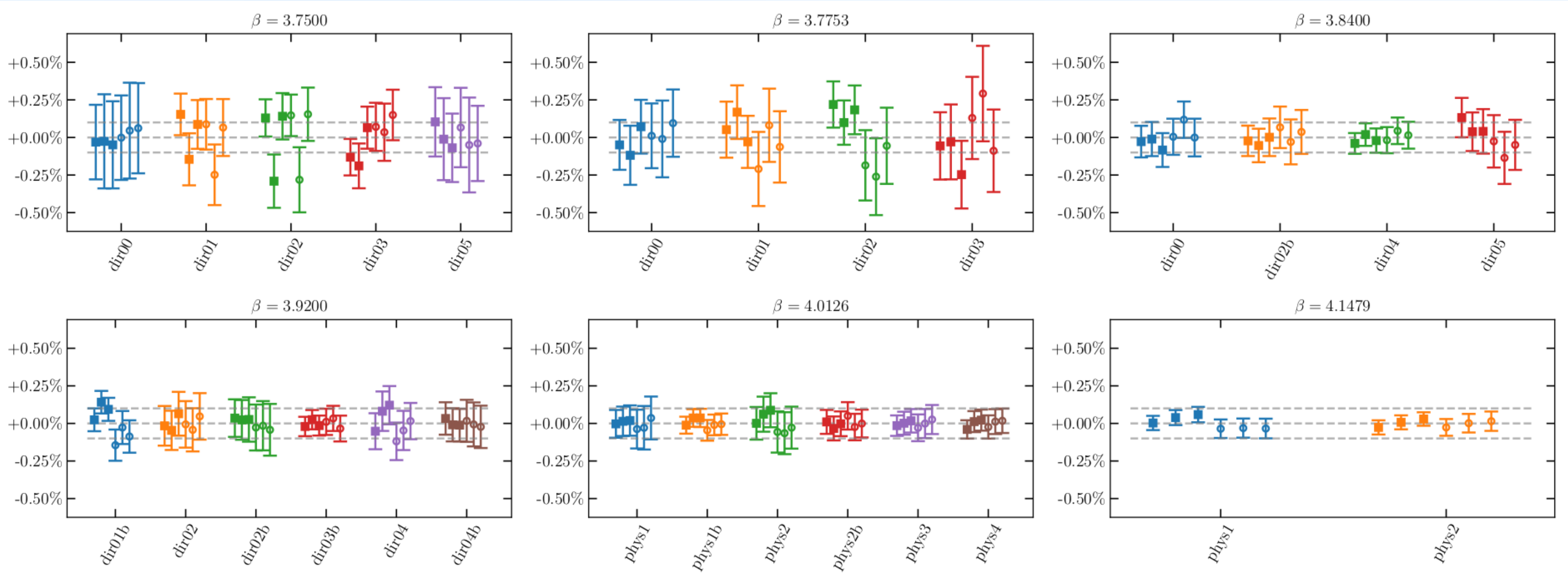
Ω measurements

β	#conf	N_{Wptl}	$N_{3\text{d}}$	t_p	t_a	t_b	range #1	range #2	# pt, sm sources
3.7000	904	24	32	1	4	7	$7\ldots15$	$8\ldots15$	28928, 229376
3.7500	2072	30	40	1	4	7	$8\ldots18$	$9\ldots18$	66208, 530176
3.7553	1907	34	46	1	4	7	$9\ldots19$	$10\ldots19$	61024, 488192
3.8400	2949	46	62	2	4	9	$10\ldots20$	$11\ldots20$	125440, 2807552
3.9200	4296	67	90	2	6	9	$12\ldots25$	$13\ldots25$	137472, 3038720
4.0126	6980	101	135	3	6	9	$15\ldots30$	$16\ldots30$	223360, 4235520
4.1479	5017	178	238	5	6	11	$19\ldots40$	$21\ldots40$	160544, 2068736

- **Over 30,000 gauge configurations**
- **10's of millions measurements**
- **Measurements on GPUs based on Quda [1] and Qlattice [2]**

[1] https://github.com/lattice/quda
[2] https://github.com/jinluchang/Qlattice

Ω masses



- Correlated (first 3 points) and uncorrelated (second 3 points) are consistent
- Taste breaking effects not observed between different Omega operators
- Reached less than 0.1% error on 0.048 fm lattices

Continuum extrapolation formula

- The logarithmic derivative of the gauge-action density along the gradient flow time

$$W_\tau[U] \equiv \frac{d(\tau^2 E[U,\tau])}{d log\tau}, \langle W_{\tau=w_0^2} \rangle = 0.3$$

- Continuum extrapolations as a taylor expansion of a^2

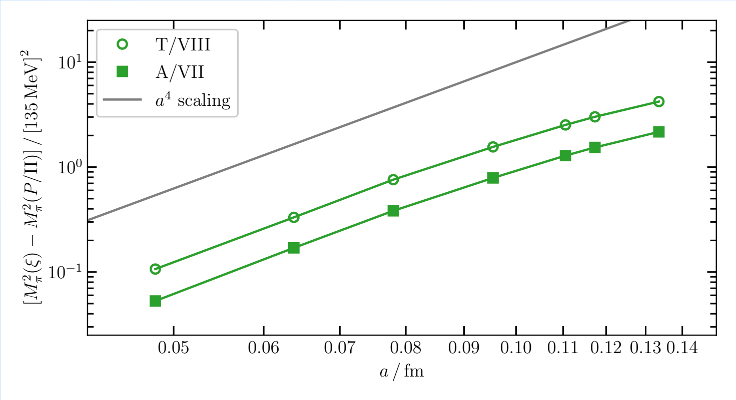
$$Y = Y_0 + A_1 a^2 + A_2 a^4 + A_3 a^6 + \cdots$$

- Or non-analytic from the Symanzik effective theory [1] with \`n\` unknown

$$a^2 \to \alpha_s(a)^n a^2$$

- Major staggered artifact (taste violation) scale with a power of n = 3

$$\Delta_{KS}(\xi) \equiv M_\pi^2(\xi) - M_{ll}^2$$



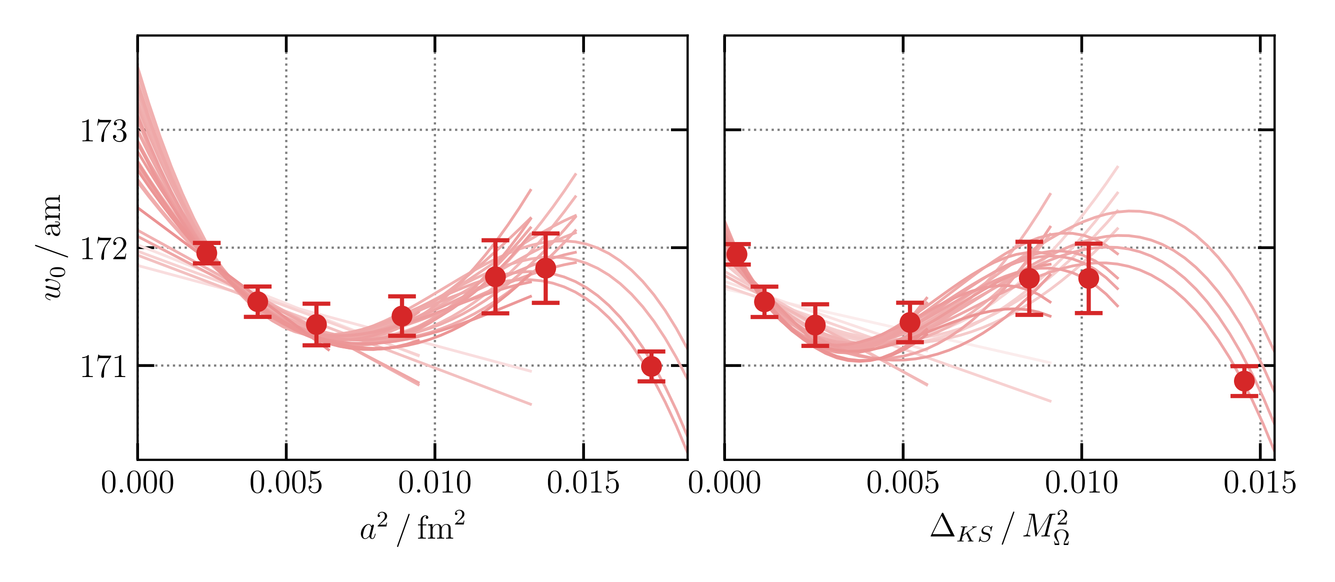
[1] N. Husung, P. Marquard, R. Sommer, Eur.Phys.J.C 80 (2020) 3, 200

Continuum extrapolation

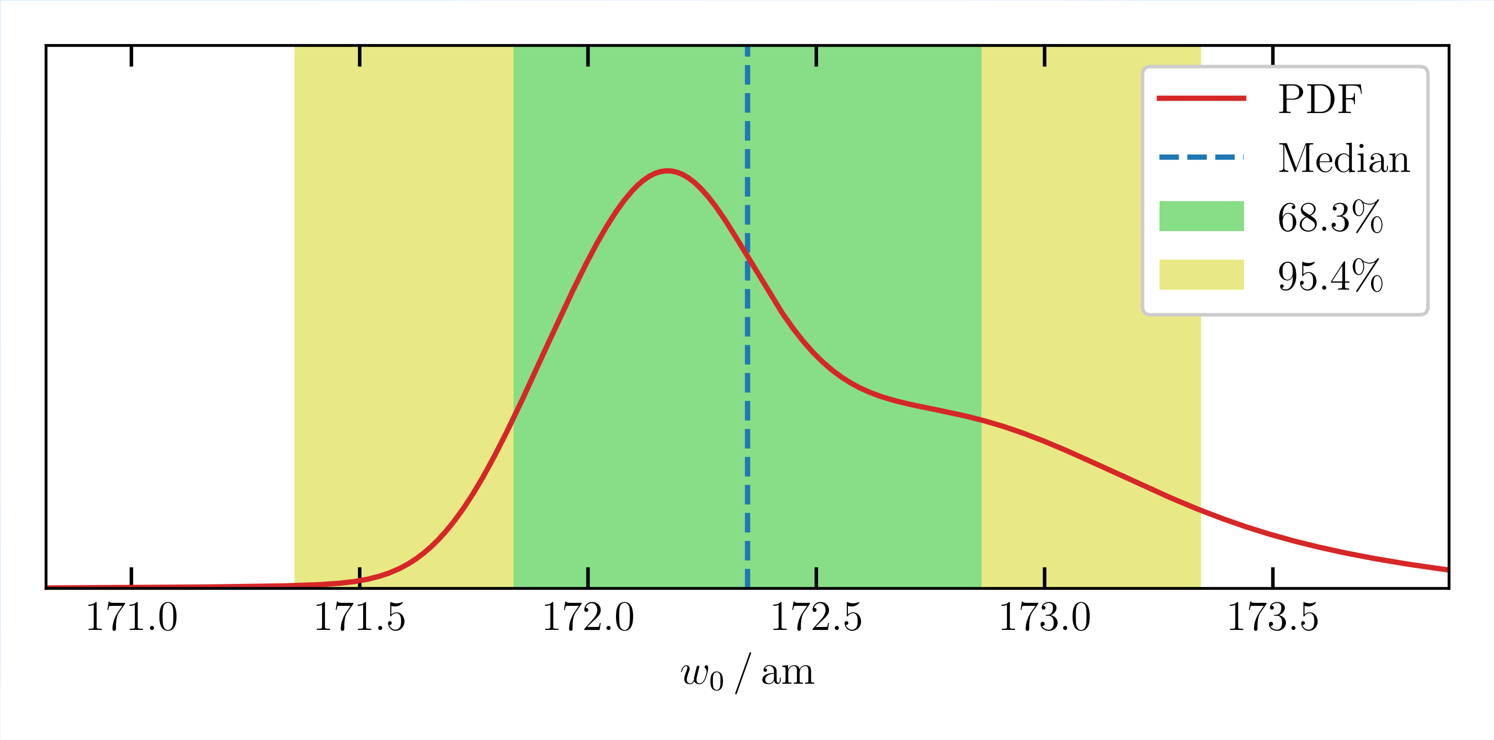
- Observable $Y = w_0 M_\Omega$

$$Y = A(a^2) + A'(\Delta_{KS}) + (B_0 + B_1 a^2) X_l + (C_0 + C_1 a^2) X_s$$

- ➢ $A(a^2)$ or $A'(\Delta_{KS})$
- ➢ beta cuts 0, 1, 2, 3, 4
- ➢ B1 or C1 included or not
- ➢ different Omega fits
- ➢ different meson fits



Fits distribution



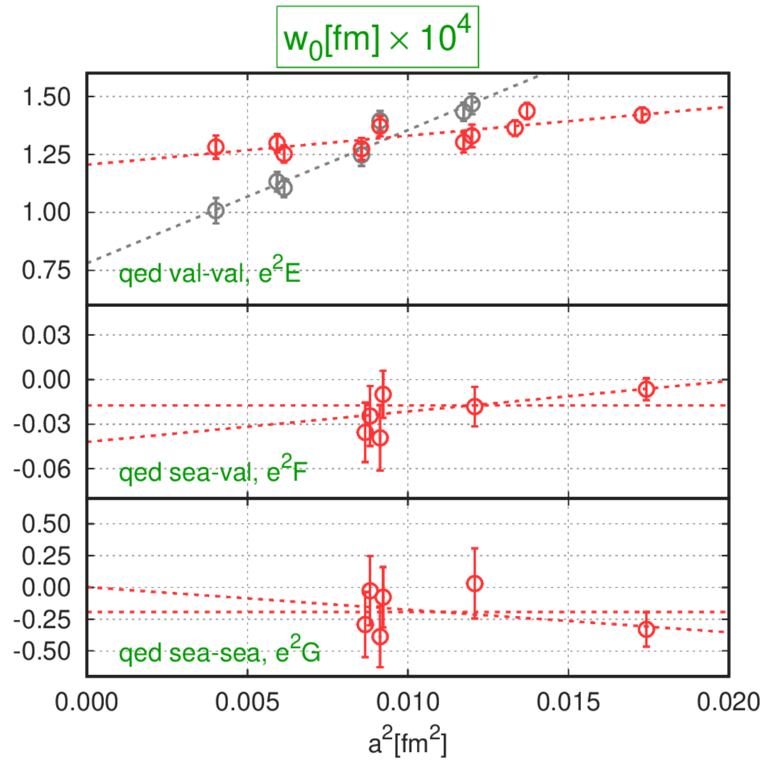
Electromagnetic effects

- Observable $Y = w_0 M_\Omega$

$$Y = A + BX_l + CX_s + DX_{\delta m} + Ee_v^2 + Fe_v e_s + Ge_s^2$$

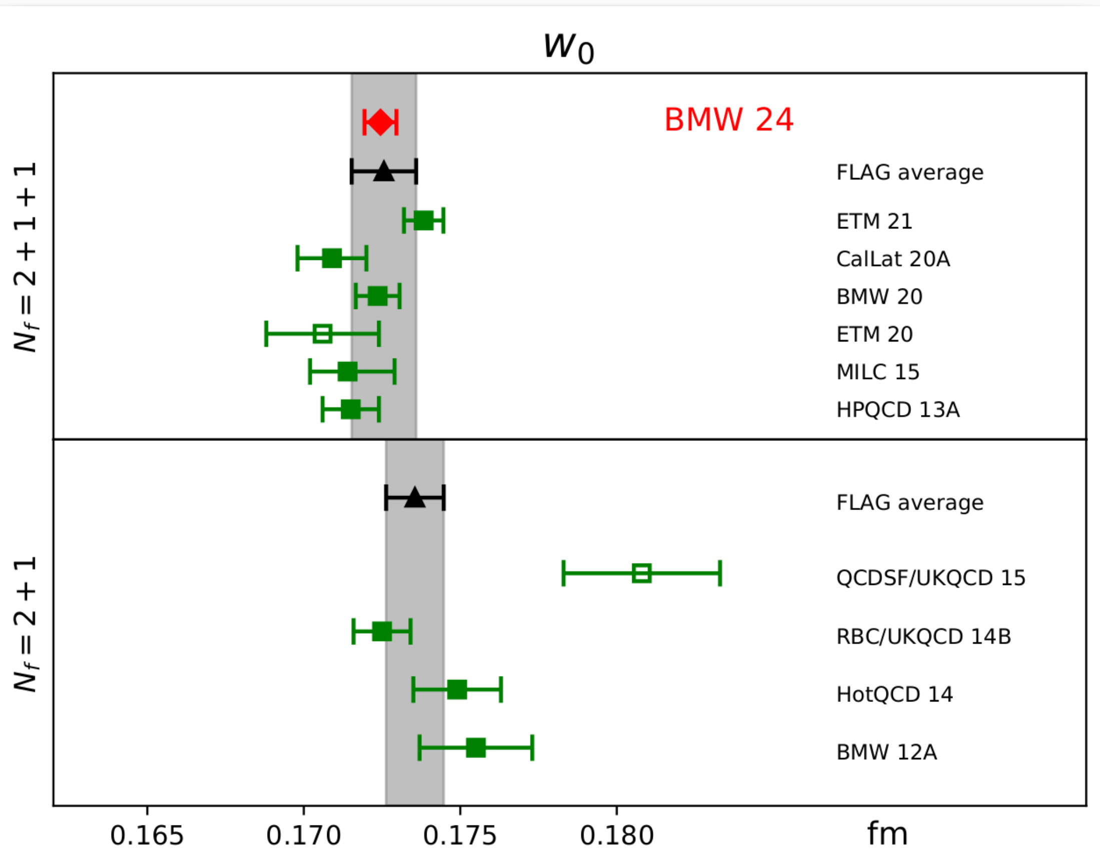
- Fit to a system of equations [1]

$$[Y]_0 = [A + BX_l + CX_s]_0$$
$$[Y]'_m = [DX_{\delta m}]'_m$$
$$[Y]''_{20} = [A + BX_l + CX_s + DX_{\delta m}]''_{20} + [E]_0$$
$$[Y]''_{11} = [A + BX_l + CX_s + DX_{\delta m}]''_{11} + [F]_0$$
$$[Y]''_{02} = [A + BX_l + CX_s + DX_{\delta m}]''_{02} + [G]_0$$



[1] Sz. Borsanyi, and et. al., BMWc, Nature 593 (2021) 7857, 51-55

Final results



- 7 lattice spacings all at physical pion mass
- Omega baryon statistics errors well under control
- Electromagnetic effects included

$$[w_0]_{\mathrm{phys}} = 0.17245(22)(46)[51]\ \mathrm{fm}$$

Thank You