Progress in lattice simulations for two Higgs doublet models

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SU(2) Two Higgs Doublet model – Motivation

4D SU(2) gauge theory with 2 fundamental Higgs

- Single Scalar – simplest way to generate EWSB
- Minimal SM extension – possible features
  - First-order EWPT (?) – Baryogenesis
  - New source of CP violation
- Enlarged spectrum
- Mimics the SM at low energies

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Fundamental Representation and Gauge Fields

$$\Phi_i(x) = \begin{pmatrix} \phi_i^+(x) \\ \phi_i^0(x) \end{pmatrix} \quad i = 1, 2,$$

$$D_\mu = \partial_\mu + \mathbb{A}_\mu, \qquad \mathbb{A}_\mu = -igA_\mu^a \sigma_a/2,$$

$$G_{\mu\nu} = \partial_\mu \mathbb{A}_\nu - \partial_\nu \mathbb{A}_\mu + [\mathbb{A}_\mu, \mathbb{A}_\nu]$$

SU(2) Two Higgs Doublet model – Scalar Potential

$$\mathcal{L}_{\text{2HDM}} = (D_\mu \Phi_1)^\dagger (D_\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D_\mu \Phi_2) + V_{\text{2HDM}} - \frac{1}{2g^2} \text{Tr}[G_{\mu\nu} G_{\mu\nu}]$$

$$V_{\text{2HDM}} = \mu_{11}^2 (\Phi_1^\dagger \Phi_1) + \mu_{22}^2 (\Phi_2^\dagger \Phi_2) + \mu_{12}^2 \,\text{Re}(\Phi_1^\dagger \Phi_2) + \eta_1 (\Phi_1^\dagger \Phi_1)^2 + \eta_2 (\Phi_2^\dagger \Phi_2) + \eta_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \eta_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \eta_5 \,\text{Re}(\Phi_1^\dagger \Phi_2)^2 + \text{Re}(\Phi_1^\dagger \Phi_2) \left[\eta_6 (\Phi_1^\dagger \Phi_1) + \eta_7 (\Phi_2^\dagger \Phi_2)\right]$$

Global Symmetries

$$V_{\text{2HDM}} =$$
$$\mu_{11}^2(\Phi_1^\dagger\Phi_1) + \mu_{22}^2(\Phi_2^\dagger\Phi_2) + \mu_{12}^2\,\text{Re}(\Phi_1^\dagger\Phi_2)$$
$$+\,\eta_1(\Phi_1^\dagger\Phi_1)^2 + \eta_2(\Phi_2^\dagger\Phi_2) + \eta_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \eta_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\,\eta_5\,\text{Re}(\Phi_1^\dagger\Phi_2)^2 + \text{Re}(\Phi_1^\dagger\Phi_2)\left[\eta_6(\Phi_1^\dagger\Phi_1) + \eta_7(\Phi_2^\dagger\Phi_2)\right]$$

- Most general case: $SU(2)$ global symmetry
- Previous lattice studies – $SU(2) \times SU(2)$ global symmetry:
  - Phase structure & SSB
  - [Lewis and Woloshyn 2010]
  - [Wurtz, Lewis, and Steele 2009]
- No previous study of the spectrum

$$V_{\text{2HDM}} =$$
$$\mu_{11}^2(\Phi_1^\dagger\Phi_1) + \mu_{22}^2(\Phi_2^\dagger\Phi_2) + \mu_{12}^2\,\mathrm{Re}(\Phi_1^\dagger\Phi_2)$$
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$$+\,\eta_5\,\mathrm{Re}(\Phi_1^\dagger\Phi_2)^2 + \mathrm{Re}(\Phi_1^\dagger\Phi_2)\left[\eta_6(\Phi_1^\dagger\Phi_1) + \eta_7(\Phi_2^\dagger\Phi_2)\right]$$

- $O(4) \sim SU(2)_L \times SU(2)_R$ custodial symmetry
  - [Haber and O'Neil 2011]
  - $\eta_4 = \eta_5$
- Same symmetry as the SM

$$
\begin{aligned}
V_{\text{2HDM}} =& \\
&\mu_{11}^2(\Phi_1^\dagger\Phi_1) + \mu_{22}^2(\Phi_2^\dagger\Phi_2) + \mu_{12}^2\,\mathrm{Re}(\Phi_1^\dagger\Phi_2) \\
&+ \eta_1(\Phi_1^\dagger\Phi_1)^2 + \eta_2(\Phi_2^\dagger\Phi_2) + \eta_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \eta_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
&+ \eta_5\,\mathrm{Re}(\Phi_1^\dagger\Phi_2)^2 + \mathrm{Re}(\Phi_1^\dagger\Phi_2)\left[\eta_6(\Phi_1^\dagger\Phi_1) + \eta_7(\Phi_2^\dagger\Phi_2)\right]
\end{aligned}
$$

- Discrete $\mathbb{Z}_2$ symmetries: $\mu_{12} = \eta_6 = \eta_7 = 0$
  - $\Phi_1 \longrightarrow -\Phi_1$
  - $\Phi_2 \longrightarrow -\Phi_2$
- Inert Model: $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetric [Deshpande and Ma 1978]
  - $\mathbb{Z}_2$ and FCNC [Hou and Kikuchi 2018]
  - Dark matter model [Honorez et al. 2007]

Lattice action & Simulation details

Quaternion representation:

$$\Phi_n(x) = \frac{1}{\sqrt{2}} \sum_{\alpha=0}^{N} \theta_\alpha \phi_\alpha^{(n)}(x),$$

$$\theta_0 = 1_{2\times 2}, \quad \theta_i = i\sigma_i$$

$$S_{\rm 2HDM}^{O(4)} = S_{\rm Wilson} + \sum_x \sum_{n=1}^{2} \Big\{ \sum_\mu -2\kappa_n \,{\rm Tr}\left(\hat\Phi_n^\dagger U_\mu \hat\Phi_n(x+\hat\mu)\right)$$
$$+ {\rm Tr}\left(\hat\Phi_n^\dagger \hat\Phi_n\right) + \hat\eta_n \left[{\rm Tr}\left(\hat\Phi_n^\dagger \hat\Phi_n\right) - 1\right]^2 \Big\} + 2\mu^2 \,{\rm Tr}\left(\hat\Phi_1^\dagger \hat\Phi_2\right)$$
$$+ \hat\eta_3 \,{\rm Tr}\left(\hat\Phi_1^\dagger \hat\Phi_1\right) {\rm Tr}\left(\hat\Phi_2^\dagger \hat\Phi_2\right) + \hat\eta_4 \,{\rm Tr}\left(\hat\Phi_1^\dagger \hat\Phi_2\right)^2$$

- HMC w/ GPU [igit.ific.uv.es/gtelo/latticegpu.jl/-/tree/su2-higgs]
- Error analysis [igit.ific.uv.es/alramos/aderrors.jl]

$\langle \Phi_1 \rangle = 0$
$\langle \Phi_2 \rangle \neq 0$

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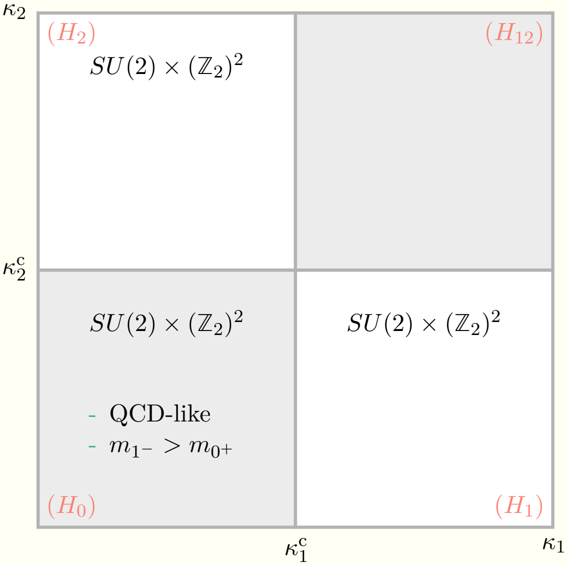
$\langle \Phi_2 \rangle \neq 0$

Tree Level:
$$\frac{1}{8} - \frac{\hat{\eta}_2}{4}$$



$\langle \Phi_2 \rangle = 0$
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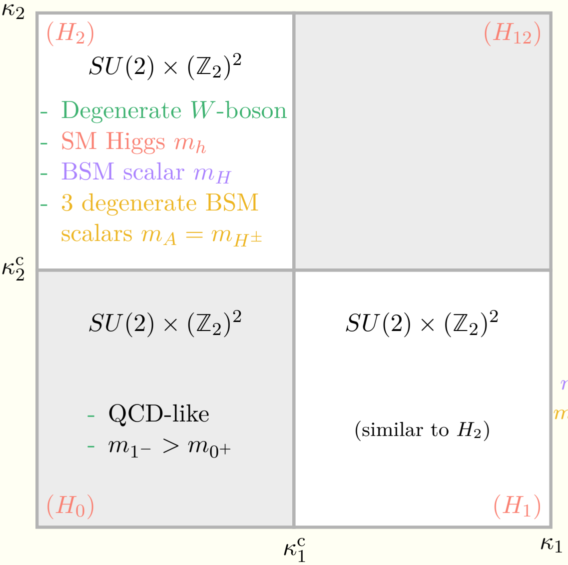

$\langle\Phi_2\rangle = 0$
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$$m_W^2 = g^2 v^2/4$$
$$m_h^2 = \eta_1 v^2$$
$$m_H^2 = m_{H^\pm} + \eta_3 v^2$$
$$m_{H^\pm}^2 = \mu_{22}^2 + \tfrac{1}{2}\lambda_3 v^2$$
$$m_a^2 = m_{H^\pm}$$

Phase structure & Spectrum

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Tree Level: $\frac{1}{8} - \frac{\hat{\eta}_2}{4}$

$\langle \Phi_2 \rangle = 0$
$\langle \Phi_1 \rangle = 0$

κ_2	$\kappa < \kappa_1^c$	$\kappa > \kappa_1^c$
$\kappa_2 > \kappa_2^c$	(H_2) $SU(2) \times (\mathbb{Z}_2)^2$ - Degenerate W-boson - SM Higgs m_h - BSM scalar m_H - 3 degenerate BSM scalars $m_A = m_{H\pm}$	(H_{12}) - 3 non-degenerate 1^- - 3 Goldstone Bosons - 2 scalar states m_h, m_H
$\kappa_2 < \kappa_2^c$	$SU(2) \times (\mathbb{Z}_2)^2$ - QCD-like - $m_{1^-} > m_{0^+}$ (H_0)	$SU(2) \times (\mathbb{Z}_2)^2$ (similar to H_2) (H_1)

κ_1^c κ_1

$\langle \Phi_1 \rangle \neq 0$
$\langle \Phi_2 \rangle \neq 0$

$m_W^2 = g^2 v^2/4$
$m_h^2 = \eta_1 v^2$
$m_H^2 = m_{H\pm} + \eta_3 v^2$
$m_{H\pm}^2 = \mu_{22}^2 + \frac{1}{2}\lambda_3 v^2$
$m_a^2 = m_{H\pm}$

Observables

- Phase Diagram
  - Gauge invariant link

$$L^a_{\alpha_{nm}} = \frac{1}{8V} \sum_{x,\mu} \mathrm{Tr}\{\alpha_n^\dagger(x) U_\mu(x) \alpha_m(x+\hat{\mu}) \theta^a\}$$

$$L_{\alpha_n} \equiv L^4_{\alpha_{nn}}$$

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$$L_{\alpha_n} \equiv L^4_{\alpha_{nn}}$$

Spectrum of the theory

$$S^a_{ij}(x^4) = \sum_{\vec{x}} \mathrm{Tr}\left[\Phi_i^\dagger(x) \Phi_j(x) \theta^\alpha\right]$$

$$W^a_{ij,\mu}(x^4) = \sum_{\vec{x}} \mathrm{Tr}\left[\Phi_i^\dagger(x) U_\mu(x) \Phi_j(x+\hat{\mu}) \theta^\alpha\right]$$

$$S^4_{ii},\ W^4_{ii} \to 0^+, \qquad W^4_{ij} \to 0^+\ (i \neq j), \qquad W^k_{ii} \to 1^-.$$

- Gradient flow and Laplacian smearing for $U_\mu(x), \Phi(x)$

Constant SM physics

sector (H_2)

Standard Model physics on the lattice

$$R = \left(\frac{m_h}{m_W}\right)_{\text{latt}} = \left(\frac{m_h}{m_W}\right)_{\text{phys}} = 1.5$$

$$S = \left(\frac{m_W}{\mu_0}\right)_{\text{latt}} = \left(\frac{m_W}{\mu_0}\right)_{\text{phys}} = 1.0, \quad g^2_{\text{GF}}(\mu_0)\Big|_{m_W} = 0.5$$

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Gradient Flow – Non perturbative gauge running coupling

$$\mu_0 = \frac{1}{\sqrt{8t_0}}, \qquad g^2_{GF}(\mu) \equiv \frac{128\pi^2}{9} t^2 \langle E(t) \rangle \Big|_{t=1/8\mu^2}$$

- Flowed gauge action density (Clover)

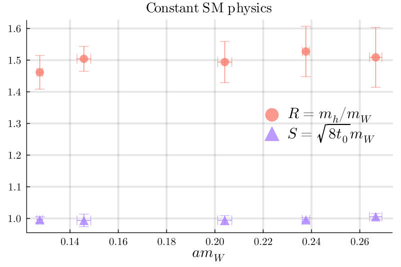
$$\langle E(x,t) \rangle = -\frac{1}{4} \left\langle G^a_{\mu\nu}(x,t) G^a_{\mu\nu}(x,t) \right\rangle$$

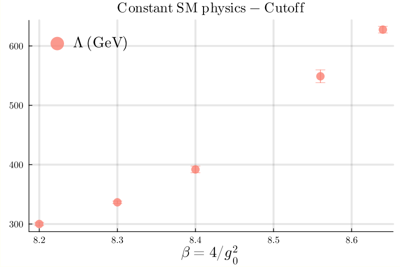
- [Lüscher and Weisz 2011]

$$t^2 \langle E(t) \rangle = \frac{9}{128\pi^2} g^2_{\overline{MS}}(\mu)(1 + \mathcal{O}(g^2)) \Big|_{\mu=1/\sqrt{8t}}$$

Tuning SM physics

- $\{\beta, \kappa_2, \eta_2\}$

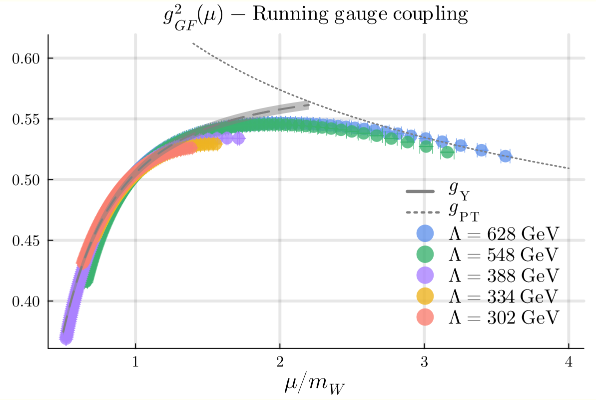




- Scalar interpolator with VEV – decreased precision
- Precise vector mass and GF scale
- Scale setting with $a = m_W^{\text{phys}}/\hat{m}_W$

$$\Lambda \sim \ 300 - 620 \text{ GeV}$$

Running Gauge coupling



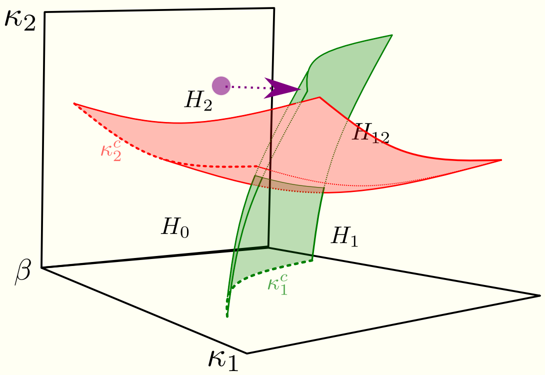
$$g^2_{\mathrm{Y}}(\mu) = r^2 \frac{\mathrm{d}V_{\mathrm{Y}}}{\mathrm{d}r}\bigg|_{\mu=1/r} \qquad V_{\mathrm{Y}}(r) \propto \frac{1}{4\pi r} e^{-mr}$$

$$\beta_{SU(N)+\text{Scalars}} = \mu \frac{\mathrm{d}g}{\mathrm{d}\mu} = -\frac{b_0 g^3}{16\pi^2} + \mathcal{O}\big(g^5\big), \quad b_0 = \frac{11N - n_s}{3}.$$

- LCP defined with $\{\beta, \kappa_2, \eta_2\}$

Scan BSM sector

- LCP defined with $\{\beta, \kappa_2, \eta_2\}$
- Remaining couplings control the BSM spectrum – scan κ_1



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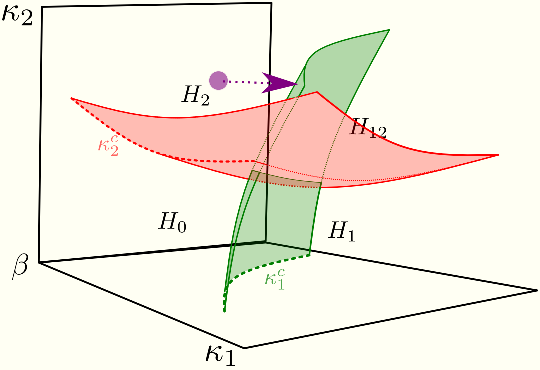
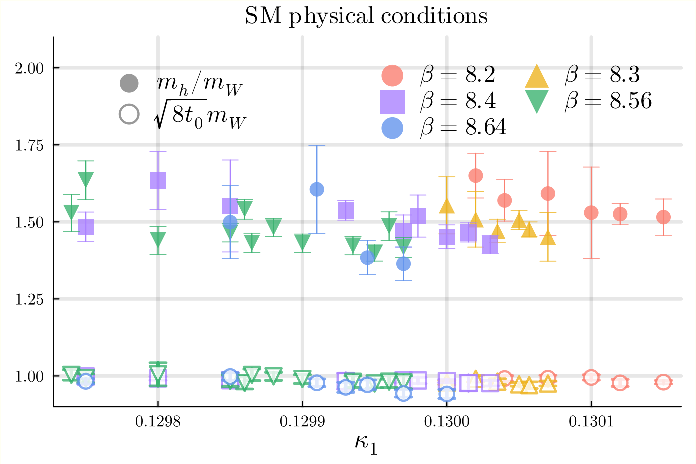


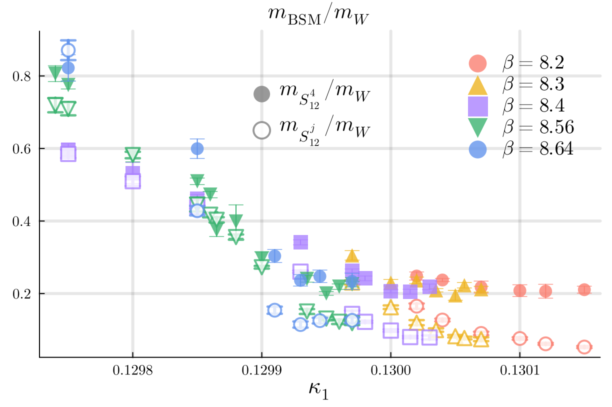
- Non-SM $SU(2)$ SSB: $(H_2) \longrightarrow (H_{12})$

Scans BSM sector – SM conditions



- Small quartics – SM physics roughly unchanged
- Large range of κ_1

Scans BSM sector – BSM masses



- Light scalar masses – roughly cutoff independent
- Near PT: $S^j_{12} \longrightarrow$ Goldstones

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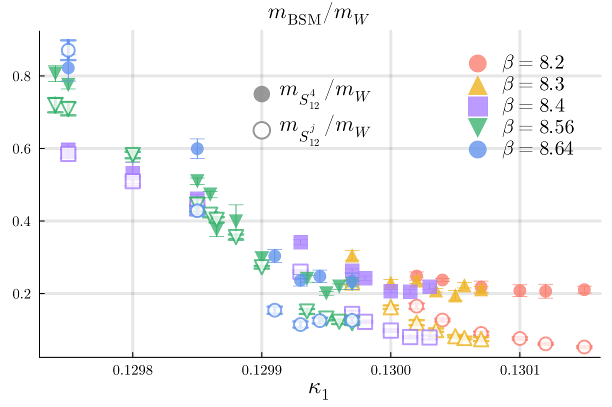

- Light scalar masses – roughly cutoff independent
- Near PT: $S^j_{12} \longrightarrow$ Goldstones
- Could there be un-detected light scalars?
  - Answer not definitive [CMS, arXiv:2405.18149] [ATLAS, arXiv:2407.07546]

Finite Temperature

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- Simulate at $N_t < L$ – $T = \frac{1}{aN_t}$
- L_{α_2} signals Higgs mechanism

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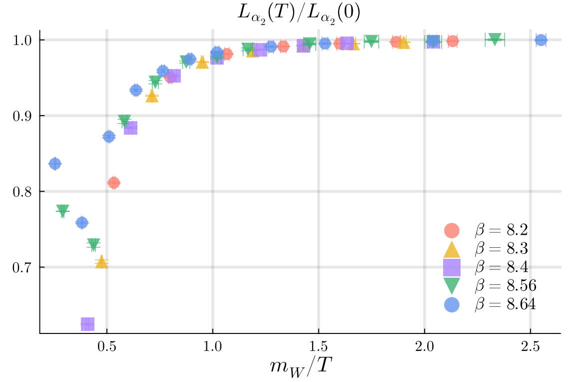
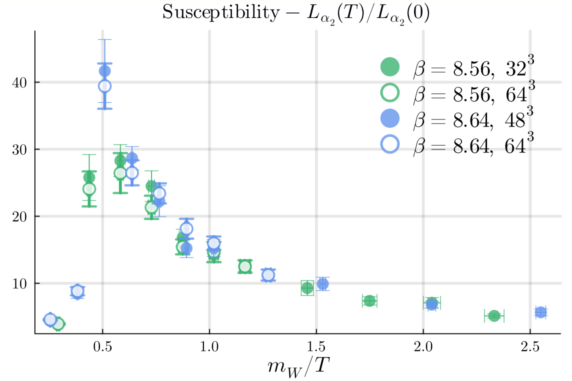
- L_{α_2} signals Higgs mechanism

$$L^R_\alpha(T) = Z(g_i)L^{\text{Latt}}_\alpha(T, g_i)$$



$$\chi_O(L) = L^3 \left(\langle O^2 \rangle - \langle O \rangle^2 \right)$$



- Susceptibility is volume independent
- Crossover behavior

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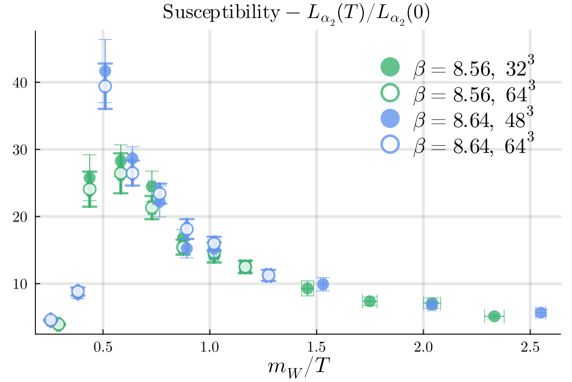


- Susceptibility is volume independent
- Crossover behavior
- Conflict w/ PT predictions (?) [Bernon, Bian, and Jiang 2018]

Conclusions

- Knowledge of the parameter space
- Framework ready to study any 2HDM
- Constant SM line for Custodial 2HDM
- Light BSM scalars realizable within SM physics
- Crossover EWPT (larger cutoff required)

Future plans

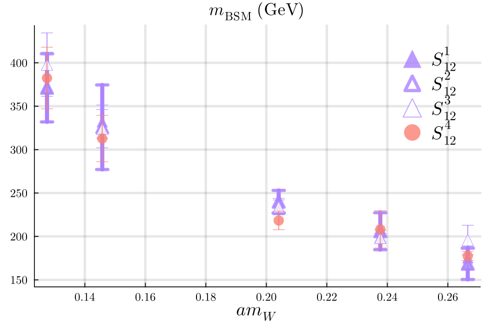
- $\mathcal{O}(1)$ quartic couplings for first-order EWPT

  [Hou and Kikuchi 2018]
- Complicates tuning – compute renoremalized quartic couplings

Details LCP

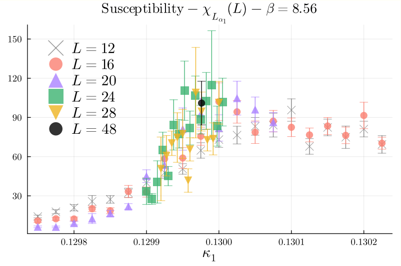
β	8.2	8.3	8.4	8.56	8.64
κ_2	0.13175	0.13104	0.1306	0.1301	0.129985
η_2	0.00338	0.003	0.00285	0.00275	0.002737
R	1.509(94)	1.527(80)	1.494(65)	1.504(39)	1.462(53)
S	1.0055(98)	0.9956(65)	0.994(14)	0.994(20)	0.9958(94)
am_h	0.402(28)	0.363(21)	0.305(16)	0.2192(84)	0.1863(75)
am_W	0.2666(26)	0.2377(15)	0.2041(29)	0.1458(29)	0.1275(11)
t_0/a^2	1.7781(58)	2.1934(74)	2.966(13)	5.810(45)	7.626(43)
Λ (GeV)	301.5(29)	338.2(21)	393.8(57)	551(11)	630.4(56)
mL	8.485(14)	7.639(13)	6.569(15)	4.694(18)	6.145(17)

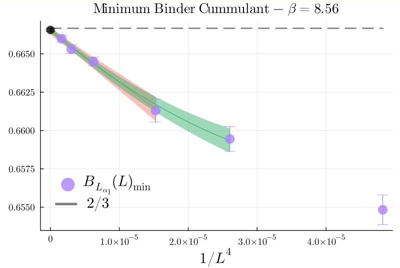
BSM masses physical units



- Degenerate scalars $m_A = m_{H^\pm} \approx m_H$
- $m_{\mathrm{BSM}} \sim 350$ GeV
- EWPT shows crossover behavior
- Contrarily to PT prediction [Bernon, Bian, and Jiang 2018]

Scan BSM sector: $(H_2) \longrightarrow (H_{12})$





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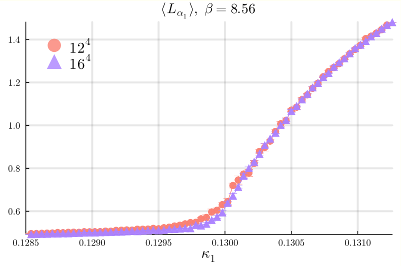
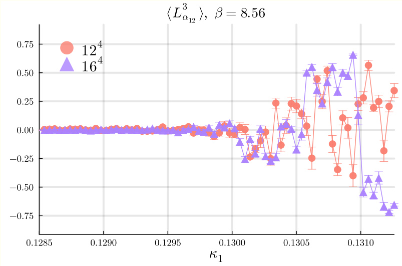
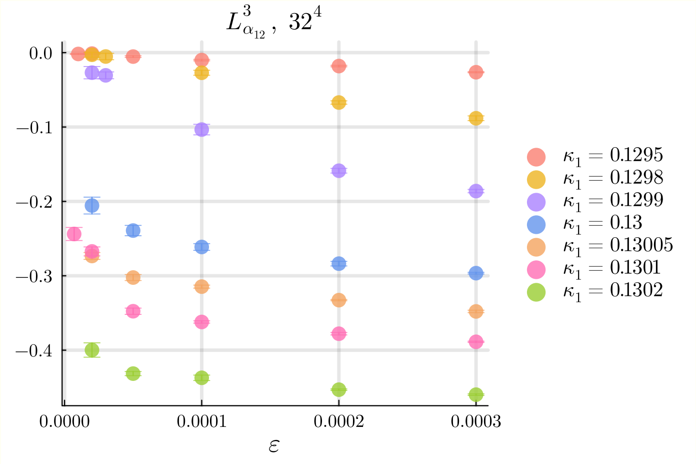




- L_{α_1} not a good observable to study SSB
- Multiple vacua (?) [Lewis and Woloshyn 2010]
- Explicit breaking & extrapolation required to evaluate SSB

$$\varepsilon \, \mathrm{Tr} \left[\Phi_1^\dagger(x) \Phi_2(x) \theta^3 \right], \quad \varepsilon \to 0$$

SSB (H_2) $\longrightarrow$ (H_{12}) (Explicit breaking)



- Evidence for SSB
- Not conclusive! Thermodynamic limit required

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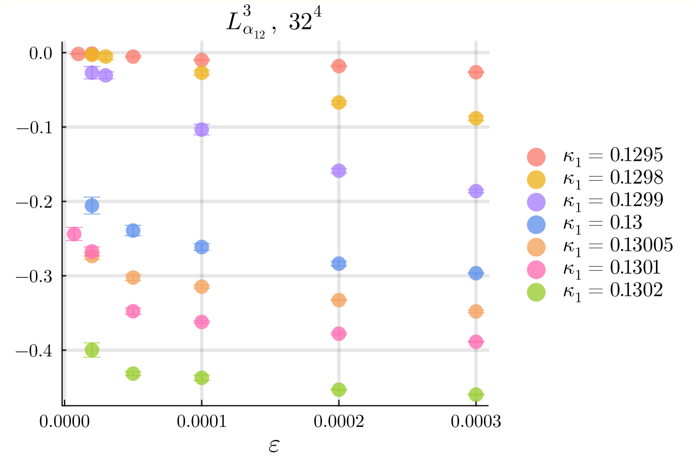

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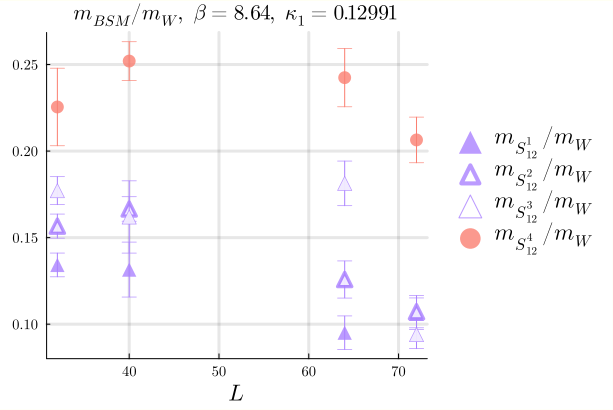
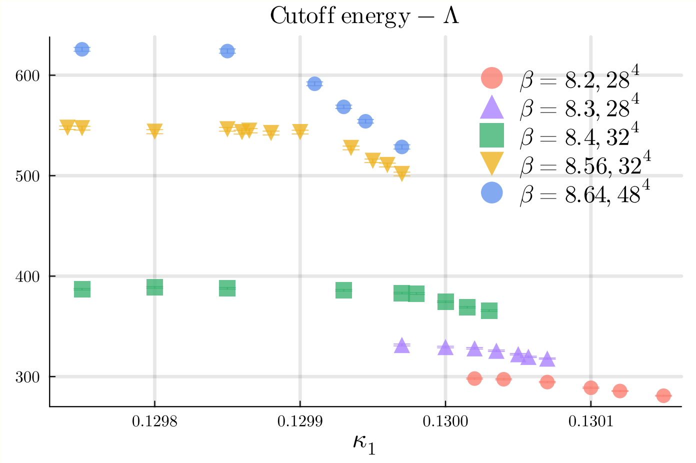
Scans BSM sector – Finite volume



- No Goldstones in this regime
- Finite volume effects under control for m_H

Scans BSM sector – Cutoff & Crossover Region

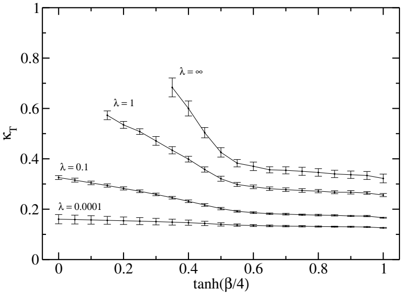


- Cutoff unaffected up to $\sim$ PT region

Lagrangian and Phase structure - Single Higgs

$$S = S_{\mathrm{YM}}[U;\beta] + \sum_x \left\{ \sum_\mu -2\kappa\,\mathrm{Tr}\left(\hat{\Phi}^\dagger(x)U_\mu(x)\hat{\Phi}(x+\hat{\mu})\right) + \mathrm{Tr}\left(\hat{\Phi}^\dagger(x)\hat{\Phi}(x)\right) + \lambda\left[\mathrm{Tr}\left(\hat{\Phi}^\dagger(x)\hat{\Phi}(x)\right) - 1\right]^2 \right\}$$

- $\beta = 4/g^2$
- $a^2\mu^2 = \frac{1-2\eta-8\kappa}{\kappa}$
- $\lambda = \eta/\kappa^2$
- Confinement & Higgs
- Analytically connected



[M. Wurtz et al, Phys.Rev.D 79 (2009) 074501]

Physical conditions & Continuum limit – Single Higgs theory

- $m_H/m_W \approx 1.5$
- $g^2_{GF}(\mu = m_W) = 0.5, \quad m_W/\mu = 1.0$

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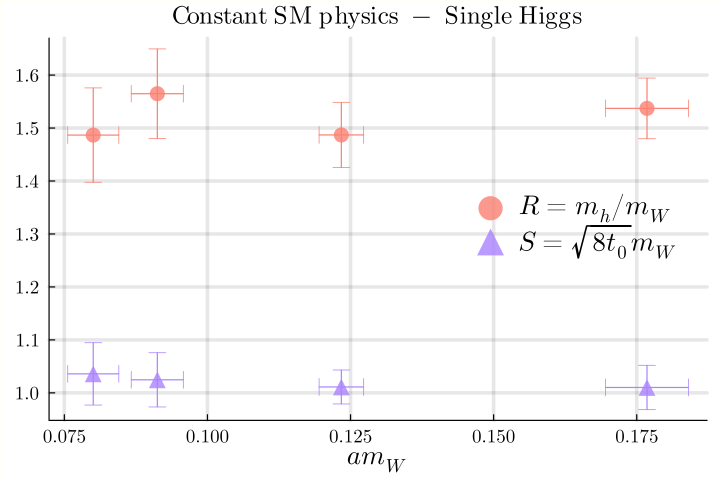
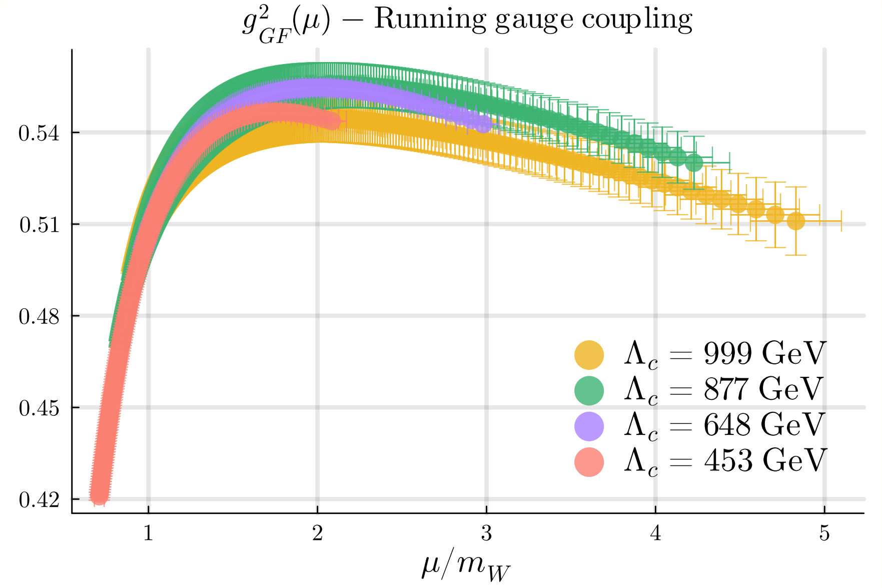
- $m_H/m_W \approx 1.5$
- $g^2_{GF}(\mu = m_W) = 0.5, \quad m_W/\mu = 1.0$



Running coupling $g^2_{GF}(\mu)$ – Single Higgs theory



Fields & Couplings – Lattice action

$$\Phi_n(x) = \frac{\hat{k}_n}{a}\hat{\Phi}_n(x), \qquad a^2\mu_{nn}^2 = \frac{1-2\hat{\eta}_n - 8\hat{k}_n}{\hat{k}_n}, \qquad \eta_n = \frac{\hat{\eta}_n}{\hat{k}_n^2}.$$

$$a^2\mu_{12}^2 = \hat{\mu}_{12}^2, \qquad \eta_n = \frac{\hat{\eta}_n}{\hat{k}_1\hat{k}_2}, \quad n = 3,4,5$$

$$\eta_6 = \frac{\hat{\eta}_6}{\hat{k}_1^{3/2}\hat{k}_2^2}, \qquad \eta_7 = \frac{\hat{\eta}_7}{\hat{k}_1^{1/2}\hat{k}_2^3}.$$

RGEs for the Inert Model

$$8\pi^2\frac{\mathrm{d}\lambda_1}{\mathrm{d}t} = (N+4)\lambda_1^2 + N\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \lambda_5^2 - \frac{3(N^2-1)}{N}\lambda_1 g^2 + \frac{3(N-1)(N^2+2N-2)}{4N^2}g^4$$

$$8\pi^2\frac{\mathrm{d}\lambda_2}{\mathrm{d}t} = (N+4)\lambda_2^2 + N\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \lambda_5^2 - \frac{3(N^2-1)}{N}\lambda_2 g^2 + \frac{3(N-1)(N^2+2N-2)}{4N^2}g^4$$

$$8\pi^2\frac{\mathrm{d}\lambda_3}{\mathrm{d}t} = [(N+1)\lambda_3 + \lambda_3](\lambda_1+\lambda_2) + 2\lambda_3^2 + \lambda_4^2 + \lambda_5^2 - \frac{3(N^2-1)}{N}\lambda_3 g^2 + \frac{3(N^2+2)}{4N^2}g^4$$

$$8\pi^2\frac{\mathrm{d}\lambda_4}{\mathrm{d}t} = \lambda_4(\lambda_1+\lambda_2) + 4\lambda_3\lambda_4 + N\lambda_4^2 + (N+2)\lambda_5^2 - \frac{3(N^2-1)}{N}\lambda_4 g^2 + \frac{3(N^2+2)}{4N^2}g^4$$

$$8\pi^2\frac{\mathrm{d}\lambda_5}{\mathrm{d}t} = \lambda_5[(\lambda_1+\lambda_2) + 4\lambda_3 + 2(N+1)\lambda_4 - \frac{3(N^2-1)}{N}\lambda_5 g^2$$

$O(4)$ condition:

$$8\pi^2\frac{\mathrm{d}(\eta_4-\eta_5)}{\mathrm{d}t} = 2(\eta_4^2 + 2\eta_5^2 - 3\eta_4\eta_5) + (\eta_4-\eta_5)\left[2\eta_1 + 2\eta_2 + 4\eta_3 - 9/2g^2\right].$$