Lattice versus perturbation theory: Testing the Abelian-Higgs model at three loops

Andreas Ekstedt

Uppsala University

Lattice 2024, Liverpool

2024.08.02

UPPSALA UNIVERSITET

The electroweak phase transition

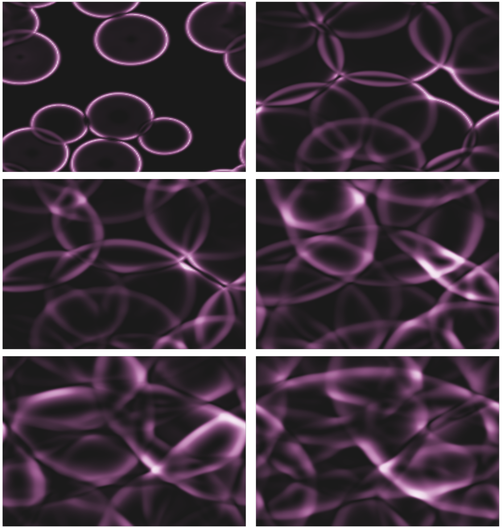
If the transition is first-order:

- Latent heat is released ← This talk
- Bubbles nucleate and expand
- Generation of gravitational waves

For this to work:

Need robust perturbative calculations

Lattice results are indispensable

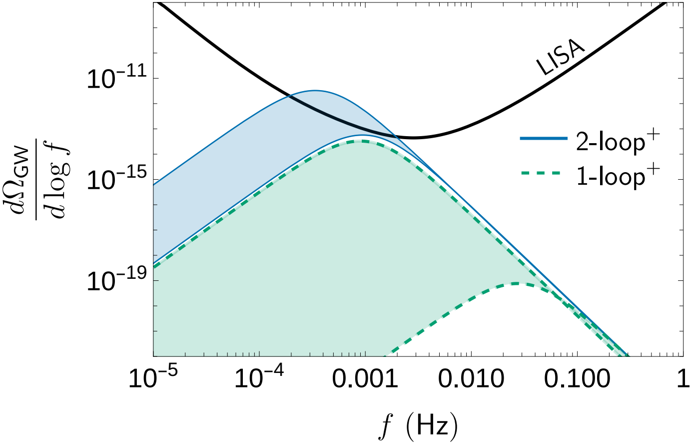


Lattice versus Perturbation theory

Lattice keeps perturbation theory honest

- Estimation of uncertainties
- Tests of various perturbative schemes
- Precision predictions for benchmark points

Alas, theory uncertainties



See 2104.04399

Green band—What most computations give
Blue band—Probably the best that we can do

An (**incomplete**) overview of previous lattice studies

Equilibrium physics:

- $SU(2)$+Higgs 2205.07238; 9605288; 9704013
- $U(1)$+Higgs 9703004; 9711048
- Real-scalar theories 0103227; 2101.05528
- 2HDM/SUSY 9804019; 1904.01329
- SM+Singlet 2405.01191
- SM+Triplet 2005.11332
- $\vdots$

Nucleation rates:

- $SU(2)$+Higgs 0009132; 2205.07238
- Real-scalar theories 2404.01876; 0103036; 2310.04206
- $\vdots$

This talk: the Abelian-Higgs model

Lagrangian

$$\mathscr{L} = -\frac{1}{4g^2}F_{ij}^2 + (D\Phi)(D\Phi)^\dagger + \left[m^2\Phi(x)\Phi^\dagger(x) + \lambda(\Phi(x)\Phi^\dagger(x))^2\right]$$

Simulation details:

- **High** temperatures $\rightarrow$ three-dimensional simulations
- λ, g^2 improved to $\mathscr{O}(a)$
- m^2 improved to $\mathscr{O}(a^0)$
- Typical lattice spacings $(ag^2)^{-1} \in [4, \ldots, 20]$
- For each λ value: 3 a values and $3-4$ different volumes for each a
- Multicanonical methods are used for all points

Everything will be expressed in terms of: $x \equiv \frac{\lambda}{g^2}, \quad y \equiv \frac{m^2}{g^4}$

Observables

Order parameter: $\langle \Phi\Phi^\dagger \rangle \to P(\langle \Phi\Phi^\dagger \rangle)$

Critical mass: $y_c : \left[\int_{\text{broken}} P(\langle \Phi\Phi^\dagger \rangle) - \int_{\text{sym}} P(\langle \Phi\Phi^\dagger \rangle) \right]_{y=y_c} = 0$

Quadratic condensate:

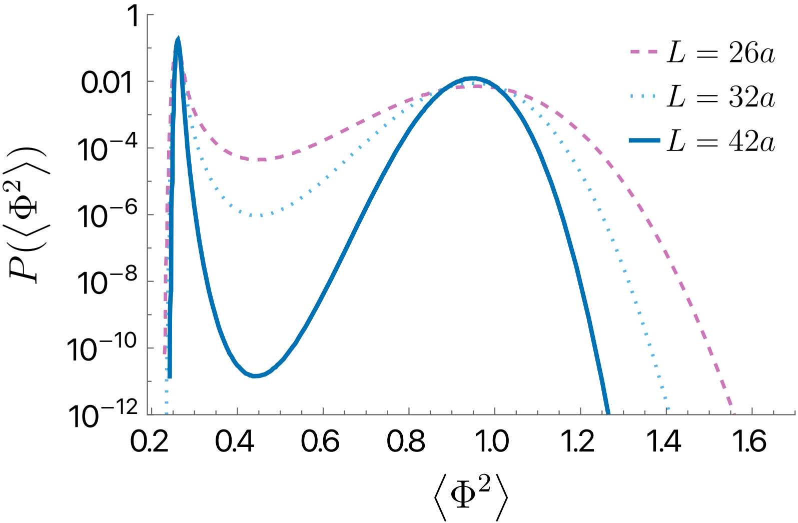
$\Delta \langle \Phi\Phi^\dagger \rangle = \mathbf{2} \left[\int_{\text{broken}} P(\langle \Phi\Phi^\dagger \rangle)\Phi\Phi^\dagger - \int_{\text{sym}} P(\langle \Phi\Phi^\dagger \rangle)\Phi\Phi^\dagger \right]_{y=y_c}$

Quartic condensate: $\Delta \langle (\Phi\Phi^\dagger)^2 \rangle$

The latent heat:

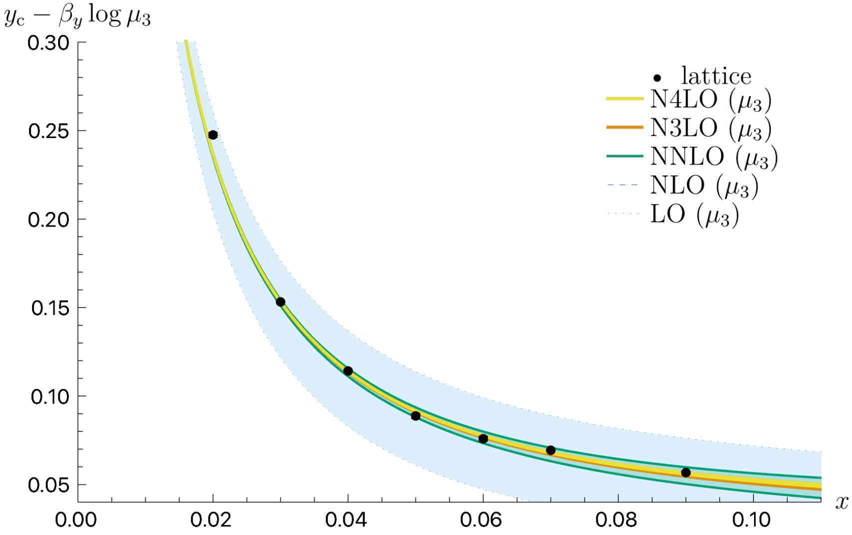
$$\Delta L = \frac{dy_c}{d\log T} \Delta \langle \Phi\Phi^\dagger \rangle + \frac{dx}{d\log T} \Delta \langle (\Phi\Phi^\dagger)^2 \rangle$$

Example at y_c; $x = 0.04$, $(ag^2) = 4^{-1}$



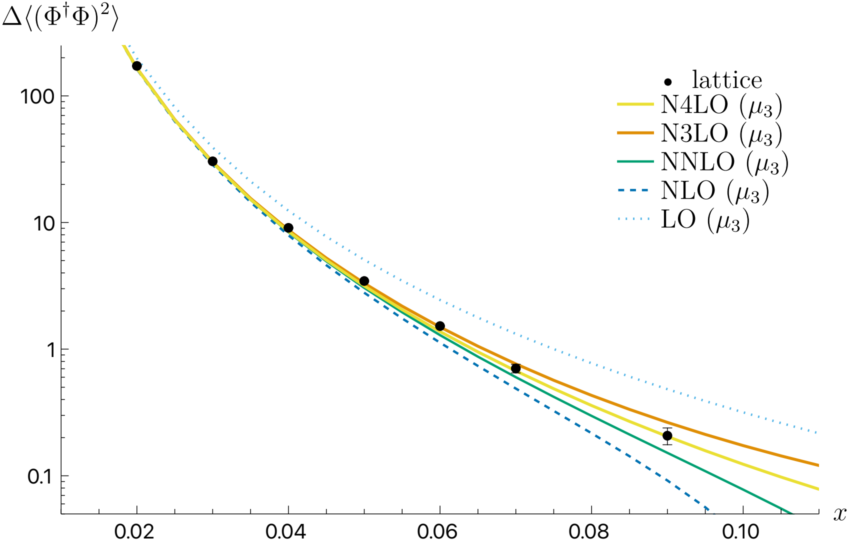
Results for the critical mass: μ_3 is the 3d RG scale

LO $\sim$ 1-Loop, NLO+NNLO $\sim$ 2-Loop, $N^3LO + N^4LO \sim$ **3-Loop**



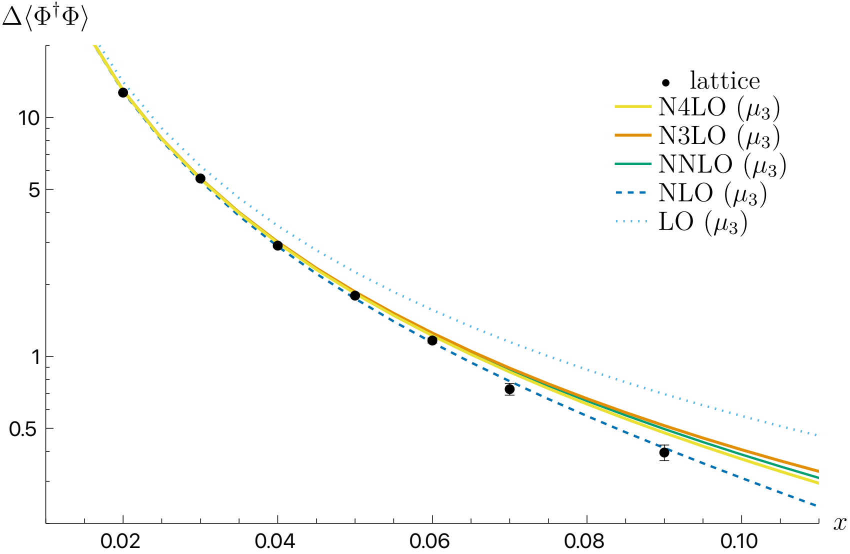
Results for the quartic condensate:

LO $\sim$ 1-Loop, NLO+NNLO $\sim$ 2-Loop, $\text{N}^3\text{LO} + \text{N}^4\text{LO} \sim$ **3-Loop**



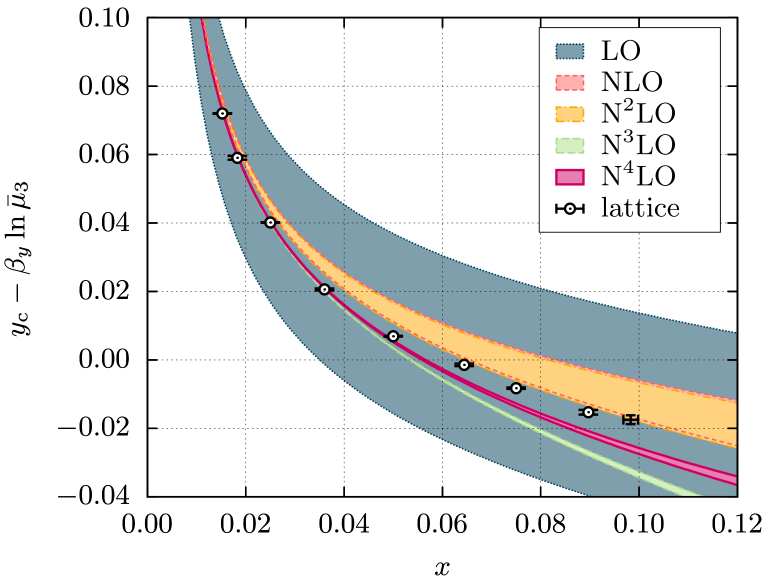
Results for the quadratic condensate:

LO ∼ 1-Loop, NLO+NNLO ∼ 2-Loop, $N^3LO + N^4LO$ ∼ **3-Loop**

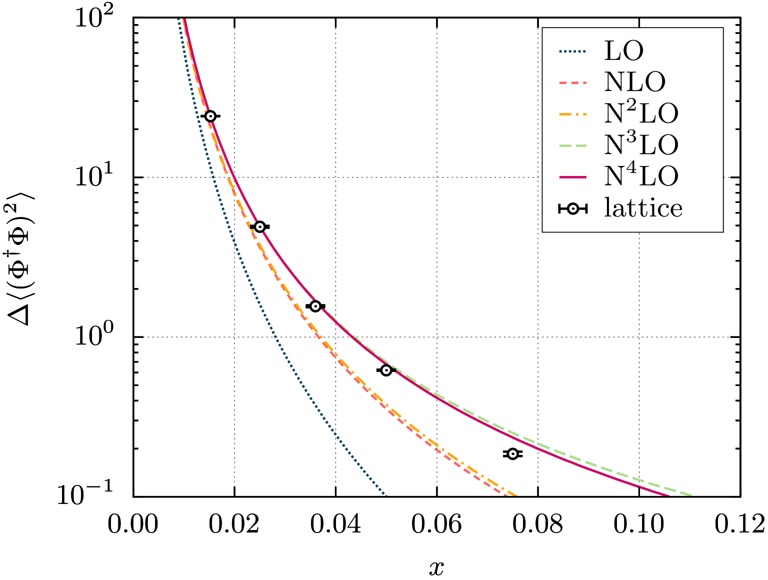


Same comparison for $\mathrm{SU}(2)$+Higgs: Lattice data from hep-lat:2205.07238

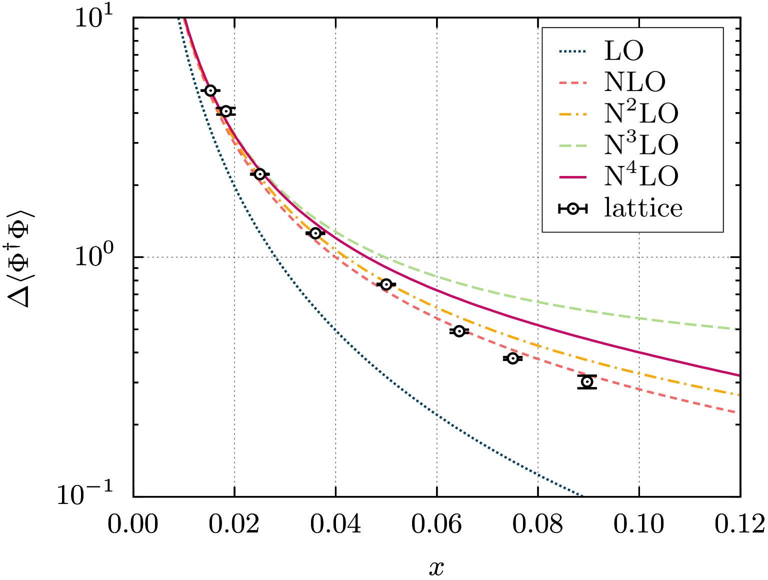
Results for the critical mass: ($x_c \sim 0.1$)



Results for the quartic condensate:



Results for the quadratic condensate:



In summary

- Lattice simulations are crucial for gravitational-wave predictions
- Perturbative calculations tend to be tricky
- Great agreement with lattice and 3-loop calculations

$\rightarrow$ The exception is the quadratic condensate $\Delta \langle \Phi\Phi^\dagger \rangle$

$\rightarrow$ Not clear if it's a problem with lattice or perturbative calculations

Future prospects:

- More lattice and higher-loop results on their way
- Many simulations of nucleation rates on the horizon

$\rightarrow$ Comparisons with perturbation theory are **indispensable**

Thanks for listening!