Dilaton Forbidden Dark Matter

arXiv:2404.07601

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August 2, 2024

Outline

Composite Dark Matter

Talk based on arXiv:2404.07601 with T. Appelquist and M. Piai.

I want to talk about a description of DM, in which the DM is a composite particle that forms in a new dark sector gauge theory.

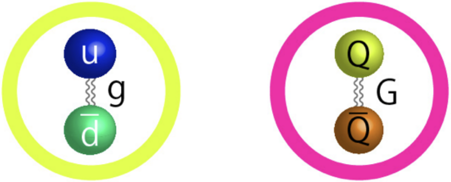

Figure: Dark pion (image: Kavli IPMU).

The dark sector gauge theory interacts *very* feebly with the standard model. Dark matter is a composite state, analogous to the pion of QCD.

The Program in a Nutshell

- Suppose the dark sector is a near–conformal gauge theory, and dark matter is its pion.
- Lattice studies indicate the low energy spectrum of these gauge theories have a light scalar. Unlike the pion, the light scalar carries no conserved charges and so can decay (slowly) to standard model.
- Nevertheless, the pions can annihilate readily into the scalars, so the freezeout of this process can set the relic density of pions.
- These low energy states may be described using dilaton EFT, which has successfully been used to fit lattice data.

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- Nevertheless, the pions can annihilate readily into the scalars, so the freezeout of this process can set the relic density of pions.
- These low energy states may be described using dilaton EFT, which has successfully been used to fit lattice data.
- In the following, specialise to $\mathrm{SU}(3)$ gauge theory with $N_f = 8$ fermions for concreteness.

The SU(3) Gauge Theory with $N_f = 8$ Fundamental Fermions

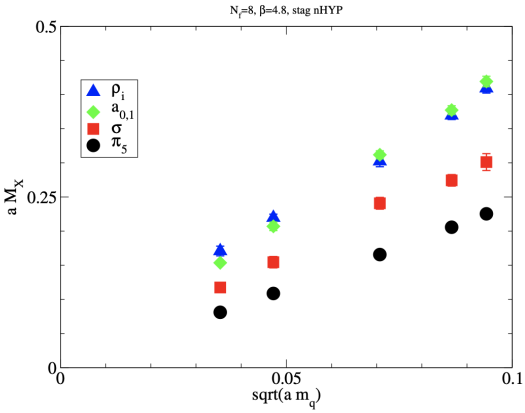


Figure: Lattice data for the spectrum from the LSD collaboration: 2306.06095

Dilaton EFT

Reviewed in Universe 9 (2023) 1, 10 with T. Appelquist and M. Piai.

Leading order Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \chi)^2 + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] + \frac{mB_\pi f_\pi^2}{2}\left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right] - V_\Delta(\chi) \,. \quad (1)$$

- pNGB terms are similar to those in chiral Lagrangian.
- Dependence on dilaton field χ is determined by scale invariance.
- See dilaton EFT of Golterman & Shamir: PRD **94** (2016).

Dilaton Potential

The dilaton field χ experiences a net potential of

$$W(\chi) \equiv V_\Delta(\chi) - \frac{M_\pi^2 F_\pi^2 N_f}{2} \left(\frac{\chi}{F_d}\right)^y . \quad (2)$$

Fits to lattice data and theoretical arguments indicate that $y \sim 2$
LSD collab: PRD **108** (2023) 9, 9, R. Zwicky: PRD **109** (2024) 3, 034009.

Expand potential around its minimum $\chi = F_d + \bar{\chi}$:

$$W(\bar{\chi}) = \text{constant} + \frac{M_d^2}{2}\bar{\chi}^2 + \frac{\gamma}{3!}\frac{M_d^2}{F_d}\bar{\chi}^3 + \dots , \quad (3)$$

where $\gamma \geq 2$ (from unitarity bound) GGS PRL.**100** 111802, (2008) and γ cannot be too large for EFT to remain weakly coupled.

Freezeout

We represent the small splitting between dilaton and pion masses with

$$\delta \equiv \frac{M_d - M_\pi}{M_\pi} \,. \tag{4}$$

We take $0 < \delta < 1/2$, as seen in lattice data.

Also make $\Gamma_{\chi \to \mathrm{SM}}$ large enough to maintain $n_\chi = n_\chi^{\mathrm{eq}}$ (more on SM couplings in backup).
The relic density is then set by $\pi\pi \to \chi\chi$ annihilations freezing out:

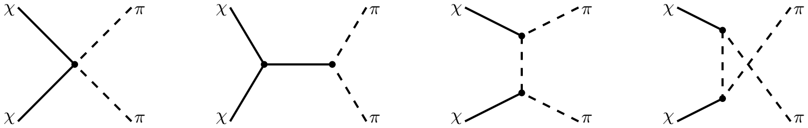
Boltzmann Equation

$$\frac{\partial n_\pi}{\partial t} + 3Hn_\pi = -\left\langle \sigma_{2\pi \to 2\chi} v \right\rangle n_\pi^2 + \left\langle \sigma_{2\chi \to 2\pi} v \right\rangle \left(n_\chi^{\mathrm{eq}}\right)^2 \,. \tag{5}$$

We solve numerically to get the relic density of pions today.

Thermally Averaged Cross Sections

The inverse annihilation process $\chi\chi \to \pi\pi$ can happen for zero kinetic energy in the initial state, because $\delta > 0$. We compute its cross section using dilaton EFT:



For $T \ll M_\pi$, the thermal averaged x-section $\approx$ x-section at $\vec{p} = 0$:

$$\langle \sigma_{2\chi \to 2\pi} v \rangle = \frac{M_\pi^2 N_\pi}{36\pi F_d^4} \sqrt{\delta(2+\delta)}(1+\delta)(5+\gamma)^2 \,. \qquad (6)$$

Forbidden Dark Matter

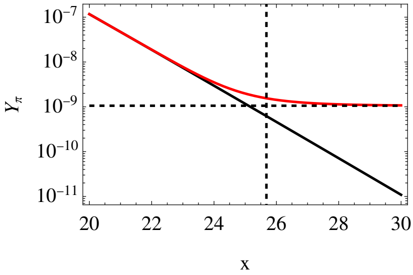
However, the calculation of the thermal average $\langle \sigma_{2\pi \to 2\chi} v \rangle$ is less straightforward, as this reaction is kinematically forbidden when pions have zero momentum (or at $T = 0$).

In this case, taking the thermal average leads to an exponential suppression of the cross section. For $x = M_\pi / T$, we have

$$\langle \sigma_{2\pi \to 2\chi} v \rangle = \frac{(1+\delta)^3}{N_\pi^2} e^{-2\delta x} \langle \sigma_{2\chi \to 2\pi} v \rangle \, . \tag{7}$$

The dark matter relic abundance is set through annihilations to heavier states that are kinematically forbidden at $T = 0$. This framework is an example of forbidden DM Griest & Seckel: PRD **43**, 3191 (1991), D'Agnolo & Ruderman: PRL **115**, 061301 (2015)

Solving the Boltzmann Equation



We plot solution taking the scale as $M_\pi = 1$ GeV, with parameters $M_\pi/F_\pi = 4$, $F_\pi^2/F_d^2 = 0.1$, $\delta = 0.3$, $\gamma = 3$ and $y = 2$.

Plot using convenient variables:

$$Y_\pi = n_\pi/s$$
$$x = M_\pi/T$$

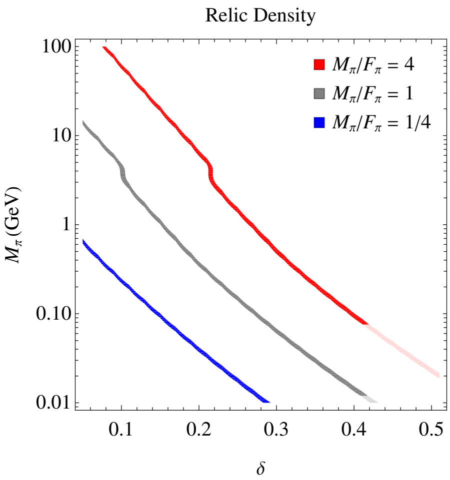
High temp boundary condition $Y_\pi(T_i) = n_\pi^{\rm eq}(T_i)/s(T_i)$.

Before freezeout, $Y_\pi \approx n_\pi^{\rm eq}(T)/s(T)$.

After freezeout Y_π roughly constant.

$$\Omega_{\rm CDM} h^2 = \frac{M_\pi s_0 Y_\pi(\infty)}{\rho_c/h^2},$$

Parameter Space



- Bands indicate parameter space for which $\Omega_{\text{CDM}}h^2$ is within 10% of its observed value.
- Range of DM masses allowed. Lighter than typical WIMPs, due to forbidden mechanism.
- Pale shaded regions excluded due to upper bounds on $\frac{\sigma}{M_\pi}(\pi\pi \to \pi\pi)$ e.g. from bullet cluster...

Freezeout Temperature

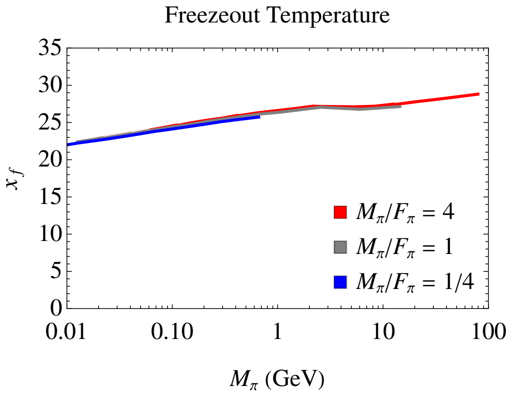


Figure: Plot of $x_f = M_\pi / T_f$ as a function of the dark-matter mass. The mass splitting δ has been adjusted to ensure that the dark matter relic density is equal to its observed value.

Coupling to Visible Sector

At the level of dilaton EFT, the necessary couplings take the form

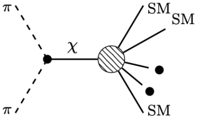
$$\mathcal{L}_{\text{int}} = \epsilon F_d^{4-d_{\text{SM}}} \left(\frac{\bar{\chi}}{F_d} \right) \mathcal{O}_{\text{SM}} \,, \tag{8}$$

where ϵ are weak, dimensionless constants, and $\mathcal{O}_{\text{SM}}$ are singlet, scalar operators involving light SM fields (e.g $\mathcal{O} = F_{\mu\nu} F^{\mu\nu}$).

1. The dilaton couplings are not constrained by the form of the SM energy momentum tensor (as dilaton is a composite of dark sector, and not SM dofs).
2. Bounds exist for specific subsets of these couplings from astrophysics, CMB, collider experiments. We leave this for future work.
3. We can however derive a more model independent constraint...

Consistency Condition

1. The inclusive decay rate $\Gamma_{\chi\to\mathrm{SM}}$ must be large enough to bring the dark sector and SM into thermal equilibrium long before freezeout.
2. The decay rate must also be small enough so that direct annihilations $\pi\pi \to$ SM do not overwhelm forbidden annihilations to dilatons.



Two-Sided Bound on the Inclusive Decay Rate

$$H_{T=M_\pi} \lesssim \Gamma_{\chi\to\mathrm{SM}} \lesssim H_{T=T_f} \frac{M_\pi N_\pi F_d^2}{n_\pi^{\mathrm{eq}}(T_f)}, \tag{9}$$

Summary and Outlook

- We have proposed a description of composite DM.
- The DM is a pion of a nearly conformal gauge theory, and the dilaton plays the role of a mediator with the standard model.
- We have used a dilaton EFT to describe these states and their scattering cross sections.
- Our framework naturally implements the forbidden dark matter mechanism. The DM is a thermal relic with abundance set by forbidden $\pi\pi \to \chi\chi$ annihilations. The framework accommodates a wide range of DM masses: $M_\pi \sim 10$ MeV $-$ 100 GeV.

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- ☞ Lattice studies can reveal qualitative features of new dark sectors.

Thank you!

Lattice Action

- Our numerical calculations use improved nHYP smeared staggered fermions with smearing parameters $\alpha = (0.5, 0.5, 0.4)$. [LSD PRD 99(2019)014509]
- $\beta_A/\beta_F = -0.25$ where $\beta_F = 4.8$.
- After taste splitting, only $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ flavor symmetry preserved in massless theory (3 exact NGBs).
- Spectral study has revealed that the taste splitting of the 63-plet masses are on the order of 20–30%. [LSD PRD 99(2019)014509]

Summary of Improvements to Lattice Dataset

Presented in 2306.06095

Since the previous LSD study of the $N_f = 8$ theory PRD **99** (2019) 014509, we have made some changes.

1. We have data for a new observable: The scalar decay constant F_S.
2. We have extrapolated the quantities M_π, F_π, M_σ (and also F_S) to the infinite volume limit.
3. We have improved our estimates of systematic uncertainties using Bayesian Model Averaging Jay, Neil PRD **103** (2021) 114502

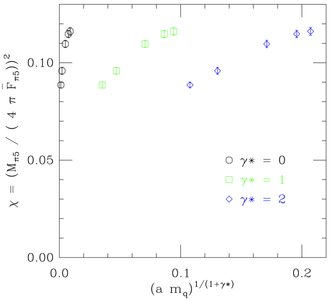
The $N_f = 8$ spectrum has also been calculated before in LatKMI PRD **96** (2017) 014508

Result Of Global Fit to dEFT

Parameter	Value and Uncertainty
y	2.091(32)
aB_π	2.45(13)
Δ	3.06(41)
$a^2 f_\pi^2$	$6.1(3.2) \times 10^{-5}$
f_π^2 / f_d^2	0.1023(35)
m_d^2 / f_d^2	1.94(65)
χ^2/dof	21.3/19=1.12

Table: Central values of fit parameters obtained in a six parameter global fit to LSD data for $M_{\pi,d}^2$, $F_{\pi,S}^2$ and scattering length.

Corrections to Scaling



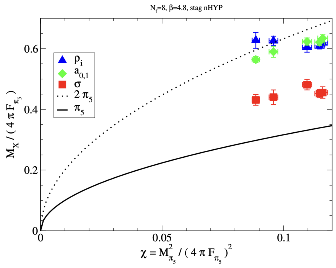


Figure: Lattice data from 2306.06095, indicating corrections to scaling. In a mass-deformed CFT (at the fixed point), dimensionless ratios of quantities should be independent of quark mass.

NLO Corrections in dEFT

We do not have complete NLO calculations for all our observables in dEFT.

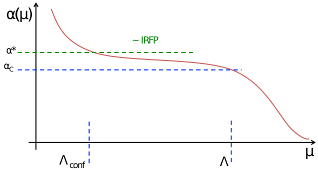
Some of these corrections will likely come suppressed by $M_\pi^2/(4\pi F_\pi)^2$.

Lets take a phenomenological approach and add a contribution to the observable that shows the largest tension in the fit:

$$M_\pi a_0^{(2)} = \frac{-M_\pi^2}{16\pi F_\pi^2}\left(1 - (y-2)^2 \frac{f_\pi^2}{f_d^2}\frac{M_\pi^2}{M_d^2} + \frac{l_a M_\pi^2}{(4\pi F_\pi)^2}\right), \tag{10}$$

We neglect potential chiral logs.

Interpretation of Δ



1. Strongly coupled over large interval of scales $\implies$ possibility of large anomalous dimensions. Note our lattice fits showed $y \approx 2$.

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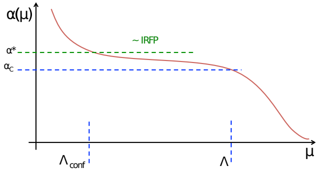

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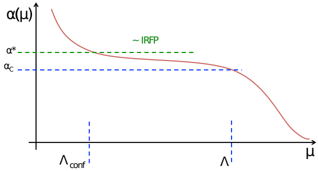

1. Strongly coupled over large interval of scales $\implies$ possibility of large anomalous dimensions. Note our lattice fits showed $y \approx 2$.
2. Allows for new relevant interactions besides (near marginal) gauge interaction.
3. Δ should be identified with the engineering plus anomalous dimension of this new relevant operator.
4. We are agnostic about the value of Δ. See theoretical arguments for $\Delta = 2$ [Zwicky] and $\Delta \to 4$ [Golterman and Shamir].