UNIVERSITY OF PLYMOUTH

The mass of the σ in a chiral ensemble in $SU(2)$ with two fundamental flavours

Laurence Sebastian Bowes [1] Vincent Drach Patrick Fritzsch
Sofie Martins Antonio Rago Fernando Romero-Lopez

[1]University of Plymouth, England

laurence.bowes@plymouth.ac.uk

August 2, 2024

Composite Higgs Models

- In order to address the naturalness and hierarchy problems, introduce a new sector into the SM, giving a dynamical origin to the electroweak spontanous symmetry breaking.
- The Higgs emerges as either a pseudo-Nambu-Goldstone boson or as a light scalar resonance. These are not mutually exclusive, and the amount of mixing between the two scenarios is controlled by the vacuum misalignment angle θ.
- Scattering processes involving a potential new strong sector are expected to be testable at the LHC.

> "...ICFA reconfirms the international consensus on the importance of a Higgs Factory as the highest priority for realizing the scientific goals of particle physics ..."
>
> — International Committee on Future Accelerators, 2022

$SU(2)$ with 2 Fundamental Flavours

Continuum Theory

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \overline{u}(i\gamma^\mu D_\mu - m)u + \overline{d}(i\gamma^\mu D_\mu - m)d$$

- The minimal model for the composite Higgs sector
- Pseudoreal fundamental representation gives rise to the flavour symmetry breaking structure $SU(4)_f \to Sp(4)_f$
- Can build testable composite Higgs models which are not excluded by experiment. (eg [1402.0233])

Research Aims

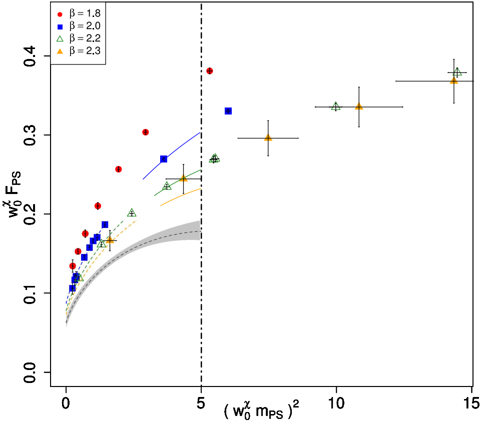
Long Term Goal

Understand how the properties of resonances in the composite Higgs scenario would change the observable Higgs boson phenomenology at the LHC.

- Specifically here the flavour singlet scalar resonance in the new strongly interacting sector ($\mathcal{O}_\sigma = \overline{u}u + \overline{d}d$), which we call the σ in analogy with QCD. In the composite Higgs scenario, the σ is predicted to be produced at the LHC similarly to the SM Higgs.
- Understanding the role of the σ in the composite Higgs sector in isolation will provide insight into the many low-energy constants of the effective theory at the EW scale, and provide valuable constraints on parameter space.

Previous Work

- Previously studied on the lattice with unimproved Wilson fermions, but exhibited significant order-a effects, prompting a move to a tuned order-a improved action in order to go more chiral. [1602.06559]
- The scattering amplitude of the σ was studied on the lattice with tree-level Wilson clover fermions and a tree-level Symanzik improved gauge action. The σ was shown to be stable up to $\frac{m_v}{m_{ps}} < 2.5$. [2107.09974]



$\Longrightarrow$ See Sofie's talk next for progress towards the continuum limit!

Lattice Setup

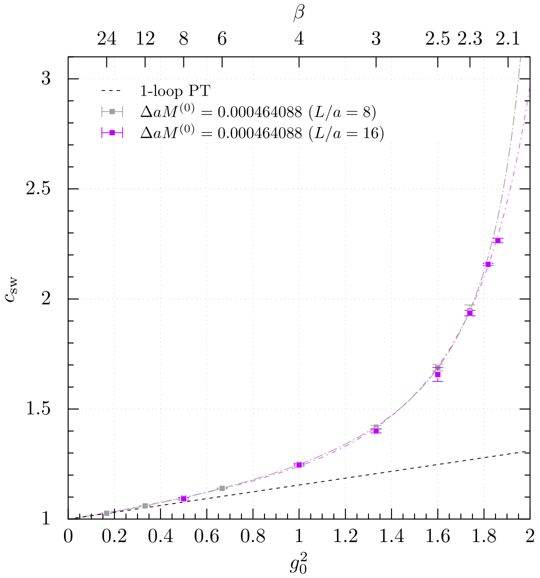
- Plaquette Gauge Action
- Exponential Clover Wilson Fermions
  Francis, Fritzsch, Lüscher, Rago [1911.04533]

$$M_0 + c_{SW}\frac{i}{4}\sigma_{\mu\nu}\hat{F}_{\mu\nu} \;\rightarrow\; M_0 \exp\left[\frac{c_{SW}}{M_0}\frac{i}{4}\sigma_{\mu\nu}\hat{F}_{\mu\nu}\right].$$

  - Enforces the diagonal part of the Wilson-Dirac operator to be positive and gapped above zero, enhancing numerical stability.
  - $O(a)$ improvement once c_{SW} is tuned non-perturbatively.
- Set the scale for the ensembles using the Wilson gauge flow.
- Simulations performed using HiRep [github.com/claudiopica/HiRep]

Tuning of c_{SW}

- c_{SW} is now tuned for all $\beta \geq 2.15$.
- **Non-perturbative** tuning of c_{SW} via Schrodinger functional simulations
- A vast amount of our time spent tuning (hundreds of thousands of trajectories)
- Find a value for κ_{crit} by tuning $M = M_0$, and then find c_{SW} at κ_{crit} by tuning $\Delta M = \Delta M_0$.



Strategy

- Our goal is to identify a setup where the σ is likely to decay into two pions and perform a Luscher scattering analysis.
- We chose the coarsest lattice spacing that we had a tuned value of c_{sw} for, which at the time was $\beta = 2.2$.
- We then ran as chiral as we could at the volume 64×32^3, where we use $\frac{m_{\mathrm{V}}}{m_{\mathrm{PS}}}$ as a measure for how chiral we are.
- As a preliminary step, we then calculate at the effective mass of the σ from a simple two-point correlation function.

Measurement of the σ state

- Flavour singlet state of positive parity

$$\mathcal{O}_\sigma(x) = \overline{\psi}_{\alpha ic}(x)\psi_{\alpha ic}(x) = \overline{u}_{\alpha c}(x)u_{\alpha c}(x) + \overline{d}_{\alpha c}(x)d_{\alpha c}(x)$$

- Correlator has a disconnected term which must be evaluated

$$\left\langle \mathcal{O}_\sigma(x)\overline{\mathcal{O}}_\sigma(0)\right\rangle_F = 4\mathrm{Tr}\left[S(x,x)\right]\mathrm{Tr}\left[S(0,0)\right] - 2\mathrm{Tr}\left[S(0,x)S(x,0)\right]$$

- Use even-odd SEMWall sources for the connected term, and pure volume sources for the disconnected term.
- $Z(2) \otimes Z(2)$ noise, $\xi \in \frac{1}{\sqrt{2}}\{1+i, 1-i, -1+i, -1-i\}$
- σ has a VEV which has to be subtracted in the analysis

$$\lim_{T\to\infty}\langle O_\sigma(t)\overline{O}_\sigma(0)\rangle_T = \sum_n \langle 0|\hat{O}_\sigma|n\rangle\langle n|\hat{O}_\sigma|0\rangle e^{-t\Delta E_n}.$$

Connected term: EO SEMWall sources

Introduce a set of N complex fields $\Xi^{\tau\sigma k}_{\alpha c}(x)$ with support on only the even sites, on a single timeslice τ and on a single spin index σ.

$$\left[\xi^{k*}_c(\vec{x})\xi^k_d(\vec{y})\right] = \delta_{cd}\delta_{\vec{x}\vec{y}}$$

Let $\varphi^{\tau\sigma k}_{\alpha c}(x) := S_{\alpha\beta cd}(x,y)\Xi^{\tau\sigma k}_{\beta d}(y)$

$$(\gamma_5)_{\rho\sigma}(\gamma_5)_{\alpha\beta}[\varphi^{*\tau\sigma k}_{\alpha c}(x)\varphi^{\tau\rho k}_{\beta c}(x)] = \frac{1}{2}\sum_{\vec{y}} \mathrm{Tr}[S((\tau,\vec{y}),x)S(x,(\tau,\vec{y}))]$$

where the γ_5 appears because we use γ_5-hermiticity.

Disconnected term: volume sources

Introduce a set of N fields $\xi^k_{\alpha c}(x)$, $k = 1...N$ such that for any $x, y, \alpha, \beta, c, d$

$$\left[\xi^{k*}_{\alpha c}(x)\xi^k_{\beta d}(y)\right] = \delta_{\alpha\beta}\delta_{cd}\delta_{xy}$$

and let $\varphi^k_{\alpha c}(x) := S_{\alpha\beta cd}(x, y)\xi^k_{\beta d}(y)$. Then

$$[\xi^{*k}_{\alpha c}(x)\varphi^k_{\alpha c}(x)] = \mathrm{Tr}[S(x, x)].$$

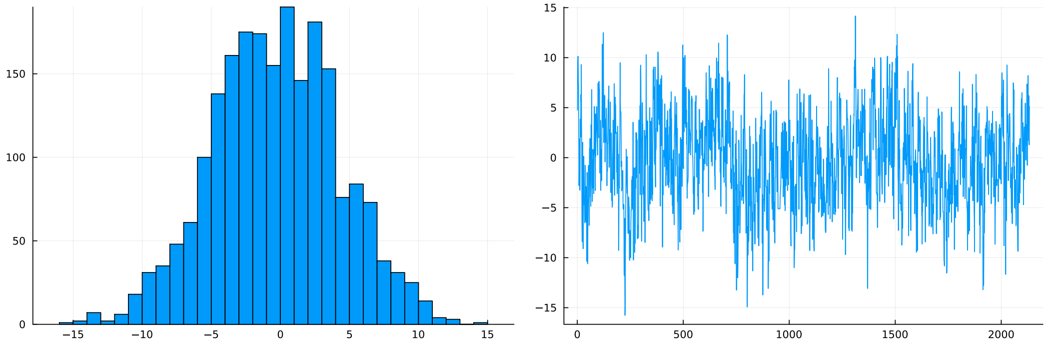
Use two copies of this (with independent stochastic sources) then sum over time separations and subtract the gauge-averaged VEV to obtain the correlator.

Ensemble

Geometry	β	m_{PS}	m_{V}	$\frac{m_{\mathrm{V}}}{m_{\mathrm{PS}}}$	N_{confs}	w_0
64×32^3	2.2	0.120(2)	0.29(2)	2.46(8)	3255	4.50(3)

- To speed up the simulations, we tuned two levels of Hasenbusch splitting
- used 4 SEMWall hits per conf for the connected term and 56 (28 for each trace) volume hits per conf for the disconnected term.
- $m_{\mathrm{PS}}L = 3.84$, smearing radius $= 0.39L$

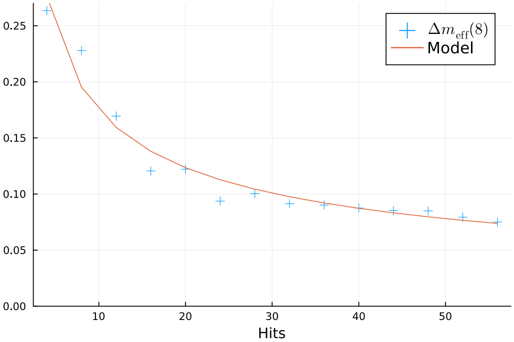
Topological Charge



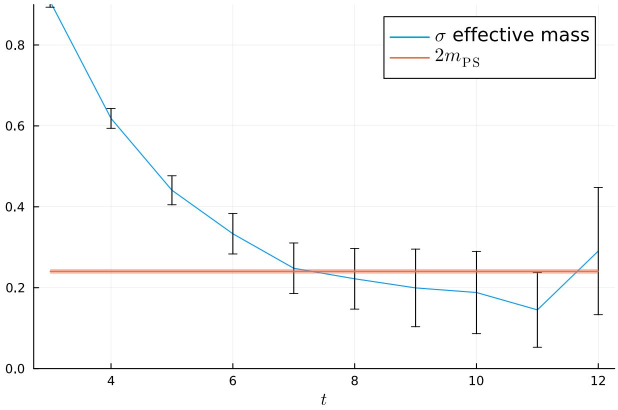
The topological charge is fluctuating and not frozen, although the autocorrelation time looks quite long

Results

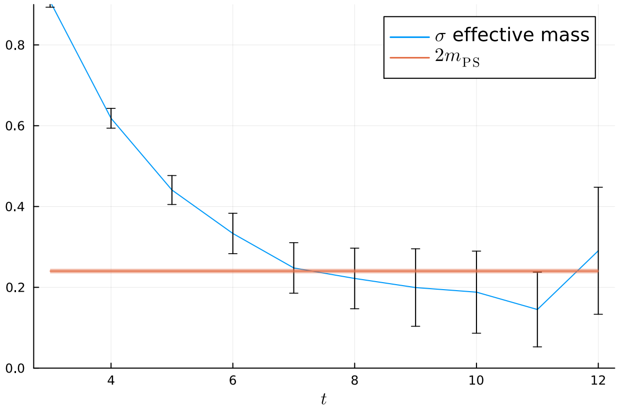
- Want to understand whether the error in the effective mass is limited by the gauge or the stochastic noise.
- Fit the simple model $\mathrm{Error}(\mathrm{hits}) = A + \frac{B}{\sqrt{\mathrm{hits}}}$
- $A = 0 \Rightarrow$ Could probably do with more hits!



Results

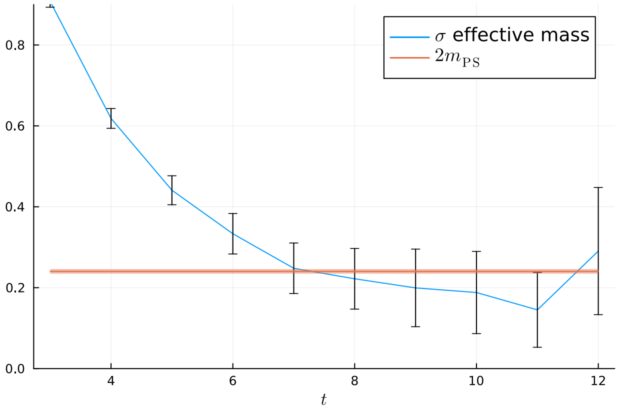


Results



🤗 The effective mass of the σ tends to the mass of two pions as expected.

Results



🤗 The effective mass of the σ tends to the mass of two pions as expected.

😬 Can't distinguish the σ from the 2 pion state, and we know that the ensemble isn't chiral enough.

Conclusion

- In the first instance we reached the chirality of the previous setup.
- Now that we have the setup, we can go more chiral than previously into the unexplored region where we expect the σ to decay.
- Can go to larger lattices, GPUs, $\beta = 2.15$.
- Stay tuned! Thanks for listening!

Backup: Connected term: EO SEMWall sources

Introduce a set of N fields $\Xi^{\tau\sigma k}_{\alpha c}(x) = \delta_{x \ mod \ 2}\delta_{t\tau}\delta_{\alpha\sigma}\xi^{k}_{c}(\vec{x})$, $k = 1 \dots N$ such that for any $\vec{x}, \vec{y}, c, d$

$$\left[\xi^{k*}_{c}(\vec{x})\xi^{k}_{d}(\vec{y})\right] = \delta_{cd}\delta_{\vec{x}\vec{y}}$$

where

$$[\cdot] := \lim_{N\to\infty} \frac{1}{N} \sum_{k} \cdot$$

Let $\varphi^{\tau\sigma k}_{\alpha c}(x) := S_{\alpha\beta cd}(x, y)\Xi^{\tau\sigma k}_{\beta d}(y)$

$$(\gamma_5)_{\rho\sigma}(\gamma_5)_{\alpha\beta}[\varphi^{*\tau\sigma k}_{\alpha c}(x)\varphi^{\tau\rho k}_{\beta c}(x)] = \frac{1}{2}\sum_{\vec{y}} \mathrm{Tr}[S((\tau, \vec{y}), x)S(x, (\tau, \vec{y}))]$$

where the γ_5 appears because we use γ_5-hermiticity.

Backup: Subtracting the VEV

$$\Gamma_\sigma(t,0)_{\text{disc}} = 2\left(\frac{1}{N}\sum_{k=1}^{N}\sum_{\vec{x},\alpha,c}\xi^{*k}_{\alpha c}(t,\vec{x})\phi^{k}_{\alpha c}(t,\vec{x})\right)\left(\frac{1}{N}\sum_{l=N+1}^{2N}\sum_{\vec{y},\beta,d}\xi^{*l}_{\beta d}(0,\vec{y})\phi^{l}_{\beta d}(0,\vec{y})\right)$$

Define

$$\Gamma_\sigma(\Delta t)_{\text{disc}} = \frac{1}{T}\sum_{t_1-t_2=\Delta t}\Gamma_\sigma(t_1,t_2)$$

and

$$\overline{\Gamma}_\sigma(t)_{\text{disc}} = \Gamma_\sigma(\Delta t)_{\text{disc}} - \left\langle \frac{1}{2NT}\sum_{k=1}^{2N}\sum_{t=1}^{T-1}\sum_{\vec{x},\alpha,c}\xi^{*k}_{\alpha c}(t,\vec{x})\phi^{k}_{\alpha c}(t,\vec{x})\right\rangle^2 .$$

Then combine it with the connected part (and check the normalisation).