

# Determination of the pseudoscalar decay constant from $SU(2)$ with two fundamental flavors

Laurence Bowes, Vincent Drach, Patrick Fritzsche, **Sofie Martins\***,  
Antonio Rago, Fernando Romero-López

\*University of Southern Denmark, [martinss@imada.sdu.dk](mailto:martinss@imada.sdu.dk)

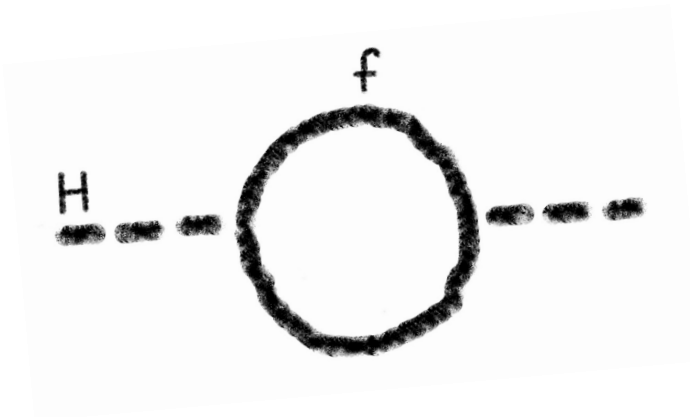
# Structure

1. SU(2)/fundamental: Motivation from phenomenology  
Criteria on a composite Higgs theory
2. Lattice Setup  
Software, action, non-perturbative improvement
3. Analysis  
Evaluation of  $f_{PS}$ , renormalization, continuum extrapolation
4. Results  
Summary & Outlook

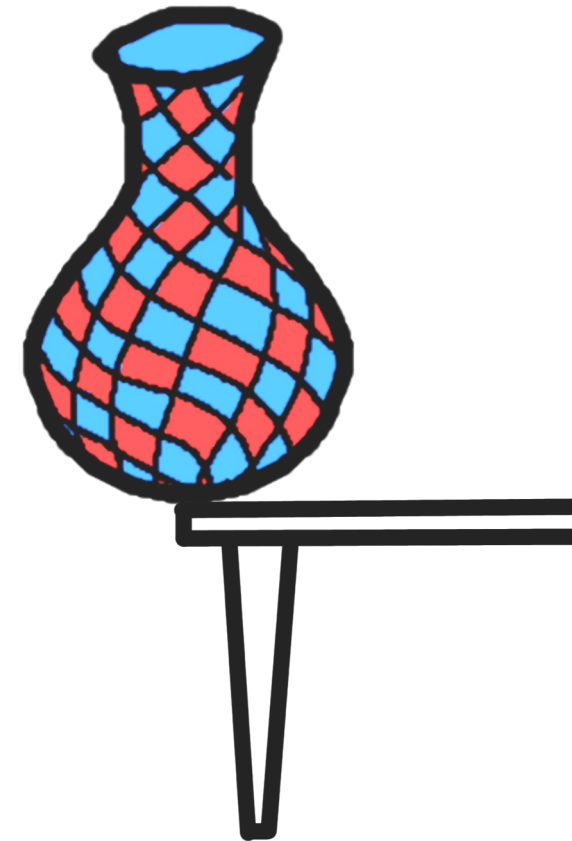
# Motivation from phenomenology

# Naturalness

Radiative corrections to the Higgs-mass



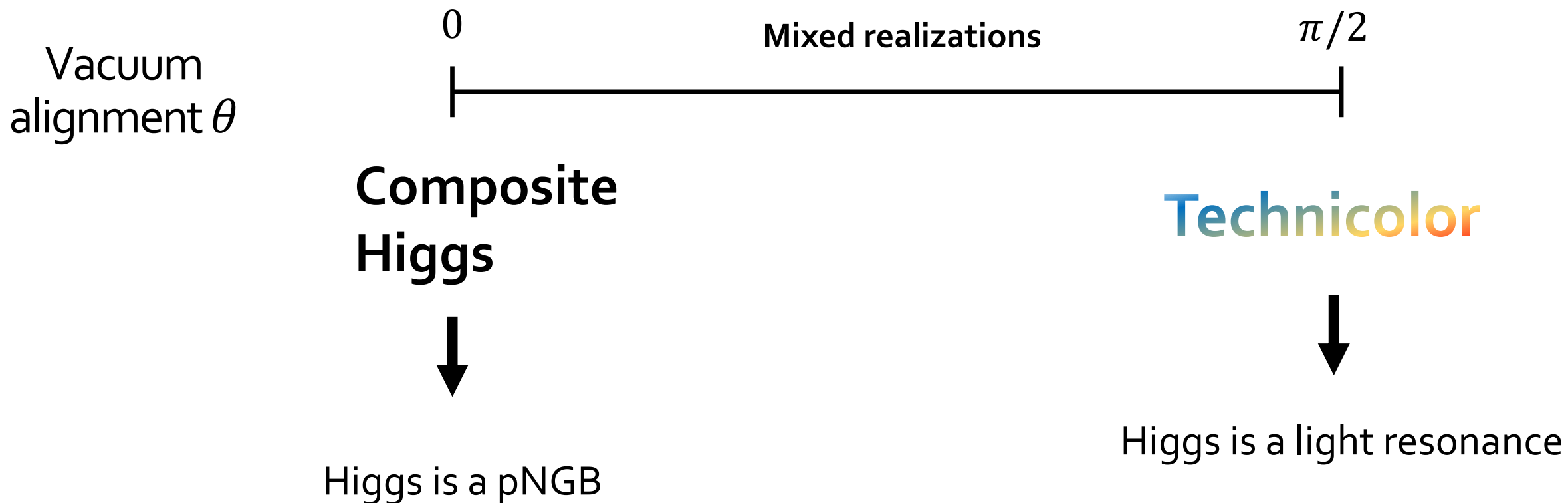
$$\Delta m_H^2 = \frac{|\lambda_f|^2}{8\pi} \Lambda_{UV}^2$$



Is the Higgs really **elementary**?

See for example [S. P. Martin, 1998, hep-ph/9709356, Adv. Ser. Direct. High Energy Phys.] and references therein

# Important inputs for experimentalists



The vacuum alignment is important to determine **oblique parameters**

See for example [Cacciapaglia, Pica, Sannino, 2020, 2002.04914, Phys. Rept.] and references therein

# SU(2) with two fundamental flavors



Asymptotic freedom



Singlet state consistent with the Higgs



Preservation of custodial symmetry

$$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$\rho = \left( \frac{M_W}{M_Z \cos(\theta_W)} \right)^2 + \dots$$

Protected by symmetry  $SU(2)_L$ . But

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$$

What protects the  $\rho$ ? A **custodial**  $SU(2)_R$ !

# Lattice Setup

Exp. Clover: [A. Francis et. al., 2020, 1911.04533, Comput. Phys. Commun.]

HiRep: [Del Debbio, Patella, Pica, 2010, 0805.2058, Phys. Rev. D]

GPUs: [Martins et. al., 2024, 2405.19294, EuroPLEx2023]



Non-perturbative  
improvement:  
Exponential clover

Improved stability  
SU(3) suggests  
smaller  $O(a^2)$  -effects



SU(2) gauge group with 2 mass  
degenerate Wilson fermions



HiRep on CPUs and GPUs

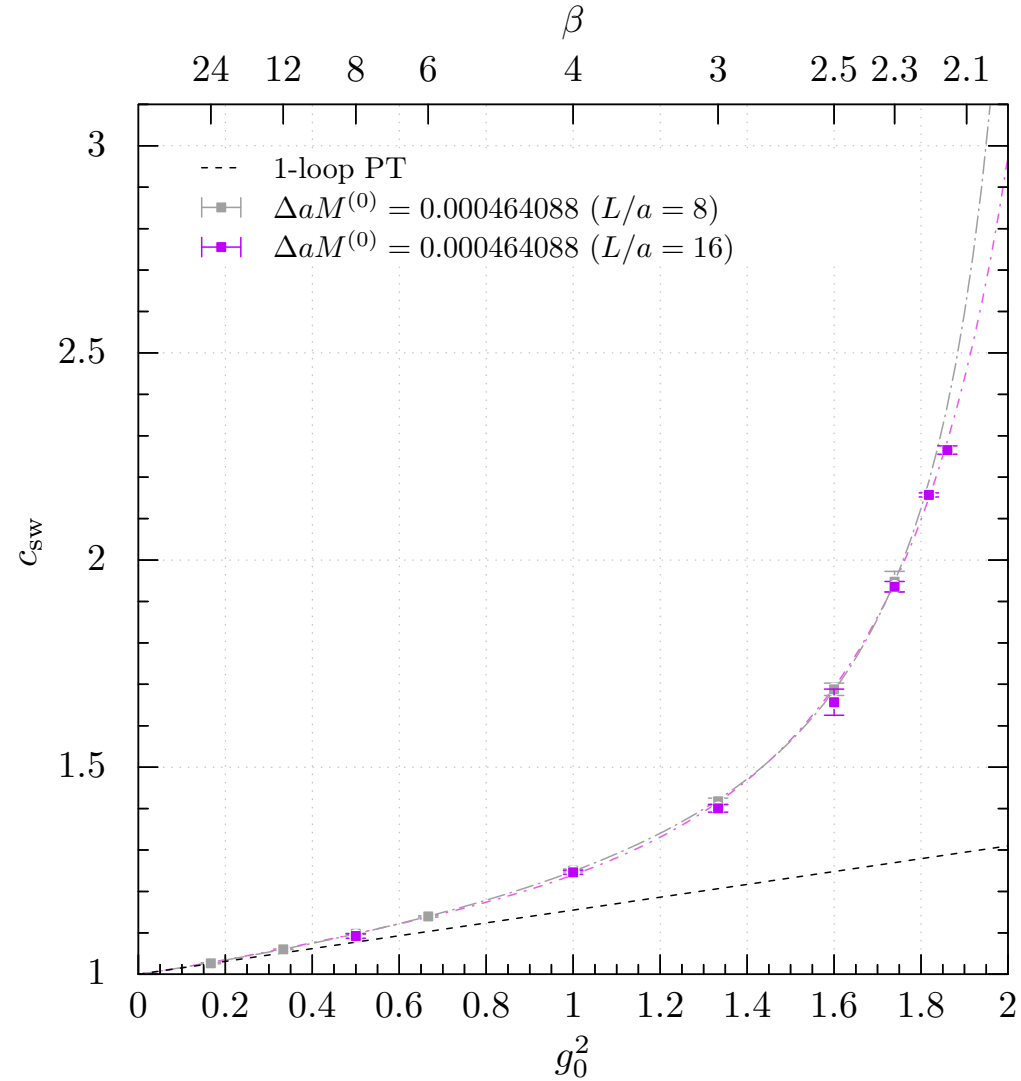
# Lattice Simulation Setup



# Non-perturbative Improvement

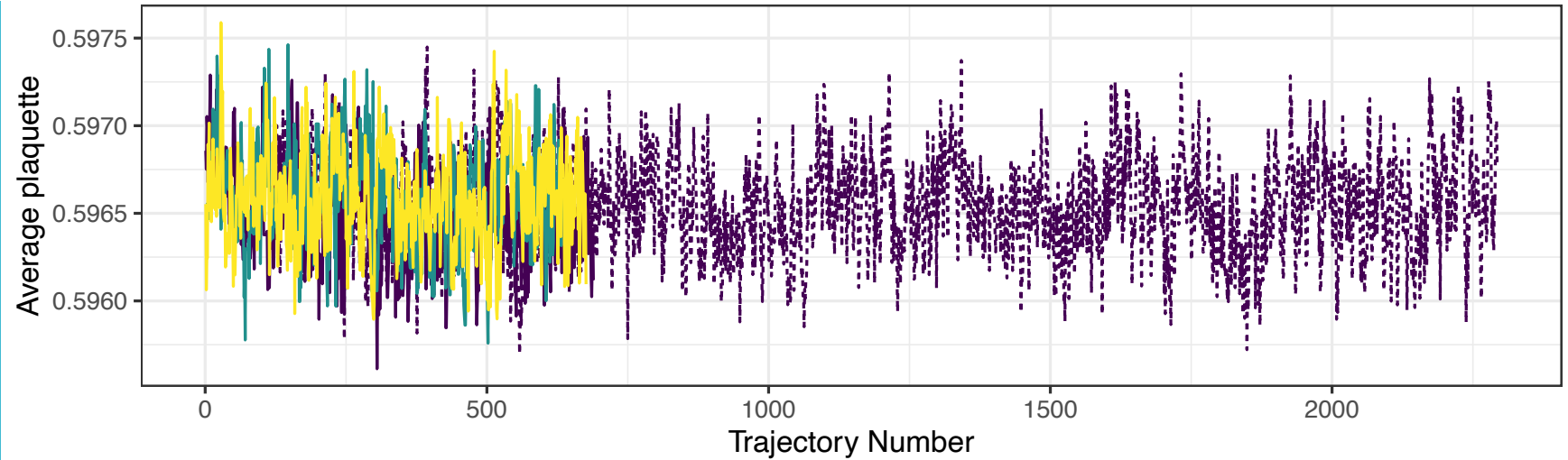
Using Schrödinger-Functional ensembles

[L. S. Bowes et. al., 2024, 2401.00589, PoS] + one additional point



# GPU ensembles

at  $M_V/M_{PS} \approx 1.5$



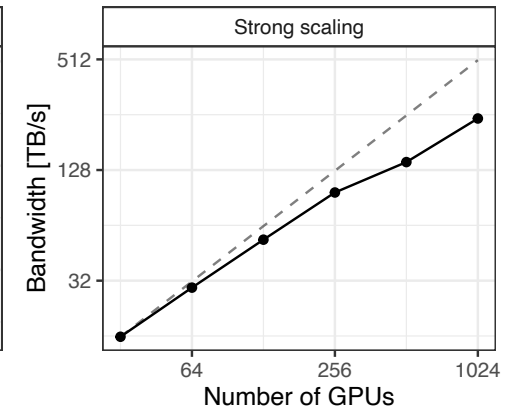
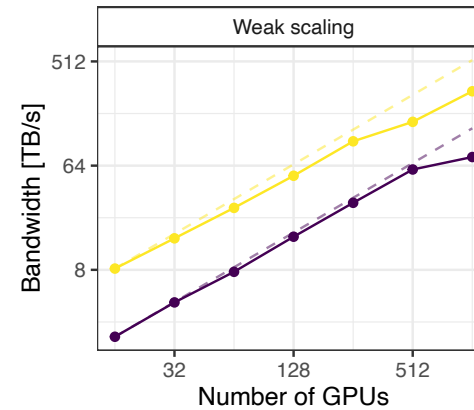
Replica — 0 — 1 — 2 GPU acceleration — FALSE - - - - TRUE

## Computational Budget CPU (DlaL3, AMD EPYC 7742)

1 node (128 cores)  
~ 1h / traj

## GPU (LUMI-G, AMD MI250x)

1 node (4 GPUs)  
~ 6 mins / traj



Local lattice size — 16<sup>4</sup> — 32<sup>4</sup>

See poster on HiRep on GPUs

# Analysis

## Twisted mass

During measurements include a chirally rotated  
(*twisted mass*) term

$$D_{TM}(m, \mu_0) = D_{expclover}(m) + i\gamma_5\mu_0$$

Tuning  $M_{PS}^{sea} = M_{PS}^{valence}$  and  $m_{PCAC} = 0$  allows omission of renormalization constants in  $f_{PS}$

Bare tm mass parameter  
↓

$$f_{PS} = \frac{2\mu_0 \langle 0 | P | \pi \rangle_{bare}}{M_{PS}^2}$$

# Twisted mass

[Hernández, Pena, Romero-López, 2019, 1907.11511, Eur. Phys. J. C]  
[Shindler, 2008, 0707.4093, Phys. Rept.] and references therein

$$D_{TM} = D_{expclover} + i\gamma_5\mu_0, \quad f_{PS} = \frac{2\mu_0 \langle 0|P|\pi \rangle_{bare}}{M_{PS}^2}$$

Bare tm mass parameter  
↓

Renormalization factor  $Z_A$  only necessary for slightly mistuned ensembles

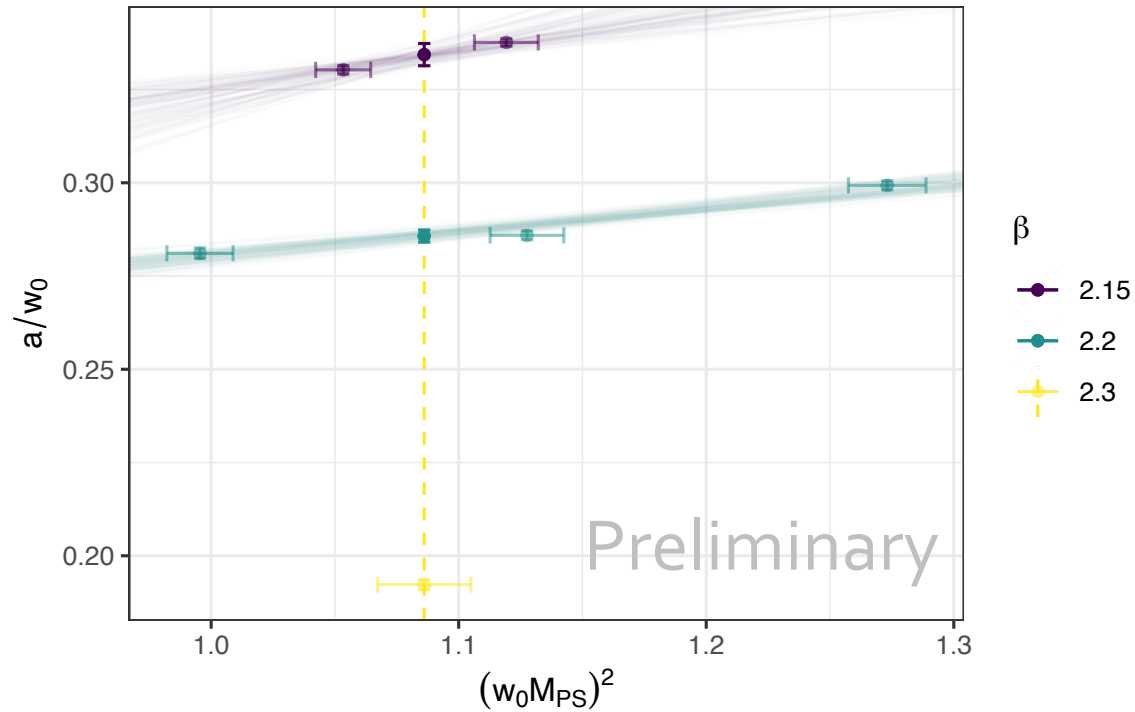
$$af_{PS} \rightarrow af_{PS} \sqrt{1 + \left(\frac{Z_A m_{PCAC}}{a\mu_0}\right)^2}$$



Renormalization

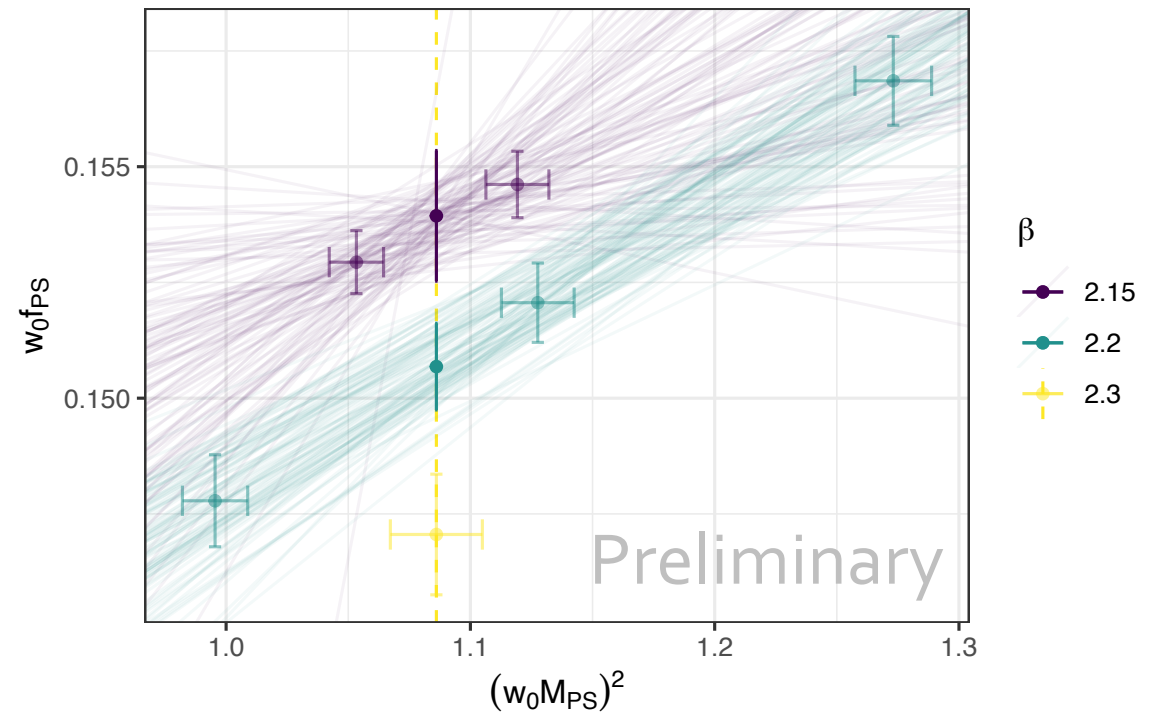


Automatic  $O(a)$ -improvement



Interpolation to reference mass:  
Lattice spacing

## Pseudoscalar Decay Constant $f_{PS}$

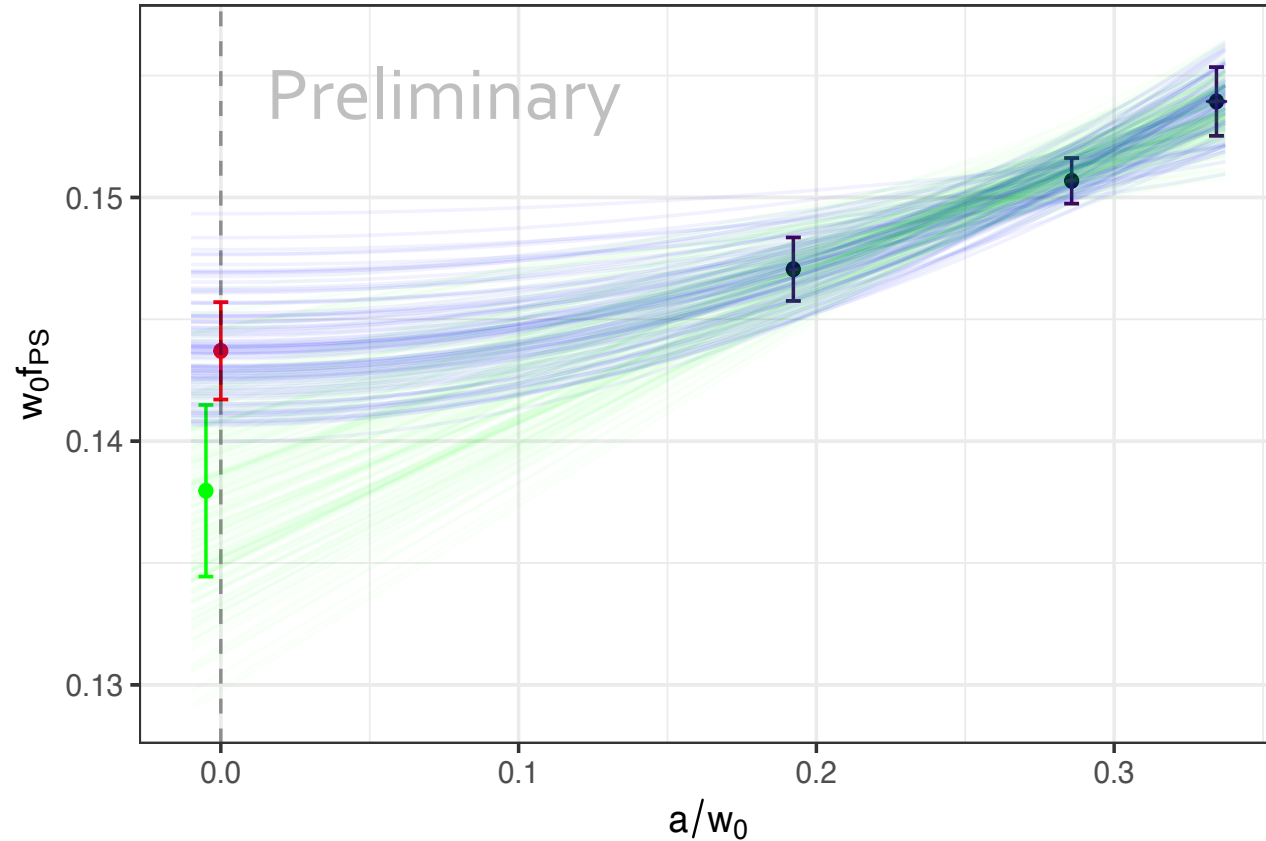


Lattice  $48 \times 24^3$ ,  $M_{PS}L = 6.7$  to  $8.4$   
Reference lattice  $36^4$ ,  $M_{PS}L = 7.2$   
Smearing radius  $r \leq 0.4L$

# Results

# Continuum limit

- Another point missing for proving that the effects are  $O(a^2)$
- Result from linear behavior almost agrees with the result from quadratic fits
- Discretization effects are  $<10\%$
- Previously: NP-renormalized  $f_{PS}$  + Wilson fermions  $\approx 30\%$



$$w_0 f_{PS} = 0.1436(19) \text{ at } (w_0 M_{PS})^2 = 1.09(2)$$

$$O(a^2) = 0.089(20) \frac{a^2}{w_0^2}$$



# Summary & Outlook

- We took the continuum limit for  $f_{PS}$  at fixed  $w_0 M_{PS}$
- We see only small  $O(a^2)$ -effects and achieved high precision

## Questions

- One more point to really understand the continuum limit scaling?
- Larger and chiral ensembles on the GPU?

## Outlook

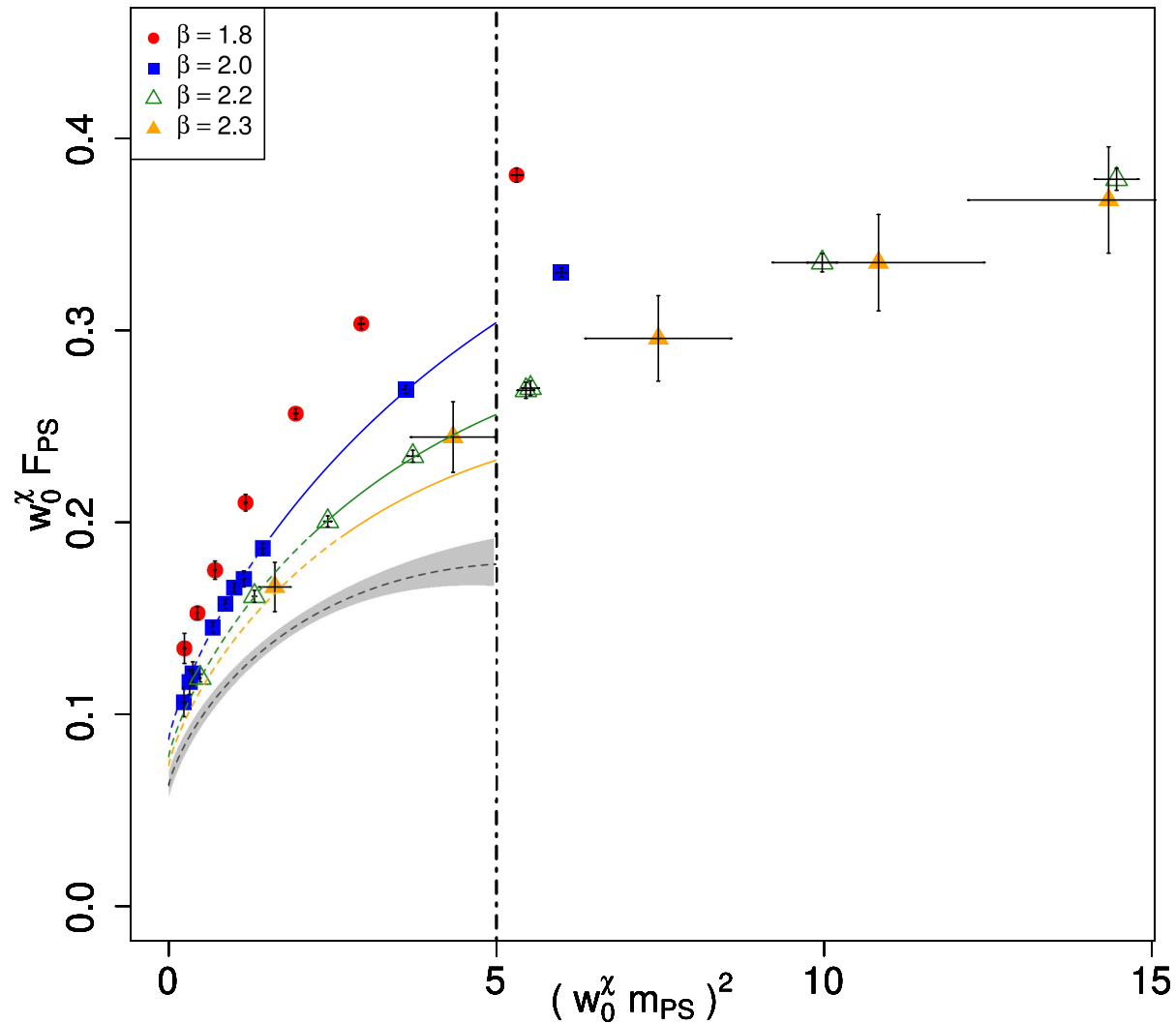
- Stay tuned for more really chiral simulations and a full scattering analysis

# Backup

# Ensemble Overview

Lattice	$\beta$	$m$	$w_0/a$	$aM_{PS}$	$af_{PS}$
$48 \times 24^3$	2.15	-0.2645	3.028(11)	0.3390(14)	0.05051(14)
$48 \times 24^3$	2.15	-0.2624	2.962(9)	0.3572(18)	0.05220(19)
$48 \times 24^3$	2.2	-0.269	3.558(18)	0.2804(13)	0.04154(19)
$48 \times 24^3$	2.2	-0.2657	3.498(15)	0.3036(16)	0.04348(17)
$48 \times 24^3$	2.2	-0.26	3.341(14)	0.3377(16)	0.04695(21)
$36^4$	2.3	-0.29	5.201(39)	0.2004(9)	0.02827(14)

# Previous results from [1602.06559]



In the given region, the discretization effects are at 30% of the total  $f_{\text{PS}}$ , now we achieve  $< 10\%$

