Symmetric mass generation and staggered fermions

Simon Catterall, Syracuse University

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Plan

- Mass without symmetry breaking (SMG). What, why and how
- Exact 't Hooft anomalies for staggered fermions
- Mirror models and chiral fermions
- Simulation results - $SO(4)$ gauge theory

(arxiv:2209.03828, 2101.01026, 2111.01001)

Nouman Butt (UIUC), Goksu Can Toga (SU $\rightarrow$ NC state), Arnab Pradhan (Syracuse)

Symmetric Mass Generation (SMG)

Typically ...

Fermions acquire mass thru symmetry breaking
eg Dirac mass, Higgs mechanism, χ-symmetry breaking in QCD

Is this always true ?

No! - Possible to generate mass without breaking symmetries.
Requires non-perturbative physics and
cancellation of all 't Hooft anomalies

't Hooft anomaly

Obstruction to gauging a global symmetry

Key observation: anomalies are RG invariants: non-zero anomaly in UV $\rightarrow$ impossible to gap all states in I.R
SMG: anomalies must cancel

Why is SMG interesting ?

Chiral gauge theories

Admit no gauge invariant mass term.
But eg symmetric four fermion terms possible
Such interactions may gap all states in I.R if no 't Hooft anomalies

Allows construction of "mirror models" eg

χ theory: $\psi_L, \text{rep r of G}$
lattice problematic - Nielsen-Ninomiya

$\downarrow$ vector-like theory

$\psi_L, \text{rep r of G} \quad \oplus \quad \psi_R, \text{rep r of G}$

target — mirror sector $\leftarrow$ drive into SMG phase

Lattice

Can we realize SMG with staggered fermions ?

$$S = \sum_{x,\mu} \eta_\mu(x)\overline{\chi}(x)\Delta^\mu \chi(x) + G\phi(x)\overline{\chi}(x)\chi(x) + \frac{1}{2}\phi^2(x)$$

Δ_μ is a gauge covariant difference and $\eta_\mu(x) = (-1)^{\sum_{i=1}^{\mu-1} x_i}$

Symmetries:

- Z_4 symmetry: $\chi(x) \to e^{i\frac{n\pi}{2}\epsilon(x)}\chi(x)$ and $\phi(x) \to -\phi(x)$.
- Shifts. $\chi(x) \to \xi_\lambda(x)\chi(x+\lambda)$ with $\xi_\lambda(x) = (-1)^{\sum_{i=\mu+1}^{4} x_i}$ and $\epsilon(x) = (-1)^{\sum_{i=1}^{4} x_i}$
- Gauge symmetry eg $SU(N)$

Symmetries

Protect theory from all bilinear mass terms

't Hooft anomalies

SMG requires vanishing 't Hooft anomalies ...

- Trivial to insert Z_4 gauge links to render S gauge invariant

Measure $\mathcal{M} = \prod_x (d\chi(x) d\overline{\chi}(x))^{2N}$

$$\mathcal{M} \to e^{i\pi n(x)N} \mathcal{M} - \text{invariant} \quad \text{iff N=2k}$$

condition to cancel Z_4't Hooft anomaly

(anomaly also visible if couple to gravity arXiv: 2209.03828)

- Assume G anomaly free.
- Shifts ? ... unclear but work in progress ... arXiv:2405.0307

Continuum and reduced fermions

Continuum fermion (spin-taste basis):

$$\Psi_\alpha^a(\mathbf{x}) = \sum_{\hat{\mathbf{b}}} \chi(\mathbf{x}+\hat{\mathbf{b}}) \left(\gamma^{\mathbf{x}+\hat{\mathbf{b}}}\right)^{\mathbf{a}\alpha}$$

where $\hat{\mathbf{b}}$ vector in unit hypercube with components $b_i \in 0,1.$ and $\gamma^{\mathbf{x}} = \gamma_1^{x_1} \cdots \gamma_4^{x_4}$

Consider **reduced** staggered field:

$$\chi_+(x) = \frac{1}{2}(1+\epsilon(x))\chi(x) \equiv \frac{1}{2}(\Psi + \gamma_5\Psi\gamma_5) = \Psi_+$$
$$\overline{\chi}_-(x) = \frac{1}{2}(1-\epsilon(x))\overline{\chi}(x) \equiv \frac{1}{2}(\overline{\Psi} - \gamma_5\overline{\Psi}\gamma_5) = \overline{\Psi}_-$$

Carries $1/2$ dof $\rightarrow Z_4$ anomaly cancellation requires $N = 4k$.

Connection with chiral fermions ?

In Euclidean chiral basis $\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \overline{\sigma}_\mu & 0 \end{pmatrix}$ where $\sigma_\mu = (I, i\sigma_i)$.

$$\text{reduced field } \Psi_+ = \begin{pmatrix} \psi_R & 0 \\ 0 & \psi_L \end{pmatrix}$$

L and R handed doublet of Weyl fields transforming as $(1,2)$ and $(2,1)$ under an $SU(2) \times SU(2)$ flavor symmetry.

no invariant mass term - chiral!

Minimal anomaly free model has $SU(4) \times SU(2) \times SU(2)$ symmetry

Matter reps ? replace $\psi_R = i\sigma_2\psi_L^*$.

Get reps $(\mathbf{4},\mathbf{2},\mathbf{1})_\mathbf{L} \oplus (\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_\mathbf{L}$ - Pati-Salam GUT !

Pati-Salam - quick summary

leptons (e,ν) as fourth color
left-right symmetric weak interaction
Symmetry: $SU(4) \otimes SU_L(2) \otimes SU_R(2)$

One generation:

$$\begin{pmatrix} u_r & u_b & u_g & \nu \\ d_r & d_b & d_g & e \end{pmatrix}_L \quad \oplus \quad \begin{pmatrix} u^c_r & u^c_b & u^c_g & \nu^c \\ d^c_r & d^c_b & d^c_g & e^c \end{pmatrix}_L$$

SM: Higgs $SU(4) \rightarrow SU(3)$ and $SU_L(2) \otimes SU_R(2) \rightarrow SU_L(2)$

$$(4,2,1) \rightarrow (3,2)_{\frac{1}{6}} \oplus (1,2)_{-\frac{1}{2}} \ [q_L \text{ and } l_L\ ]$$

$$(\overline{4},1,2) \rightarrow (\overline{3},1)_{\frac{1}{3}} \oplus (\overline{3},1)_{-\frac{2}{3}} \oplus (1,1)_1 \oplus (1,1)_0 \ [d_R, u_R,\ e_R \text{ and } \nu_R]$$

1 family of standard model !

Summary so far

Minimal anomaly free reduced staggered fermion model yields a chiral GUT theory in naive continuum limit

So:

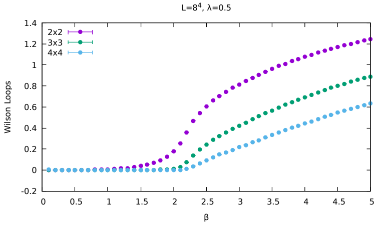
- Build lattice mirror model with this low energy sector
- Use gauge interactions to generate SMG phase for mirror fermions. Mirrors defined wrt $\epsilon(x)$ not γ^5 but chiral fermions nevertheless in (naive) continuum limit.

Problem:

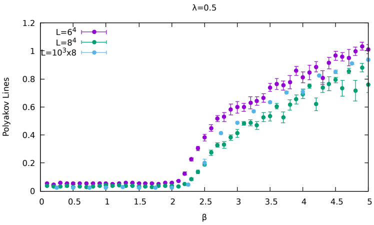
$SU(4)$ gauge theory with reduced staggered fermions has a sign problem...

Toy example: $SU(4) \to SO(4)$ mirror sector

Pfaffian that arises after integrating fermions is **real, positive definite** $\to$ can use RHMC alg.



(a) Wilson loops



(b) Polyakov line

Four fermion condensate

Confining regime - system chooses four fermion rather than bilinear condensate. No Goldstones.

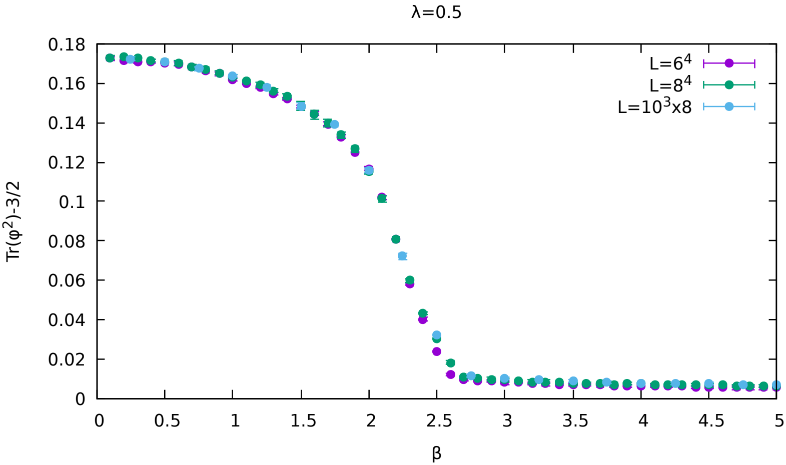


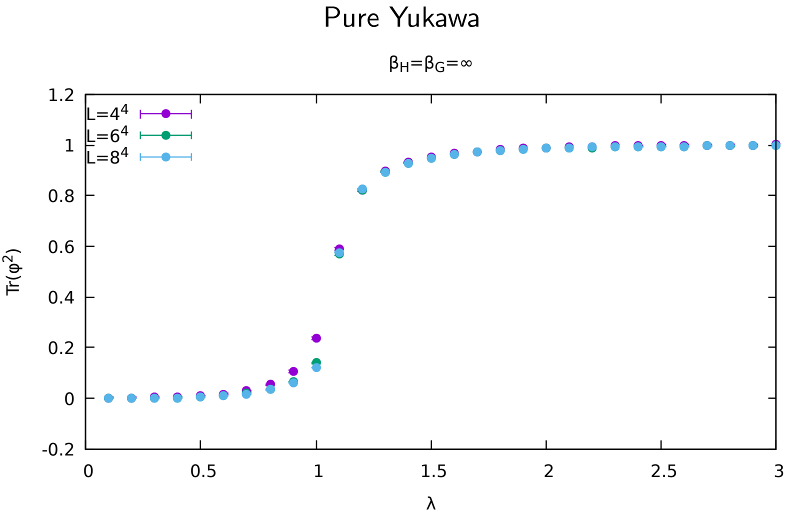
Figure: Four fermion condensate

Conclusions

- SMG is a novel non-perturbative mechanism for giving fermions masses without breaking symmetries. Needs all 't Hooft anomalies to cancel.
- May be important in constructing mirror models for chiral gauge theories
- SMG has been seen in staggered fermion models where important Z_4 anomaly cancels even on lattice.
- Simplest model involving **reduced** staggered fermions targets Pati-Salam in (naive) continuum limit.
- $SO(4)$ LGT forms a four fermion rather than bilinear condensate when it confines ...
- Need to understand better anomaly constraints coming from shift symmetries ... non-onsite symmetry and (maybe) non-invertible ...

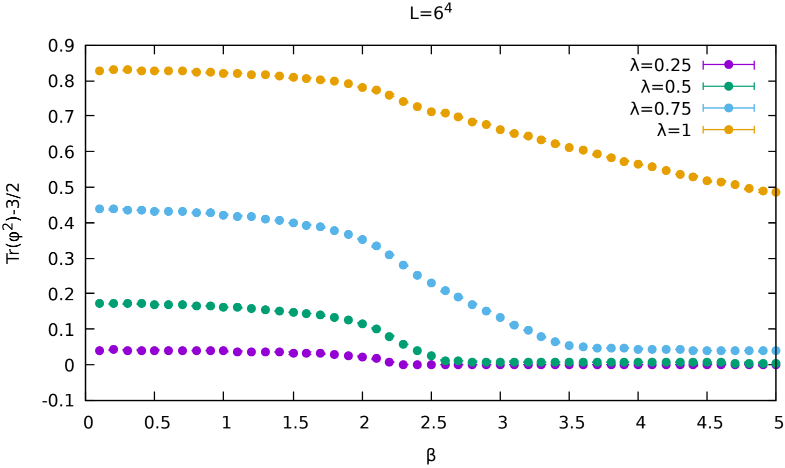
thanks !

BACKUPS



Pure Yukawa
βH=βG=∞
L=4^4
L=6^4
L=8^4
Tr(φ^2)
λ
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.5
1
1.5
2
2.5
3

Dependence of condensate on G



Link $\sum_{x,\mu} \psi(x)\epsilon(x)\xi_\mu(x)\psi(x+\mu)$ vev