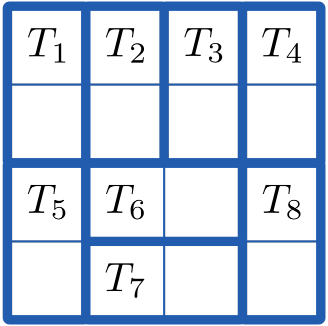
ETH zürich

ETH, Institute for Theoretical Physics
High Performance Computational Physics group

Quantum Many-Body Scars in 2+1D Gauge Theories

Joao C. Pinto Barros
Thea Budde, Marina Krstić Marinković

Lattice 2024
29th of July | Liverpool



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- Well-suited for quantum simulators.

E.g. Heavy Ion Collision (J. Berges, M. P. Heller, A. Mazeliauskas, R. Venugopalan. Rev. Mod. Phys (2021))

Outline

1. Hamiltonian (formulation) for $U(1)$ Pure Gauge Theories
2. Thermalization and Scars in Many-Body Systems
3. Constructing and Isolating Quantum Scars

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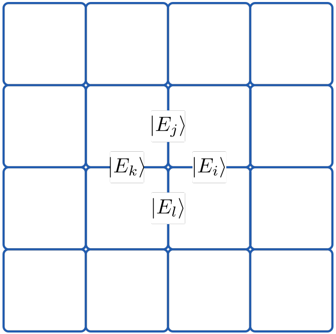
Hamiltonian for $U(1)$ Pure Gauge Theories

$$H = -t \sum_n \underbrace{U_{n1}^\dagger U_{n+\hat{1}2}^\dagger U_{n2} U_{n+\hat{2}1}}_{U_\square} + \text{h.c.} + \kappa \sum_n E_n^2$$



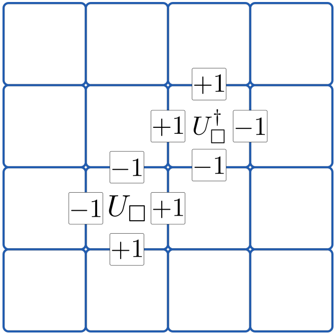
$E_n \in \mathbb{Z}$

U_n unitary raising operator

$U_i |E_i\rangle = |E_i + 1\rangle$

Gauss' Law

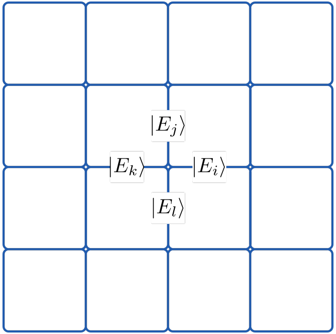
$E_i - E_k + E_j - E_l = 0$



Truncation: $E_n \in \{-S, \ldots, S\}$

Finite Hilbert space per link

$$H = -t \sum_n \underbrace{U^\dagger_{n1} U^\dagger_{n+\hat{1}2} U_{n2} U_{n+\hat{2}1}}_{U_\square} + \text{h.c.} + \kappa \sum_n E_n^2$$



U_n **non**-unitary raising operator

$$U_i \left|E_i\right\rangle = \left|E_i + 1\right\rangle$$
$$U_n \left|S\right\rangle = 0$$

Gauss' Law

$$E_i - E_k + E_j - E_l = 0$$

- $S = 1/2$ and $S = 1$ are equivalent to spins (Quantum Link Models);

  S. Chandrasekharan, U.-J. Wiese NPB (1997)

- S large enough $\Leftrightarrow S = \infty$ (at least in $1 + 1D$: see **T. Budde, Tuesday, 4:15 PM**, Quantum Computing track)

- Here: arbitrary integer S.

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What Happens to an Isolated Quantum System When Left Alone?

Prepare Quantum State

What Happens to an Isolated Quantum System When Left Alone?

Prepare Quantum State

Go away

What Happens to an Isolated Quantum System When Left Alone?

Prepare Quantum State

Go away

Come back

What Happens to an Isolated Quantum System When Left Alone?

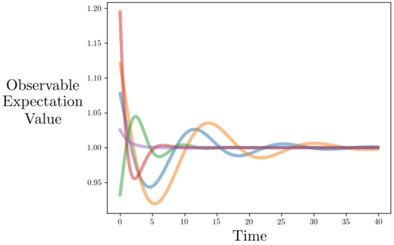
Prepare Quantum State

Go away

Measure

Come back

How Do Quantum Systems Thermalize?



Thermalization: observable converge to values independent of the initial details.

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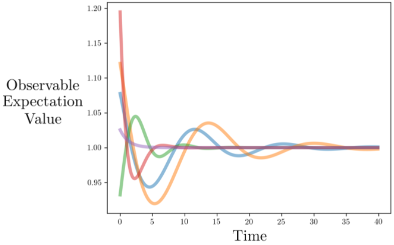

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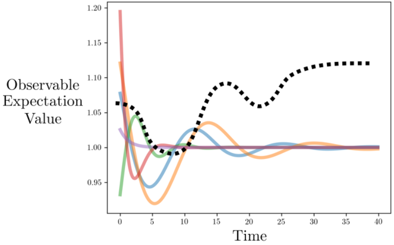

Scar: special initial conditions avoid thermalization.
(For a review: S. Moudgalya, B. A. Bernevig, N. Regnault. RPP (2022))

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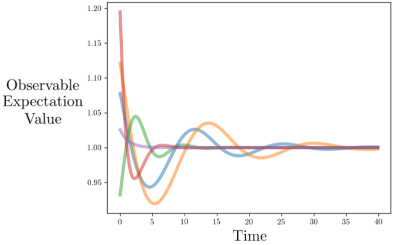

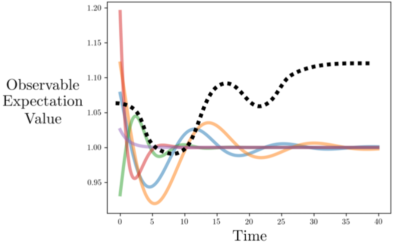

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Scar: special initial conditions avoid thermalization.
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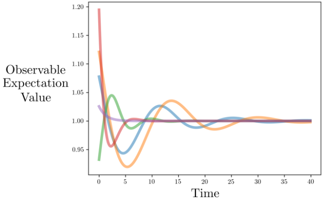
- Involves real-time evolution;
- Generically afflicted by severe sign problems;
- Well suited for quantum simulators.

H. Bernien et al. Nature (2017)

Eigenstate Thermalization Hypothesis (ETH)

Non-integrable models are expected to thermalize



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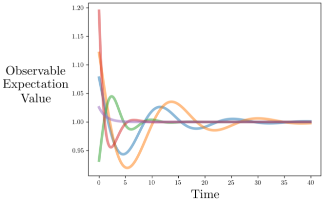
1. Mid-spectrum states are highly entangled.



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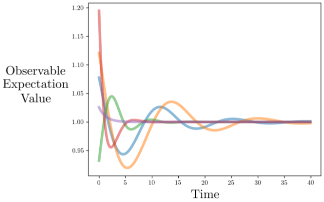
1. Mid-spectrum states are highly entangled.
2. Thermalization applies **to local observables**

$$\underbrace{\lim_{t\to\infty} \langle\psi(t)|\, O\, |\psi(t)\rangle}_{\textbf{Long time expectation value}} = O\left(\langle\psi(0)|\, H\, |\psi(0)\rangle\right) = \underbrace{\frac{1}{\mathrm{tr}\left(e^{-\beta H}\right)} \mathrm{tr}\left(O e^{-\beta H}\right)}_{\textbf{Canonical Ensemble}}$$

Temperature fixed by the energy

For a review: L. D'Alessio, Y. Kafri, A. Polkovnikov, M. Rigol. Adv. Phys. (2016)



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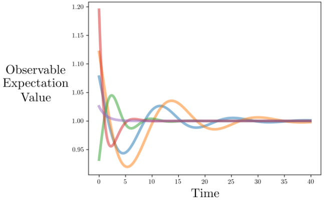

Our model

- Spectrum is symmetric (eigenstates come in pairs $|E\rangle$ and $|-E\rangle$;
- States with $E = 0$ are mid-spectrum;

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<u>HERE</u>

New mechanism for the formation of scars in 2+1D for arbitrary spin

 

T. Budde, M. Marinkovic, **JPB** - arXiv:2403.08892

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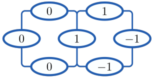
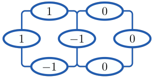
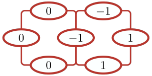
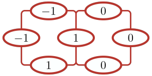
Spin-1 QLM: Two Plaquettes Zero Mode

We construct two-plaquette states:

$|\text{Blue}\rangle = \frac{1}{\sqrt{2}}$ 

$-$ 

$|\text{Red}\rangle = \frac{1}{\sqrt{2}}$ 

$-$ 

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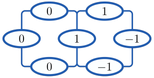
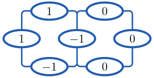
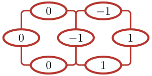
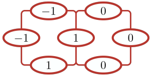
$$|\mathrm{Blue}\rangle = \frac{1}{\sqrt{2}}$$ 

$-$ 

$$|\mathrm{Red}\rangle = \frac{1}{\sqrt{2}}$$ 

$-$ 

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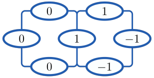
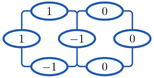
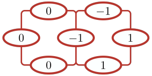
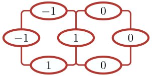
$$|\mathrm{Blue}\rangle = \frac{1}{\sqrt{2}}$$ 

– 

$$|\mathrm{Red}\rangle = \frac{1}{\sqrt{2}}$$ 

– 

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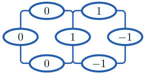
Tiling the blocks



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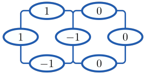

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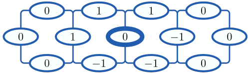

Still zero energy

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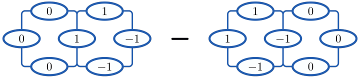

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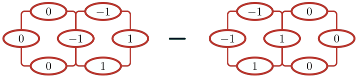

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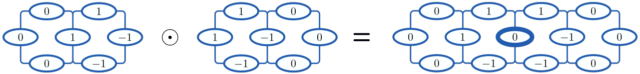
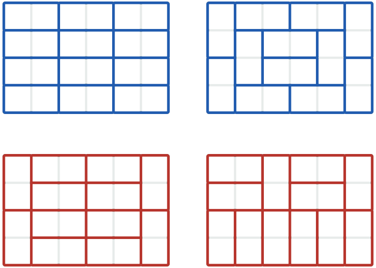


Still zero energy

We can use this to construct many "special" zero-energy states



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The Periodic Ladder

$$H = \sum_n \left(U_\Box + U_\Box^\dagger \right) + \lambda \sum_{l \text{ top row}} S_l^z$$

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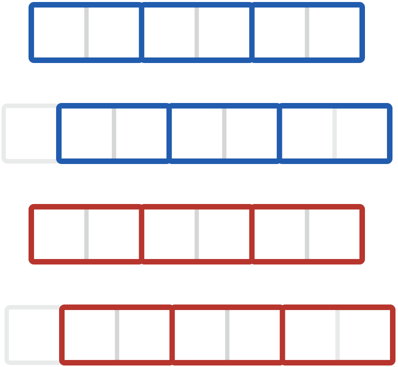

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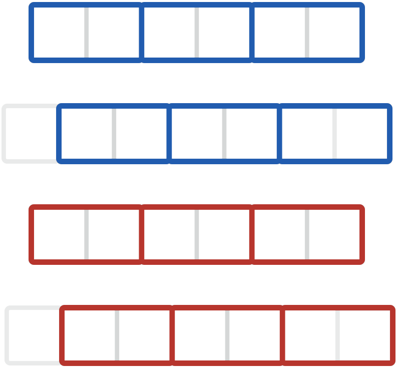
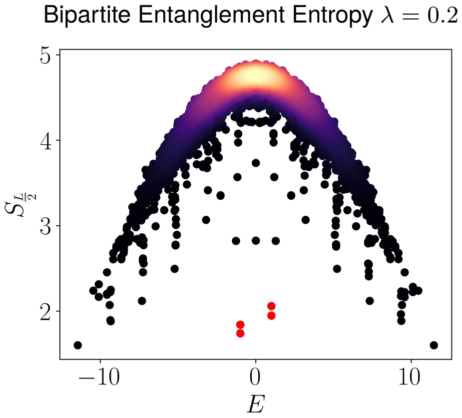
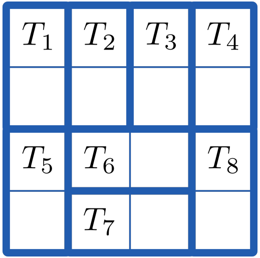




Scars for Arbitrary Volumes and Spins

$$|\psi_s^{(i,T)}\rangle = \frac{1}{(S+1)^{|T|/2}} \prod_{(n,n')\in T} \left(\sum_{k=0}^{S} (-1)^k (U_{\square n})^{i-S+k} (U_{\square n'})^{i-k} \right) |\mathbf{0}\rangle$$

$|\mathbf{0}\rangle \equiv$ State where all links are zero



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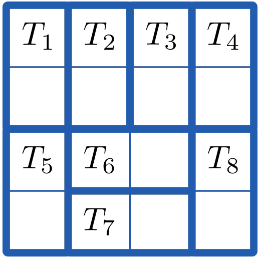
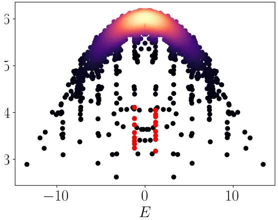
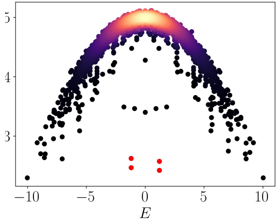


Entanglement entropy for $S = 1$ and 6×2 volume



Entanglement entropy for a $S = 2$ ladder



For details and other types of scars see T. Budde, M. Marinkovic, **JPB** - arXiv:2403.08892

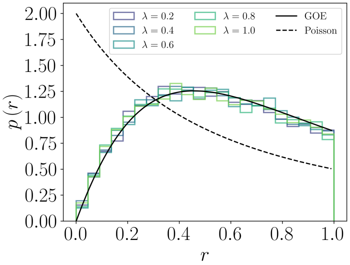
Non-Integrability of the Ladder

Address integrability:

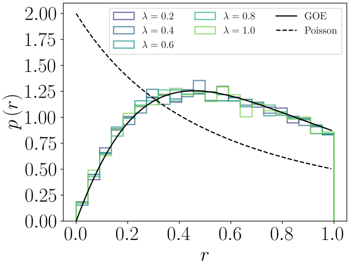
- Resolve symmetries;
- Compute level spacing distribution.

$$p(r)\,,\ r_n = \min\left\{\frac{E_{n+1}-E_n}{E_n-E_{n-1}},\frac{E_n-E_{n-1}}{E_{n+1}-E_n}\right\}$$

$-$ Parity Symmetry Sector



$+$ Parity Symmetry Sector



Integrable systems: expected Poisson

Non-integrable: expected Gaussian Orthogonal Ensemble (GOE)

Outlook

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- **Zero energy** states can combine to form **Quantum Many-Body Scars** for any spin;
- Single leg ladder scars facilitate experimental realization (not present for spin $1/2$).

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Thank You!