Lattice 2024, University of Liverpool

Euclidean Monte Carlo informed ground state preparation for quantum simulation

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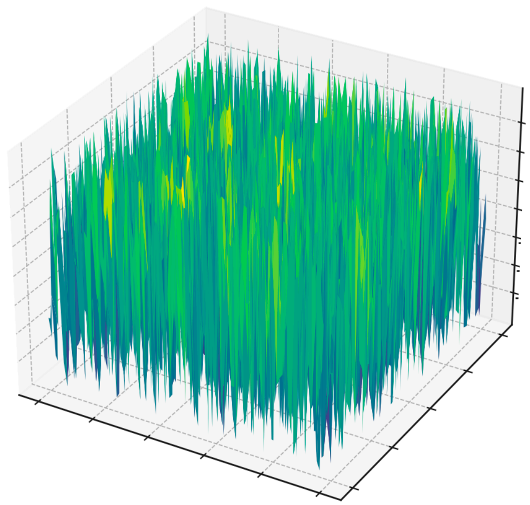
University of Maryland, College Park

(Manuscript in preparation)

Lattice QCD gives us static correlation functions.



$$\langle O \rangle = \frac{1}{Z} \int \mathscr{D}\phi \; e^{-S[\phi]} \; O$$

Hamiltonian methods can compute dynamical correlation functions.

$|\psi\rangle_{\text{in}} \rightarrow e^{-i\hat{H}t} \rightarrow$ Measure

$$\langle O(t)\rangle = \langle\psi|\, e^{i\hat{H}t}\hat{O}e^{-i\hat{H}t}\,|\psi\rangle$$

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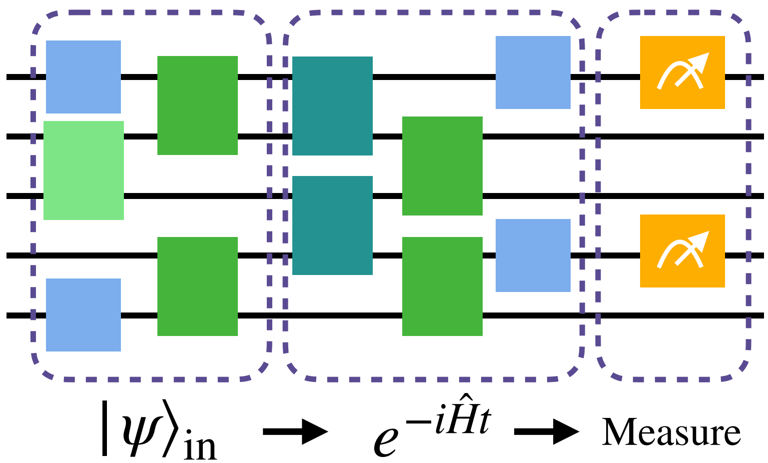

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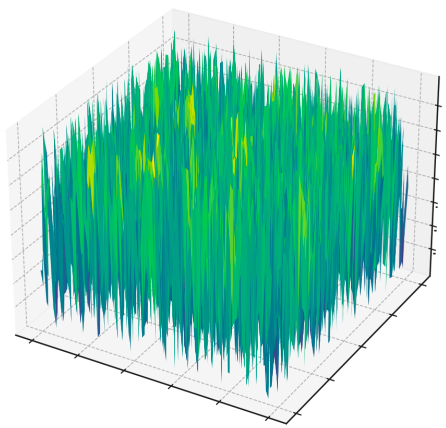
Can we bridge lattice QCD and quantum simulation?



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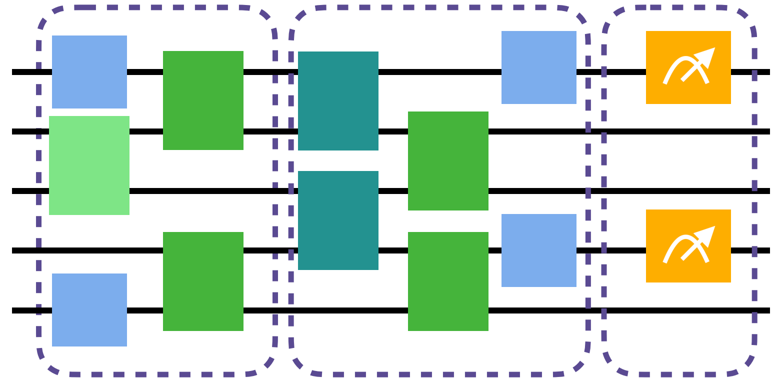
Static correlators inform state preparation.



$|\psi\rangle_{\text{in}}$ → $e^{-i\hat{H}t}$ → Measure

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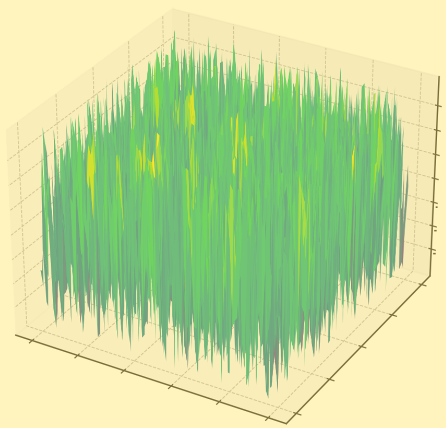

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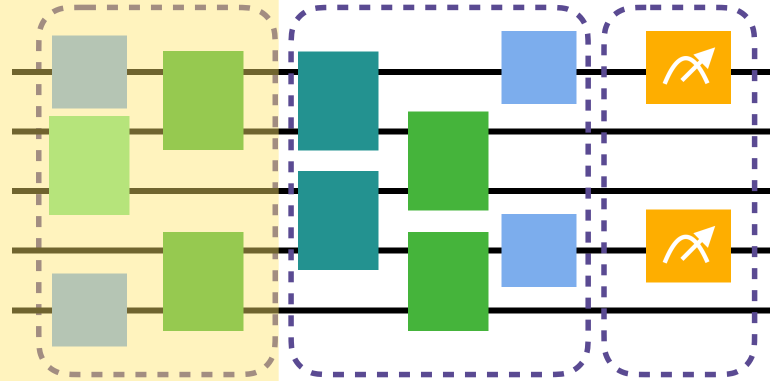

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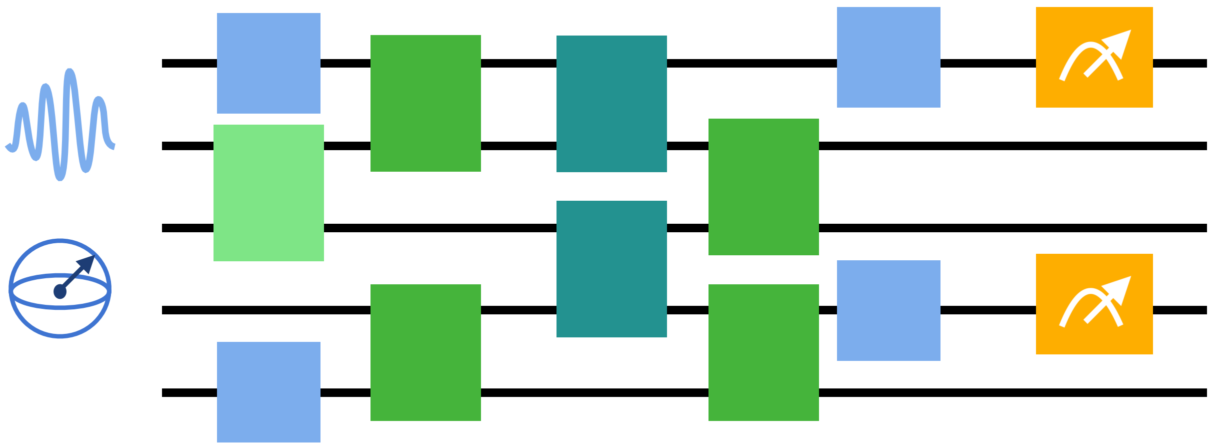

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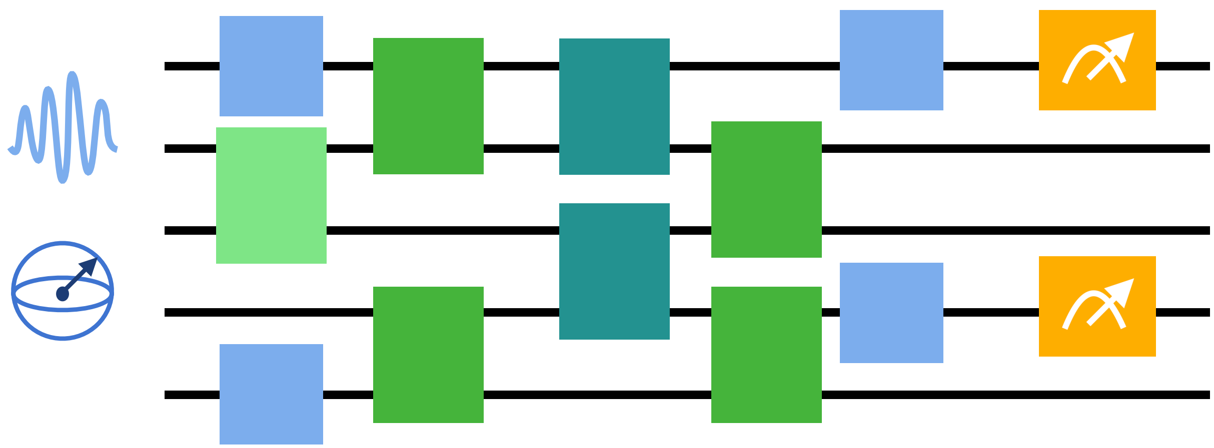

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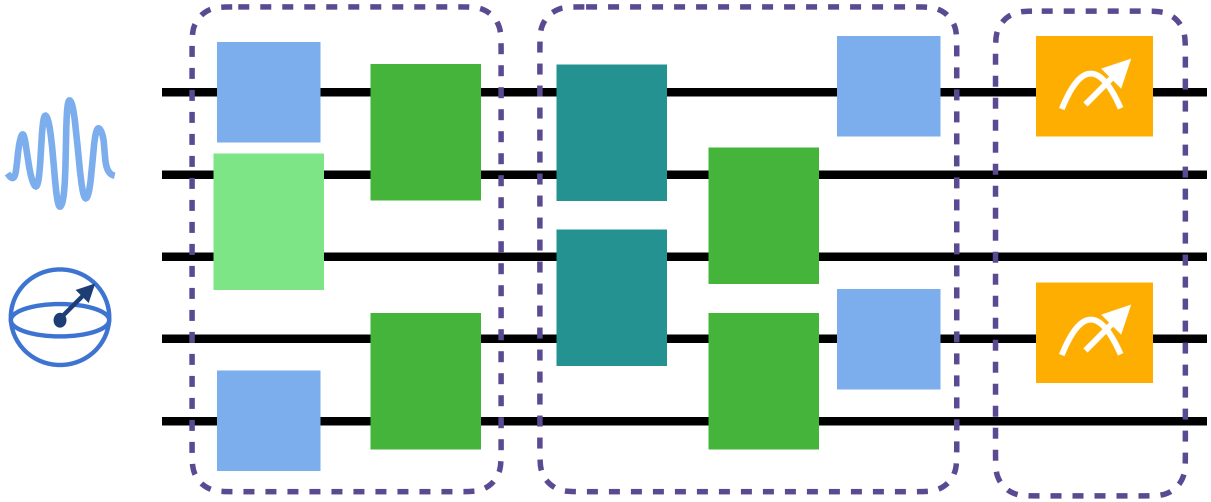

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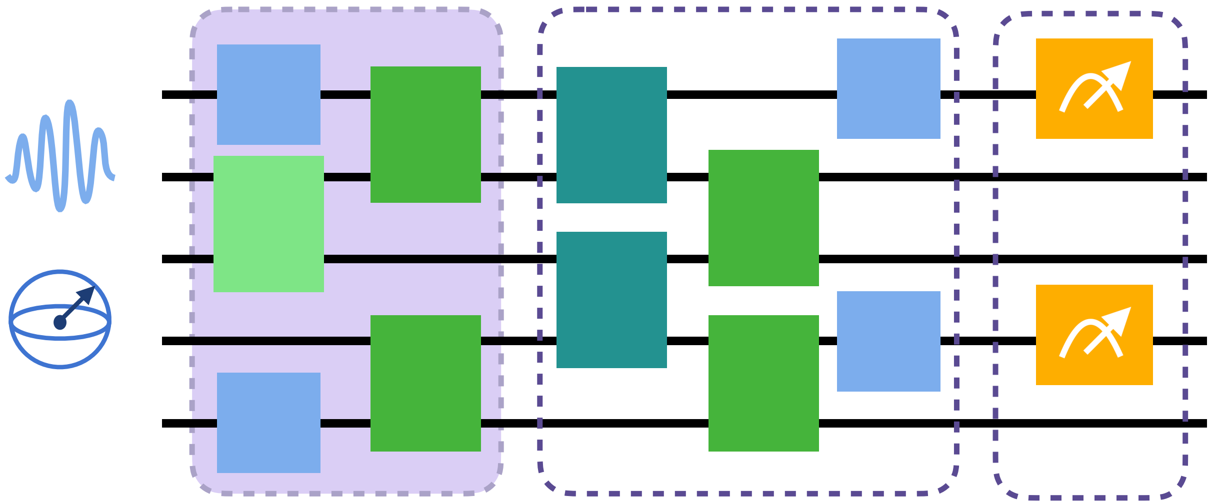

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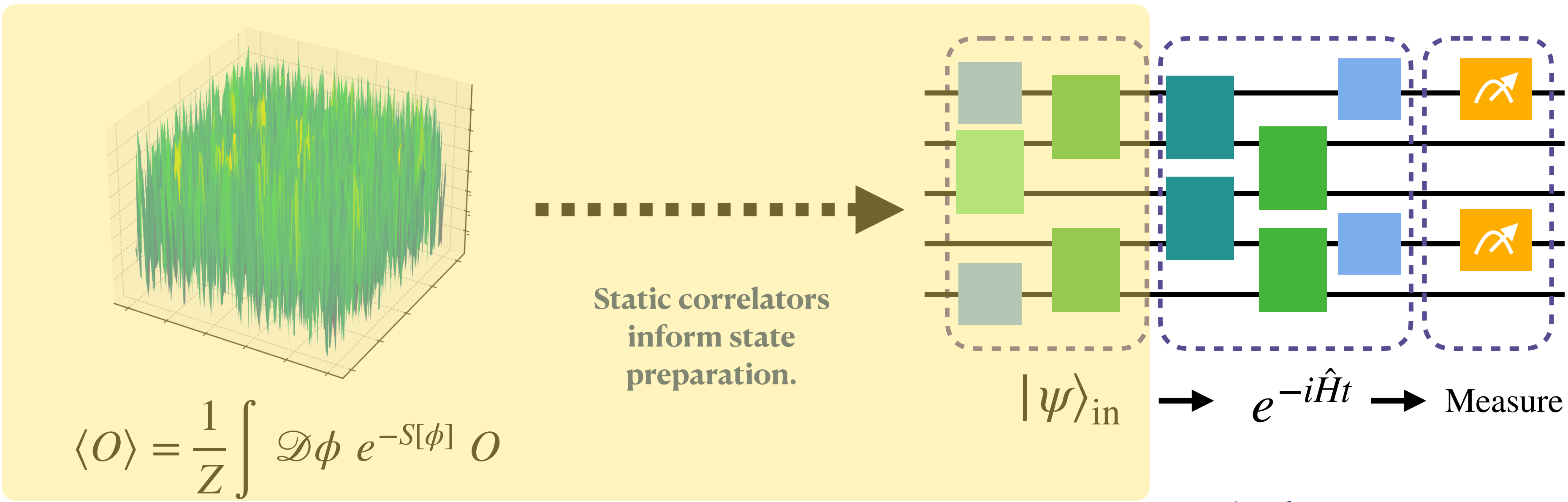

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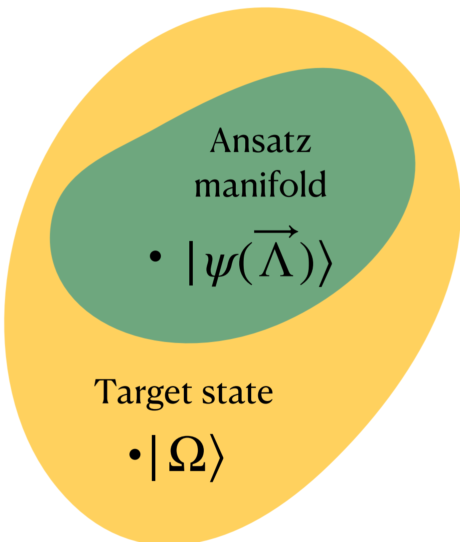
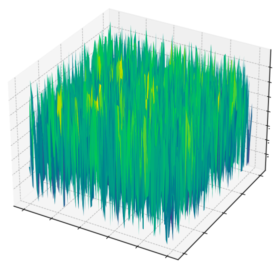
Classically determined quantum circuit for ground states

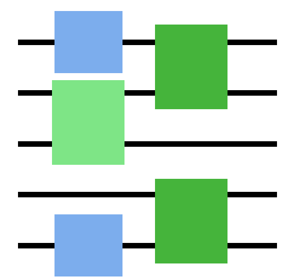


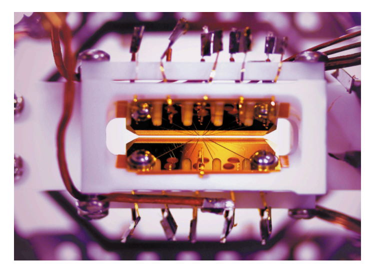


Path Integral Monte Carlo (PIMC) informed moment optimization

$|\psi(\overrightarrow{\Lambda_0})\rangle$

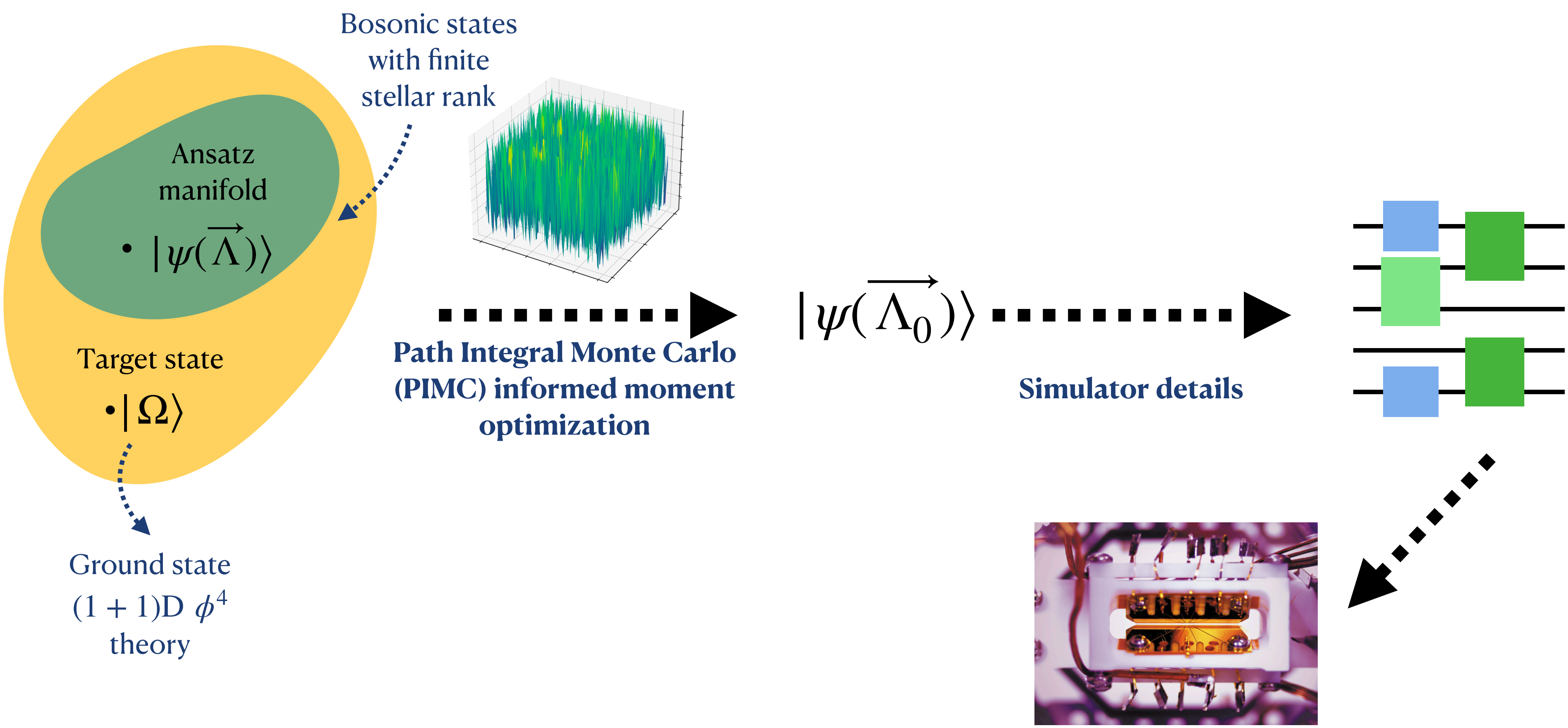
Simulator details





Trapped ion quantum computer from Monroe Lab (UMD, 2016)

Classically determined quantum circuit for ground states



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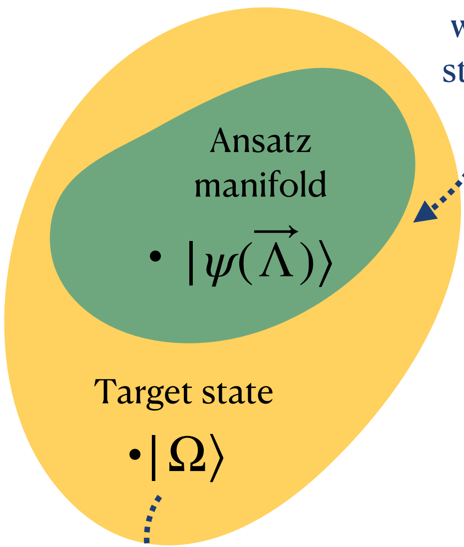
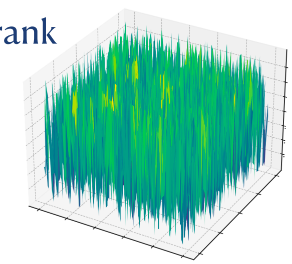
Classically determined quantum circuit for ground states

Bosonic states
with finite
stellar rank





Ground state
$(1+1)$D ϕ^4
theory

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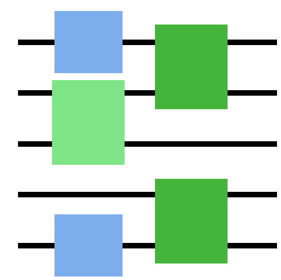

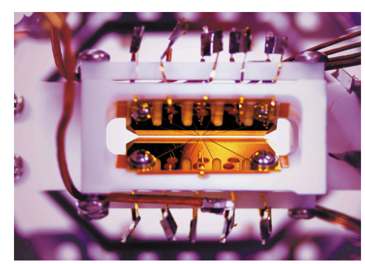

Trapped ion quantum computer from
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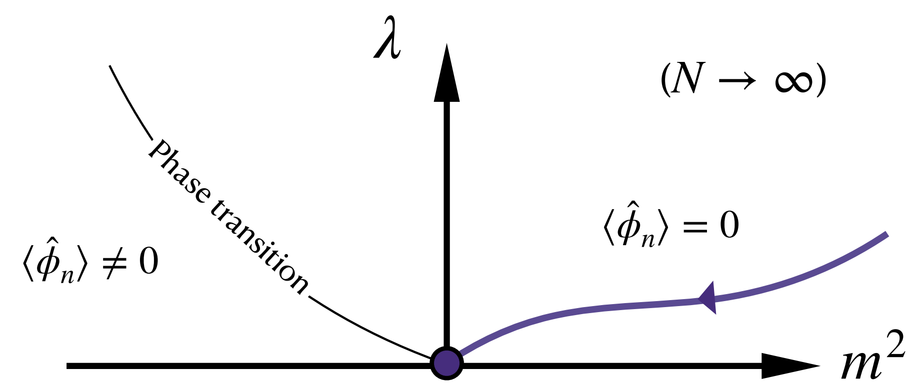
Ground state of (1 + 1)D ϕ^4 theory

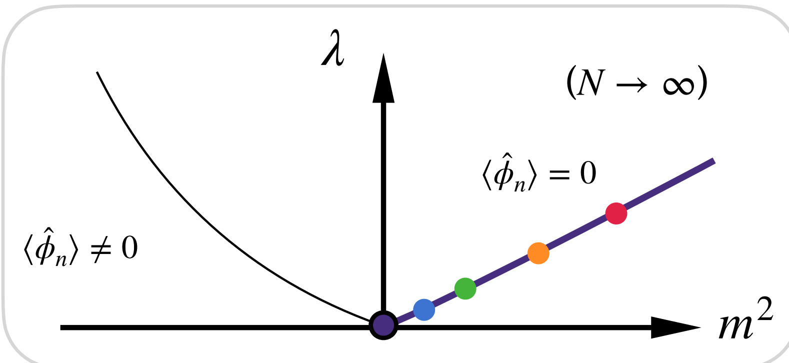
$$\hat{H} = \sum_{n=0}^{N-1} \left(\frac{\hat{\pi}_n^2}{2} + \frac{(\hat{\phi}_{n+1} - \hat{\phi}_n)^2}{2} + \frac{1}{2} m^2 \hat{\phi}_n^2 + \frac{\lambda}{4} \hat{\phi}_n^4 \right)$$

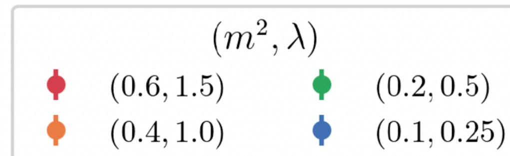


Continuum limit:

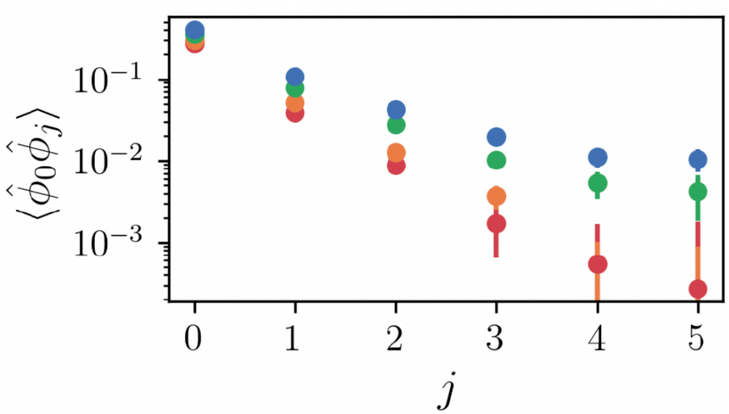
$m^2, \lambda \to 0,0$ along some line of constant physics.

Ground state of (1 + 1)D ϕ^4 theory: Monte Carlo results

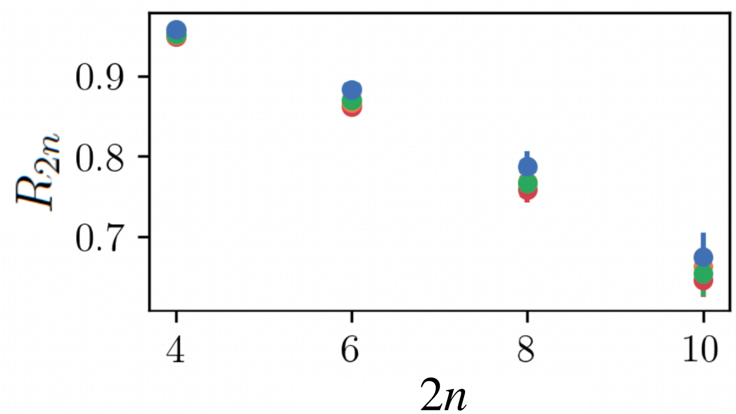




Correlations

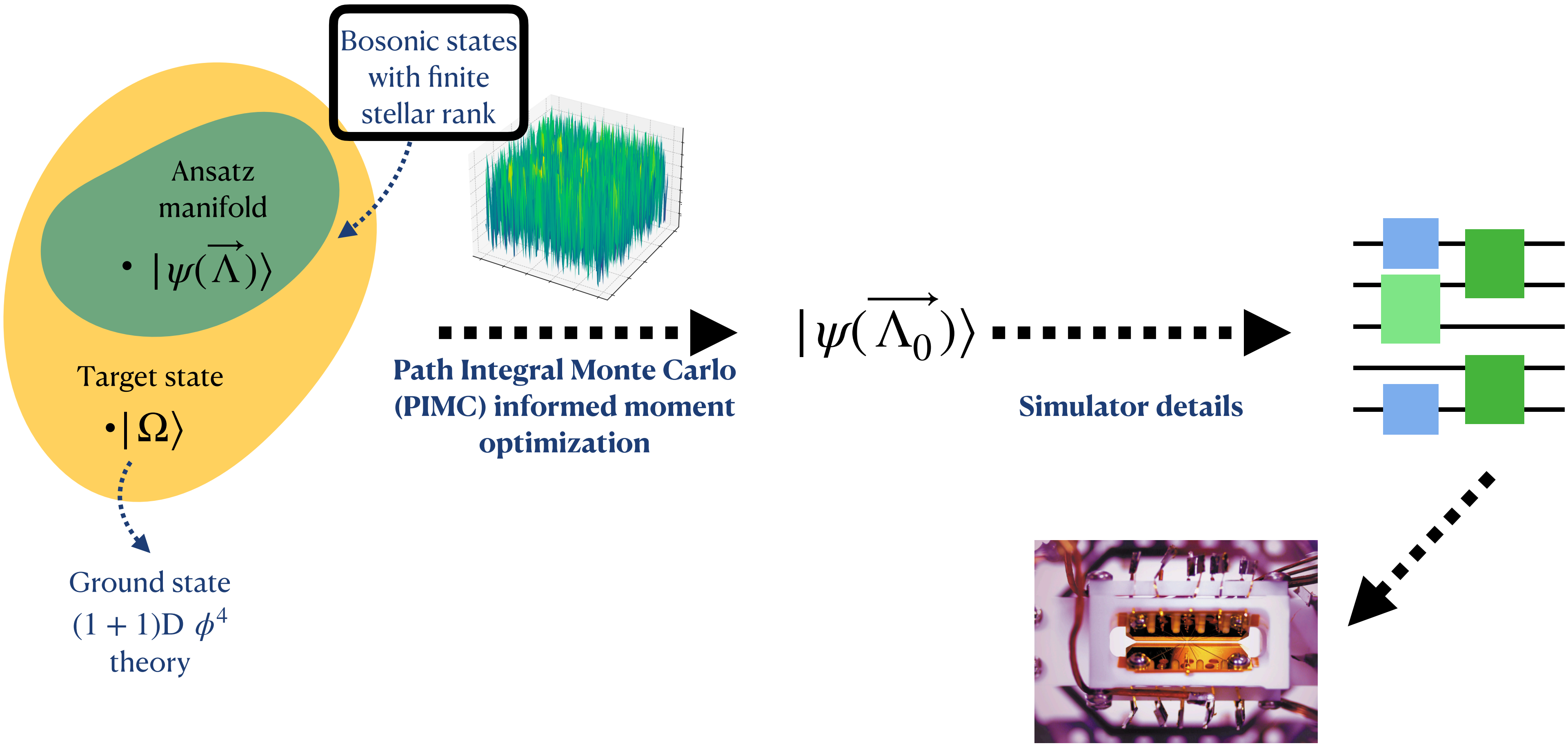


Non-Gaussianity



$$R_{2n} \equiv \frac{\langle \hat{\phi}^{2n} \rangle}{(n-1)!!\,\langle \hat{\phi}^2 \rangle^n}.$$

Classically determined quantum circuit for ground states



Trapped ion quantum computer from Monroe Lab (UMD, 2016)

Quantum states of N bosons

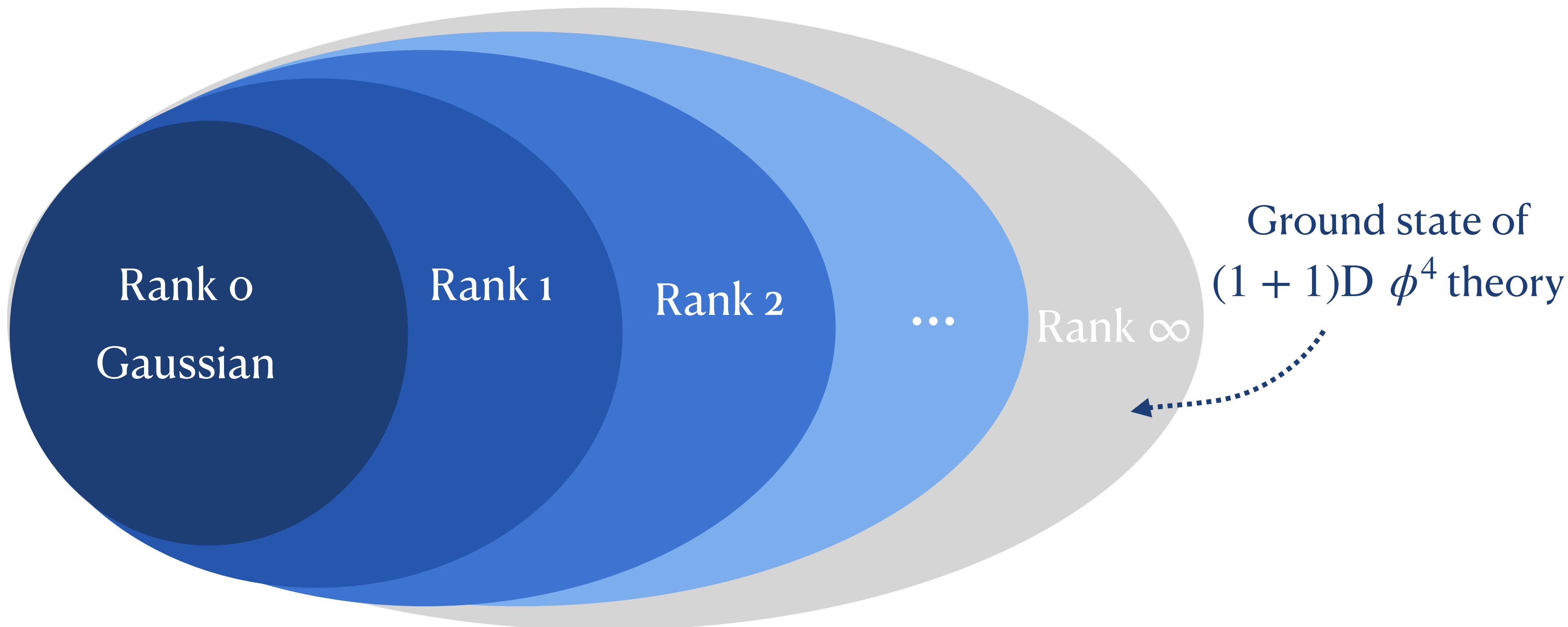
Most states of N bosons admit the decomposition

$$|\psi\rangle = \hat{U}_G \sum_{\substack{\vec{n} = n_1, \ldots, n_N \\ n_1 + \ldots + n_N \leq R}} c_{\vec{n}} \; \hat{a}_1^{\dagger n_1} \ldots \hat{a}_N^{\dagger n_N} \, |\vec{0}\rangle$$

$R \in \mathbb{N} \cup \{0\}$ is the **stellar rank** of this state.

The bosonic states which do not admit the above decomposition are said to have an infinite rank.

Chabaud, U., Markham, D., & Grosshans, F. (2020). Stellar representation of non-Gaussian quantum states. *Physical Review Letters*, *124*(6), 063605.



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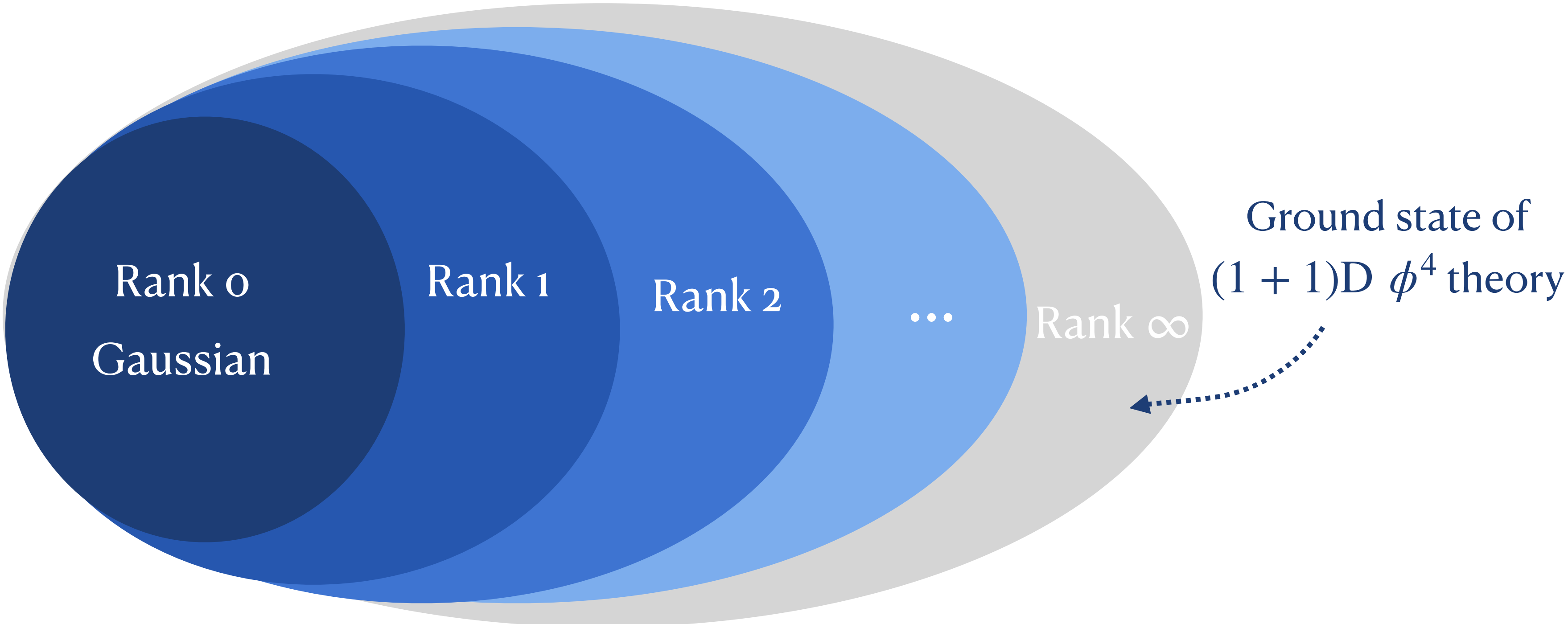


Finite rank states can get arbitrarily close to infinite-rank states (in trace distance).

Chabaud, U., Markham, D., & Grosshans, F. (2020). Stellar representation of non-Gaussian quantum states. *Physical Review Letters*, *124*(6), 063605.

Simpler choices of finite rank states

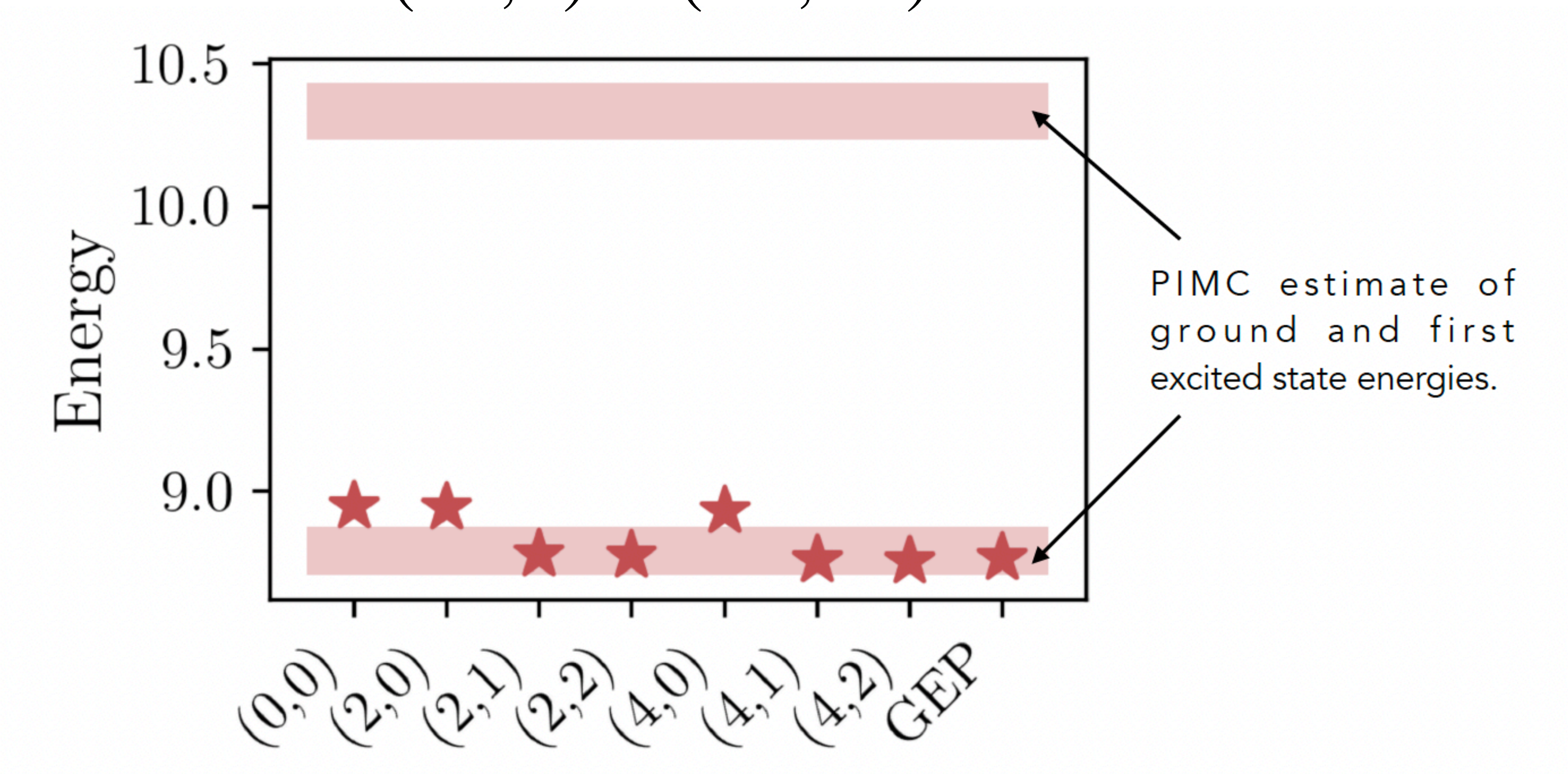
$$|\psi\rangle \; = \; \hat{U}_G \sum_{\substack{\vec{n} = n_1, \ldots, n_N \\ n_1 + \ldots + n_N \leq R}} c_{\vec{n}} \;\; \hat{a}_1^{\dagger n_1} \ldots \hat{a}_N^{\dagger n_N} \; |\vec{0}\rangle$$

$$|\psi\rangle_{GEP} = \hat{U}_G |0\rangle$$

$$|\psi\rangle_{(R,Q)} = \otimes_{i=0}^{N} \hat{S}_i(r) \sum_{\substack{\vec{n} = n_1, \ldots, n_q \\ n_1 + \ldots + n_q \leq R \\ q \leq Q}} d_{\vec{n}} \; \hat{a}_i^{\dagger n_0} \ldots \hat{a}_{i+q}^{\dagger n_q} |0\rangle$$

Energy minimization results in comparable ansatz energies...

$(m^2, \lambda) = (0.6, 1.5)$



...but exhibit distinct levels of non-local correlations and non-gaussianity.

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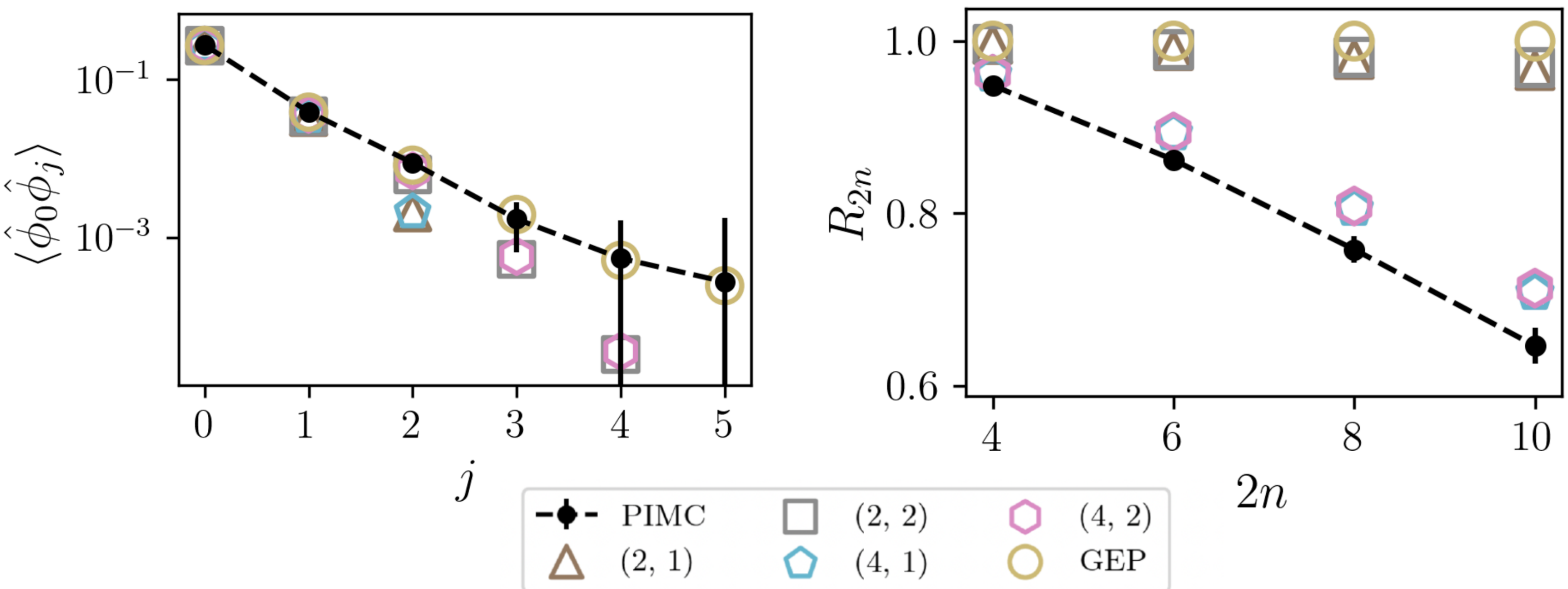
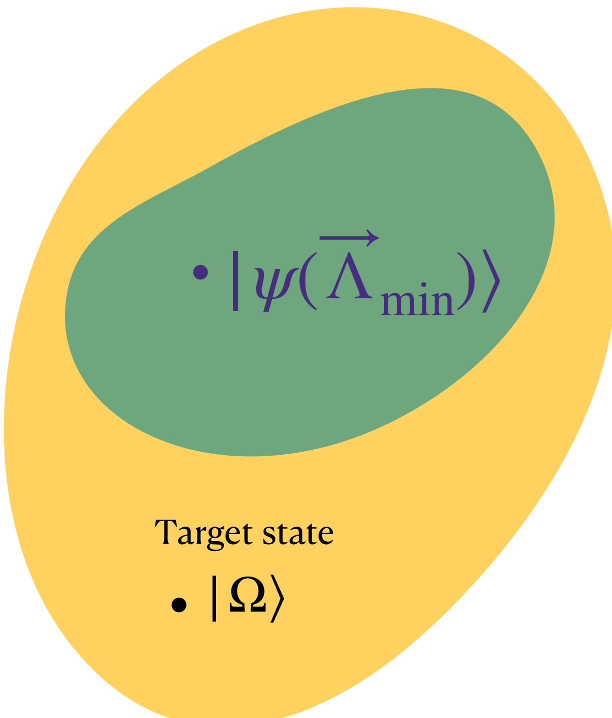
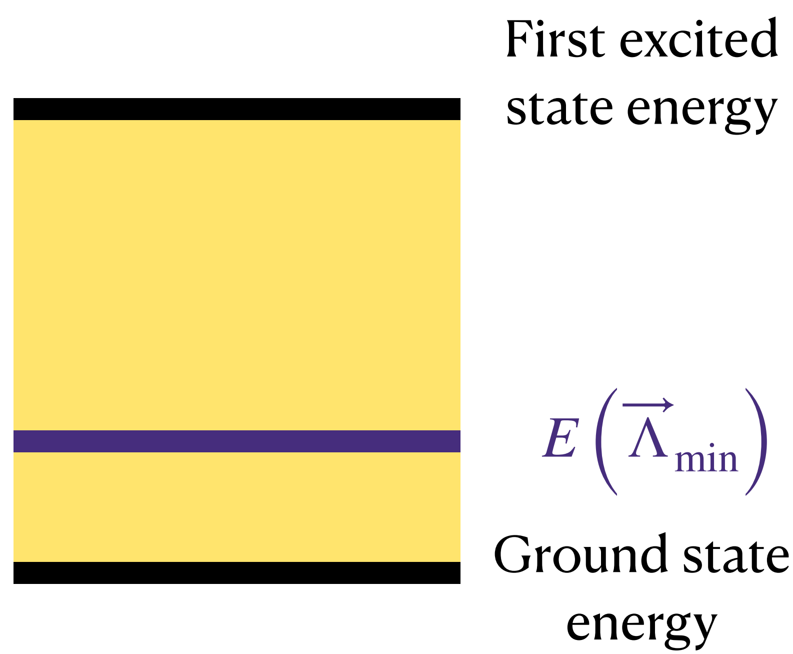


It is desirable to go beyond the single-dimensional metric of energy.





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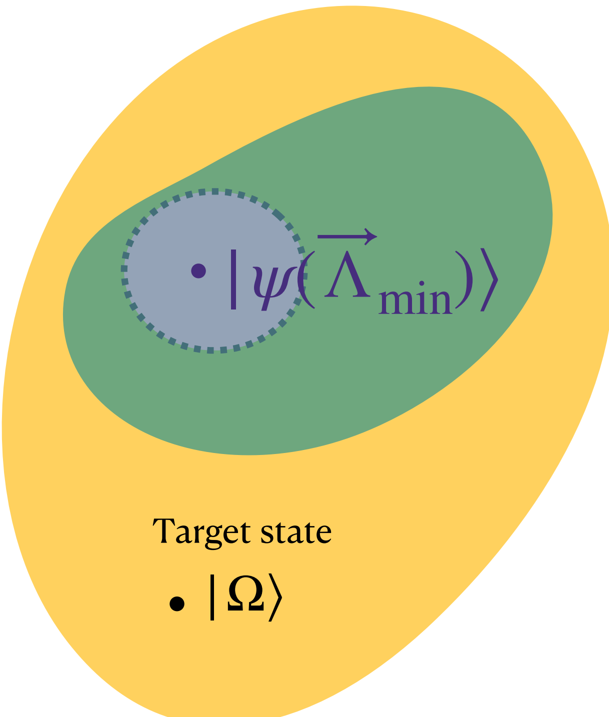
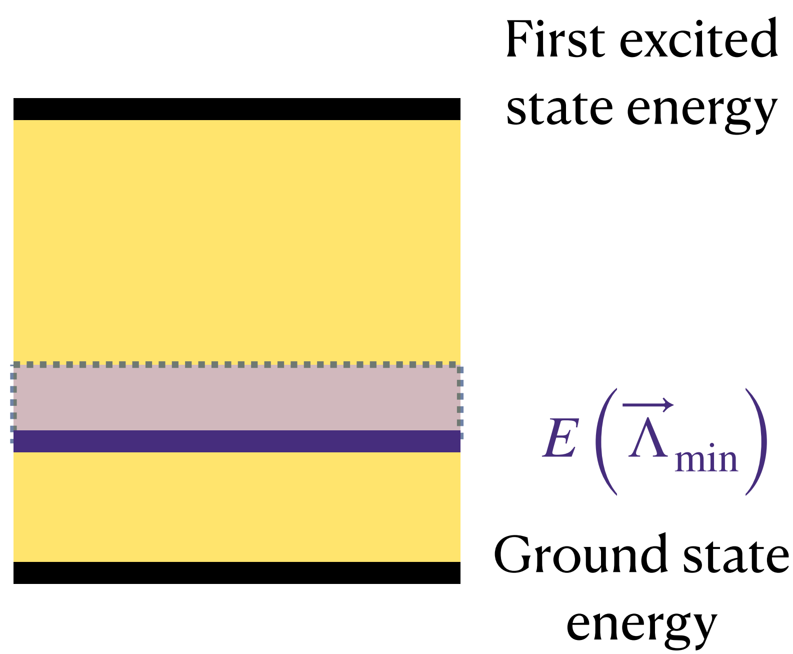
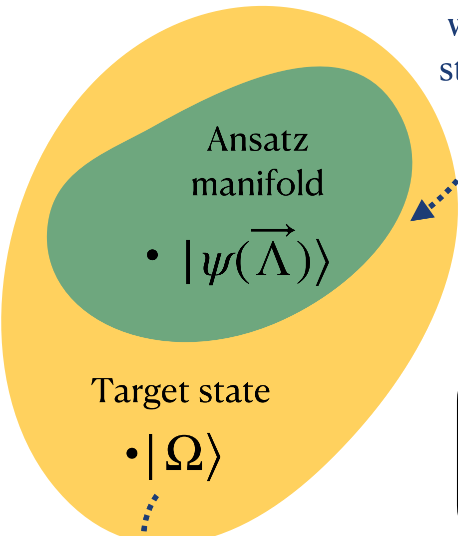
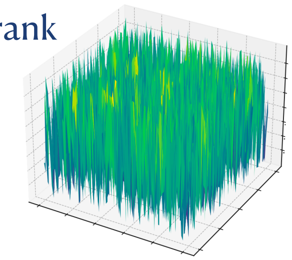




Classically determined quantum circuit for ground states

Bosonic states with finite stellar rank



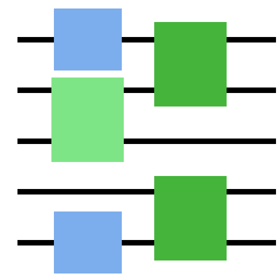


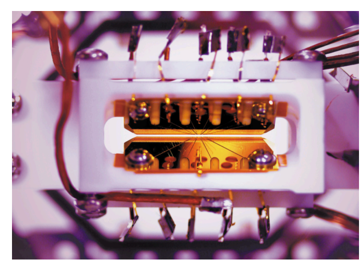
Ground state $(1+1)$D ϕ^4 theory

Path Integral Monte Carlo (PIMC) informed moment optimization

$|\psi(\vec{\Lambda_0})\rangle$

Simulator details

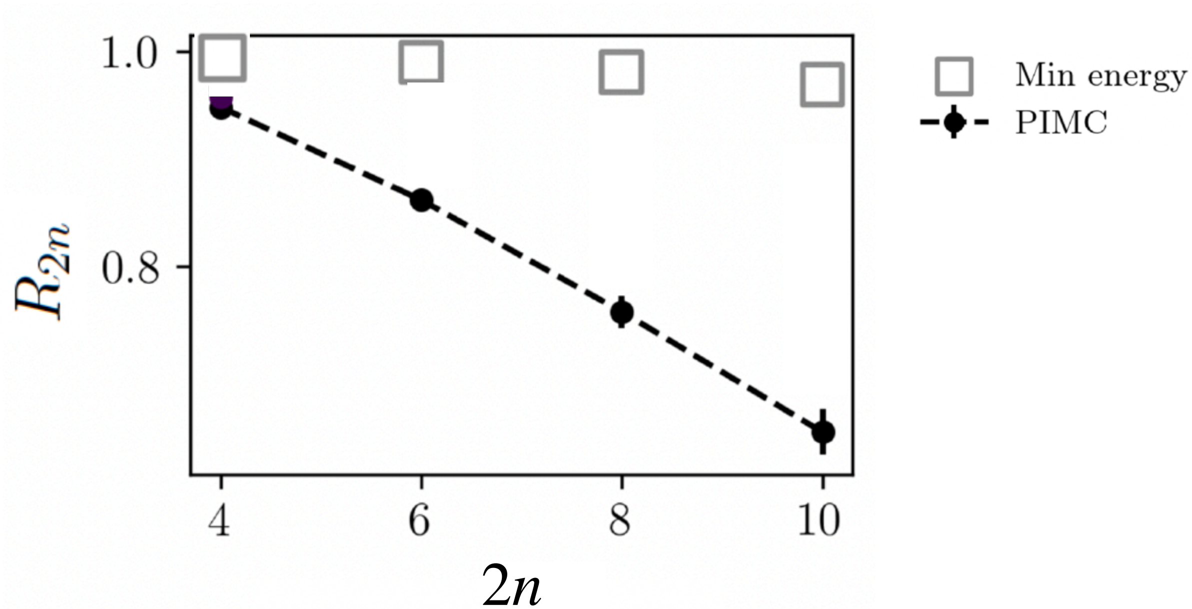




Trapped ion quantum computer from Monroe Lab (UMD, 2016)

PIMC informed moment optimization

$(m^2, \lambda) = (0.6, 1.5),\ (R, Q) = (2,2)$



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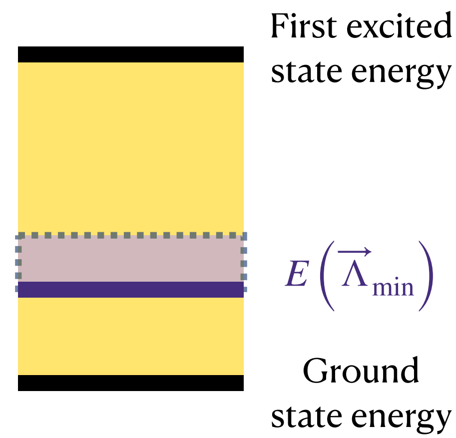

Minimize:

$$\langle \hat{H} \rangle_{\vec{\Lambda}} \quad + \quad w \sum_{2n=6,8,10} \left(\langle \hat{\phi}^{2n} \rangle_{\vec{\Lambda}} - \langle \hat{\phi}^{2n} \rangle_{\text{PIMC}} \right)^2$$

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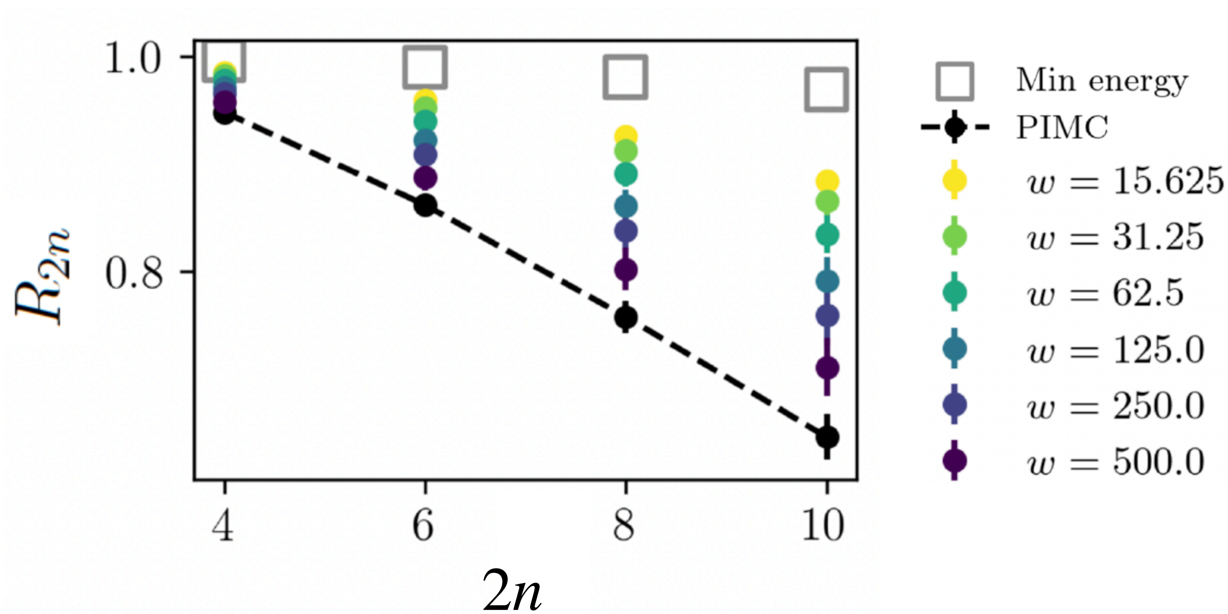

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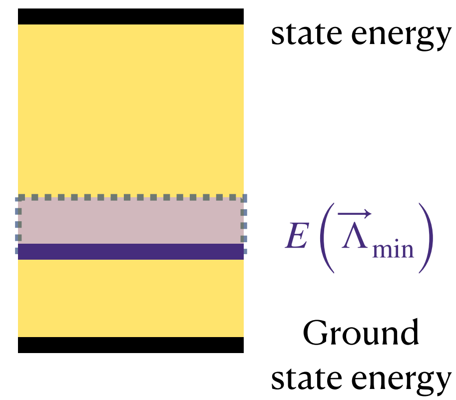

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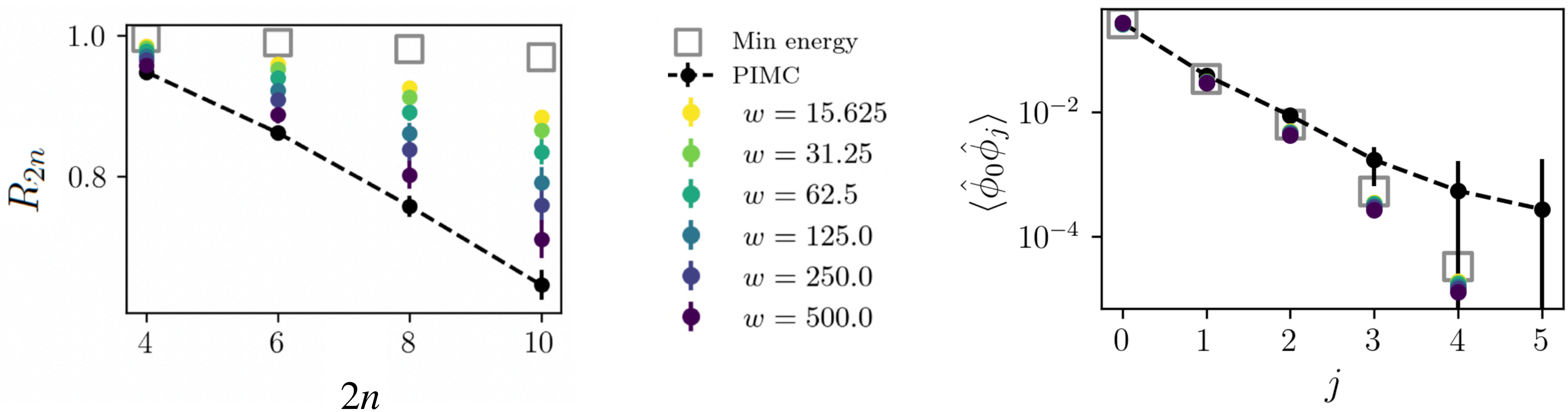

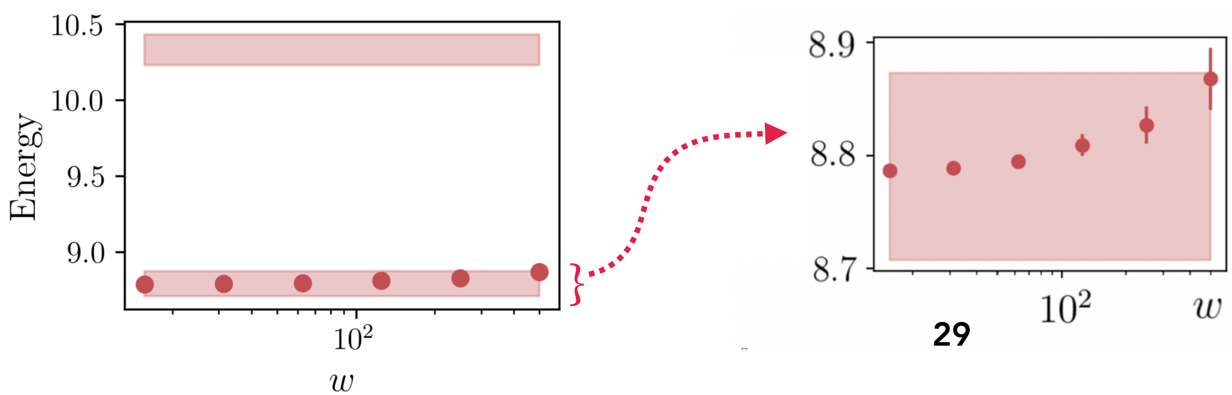

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Classically determined quantum circuit for ground states

Bosonic states with finite stellar rank

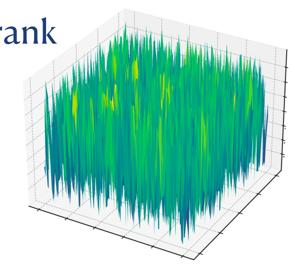
Ansatz manifold

• $|\psi(\overrightarrow{\Lambda})\rangle$

Target state

• $|\Omega\rangle$

Ground state $(1+1)$D ϕ^4 theory



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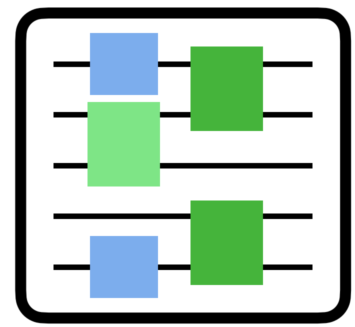
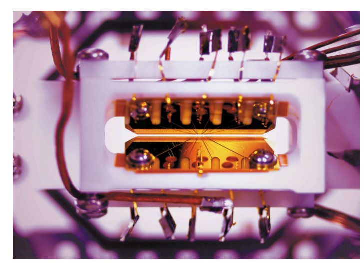
Simulator details

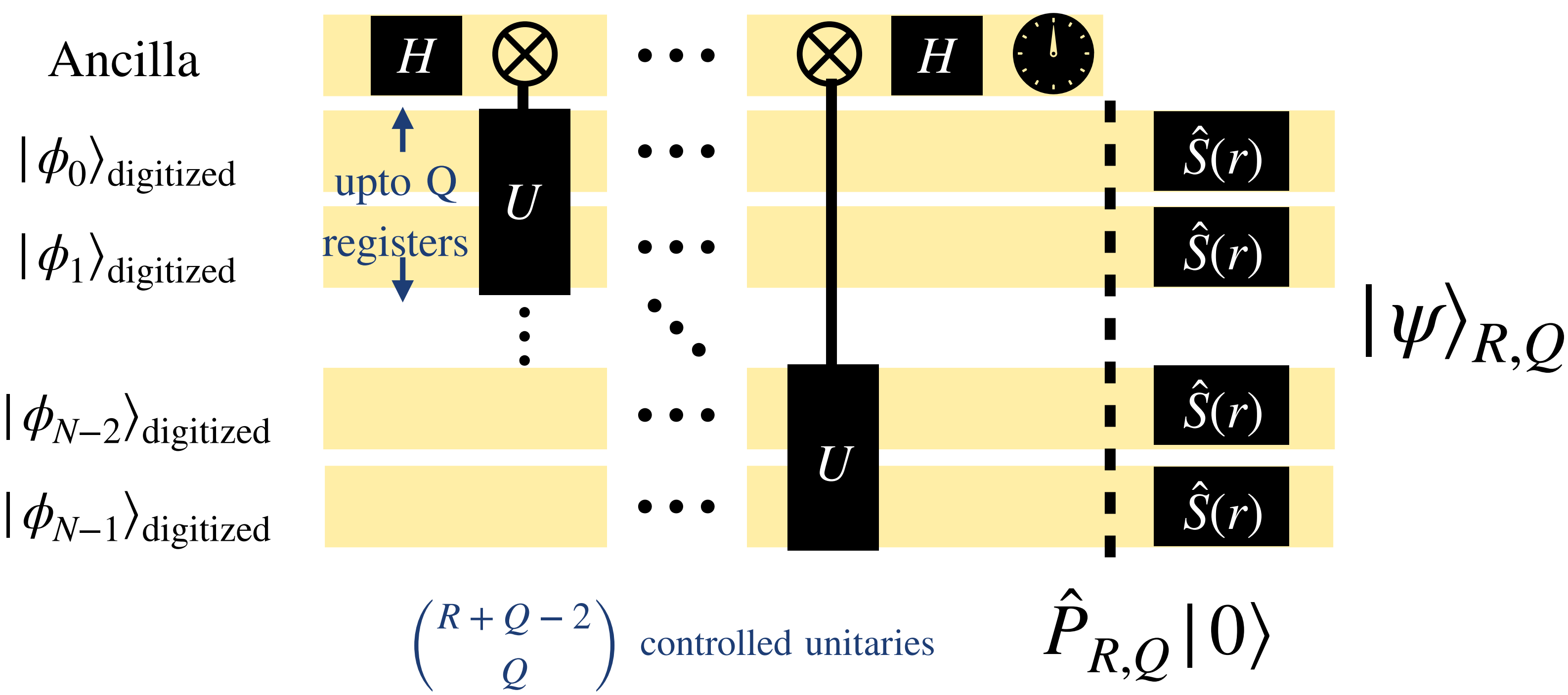




Trapped ion quantum computer from Monroe Lab (UMD, 2016)

Circuit blueprint

$$|\psi\rangle_{(R,Q)} = \hat{U}_G \sum_{\substack{\vec{n}=n_1,\ldots,n_q \\ n_1+\ldots+n_q \le R \\ q \le N/2}} d_{\vec{n}}\ \hat{a}_i^{\dagger n_0} \ldots \hat{a}_{i+q}^{\dagger n_q} |0\rangle_R$$



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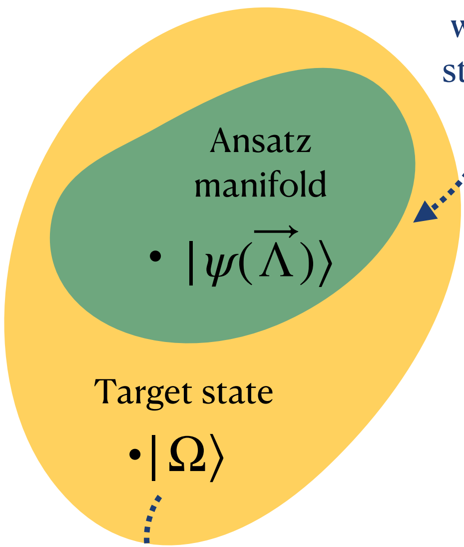

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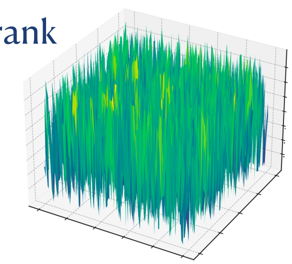

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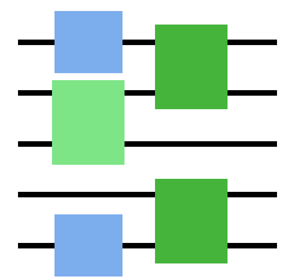

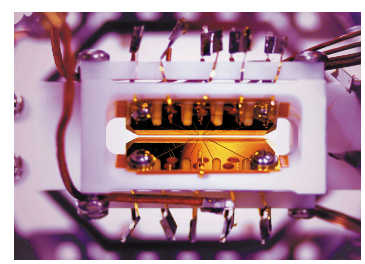

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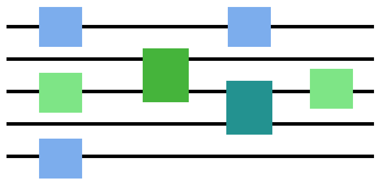
Outlook

- Systematic study of the moment optimization landscape.
- Development of continuous variable state preparation strategies.
- Application of moment optimization to gauge theories.
- Utility of moment optimization in simulations of dynamics.
- Implementation on actual quantum hardware.

Supplementary slides

	Input	Computation		
Variational Quantum Eigensolver	Quantum circuit ansatz 	Minimizing the energy of the state represented by the circuit using hybrid classical and quantum computing.	Optimized state misses certain ground state moments.	Minimal
Path-integral Monte Carlo assisted moment optimization	• Wavefunction ansatz: $\lvert\psi(\vec{\Lambda})\rangle$ • Spectral gap and ground state moments (determined by PIMC)	Optimize $\lvert\psi(\vec{\Lambda})\rangle$ to accurately represent a set of target ground state moments. Determine circuit encoding of moment optimized ansatz.		GROUND STATE KNOWLEDGE NEEDED
Direct circuit encoding	Ground state wavefunction.	Determining the quantum circuit which encodes this known wavefunction.	Wavefunction known only for a limited set of cases.	Maximal