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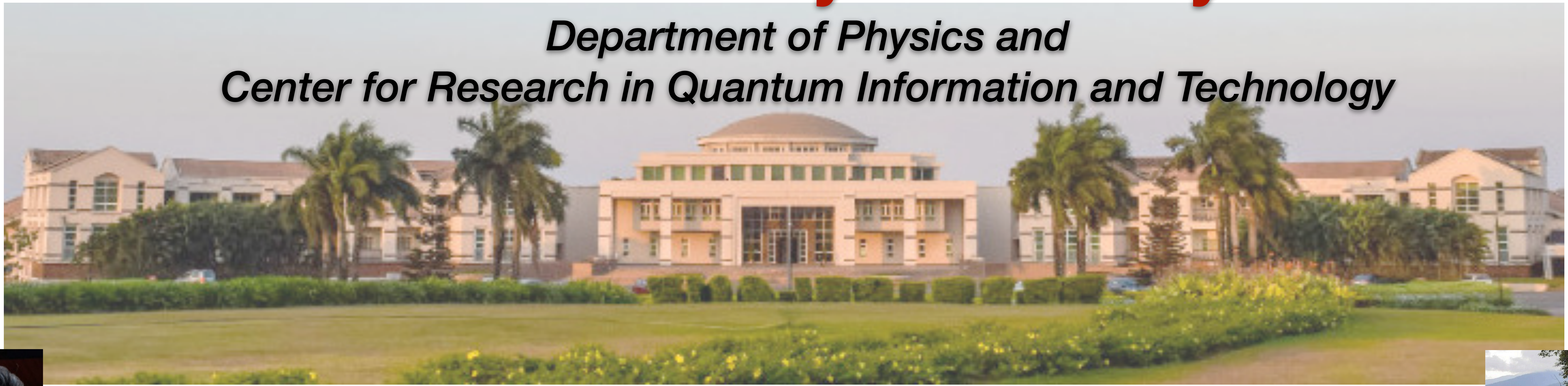
IQuS InQubator for Quantum Simulation

Symmetries of the Loop-string-hadron Framework: Towards Quantum Simulating Gauge Theories

Indrakshi Raychowdhury

Department of Physics and
Center for Research in Quantum Information and Technology



Emil Mathew

BITS Pilani, K K Birla Goa Campus

Saurabh Kadam

July 29, 2024

Ultimate goal: performing LATTICE-QCD calculations using Quantum Computer

Intermediate steps:

- Suitable development and choice of framework.
- Suitable choice of variables/basis.
- Algorithm development for various tasks- classical/quantum/hybrid.
- Quantum information theoretic understanding - connection to physics of QCD
- Quantum advantage - knowledge generation in fundamental laws of nature.

Ultimate goal: performing LATTICE-QCD calculations using Quantum Computer

Current Efforts: For simpler models such as Schwinger model, discrete gauge groups, low dimensional SU(2)/SU(3) gauge theory

Intermediate steps:

- Suitable development and choice of framework. ✓
- Suitable choice of variables/basis.
- Algorithm development for various tasks- classical/quantum/hybrid. ✓
- Quantum information theoretic understanding - connection to physics of QCD
- Quantum advantage - knowledge generation in fundamental laws of nature.

Ultimate goal: performing LATTICE-QCD calculations using Quantum Computer

Current Efforts: For arbitrary dimensional SU(2)/SU(3) gauge theories

Intermediate steps:

- Suitable development and choice of framework. ✓
- Suitable choice of variables/basis. ✓
- Algorithm development for various tasks- classical/quantum/hybrid. ✓
- Quantum information theoretic understanding - connection to physics of QCD ✓
- Quantum advantage - knowledge generation in fundamental laws of nature.

These tasks are difficult for non-Abelian gauge theories

Gauge invariance governed by the local Gauss' law constraints

$$G^{\mathrm{a}}(x)\,|\,\Psi_{phys}\rangle = 0$$

The constraints are preserved in dynamics

$$[H, G^{\mathrm{a}}(x)] = 0 \qquad \forall x, \mathrm{a}$$

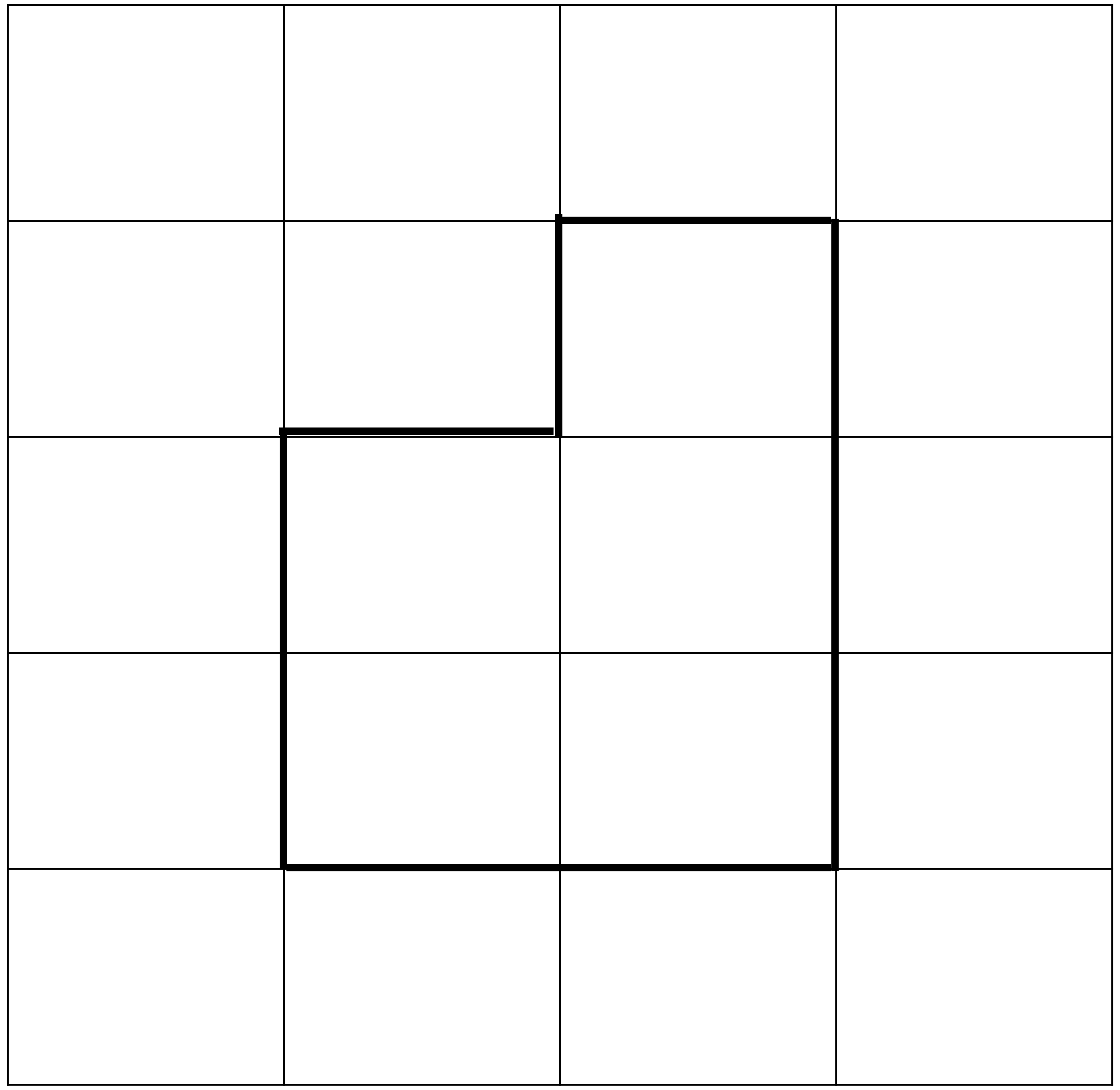
Satisfy non-Abelian SU(2)/SU(3) algebra: mutually non-commuting

SU(2): $\mathrm{a} = 1,2,3.$

SU(3): $\mathrm{a} = 1,2,3,...,8.$

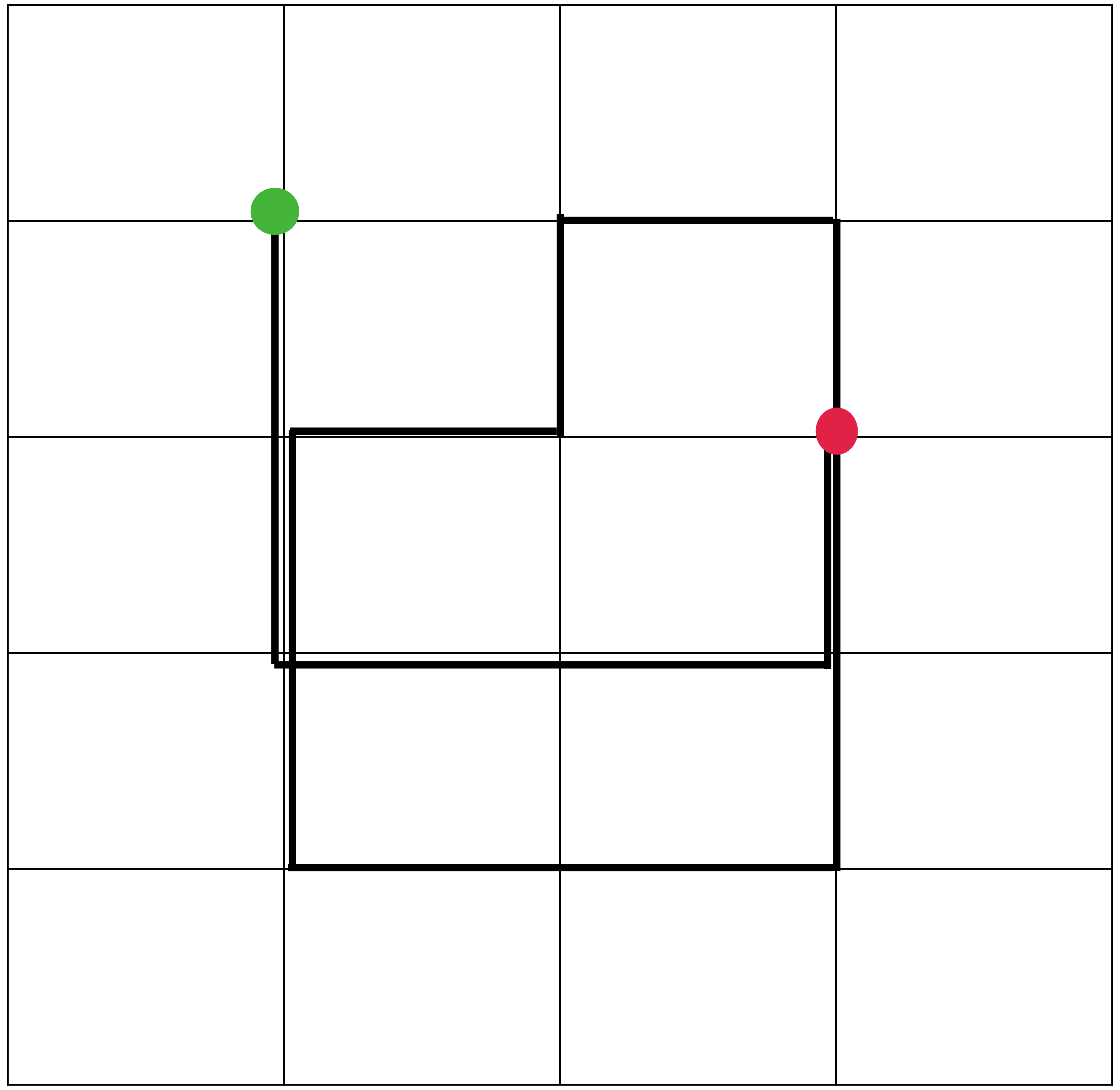
Global Symmetries: global SU(2)/SU(3) charges; discrete symmetries

Loops-Strings-Hadrons : SU(2)



Wilson loops

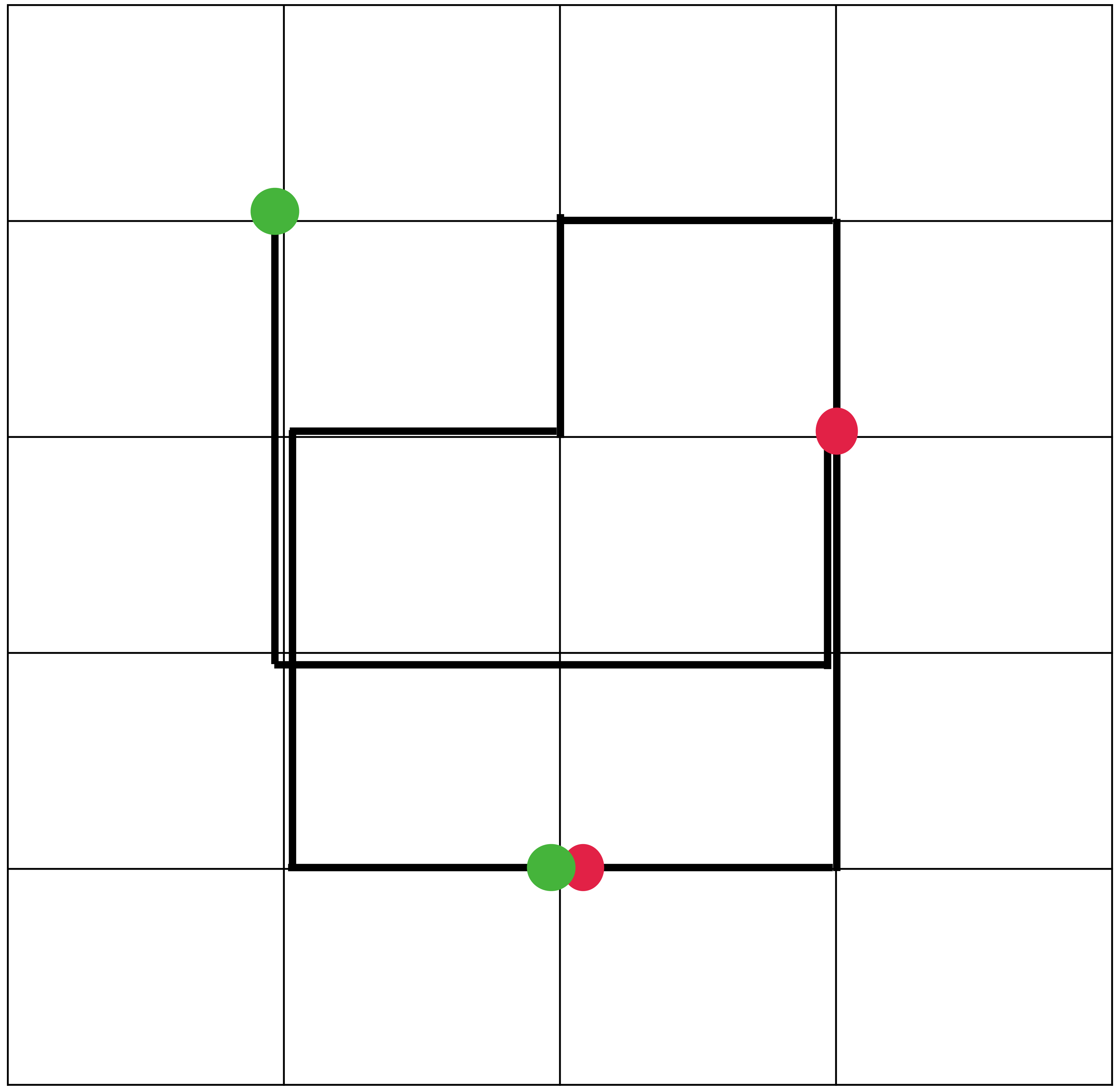
Loops-Strings-Hadrons : SU(2)



Wilson loops

Strings/mesons

Loops-Strings-Hadrons : SU(2)

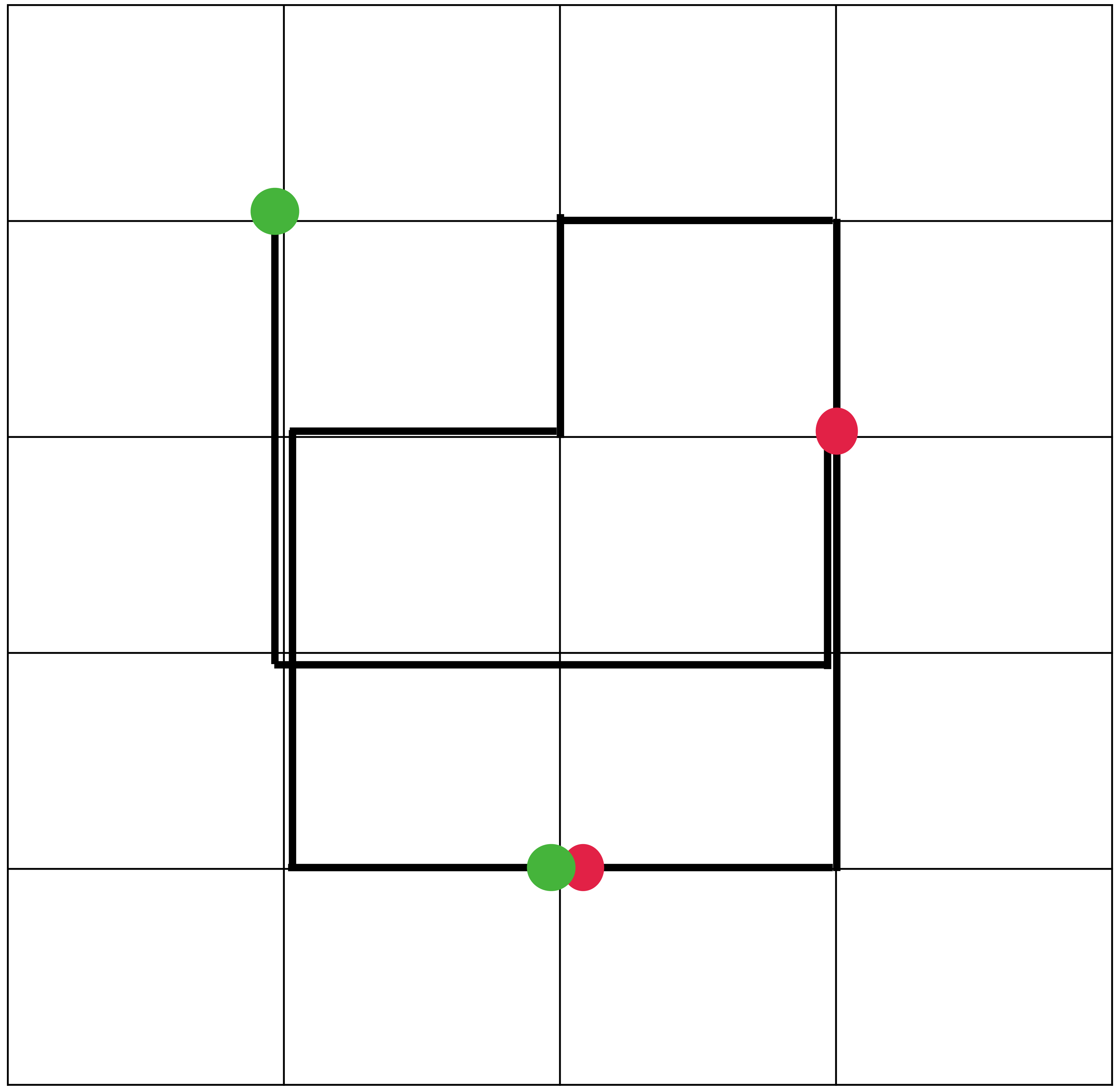


Wilson loops

Strings/mesons

Hadrons

Loops-Strings-Hadrons : SU(2)

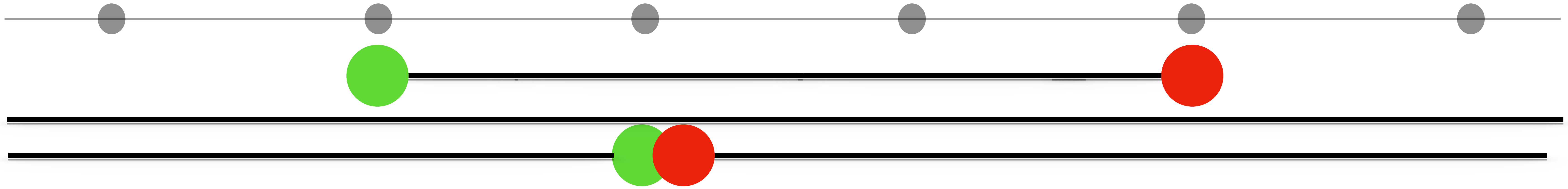


Wilson loops

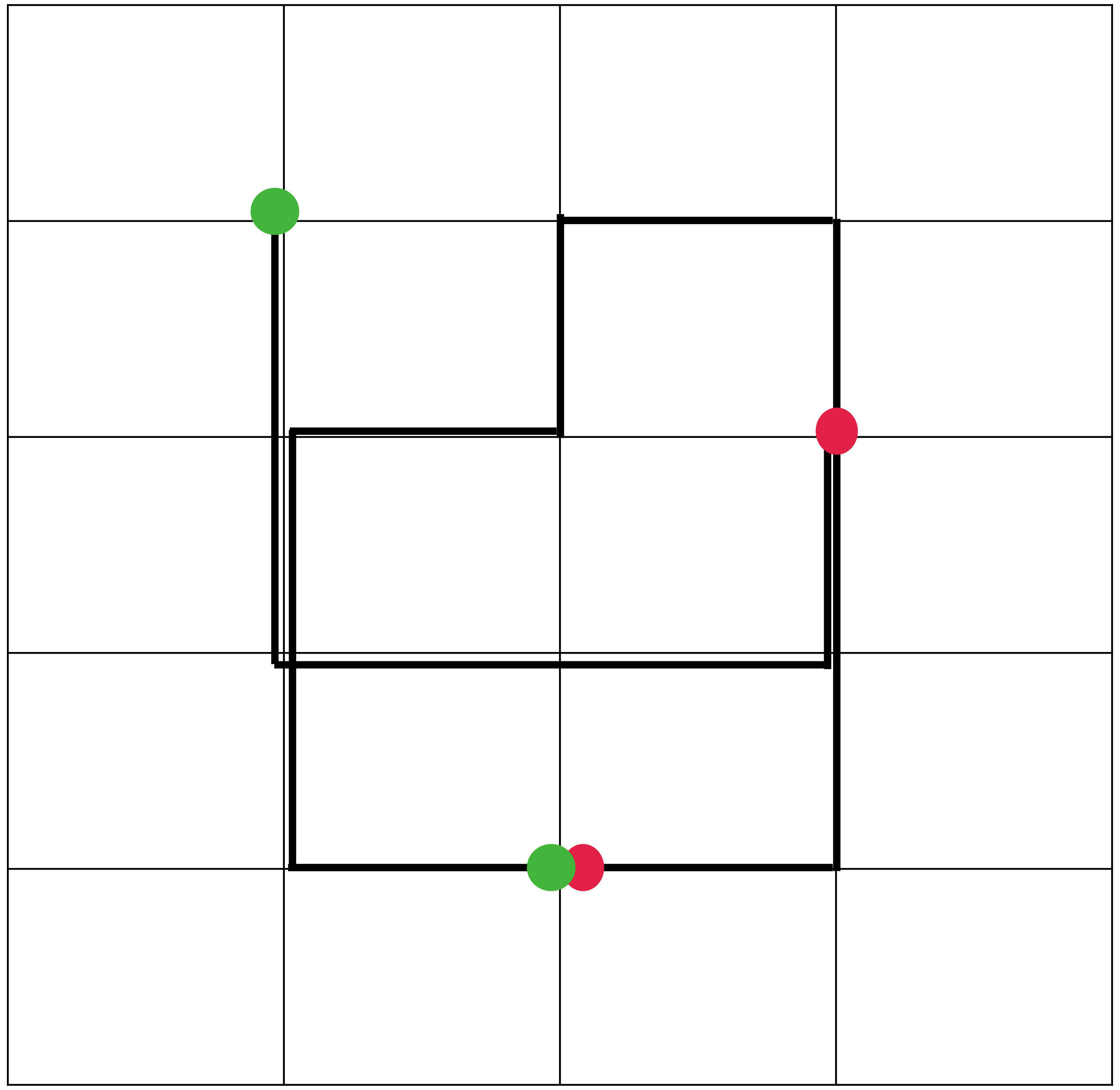
Strings

Hadrons

Loops-Strings-Hadrons : SU(2) in 1+1 d



Loops-Strings-Hadrons : SU(2)



Wilson loops

Strings

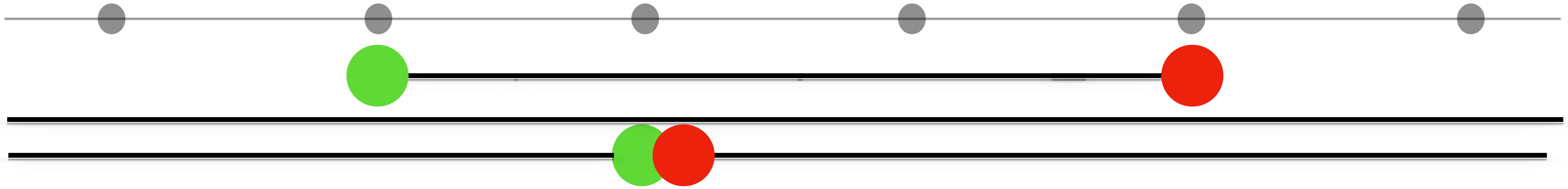
Hadrons

Gauge invariance leads to non-locality

On site snapshots of gauge invariant configurations

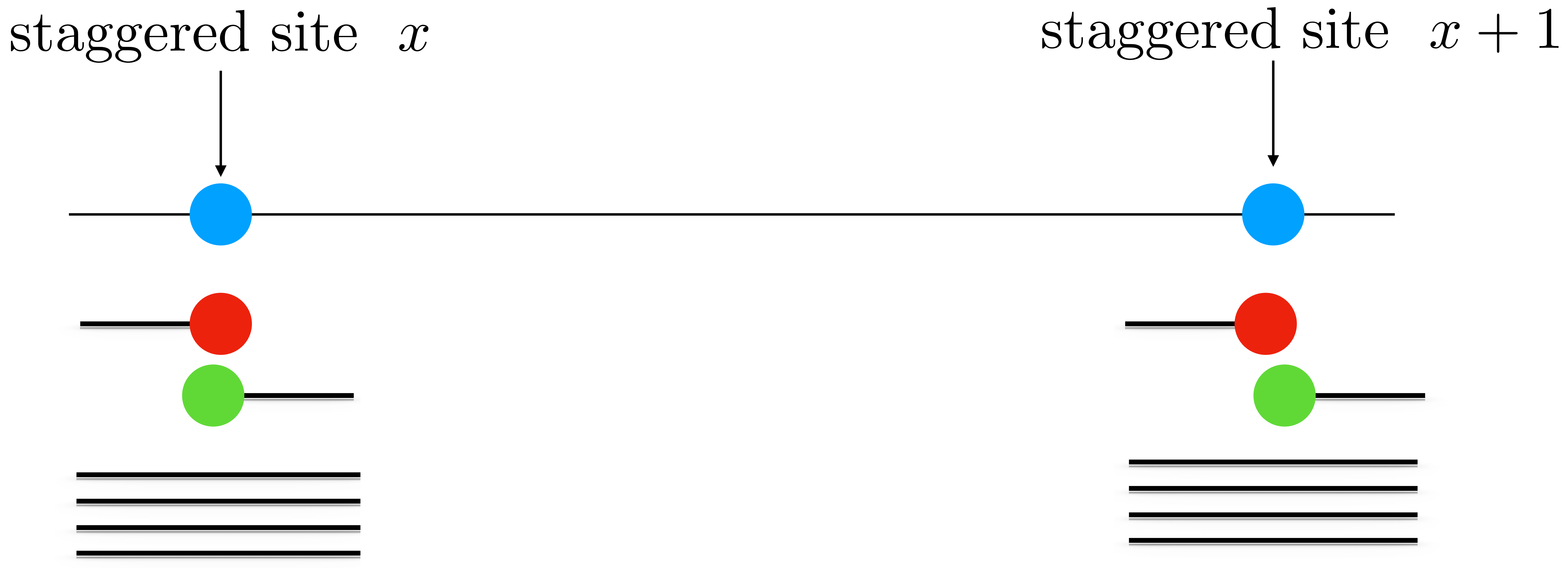
LOOP STRING HADRON
(LSH framework)

Loops-Strings-Hadrons : SU(2) in 1+1 d



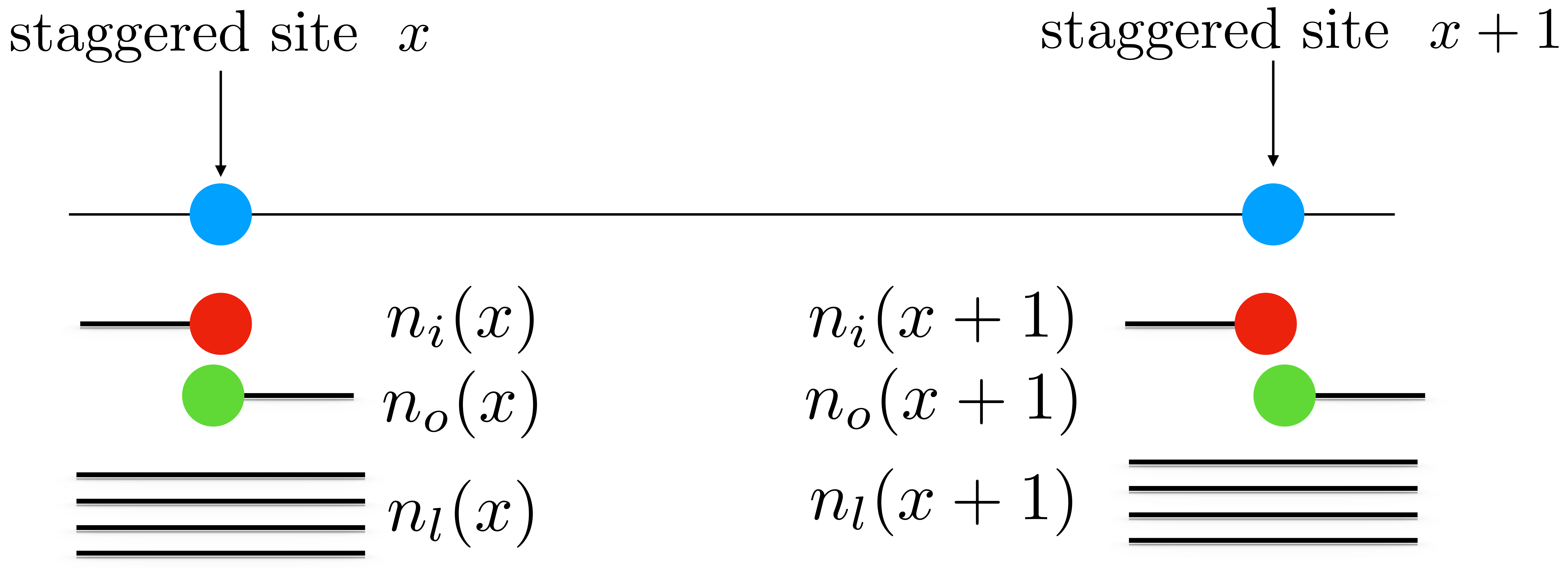
Loops-Strings-Hadrons : SU(2) in 1+1 d

On site snapshots of gauge invariant configurations



Loops-Strings-Hadrons : SU(2) in 1+1 d

On site snapshots of gauge invariant configurations



Loops-Strings-Hadrons : SU(2) in 1+1 d

Global LSH states are constructed by imposing Abelian Gauss Law constraints

Non-locality remains crucial, but is then care by Abelian constraints

Loops-Strings-Hadrons Framework

Local non-Abelian constraints are solved analytically by construction: LSH formalism is manifestly SU(2)/SU(3) invariant

Local constraint structure: U(1), always as in 1d even for higher dimensional LSH

↓

Global constraint structure: Multiple U(1)

Loops-Strings-Hadrons Framework

Local non-Abelian constraints are solved analytically by construction: LSH formalism is manifestly SU(2)/SU(3) invariant

Local constraint structure: U(1), always as in 1d even for higher dimensional LSH

↓

Global constraint structure: Multiple U(1)

Useful for both theoretical analysis and classical/quantum computation

Global constraint structure: SU(2) in 1 spatial dimension

$$H_I^{(\mathrm{LSH})} = \frac{1}{2a}\sum_n \left\{ \frac{1}{\sqrt{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x)) + 1}} \times \left[\hat{S}_o^{++}(x)\hat{S}_i^{+-}(x+1) + \hat{S}_o^{+-}(x)\hat{S}_i^{--}(x+1) \right] \times \frac{1}{\sqrt{\hat{n}_l(x+1) + \hat{n}_i(x+1)(1-\hat{n}_o(x+1)) + 1}} + \mathrm{h.c.} \right\},$$

$$\begin{aligned}
\hat{S}_o^{++} &= \hat{\chi}_o^{+}(\lambda^{+})^{\hat{n}_i}\sqrt{\hat{n}_l + 2 - \hat{n}_i}, & \hat{S}_i^{+-} &= \hat{\chi}_o^{-}(\lambda^{+})^{1-\hat{n}_i}\sqrt{\hat{n}_l + 1 + \hat{n}_i}, \\
\hat{S}_o^{--} &= \hat{\chi}_o^{-}(\lambda^{-})^{\hat{n}_i}\sqrt{\hat{n}_l + 2(1 - \hat{n}_i)}, & \hat{S}_i^{-+} &= \hat{\chi}_o^{+}(\lambda^{-})^{1-\hat{n}_i}\sqrt{\hat{n}_l + 2\hat{n}_i}, \\
\hat{S}_o^{+-} &= \hat{\chi}_i^{+}(\lambda^{-})^{1-\hat{n}_o}\sqrt{\hat{n}_l + 2\hat{n}_o}, & \hat{S}_i^{--} &= \hat{\chi}_i^{-}(\lambda^{-})^{\hat{n}_o}\sqrt{\hat{n}_l + 2(1 - \hat{n}_o)}, \\
\hat{S}_o^{-+} &= \hat{\chi}_i^{-}(\lambda^{+})^{1-\hat{n}_o}\sqrt{\hat{n}_l + 1 + \hat{n}_o}, & \hat{S}_i^{++} &= \hat{\chi}_i^{+}(\lambda^{+})^{\hat{n}_o}\sqrt{\hat{n}_l + 2 - \hat{n}_o}.
\end{aligned}$$

Ladder operator for n_o

Ladder operator for n_i

→ Global U(1) charges

$$\sum n_i \qquad \sum n_o$$

or,

$$Q = \sum_{x=0}^{N-1}[n_i(x) + n_o(x)]$$

$$q = \sum_{x=0}^{N-1}[n_o(x) - n_i(x)]$$

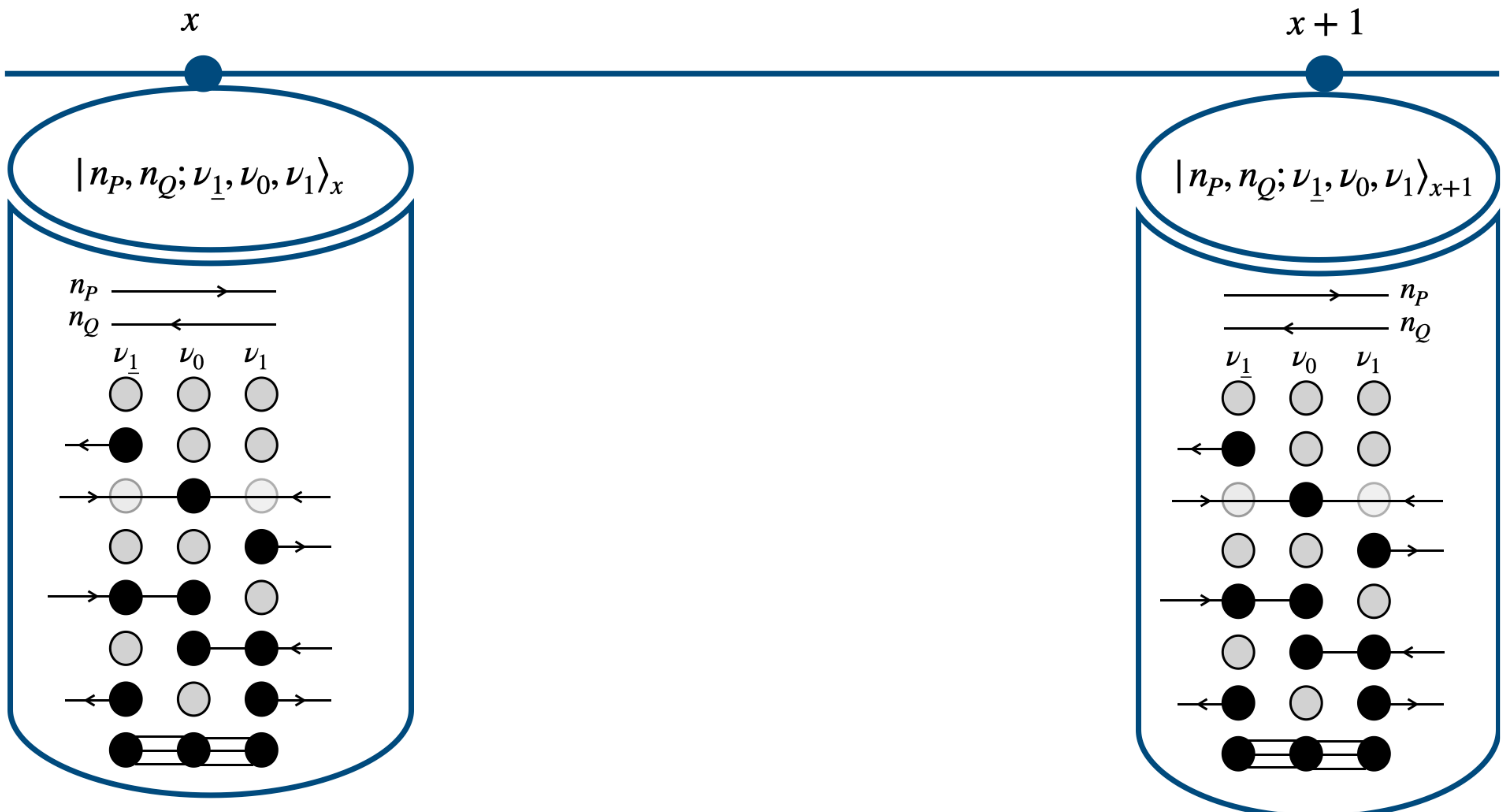
For a particular Q value, q can take any value from $-Q$ to $+Q$ and defines different disconnected sectors of the larger gauge-invariant LSH Hilbert space.

Loops-Strings-Hadrons : SU(3) in 1+1 d

PHYSICAL REVIEW D **107**, 094513 (2023)

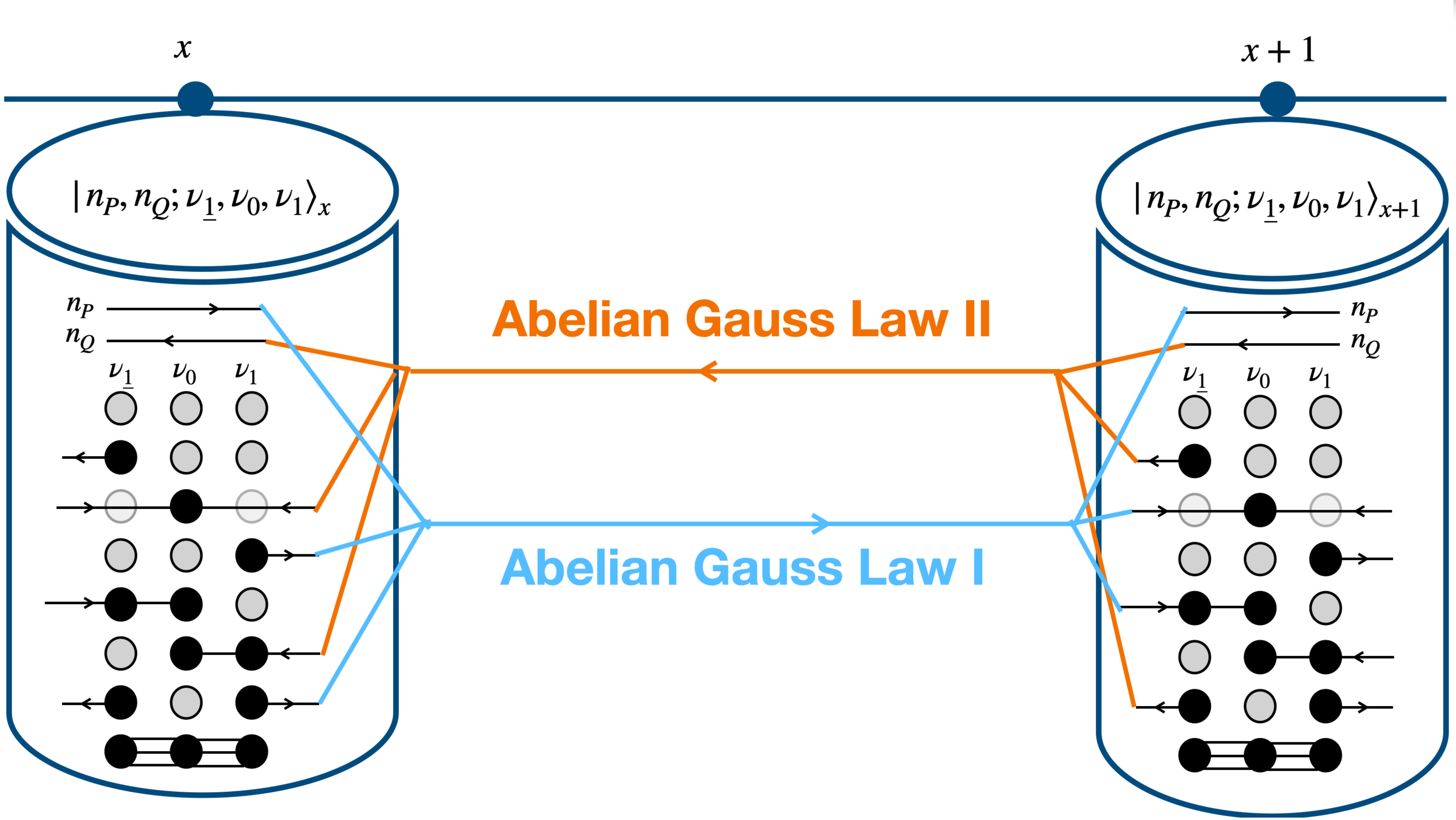
Loop-string-hadron formulation of an SU(3) gauge theory with dynamical quarks

Saurabh V. Kadam,[1,*] Indrakshi Raychowdhury,[2,†] and Jesse R. Stryker[1,‡]
[1]*Maryland Center for Fundamental Physics, University of Maryland, College Park, Maryland 20742, USA*
[2]*Department of Physics, BITS-Pilani, K K Birla Goa Campus, Zuarinagar, Goa 403726, India*



Loops-Strings-Hadrons : SU(2) in 1+1 d

Global LSH states are constructed by imposing a pair of Abelian Gauss Law constraints



Global constraint structure: SU(3) in 1 spatial dimension

$$\psi^\dagger_\alpha(r)\, U^\alpha{}_\beta(r)\, \psi^\beta(r+1)$$

H_I

A set of non-trivial constructions are involved

$$x\left[\hat\chi_1^\dagger(\hat\Gamma_P^\dagger)^{\hat\nu_0}\sqrt{1-\hat\nu_0/(\hat n_P+2)}\sqrt{1-\hat\nu_{\underline{1}}/(\hat n_P+\hat n_Q+3)}\,\right]_r \otimes \left[\sqrt{1+\hat\nu_0/(\hat n_P+1)}\sqrt{1+\hat\nu_{\underline{1}}/(\hat n_P+\hat n_Q+2)}\,\hat\chi_1(\hat\Gamma_P^\dagger)^{1-\hat\nu_0}\right]_{r+1}$$

$$+\,x\left[\hat\chi_{\underline{1}}^\dagger(\hat\Gamma_Q)^{1-\hat\nu_0}\sqrt{1+\hat\nu_0/(\hat n_Q+1)}\sqrt{1+\hat\nu_1/(\hat n_P+\hat n_Q+2)}\,\right]_r \otimes \left[\sqrt{1-\hat\nu_0/(\hat n_Q+2)}\sqrt{1-\hat\nu_1/(\hat n_P+\hat n_Q+3)}\,\hat\chi_{\underline{1}}(\hat\Gamma_Q)^{\hat\nu_0}\right]_{r+1}$$

$$+\,x\left[\hat\chi_0^\dagger(\hat\Gamma_P)^{1-\hat\nu_1}(\hat\Gamma_Q^\dagger)^{\hat\nu_{\underline{1}}}\sqrt{1+\hat\nu_1/(\hat n_P+1)}\sqrt{1-\hat\nu_{\underline{1}}/(\hat n_Q+2)}\,\right]_r \otimes \left[\sqrt{1-\hat\nu_1/(\hat n_P+2)}\sqrt{1+\hat\nu_{\underline{1}}/(\hat n_Q+1)}\,\hat\chi_0(\hat\Gamma_P)^{\hat\nu_1}(\hat\Gamma_Q^\dagger)^{1-\hat\nu_{\underline{1}}}\right]_{r+1}$$

Global constraint structure:

SU(3) in 1 spatial dimension

$$H_I = \sum_{r=1}^{N'} H_I(r) \equiv \sum_r x \left[\hat{\chi}_1^\dagger (\hat{\Gamma}_P^\dagger)^{\hat{\nu}_0} \sqrt{1-\hat{\nu}_0/(\hat{n}_P+2)}\sqrt{1-\hat{\nu}_{\underline{1}}/(\hat{n}_P+\hat{n}_Q+3)}\right]_r \otimes \left[\sqrt{1+\hat{\nu}_0/(\hat{n}_P+1)}\sqrt{1+\hat{\nu}_{\underline{1}}/(\hat{n}_P+\hat{n}_Q+2)}\,\hat{\chi}_1 (\hat{\Gamma}_P^\dagger)^{1-\hat{\nu}_0}\right]_{r+1}$$

$$+ x \left[\hat{\chi}_{\underline{1}}^\dagger (\hat{\Gamma}_Q)^{1-\hat{\nu}_0} \sqrt{1+\hat{\nu}_0/(\hat{n}_Q+1)}\sqrt{1+\hat{\nu}_1/(\hat{n}_P+\hat{n}_Q+2)}\right]_r \otimes \left[\sqrt{1-\hat{\nu}_0/(\hat{n}_Q+2)}\sqrt{1-\hat{\nu}_1/(\hat{n}_P+\hat{n}_Q+3)}\,\hat{\chi}_{\underline{1}} (\hat{\Gamma}_Q)^{\hat{\nu}_0}\right]_{r+1}$$

$$+ x \left[\hat{\chi}_0^\dagger (\hat{\Gamma}_P)^{1-\hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{\hat{\nu}_{\underline{1}}} \sqrt{1+\hat{\nu}_1/(\hat{n}_P+1)}\sqrt{1-\hat{\nu}_{\underline{1}}/(\hat{n}_Q+2)}\right]_r \otimes \left[\sqrt{1-\hat{\nu}_1/(\hat{n}_P+2)}\sqrt{1+\hat{\nu}_{\underline{1}}/(\hat{n}_Q+1)}\,\hat{\chi}_0 (\hat{\Gamma}_P)^{\hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{1-\hat{\nu}_{\underline{1}}}\right]_{r+1}$$

Global conserved charges:

$$\sum_{r=1}^{N} \nu_{\underline{1}}(r), \quad \sum_{r=1}^{N} \nu_0(r), \quad \sum_{r=1}^{N} \nu_1(r)$$

Three U(1) charges

Or,

$$\mathcal{F} = \sum_{r=1}^{N} \left(\nu_{\underline{1}}(r) + \nu_0(r) + \nu_1(r)\right)$$

$$\Delta\mathcal{P} = \sum_{r=1}^{N} \left(\nu_1(r) - \nu_0(r)\right),$$

$$\Delta\mathcal{Q} = \sum_{r=1}^{N} \left(\nu_0(r) - \nu_{\underline{1}}(r)\right),$$

$$(\mathcal{P}_f, \mathcal{Q}_f) = (\mathcal{P}_0 + \Delta\mathcal{P}, \mathcal{Q}_0 + \Delta\mathcal{Q})$$

For each U(1) global symmetry sector: still remain degeneracies

...Due to Discrete Symmetries

Translation symmetry for periodic boundary condition

along with charge conjugation symmetry.

Importance of global symmetries

Identifying block diagonal structure:

working with blocks of various sizes are feasible as per available computing capacity

Blocks of varying dimensions:

Example: 12 site SU(2)

1-d spatial lattice with

open boundary condition

N = 12 and Q_in = 0 with dim(N) = 5200300

		q												
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Q	**0**	1	0	0	0	0	0	0	0	0	0	0	0	0
	1	0	12	0	0	0	0	0	0	0	0	0	0	0
	2	78	0	66	0	0	0	0	0	0	0	0	0	0
	3	0	572	0	220	0	0	0	0	0	0	0	0	0
	4	1716	0	2145	0	495	0	0	0	0	0	0	0	0
	5	0	8580	0	5148	0	792	0	0	0	0	0	0	0
	6	15730	0	23166	0	8580	0	924	0	0	0	0	0	0
	7	0	56628	0	41184	0	10296	0	792	0	0	0	0	0
	8	70785	0	113256	0	51480	0	9009	0	495	0	0	0	0
	9	0	188760	0	151008	0	46332	0	5720	0	220	0	0	0
	10	169884	0	283140	0	141570	0	30030	0	2574	0	66	0	0
	11	0	339768	0	283140	0	94380	0	13728	0	780	0	12	0
	12	226512	0	382239	0	196625	0	44044	0	4212	0	143	0	1
	13	0	339768	0	283140	0	94380	0	13728	0	780	0	12	0
	14	169884	0	283140	0	141570	0	30030	0	2574	0	66	0	0
	15	0	188760	0	151008	0	46332	0	5720	0	220	0	0	0
	16	70785	0	113256	0	51480	0	9009	0	495	0	0	0	0
	17	0	56628	0	41184	0	10296	0	792	0	0	0	0	0
	18	15730	0	23166	0	8580	0	924	0	0	0	0	0	0
	19	0	8580	0	5148	0	792	0	0	0	0	0	0	0
	20	1716	0	2145	0	495	0	0	0	0	0	0	0	0
	21	0	572	0	220	0	0	0	0	0	0	0	0	0
	22	78	0	66	0	0	0	0	0	0	0	0	0	0
	23	0	12	0	0	0	0	0	0	0	0	0	0	0
	24	1	0	0	0	0	0	0	0	0	0	0	0	0

Has been extensively used in MPS calculations

Tensor-network Toolbox for probing dynamics of non-Abelian Gauge Theories

Aug 2, 2024, 3:35 PM

20m

Talk | Theoretical Develop... | Theoretical developme...

Speaker

Emil Mathew (BITS Pilani KK Birla Goa Campus)

Collaborators at:

UNIVERSITY of WASHINGTON

UNIVERSITY OF MARYLAND 1856

IQuS InQubator for Quantum Simulation

BERKELEY LAB Lawrence Berkeley National Laboratory

Also being extensively used in an ongoing work on thermalisation properties for non-Abelian gauge theories

Collaborators at:

Universität Regensburg

UNIVERSITY of WASHINGTON

IQuS InQubator for Quantum Simulation

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

LSH specific advantage of global symmetries

PHYSICAL REVIEW D **106,** 054510 (2022)

Protecting local and global symmetries in simulating (1 + 1)D non-Abelian gauge theories

Emil Mathew[*] and Indrakshi Raychowdhury[†]

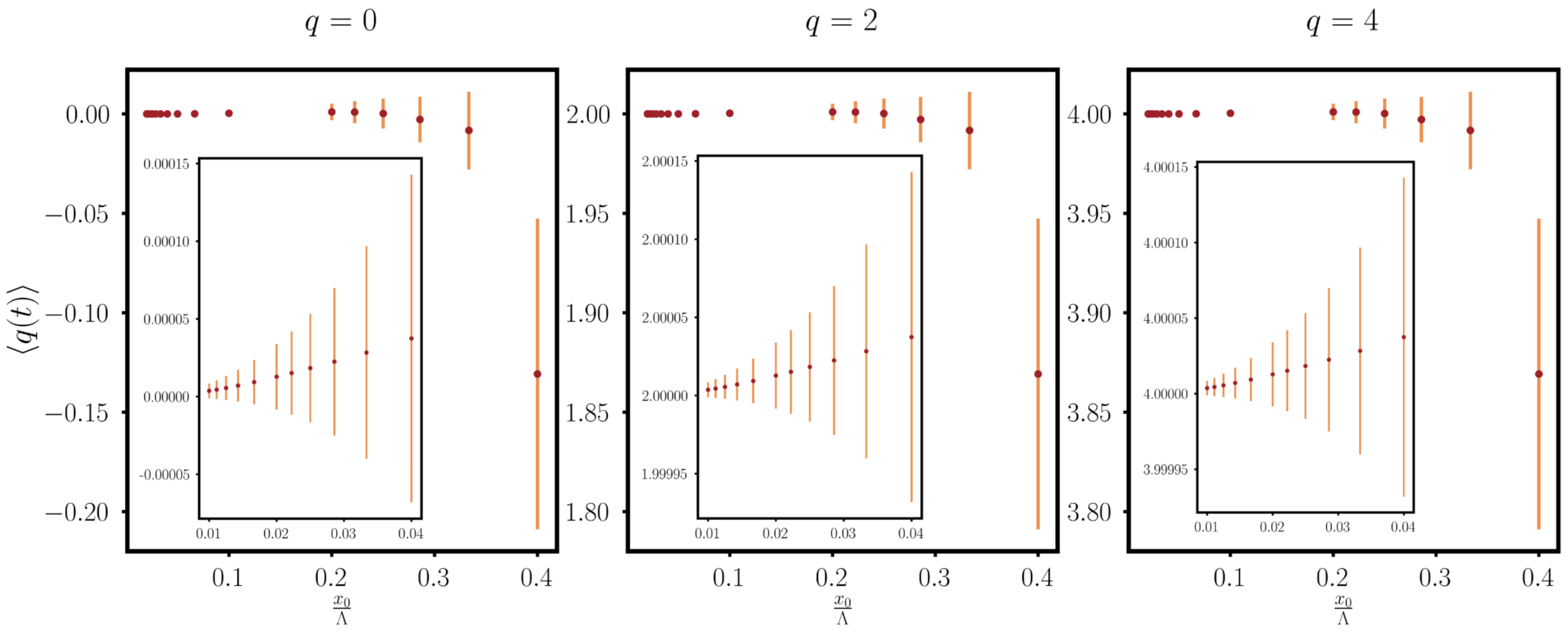
Department of Physics, BITS-Pilani, K K Birla Goa Campus, Zuarinagar, Goa 403726, India

SU(2):

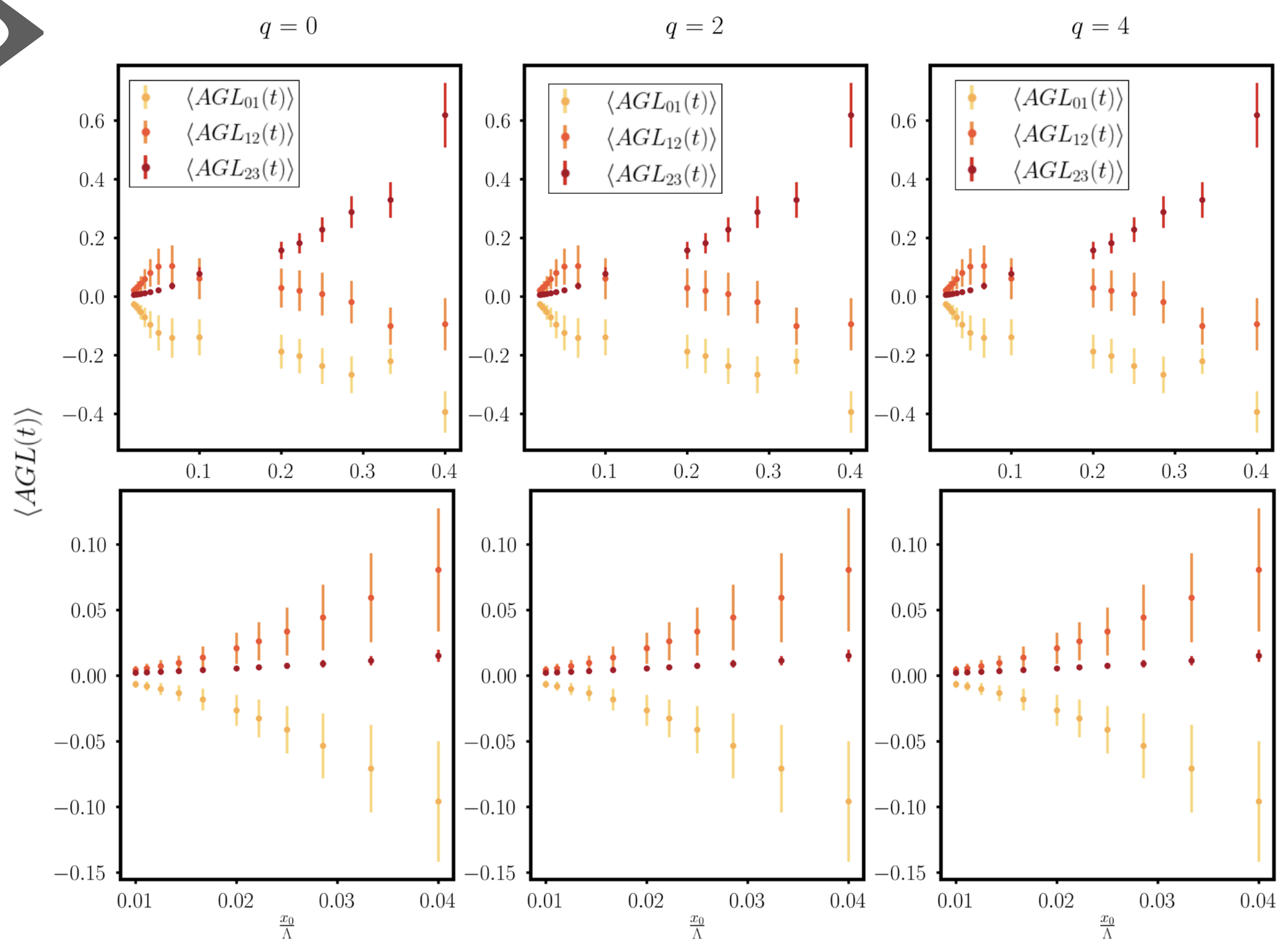
Global U(1) symmetries arise manifestly for the Hamiltonian to preserve the Abelian Gauss laws

LSH specific advantage of global symmetries

SU(2):



Complete protection of all the local symmetries



Protection of global symmetries

Quantum simulation of non-Abelian gauge theory without imposing any local constraint is possible

LSH specific advantage of global symmetries

SU(3):

~~Global U(1) symmetries arise manifestly for the Hamiltonian to preserve the Abelian Gauss laws~~

Yet, global symmetries are found to be connected to the local Abelian Gauss laws.

arXiv:2404.12158v1

Protecting gauge symmetries in the the dynamics of SU(3) lattice gauge theories

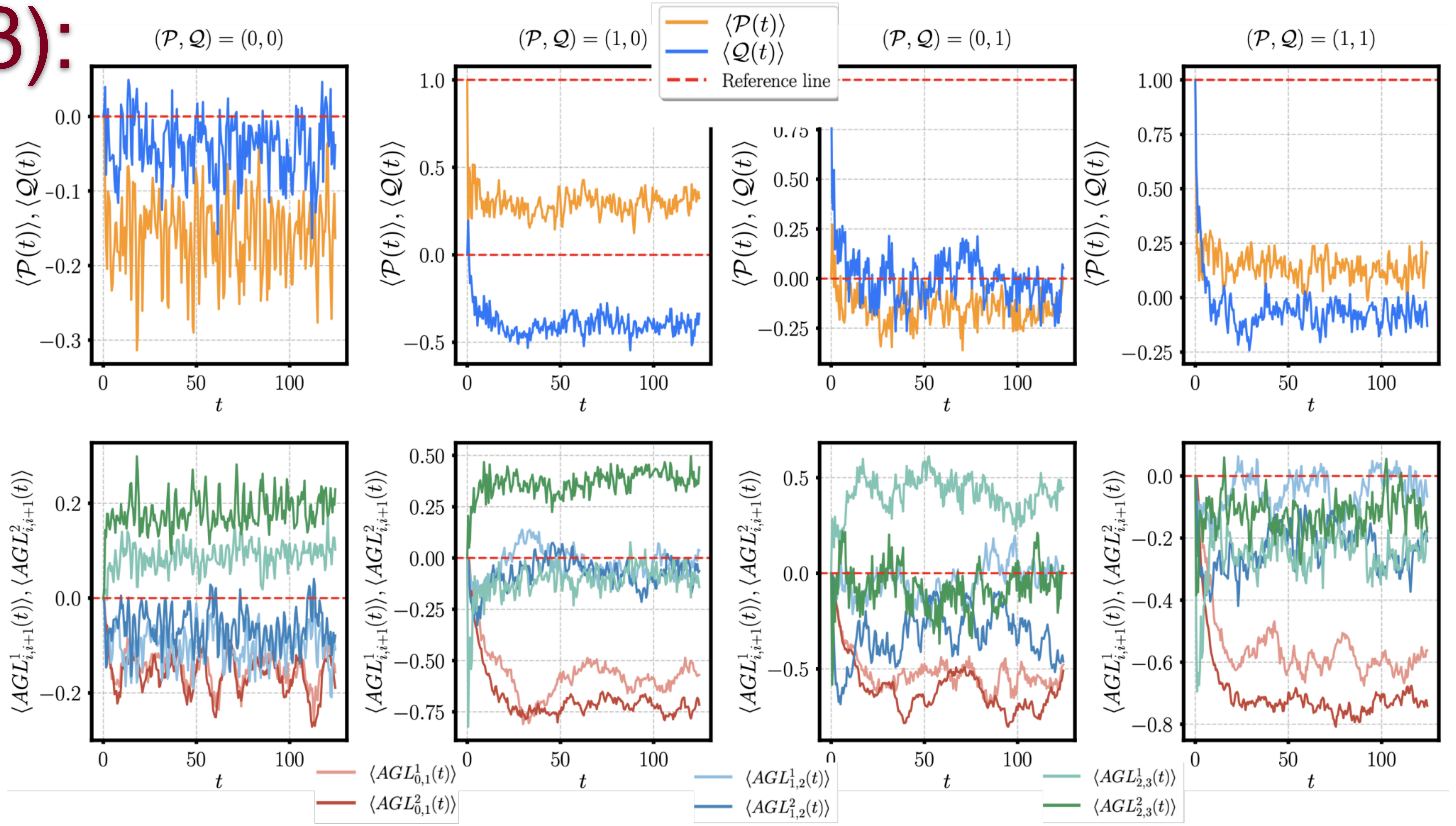
Emil Mathew[1,2,*] and Indrakshi Raychowdhury[1,2,†]

[1] *Department of Physics, BITS-Pilani, K K Birla Goa Campus, Zuarinagar, Goa 403726, India*
[2] *Center for Research in Quantum Information and Technology, Birla Institute of Technology and Science Pilani, Zuarinagar, Goa 403726, India*

(Dated: April 19, 2024)

LSH specific advantage of global symmetries

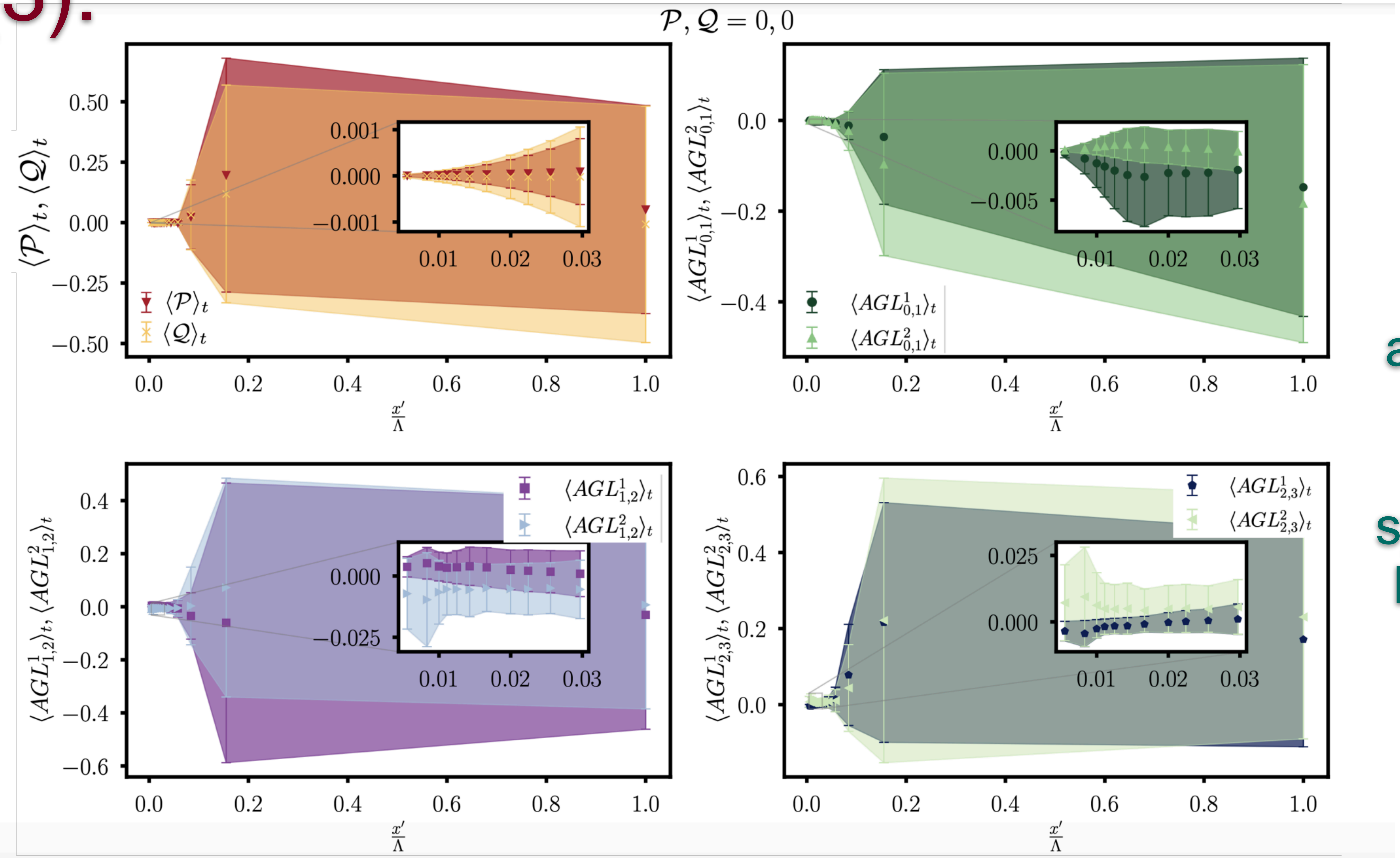
SU(3):



Manifestly violating global symmetries leads to all local symmetries to be violated

LSH specific advantage of global symmetries

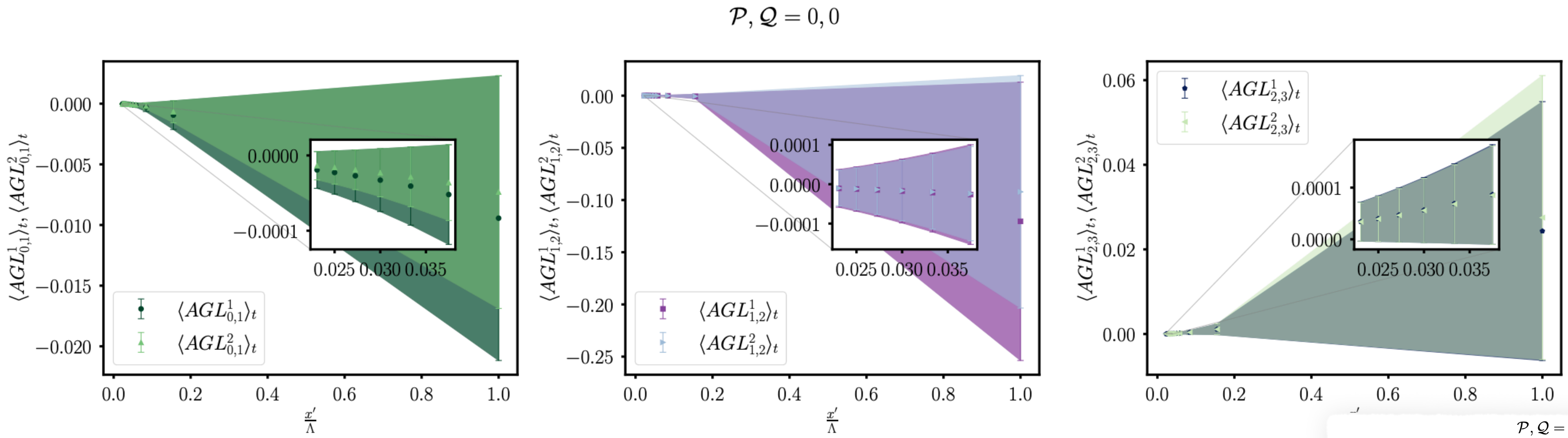
SU(3):



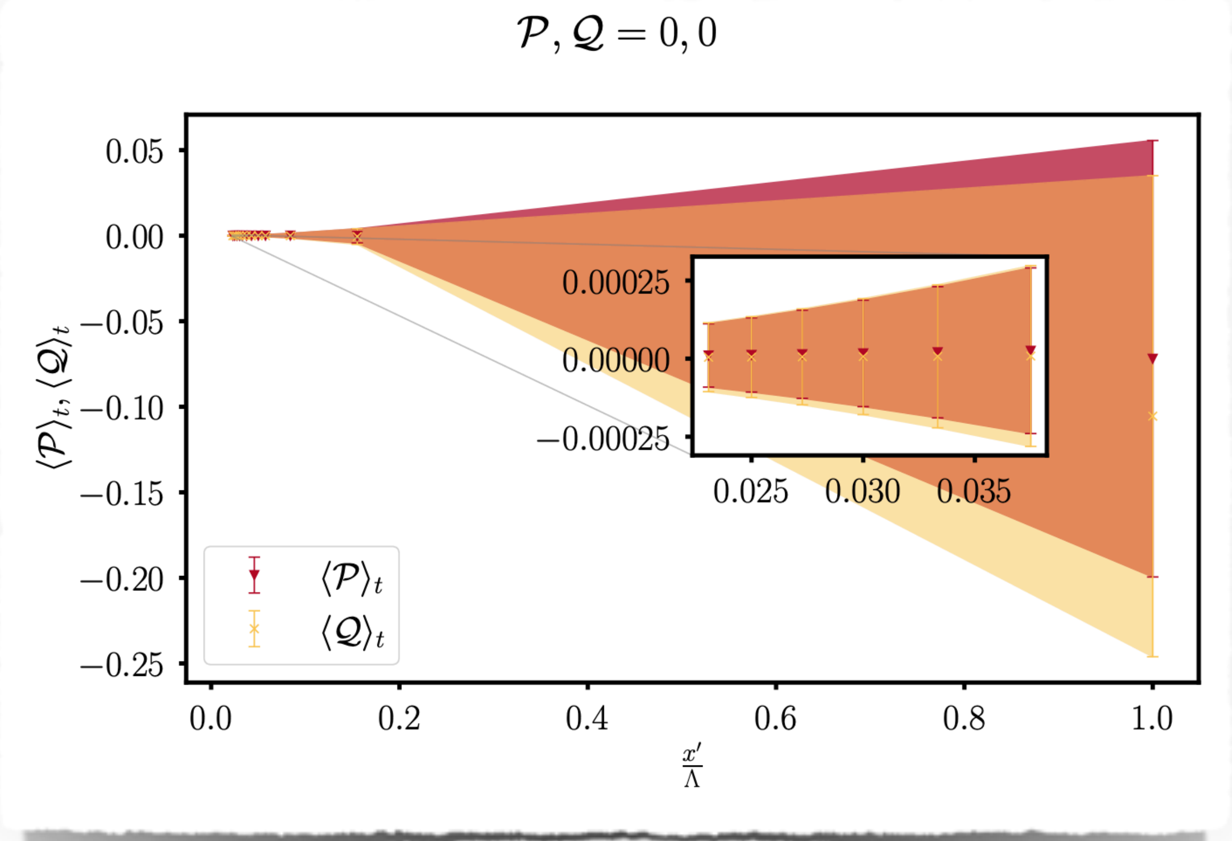
Manifestly protecting global charges automatically leads to all local symmetries to be protected

LSH specific advantage of global symmetries

All local symmetries can be protected individually by another scheme- generalized in higher dimension



Bonus in 1+1d: global symmetries are all protected



LSH specific advantage of global symmetries

SU(3):

Ongoing work on analog simulation of SU(3) lattice gauge theory, based on these facts.

Collaborators at:

LSH specific advantage of Abelian global symmetries

Understanding entanglement structure for gauge theories:

Nontrivial due to non-locality in physical states

Entanglement distillation procedures are to be performed and that is based upon global symmetry structure.

Nontrivial for non-Abelian gauge theories, specifically for SU(3).

LSH framework: Abelianized,
involves only Abelian entanglement distillation

Being explored in the context of thermalisation study and is leading to novel understanding

Towards quantum simulating QCD

Loops-Strings-Hadrons : SU(3) beyond 1+1 d

First step:

IQuS@UW-21-086

Loop-string-hadron approach to SU(3) lattice Yang-Mills theory: Gauge invariant Hilbert space of a trivalent vertex

Saurabh V. Kadam,[1,*] Aahiri Naskar,[2,†] Indrakshi Raychowdhury,[2,3,‡] and Jesse R. Stryker[4,5,§]

Loop-string-hadron approach to the SU(3) gauge invariant Hilbert space

Aug 2, 2024, 12:15 PM

20m

Talk | Theoretical Develop... | Theoretical developme...

Speaker

Jesse Stryker (Lawrence Berkeley National Laboratory (LBNL))

BITS Pilani
PILANI | DUBAI | GOA | HYDERABAD

SERB INDIA

CRQIT
Center for Research in Quantum Information and Technology

Thank You

Looking forward to
See you all again at:

Research group:

Emil Mathew
Grad student

Aahiri Naskar
Grad student

Fran Ilčić
Grad student