QUANTUM SIMULATION OF SU(3) LGT AT LEADING ORDER IN LARGE N

Anthony Ciavarella

BERKELEY LAB
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HAMILTONIAN FORMULATION OF LATTICE GAUGE THEORY

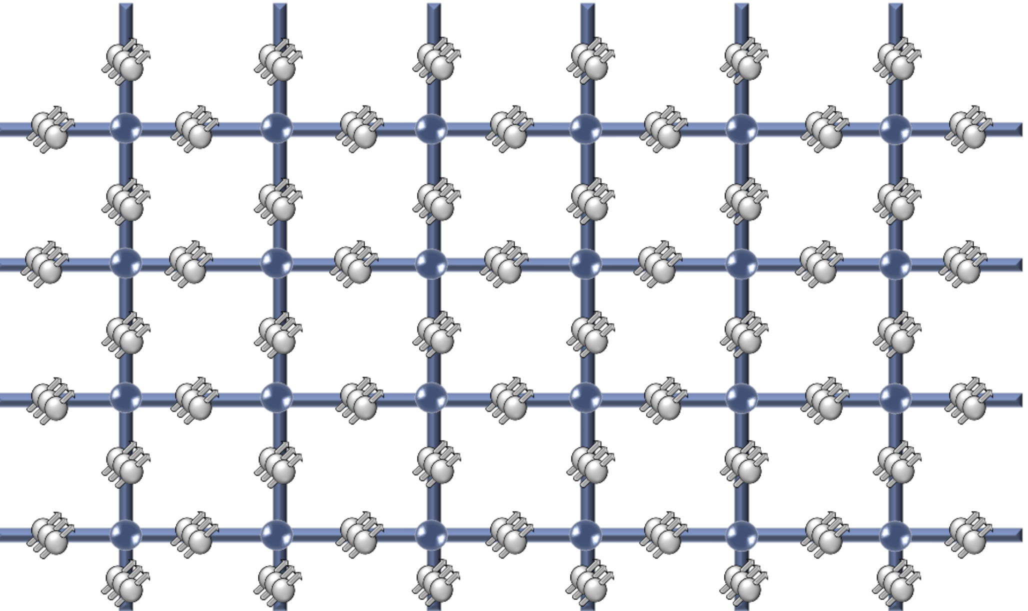
Kogut and Susskind

Phys. Rev. D 11 (1975) 395

$$|j\ m_L\rangle \otimes |j\ m_R\rangle$$

$$U^R_{s_L\ s_R}|j\ m_L\rangle\otimes|j\ m_R\rangle = \sum_{j'm'_L m'_R}\sqrt{\frac{\dim(j)}{\dim(j')}}C^{j'm'_L}_{j\ m_L;R\ s_L}C^{j'm'_R}_{j\ m_R;R\ s_R}|j'\ m'_L\rangle\otimes|j'\ m'_R\rangle$$

$$\hat{H} = \frac{g^2}{2a^{d-2}}\sum_{b,\text{links}}|\hat{\mathbf{E}}^{(b)}|^2 + \frac{1}{2a^{4-d}g^2}\sum_{\text{plaquettes}}\left[\,6 - \hat{\Box}(\mathbf{x}) - \hat{\Box}^\dagger(\mathbf{x})\,\right]$$



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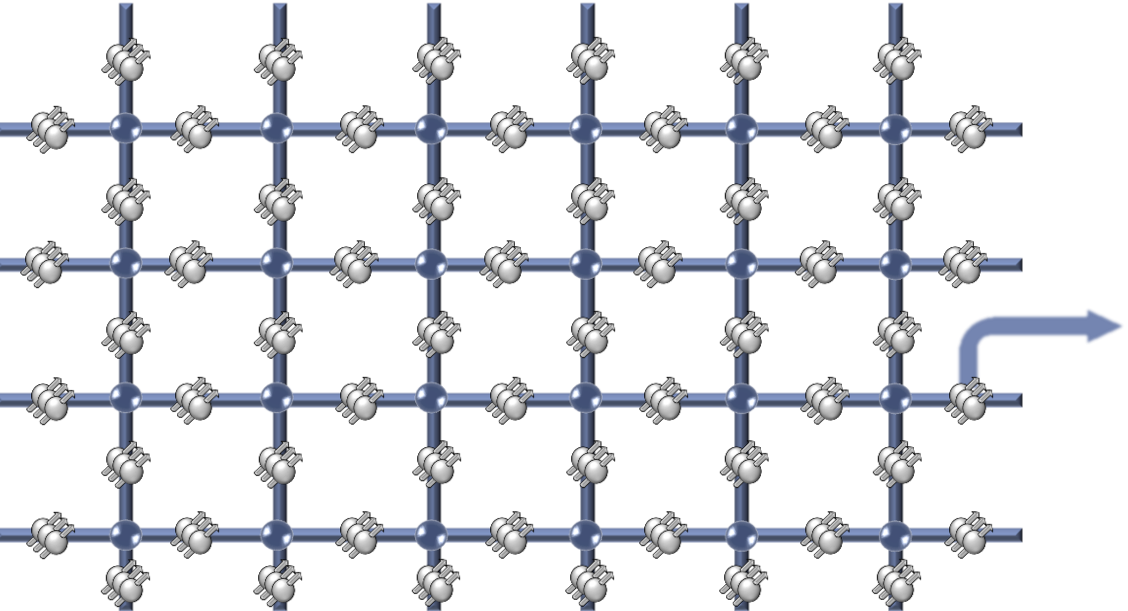
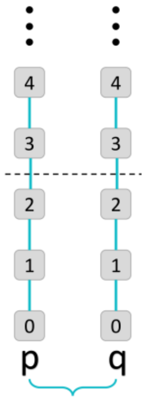


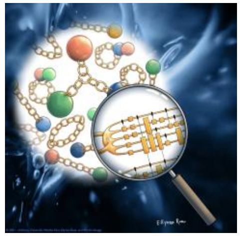


Byrnes and Yamamoto

Phys. Rev. A 73 (2006) 022328

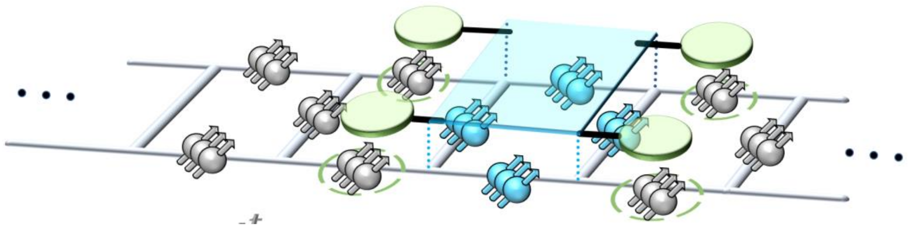
$$|p, q, T_L, T^z_L, Y_L, T_R, T^z_R, Y_R\rangle$$

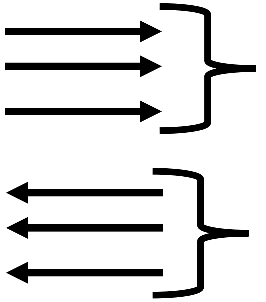
Gauss's Law



Phys. Rev. D 103 (2021) 094501

$$|p, q\rangle = |\mathbf{R}\rangle$$





OPERATORS IN THE MULTIPLET FORMULATION

Electric Matrix Element

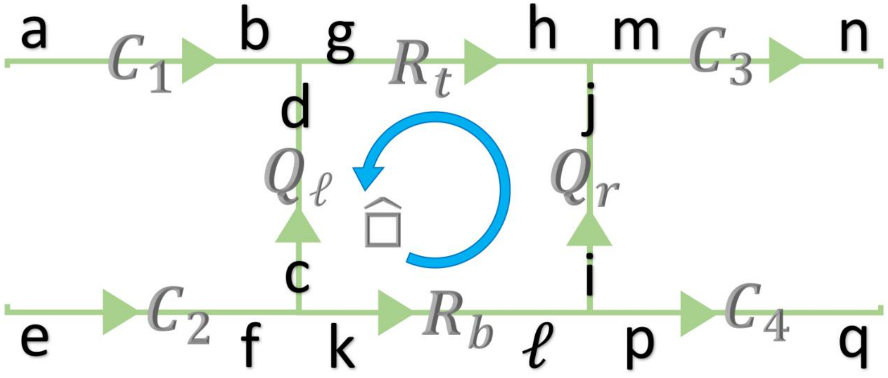
$$\sum_b |\hat{\mathbf{E}}^{(b)}|^2 \, |p,q\rangle = \frac{p^2+q^2+pq+3p+3q}{3} \, |p,q\rangle$$

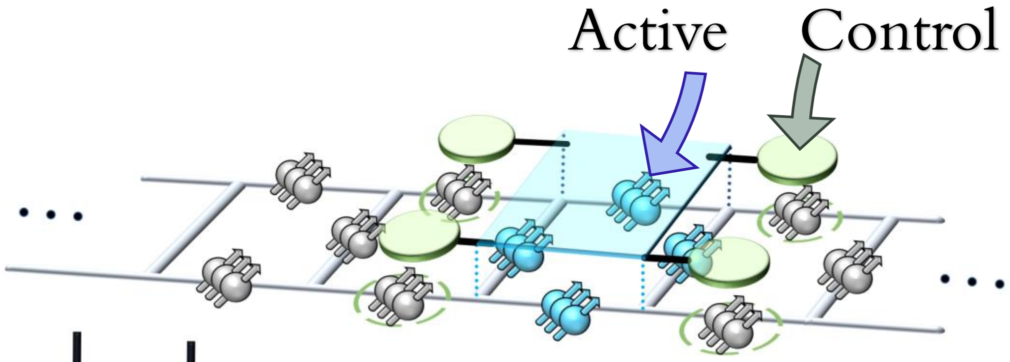
Magnetic Matrix Element

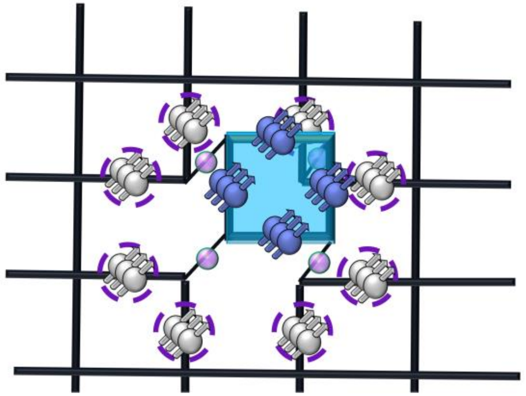
$$\left\langle \begin{pmatrix} \mathbf{C}_1, \mathbf{R}'_t, \mathbf{C}_3 \\ \mathbf{Q}'_\ell, \mathbf{Q}'_r \\ \mathbf{C}_2, \mathbf{R}'_b, \mathbf{C}_4 \end{pmatrix} \middle| \hat{\square} \middle| \begin{pmatrix} \mathbf{C}_1, \mathbf{R}_t, \mathbf{C}_3 \\ \mathbf{Q}_\ell, \mathbf{Q}_r \\ \mathbf{C}_2, \mathbf{R}_b, \mathbf{C}_4 \end{pmatrix} \right\rangle =$$

$$\sqrt{\frac{\dim(\mathbf{R}_t)\dim(\mathbf{R}_b)}{\dim(\mathbf{R}'_t)\dim(\mathbf{R}'_b)\dim(\mathbf{Q}_\ell)\dim(\mathbf{Q}_r)\dim(\mathbf{Q}'_\ell)^3\dim(\mathbf{Q}'_r)^3}} \begin{Bmatrix} \overline{\mathbf{R}}_t & \mathbf{C}_1 & \overline{\mathbf{Q}}_\ell \\ \mathbf{3} & \mathbf{1} & \mathbf{3} \\ \overline{\mathbf{R}}'_t & \mathbf{C}_1 & \overline{\mathbf{Q}}'_\ell \end{Bmatrix} \begin{Bmatrix} \mathbf{R}_t & \overline{\mathbf{C}}_3 & \overline{\mathbf{Q}}_r \\ \overline{\mathbf{3}} & \mathbf{1} & \overline{\mathbf{3}} \\ \mathbf{R}'_t & \overline{\mathbf{C}}_3 & \overline{\mathbf{Q}}'_r \end{Bmatrix} \begin{Bmatrix} \overline{\mathbf{R}}_b & \mathbf{C}_2 & \mathbf{Q}_\ell \\ \overline{\mathbf{3}} & \mathbf{1} & \overline{\mathbf{3}} \\ \overline{\mathbf{R}}'_b & \mathbf{C}_2 & \mathbf{Q}'_\ell \end{Bmatrix} \begin{Bmatrix} \mathbf{R}_b & \overline{\mathbf{C}}_4 & \mathbf{Q}_r \\ \mathbf{3} & \mathbf{1} & \mathbf{3} \\ \mathbf{R}'_b & \overline{\mathbf{C}}_4 & \mathbf{Q}'_r \end{Bmatrix}$$

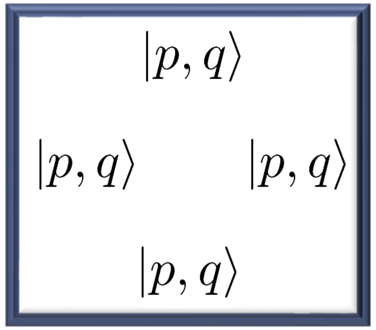
$$\begin{Bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \\ \mathbf{3} & \mathbf{1} & \mathbf{3} \\ \mathbf{D} & \mathbf{B} & \mathbf{E} \end{Bmatrix} = \sum \langle \mathbf{D}, y', \mathbf{B}, x | \mathbf{E}, q' \rangle_{\Gamma_1} \langle \mathbf{A}, y, \mathbf{B}, x | \mathbf{C}, q \rangle_{\Gamma_2} \langle \mathbf{A}, y, \mathbf{3}, c | \mathbf{D}, y' \rangle_{\Gamma_3} \langle \mathbf{C}, q, \mathbf{3}, c | \mathbf{E}, q' \rangle_{\Gamma_4}$$

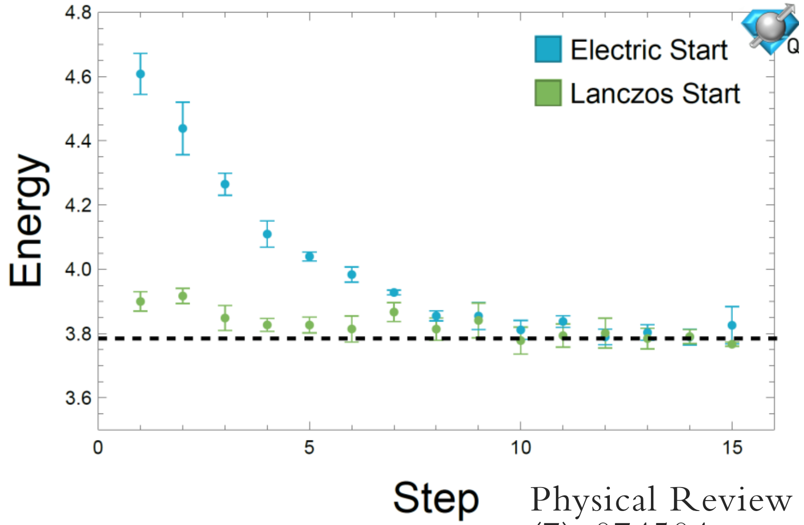






SIMULATION OF SMALL SYSTEMS

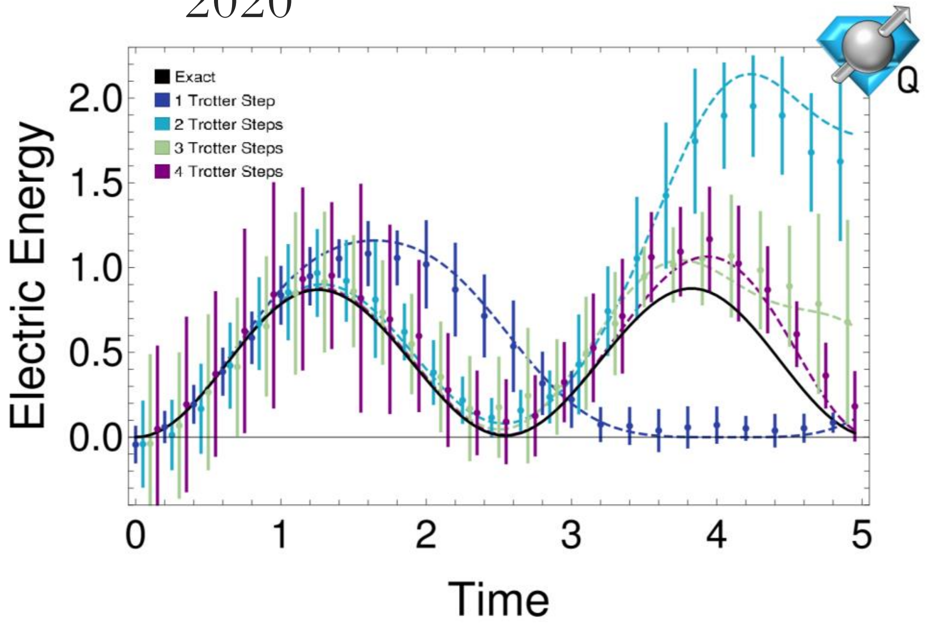




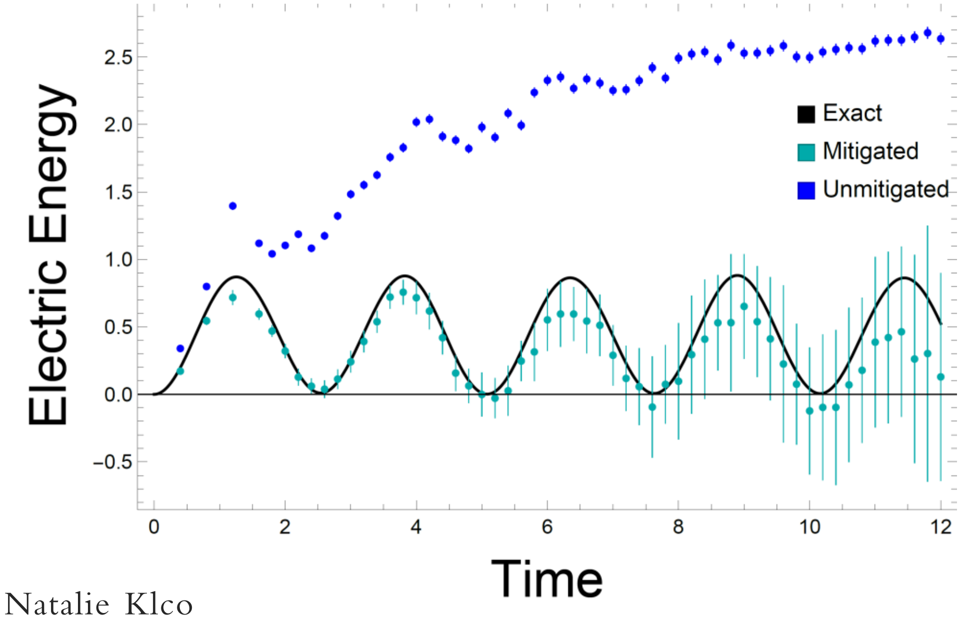
Physical Review D 105 (7), 074504

with Ivan Chernyshev

2020

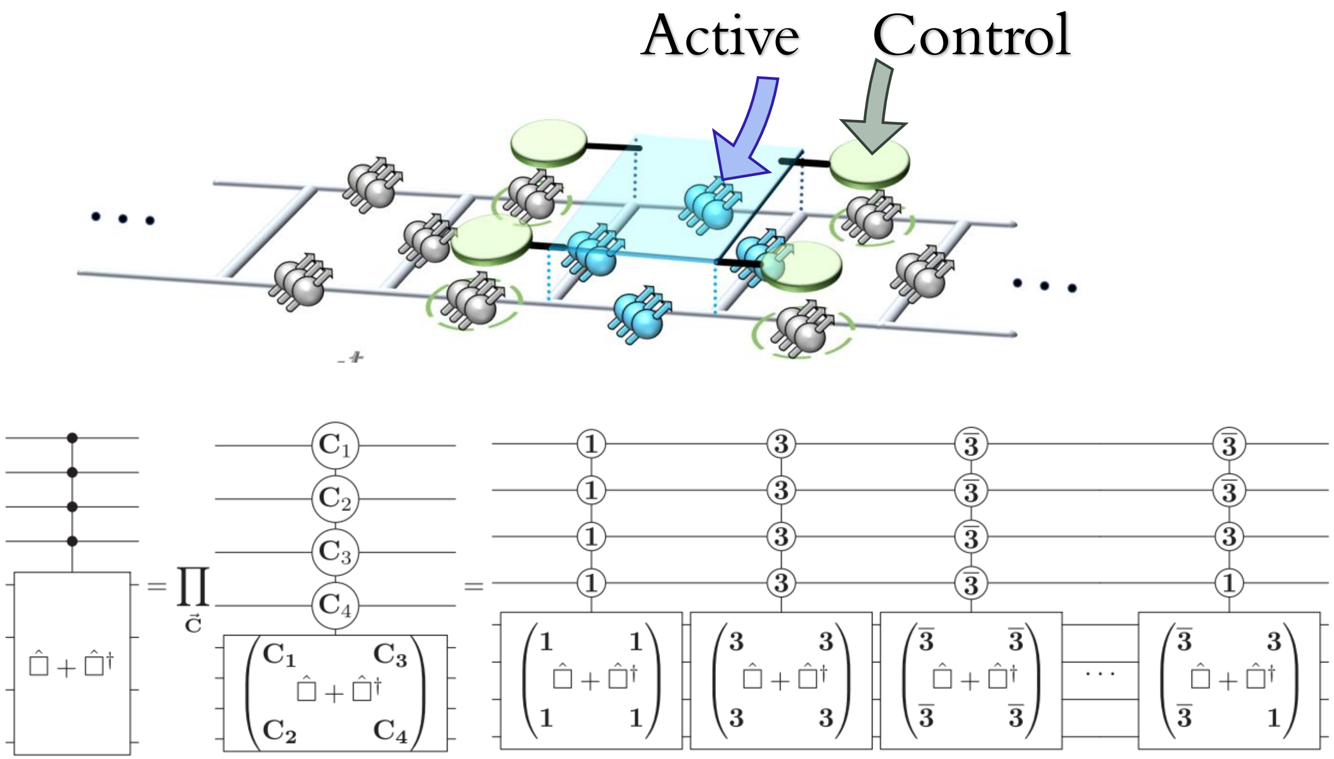


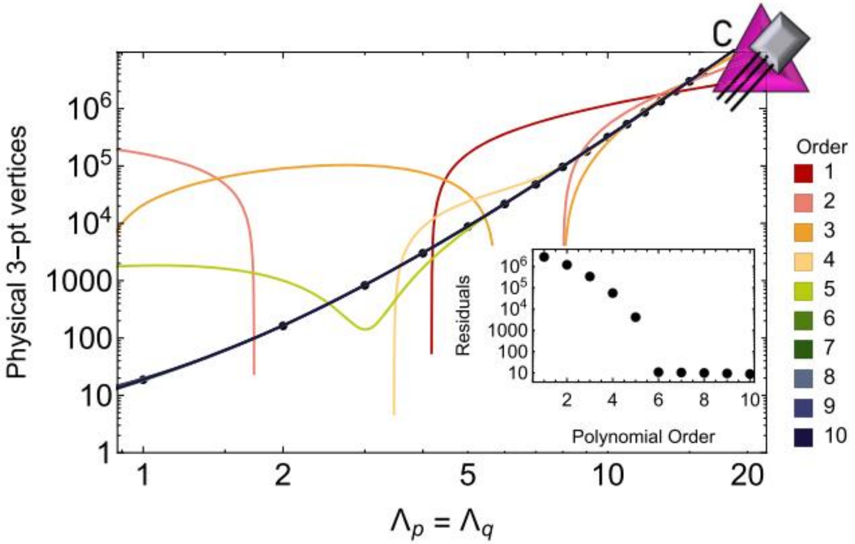
2022



Physical Review D 103 (9), 094501, with Martin Savage and Natalie Klco

CHALLENGES OF GOING TO SCALE





- Gate count for time evolution scales as Λ^{16}

LARGE N EXPANSION

SU(3) → SU(N), Expand in 1/N

- Qualitatively reproduces many aspects of QCD
- Provides a starting point for describing interactions between baryons
- Used in event generators that simulate collider physics

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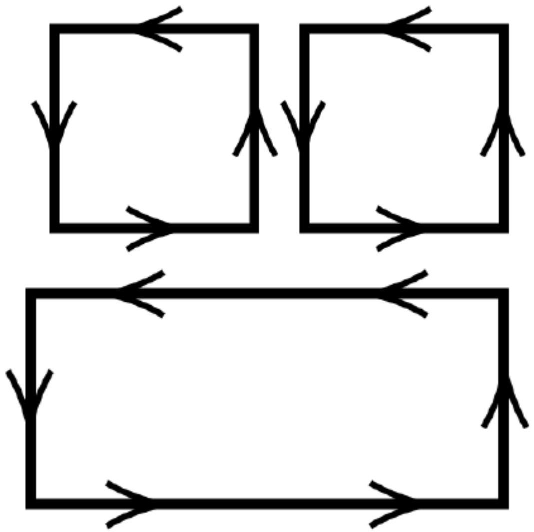

= O(1)

= O(1/N)

- Expand operators in powers of 1/N
- Truncate both in powers of 1/N and electric energy
- The large N scaling of a state is determined by the maximum overlap of the state with $|\{P_p, \bar{P}_p\}\rangle \equiv \prod_p \hat{\Box}_p^{P_p} \hat{\Box}_p^{\dagger \bar{P}_p} |0\rangle$
- Simple scaling rule

$$\langle \{L_i, a_\ell\} | \{P_p, \bar{P}_p\} \rangle \propto \prod_i N_c^{1-m_i}$$

m_i =# Plaquettes enclosed by loop i

arXiv:2402.10265

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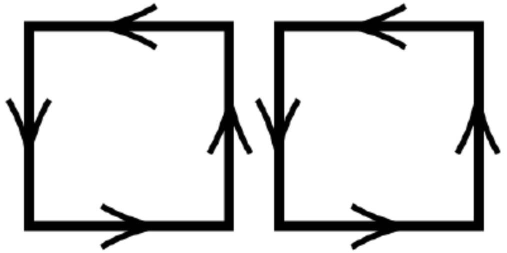

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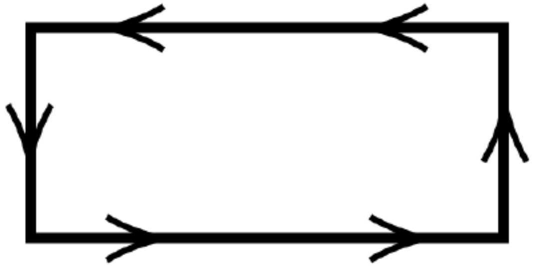

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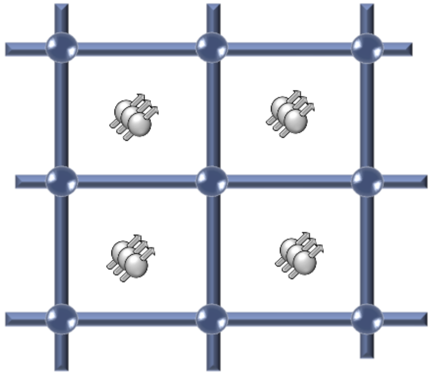
- At large N, only need to represent the number of loops running around each square



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$$\hat{H} = \sum_p \left(\frac{8}{3} g^2 - \frac{1}{2g^2} \right) \hat{P}_{1,p} - \frac{1}{g^2\sqrt{2}} \hat{P}_{0,p+\hat{x}} \hat{P}_{0,p-\hat{x}} \hat{P}_{0,p+\hat{y}} \hat{P}_{0,p-\hat{y}} \hat{X}_p$$

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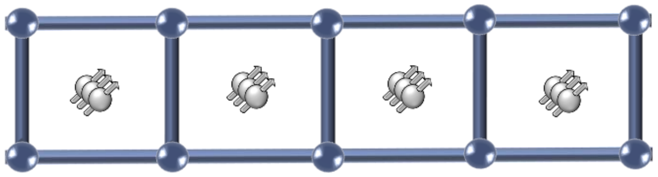
- Resources can be compared for a small lattice at this truncation (4x1)



	Naïve Encoding	Multiplet Basis	Large N_c
Qubit Count	60	24	4
Gauss's law enforced	No	Partially	Partially

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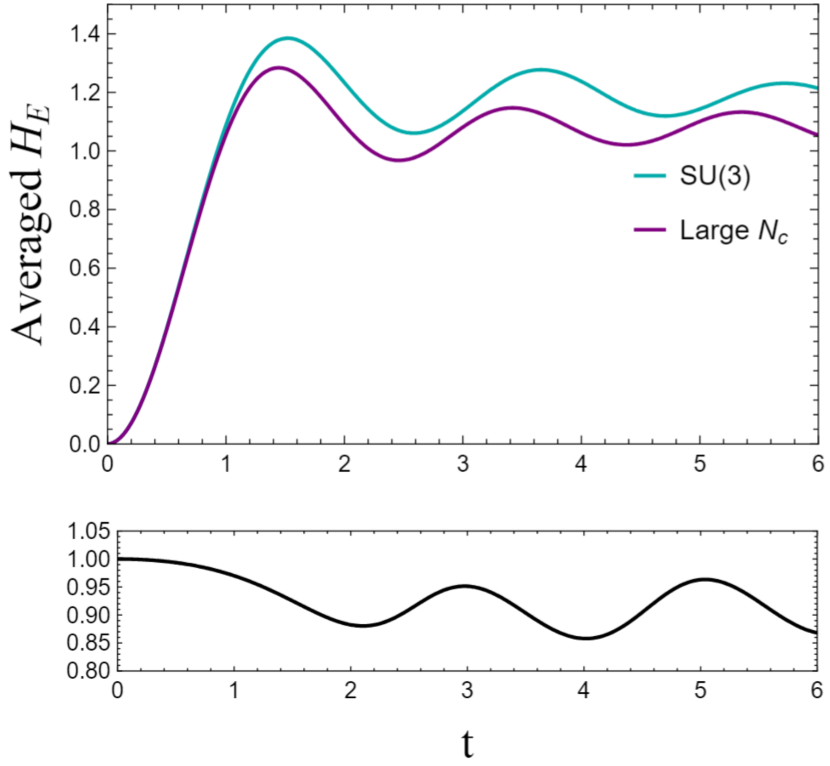

REAL TIME EVOLUTION ON IBM'S QUANTUM COMPUTERS

Interaction Picture Trotterization

$$\hat{H}_{B,I}(t) = e^{i\hat{H}_E t}\hat{H}_B e^{-i\hat{H}_E t}$$

$$e^{-i\hat{H}t} = e^{-i\hat{H}_E t}\,\mathcal{T}e^{-i\int_0^t ds\hat{H}_{B,I}(s)}$$

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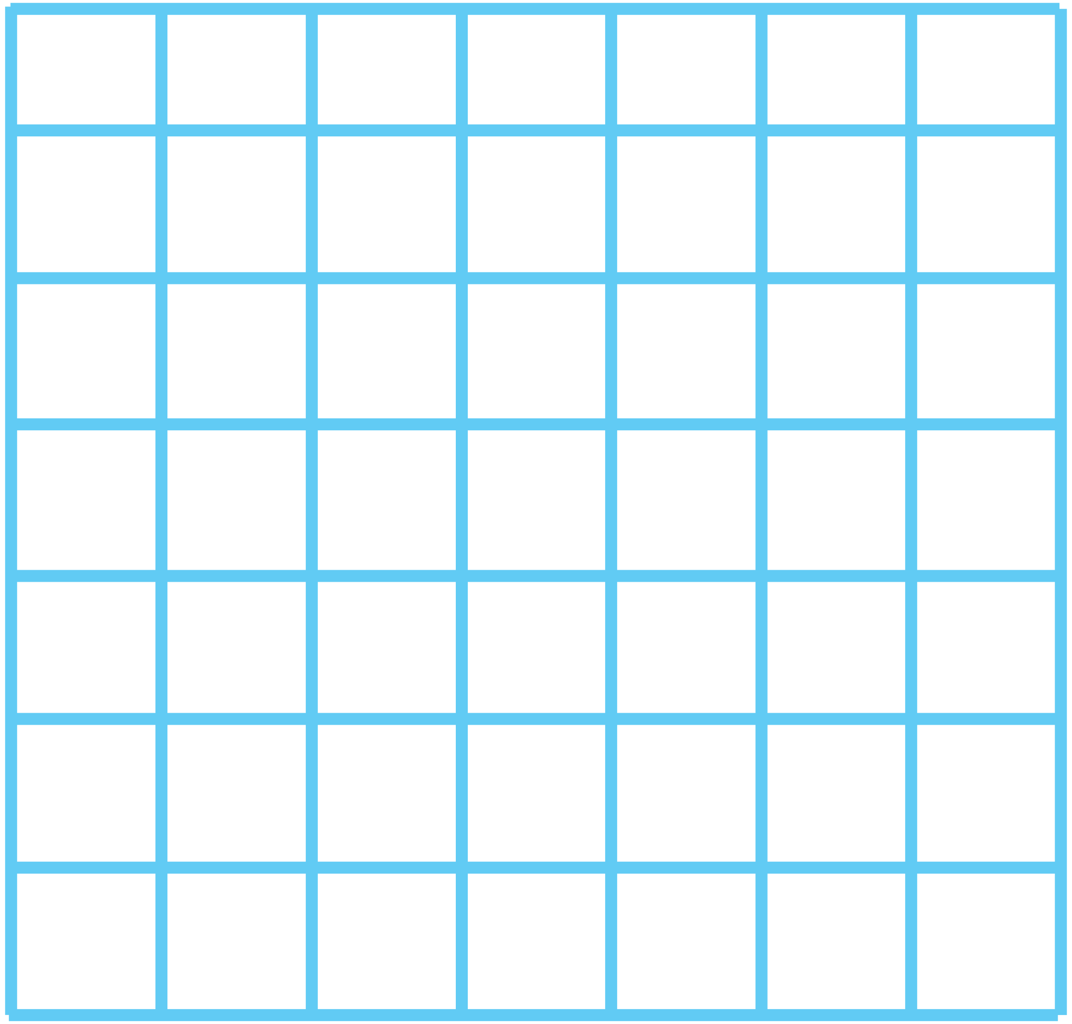
$$\left[e^{i\phi\sum_p \hat{Z}_p}\right]\left[e^{i\theta\sum_{p\in E}\hat{X}_p\prod_{q\in\partial p}\hat{P}_{0,q}}\right]\left[e^{i\theta\sum_{p\in O}\hat{X}_p\prod_{q\in\partial p}\hat{P}_{0,q}}\right]\left[e^{-i\phi\sum_p \hat{Z}_p}\right]$$



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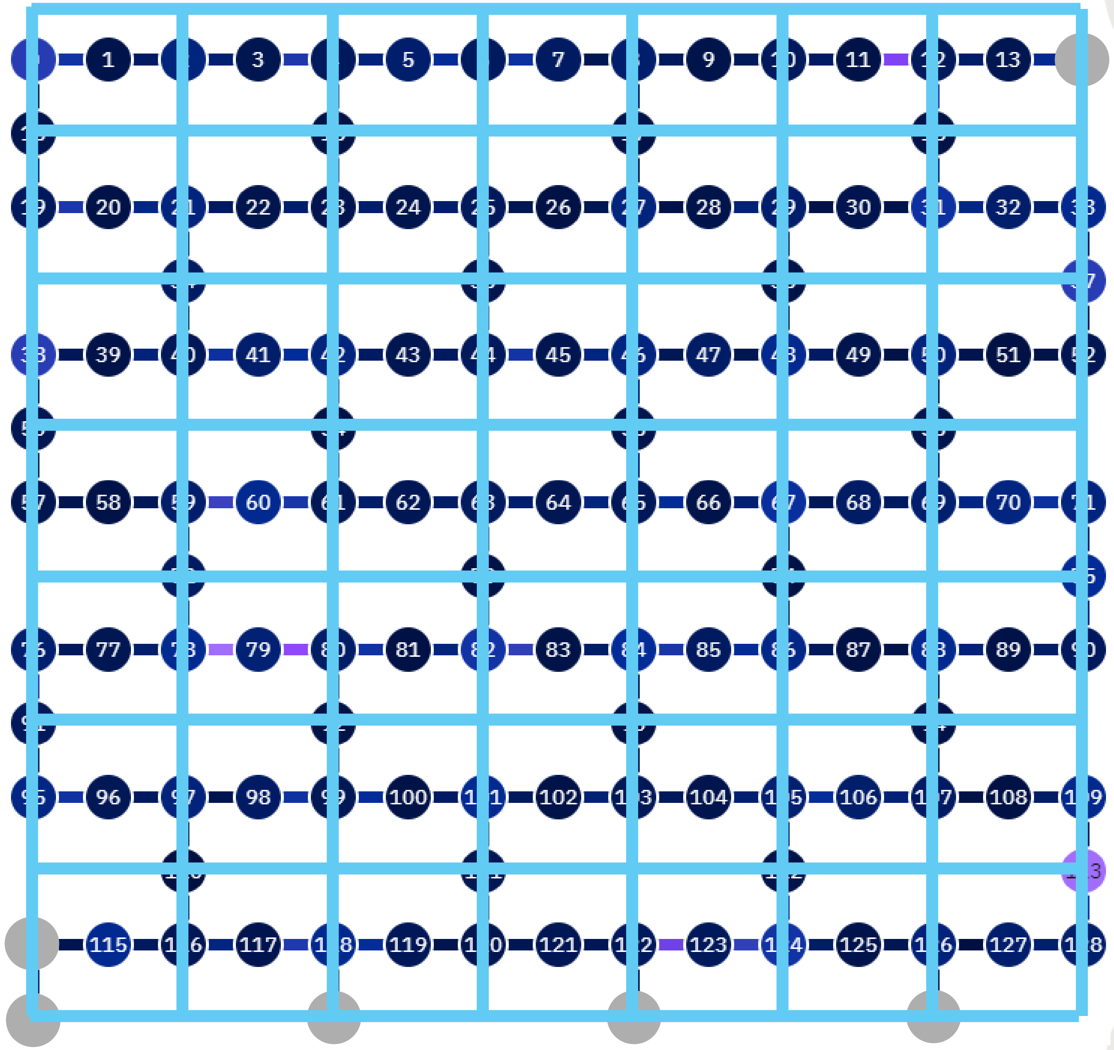

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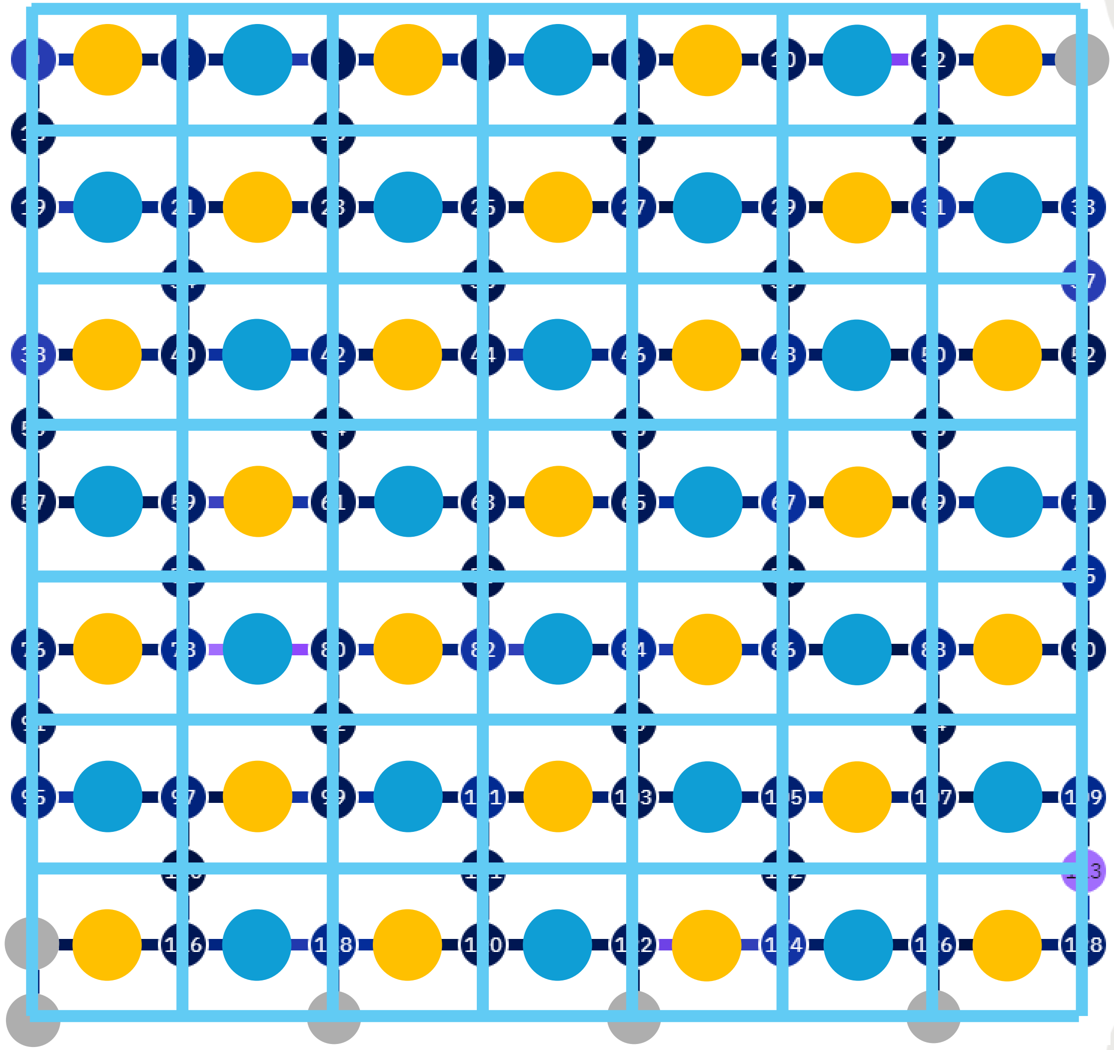
- Yellow and blue qubits are used to represent the state of the system



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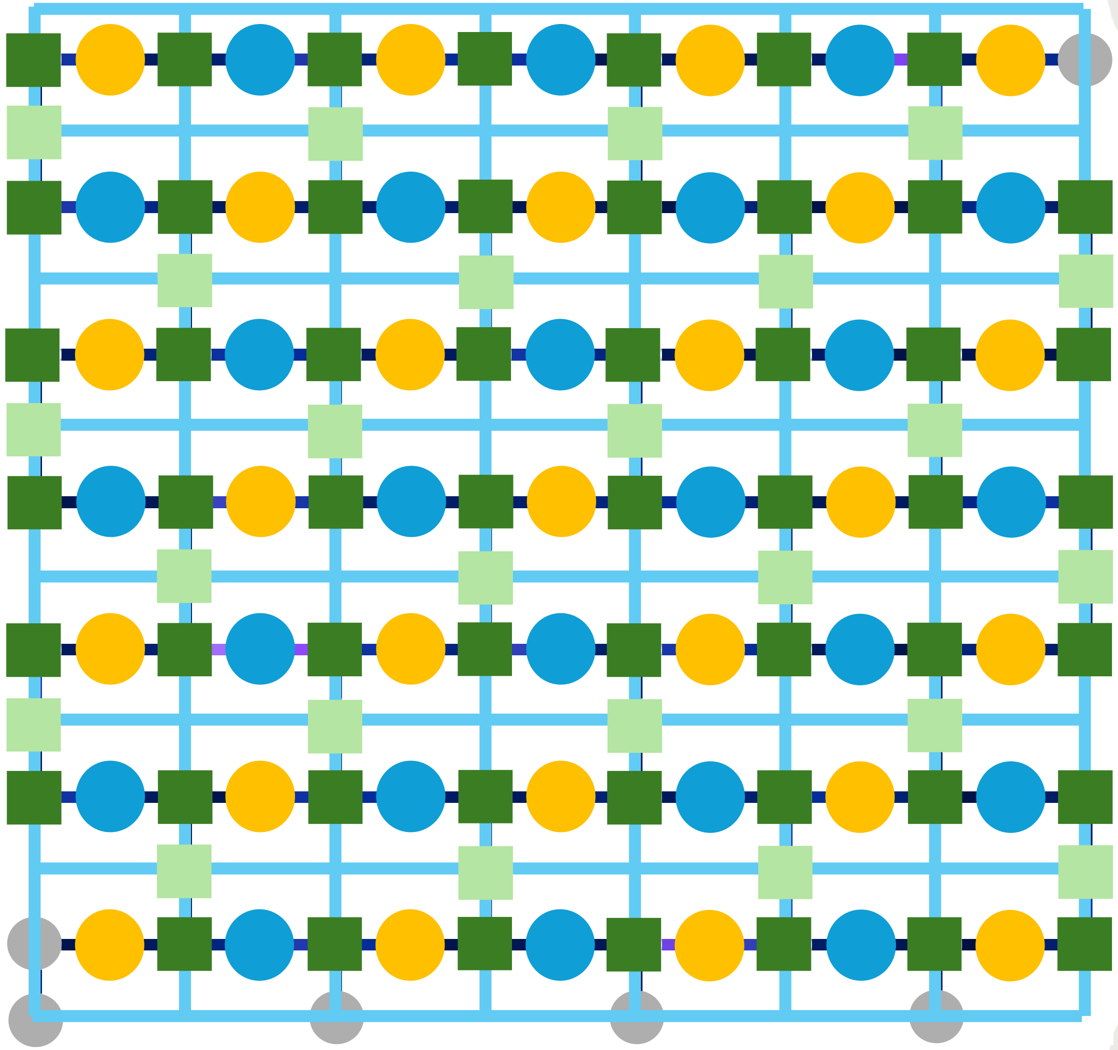
- Yellow and blue qubits are used to represent the state of the system
- Square qubits are used to enable communication between those used to represent the system
- One Trotter step = CNOT depth 45



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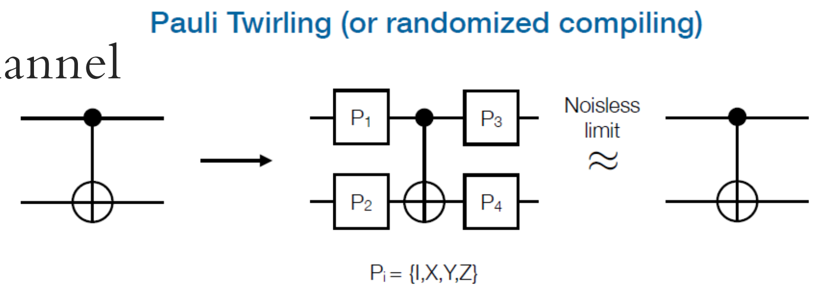

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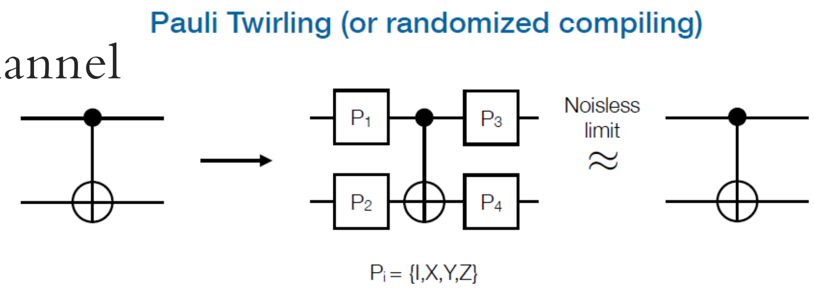
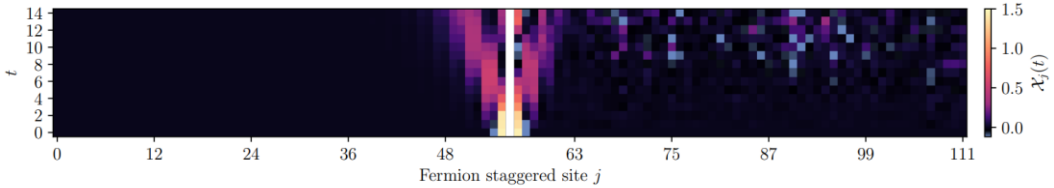
- Decoherent Pauli noise renormalizes Pauli operators

$$\langle\psi|\hat{P}|\psi\rangle \to \eta_P \langle\psi|\hat{P}|\psi\rangle$$



- This can be mitigated by running a circuit with a known answer to determine η_P

<u>Operator Decoherence Renormalization</u>
Phys. Rev. Lett. 127, 270502
arXiv:2210.11606
PRX Quantum 5, 020315
Phys. Rev. D 109, 114510



113 qubits, CNOT depth 370 (13,858 CNOTs)

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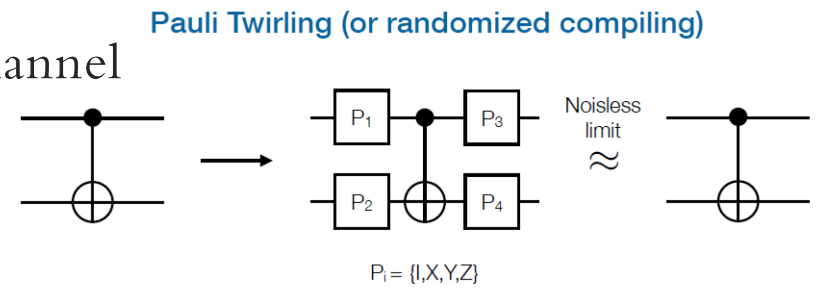

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- Other sources of hardware error can be mitigated by artificially introducing noise by applying more CNOT gates and extrapolating to zero noise.

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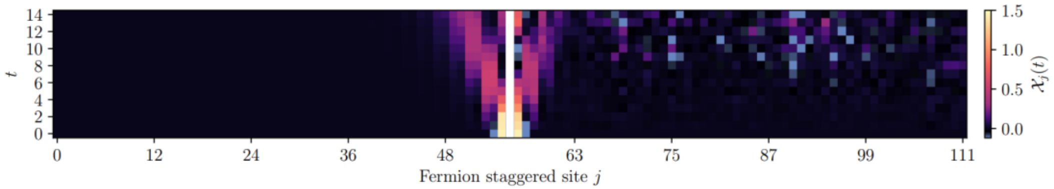

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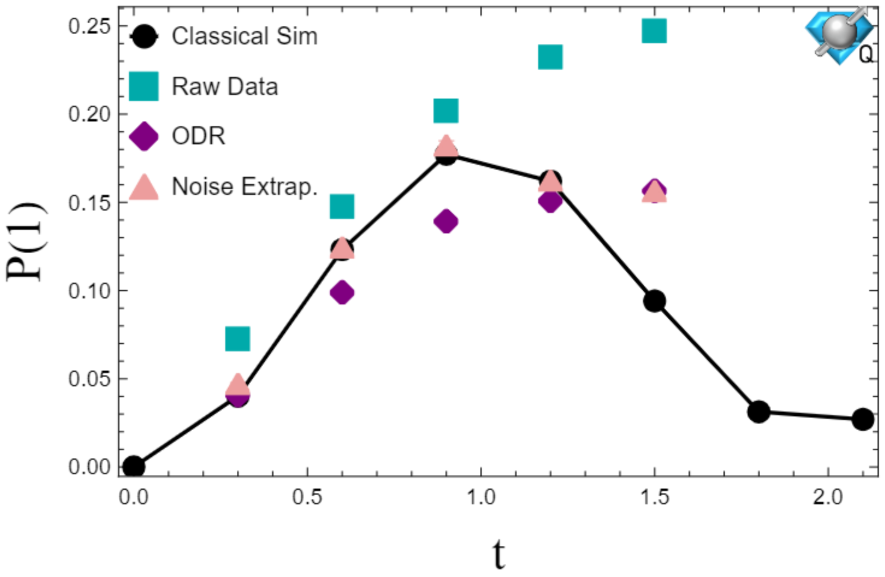
5x5



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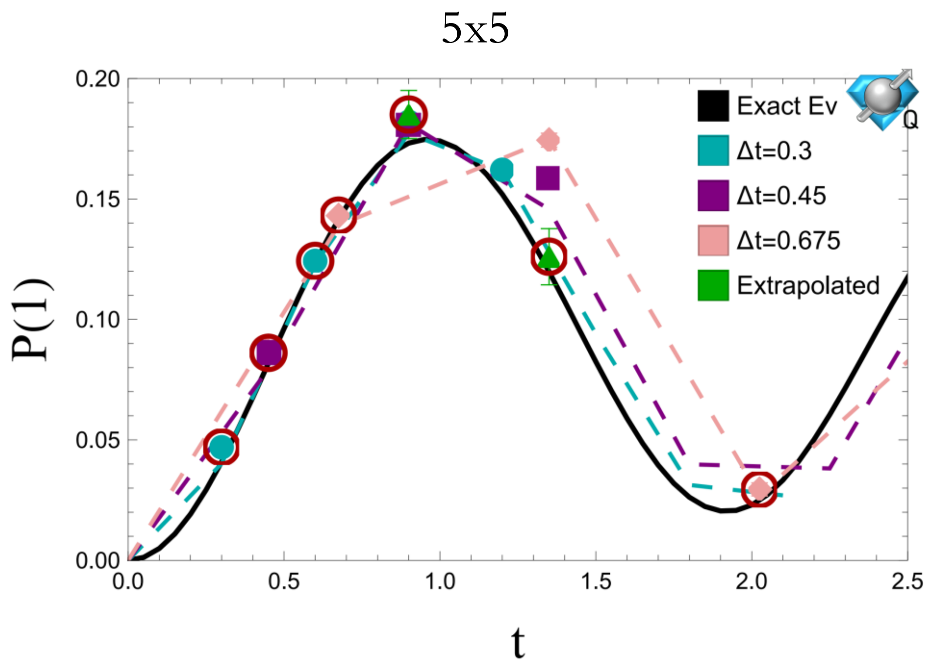
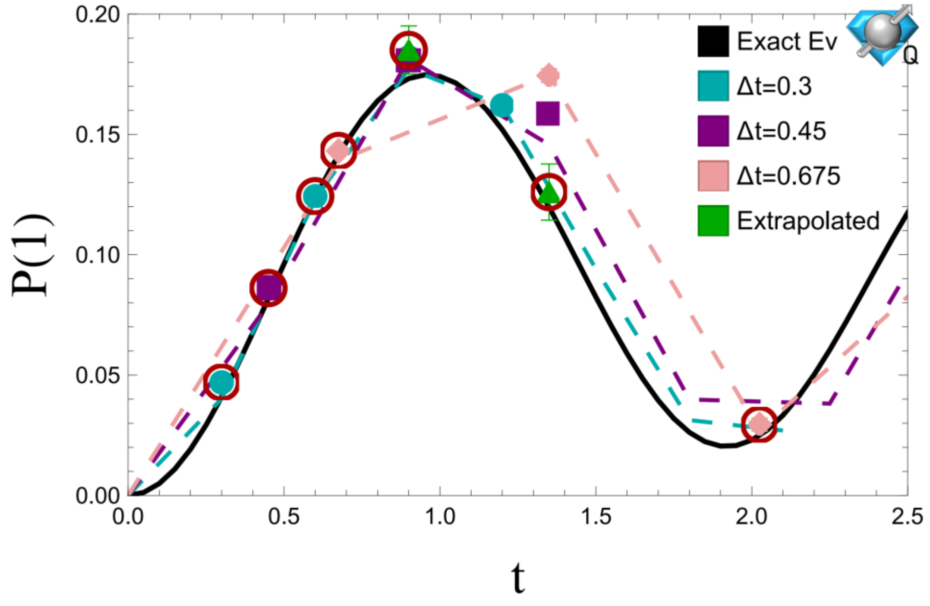


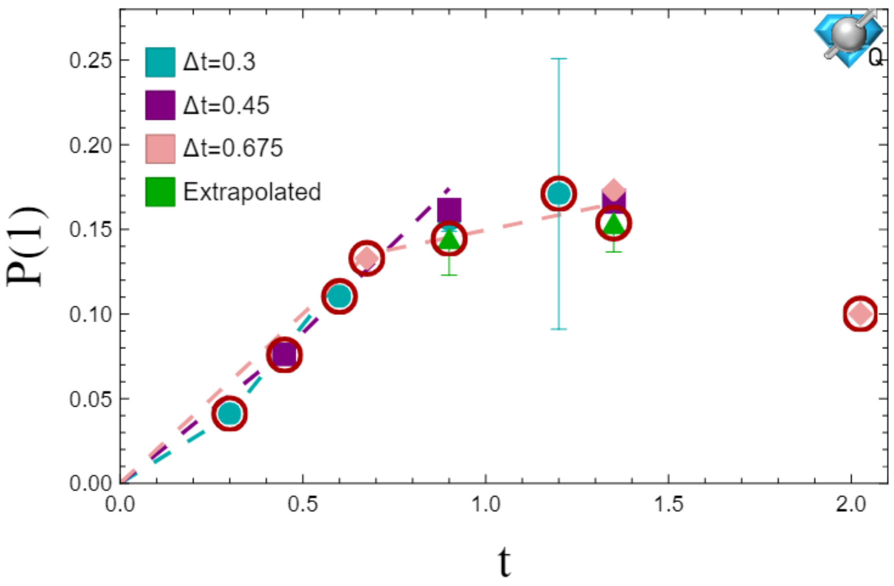
ALGORITHMIC ERRORS

- Errors also come from the Trotterization of the time evolution operator.
- This can be mitigated by performing the evolution with multiple step sizes that sample the same points in time and extrapolating.
- Noise in circuits scales with circuit depth not system size so small simulations can be used to validate the results of larger ones.
- CuQuantum was used to perform a classical simulation for a 8x8 lattice.

5x5



8x8



129 Qubits
CZ Depth 113
1500 CZ Gates
8733 1 Qubit Gates

SUMMARY & FUTURE GOALS

- The large N expansion can be used to reduce the resources needed for simulation.
- The truncated Hamiltonian is similar to the PXP model indicating there may be connections to condensed matter work on scarring and confinement in spin models.
- This also allows for straightforward implementation on neutral atom platforms.
- Future work will look at including quarks and 1/N corrections.