Towards quantum simulation of lower-dimensional supersymmetric lattice models. †

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† In collaboration with D. Schaich and continuing on previous works by C. Culver and D. Schaich:

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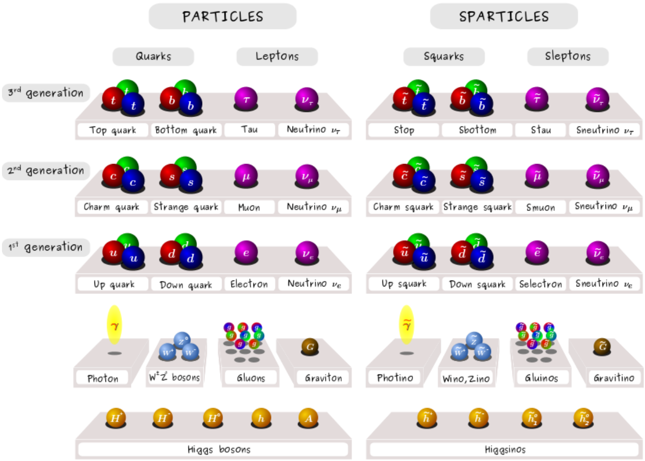
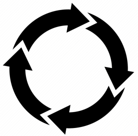
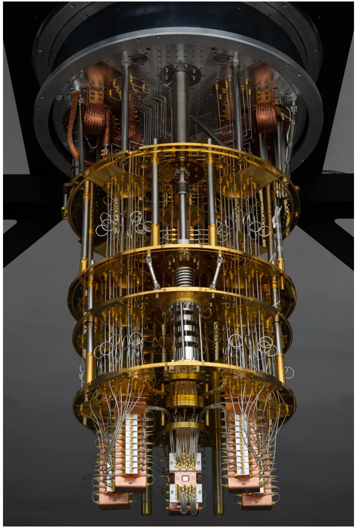
Motivations



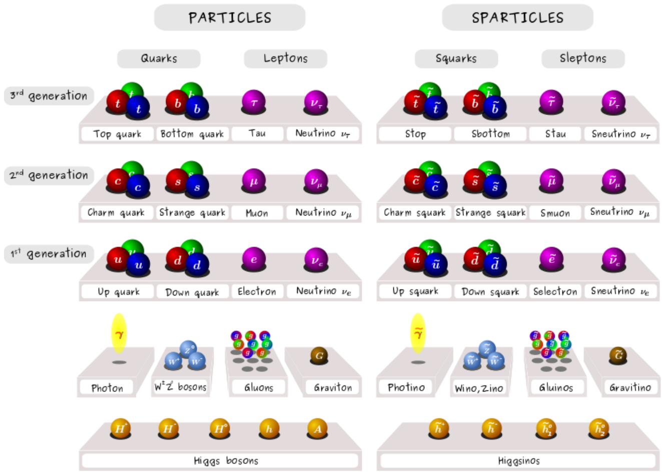
Claire David / CERN

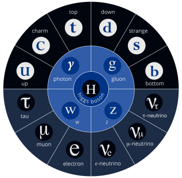




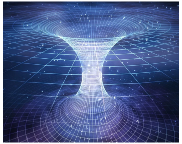
- Exploring supersymmetric model using quantum resources
- Benchmarking quantum algorithm in searching for supersymmetry breaking

Why is supersymmetry such an interesting theoretical idea?

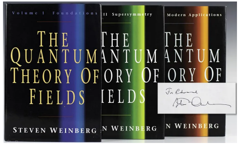




Extension of the Standard Model



Duality with Quantum gravity



Better knowledge of QFT

- Predicted sparticles have not been observed yet $\Longrightarrow$ supersymmetry must be spontaneously broken!

Essential facts about supersymmetric models

- A supercharge Q introduces a symmetry between fermion and boson

$$Q|fermion\rangle = |boson\rangle \qquad Q|boson\rangle = |fermion\rangle$$

- The Hamiltonian is $\sim Q^2$

- The energy spectrum has peculiar characteristics for broken/preserved supersymmetry:

Preserved symmetry

$$E_0 = \langle\Psi|H|\Psi\rangle = 0$$

Single ground state
Paired excited states

Spontaneously breaking

$$E_0 = \langle\Psi|H|\Psi\rangle > 0$$

All the states are paired

- The Witten index expresses a condition necessary but insufficient for supersymmetry breaking:

$$\mathcal{W} = Tr[(-1)^F e^{-iHt}] = 0 \quad \Longrightarrow \quad \mathcal{Z} = 0 \quad \Longrightarrow \quad \text{Sign problem}$$

Why should we consider quantum computing?

- Hamiltonian approach becomes feasible $\Longrightarrow$ (Hilbert space scales polynomially on qubits)
- Real-time dynamical phenomena are fully accessible $\Longrightarrow$ (No sign problem)
- Supersymmetry breaking can be directly investigated by measuring the ground state

Challenges in the use of the available quantum hardware:

- Few qubits with low connectivity
- Errors in reading the qubits
- Noisy gates
- Low computation resources
- No error correction techniques available on current hardware $\Rightarrow$ **Only error mitigation techniques**!

$$\Downarrow$$

Focusing on simple supersymmetric models on lower dimensions:

- $0+1$ Supersymmetric quantum mechanics (In this talk)
- $1+1$ Wess-Zumino model

0+1 Supersymmetric Quantum Mechanics

- Single site model with one fermion and one boson interacting

$$H = \frac{1}{2}\left\{Q, \overline{Q}\right\} = \frac{1}{2}\left(\hat{p}^2 + [W'(\hat{q})]^2 - W''(\hat{q})\left[\hat{b}^\dagger, \hat{b}\right]\right), \qquad \text{with supercharge} \quad Q = \hat{b}\,[i\hat{p} + W'(\hat{q})]$$

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Harmonic Oscillator	Anharmonic Oscillator	Double Well
$W(\hat{q}) = \frac{1}{2}m\hat{q}^2$	$W(\hat{q}) = \frac{1}{2}m\hat{q}^2 + \frac{1}{4}g\hat{q}^4$	$W(\hat{q}) = \frac{1}{2}m\hat{q}^2 + g\left(\frac{1}{3}\hat{q}^3 + \mu^2\hat{q}\right)$
[Supersymmetric]	[Supersymmetric]	[Spontaneously breaking]

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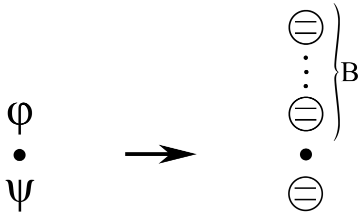
[Supersymmetric]	[Supersymmetric]	[Spontaneously breaking]

- Fermion operators $\Longrightarrow$ Jordan-Wigner transformation $\hat{b} = \frac{1}{2}(X + iY)$ and $\hat{b}^\dagger = \frac{1}{2}(X - iY)$
- Boson operators $\Longrightarrow$ $\hat{q}$ and $\hat{p}$ from the Harmonic oscillator truncated to Λ modes



$2\Lambda = 2^{B+1}$ classical resources

$(B+1)$ qubits

1+1 Wess-Zumino Model

- N-site model with N fermions and N bosons interacting at each site

$$H = Q^2 = \sum_{n=1}^{N} \left[\frac{p_n^2}{2a} + \frac{a}{2}\left(\frac{\phi_{n+1}-\phi_{n-1}}{2a}\right)^2 + \frac{a}{2}V(\phi_n)^2 + aV(\phi_n)\frac{\phi_{n+1}-\phi_{n-1}}{2a} + (-1)^n V'(\phi_n)\left(\chi_n^\dagger \chi_n - \frac{1}{2}\right) + \frac{1}{2a}\left(\chi_n^\dagger \chi_{n+1} + \chi_{n+1}^\dagger \chi_n\right)\right]$$

Linear prepotential

$$V(\phi_n) = \phi_n$$

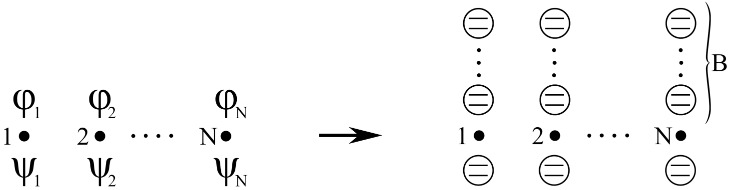
[Supersymmetric]

Quadratic prepotential

$$V(\phi_n) = \phi_n^2 + c$$

[Dynamical breaking]

- Fermion fields $\Longrightarrow$ Jordan-Wigner transformation $\chi_n = \frac{1}{2}(X_n + iY_n)$ and $\chi_n^\dagger = \frac{1}{2}(X_n - iY_n)$
- Boson fields $\Longrightarrow$ $\hat{q}$ and $\hat{p}$ from the Harmonic oscillator using Λ modes



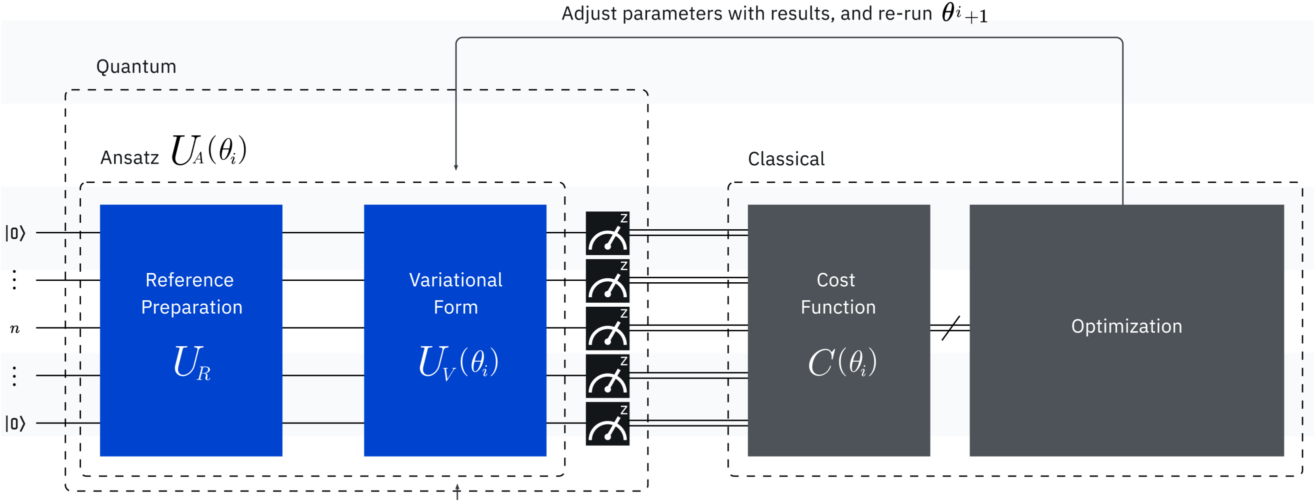
$2^N \Lambda^N = 2^N \left(2^B\right)^N = 2^{N(1+B)}$ classical resources

$N(1+B)$ qubits

Toward quantum Computing

Variational Quantum Eigensolver (VQE)

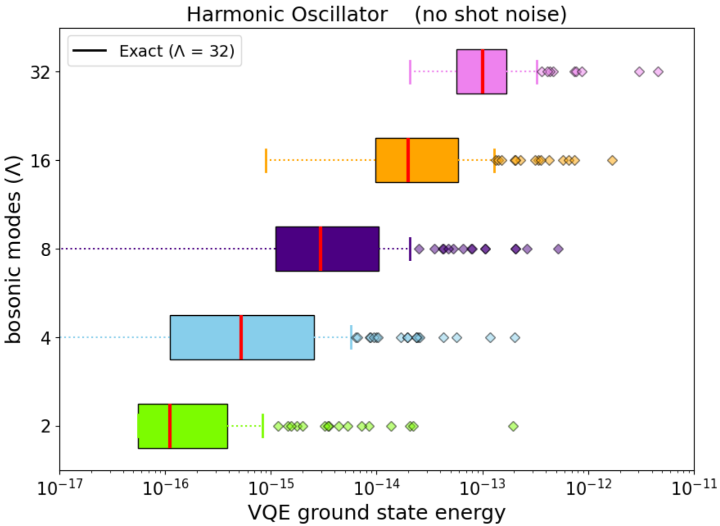
- VQE is a heuristic hybrid quantum-classical algorithm to find the ground state



- $|0\rangle \Rightarrow$ Ansatz $\Rightarrow |\psi(\theta_i)\rangle \Rightarrow$ Measurements $\Rightarrow \langle\psi(\theta_i)|H|\psi(\theta_i)\rangle \Rightarrow$ C. Optimizer $\Rightarrow$ new $(\theta_i) \Rightarrow$ Re-run

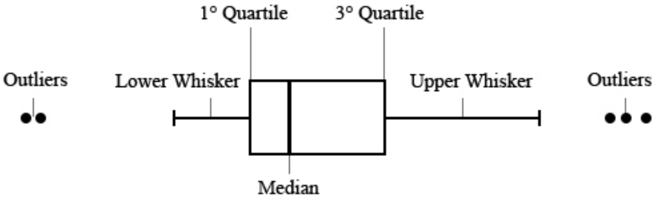
VQE preliminary results

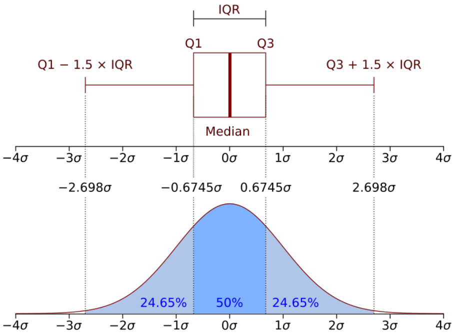
VQE results and box-and-whisker plot



- Preserved supersymmetry, VQE agrees!

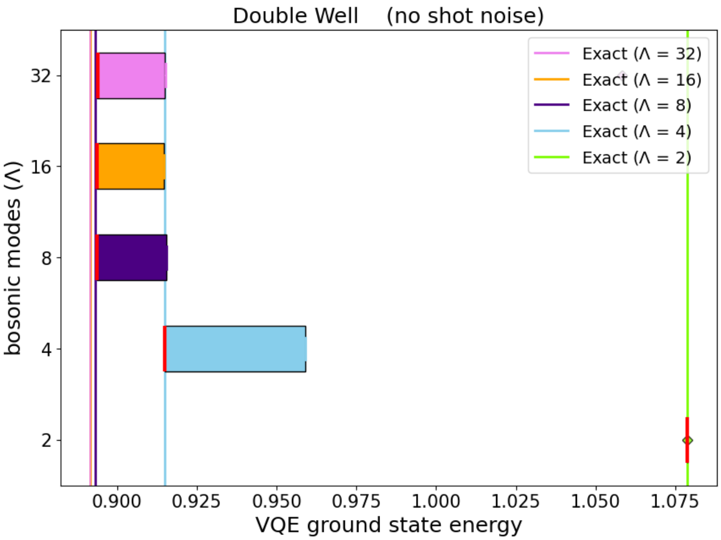
Essential on box-and-whisker plot

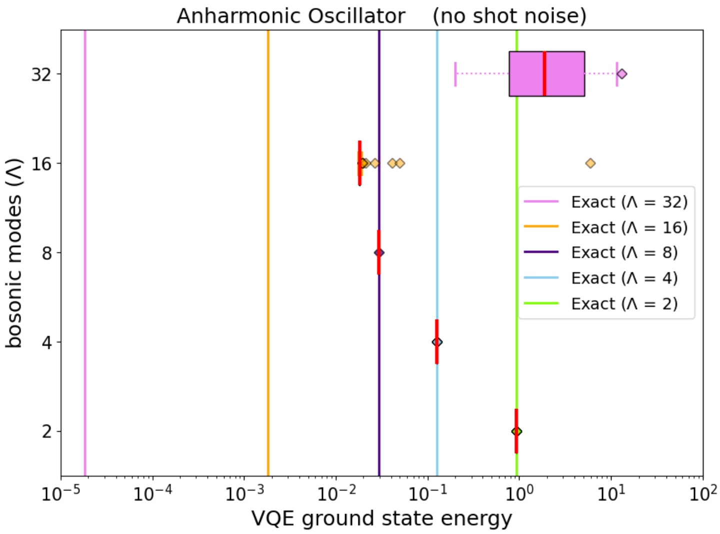




box plot for Gaussian distribution

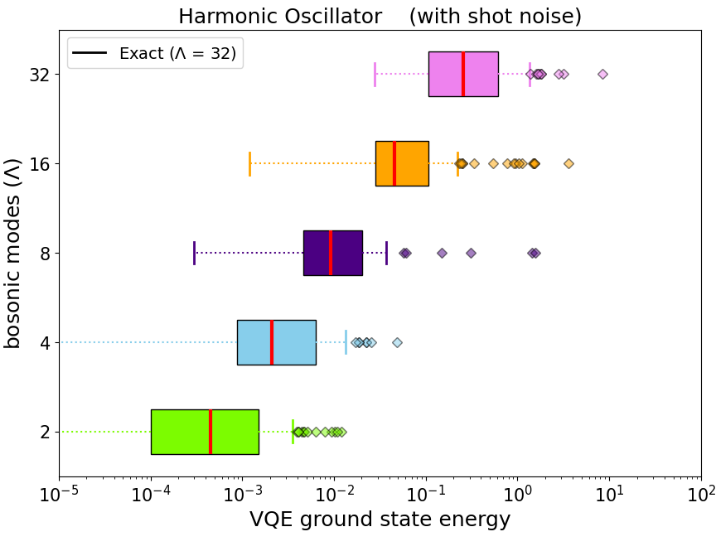
VQE without shots noise

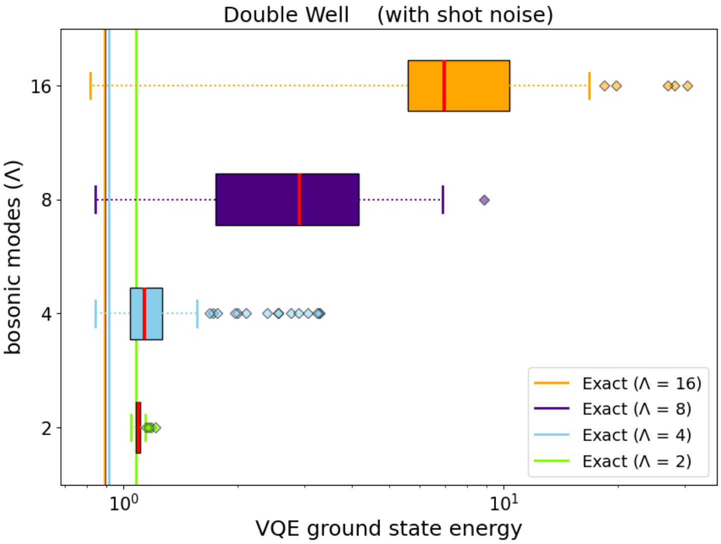




- Supersymmetry breaking, VQE agrees!
- Preserved supersymmetry, VQE disagrees!

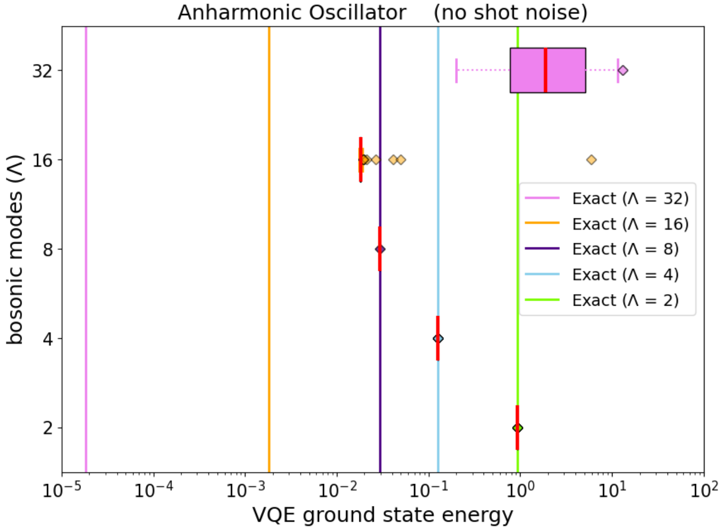
VQE with shots noise

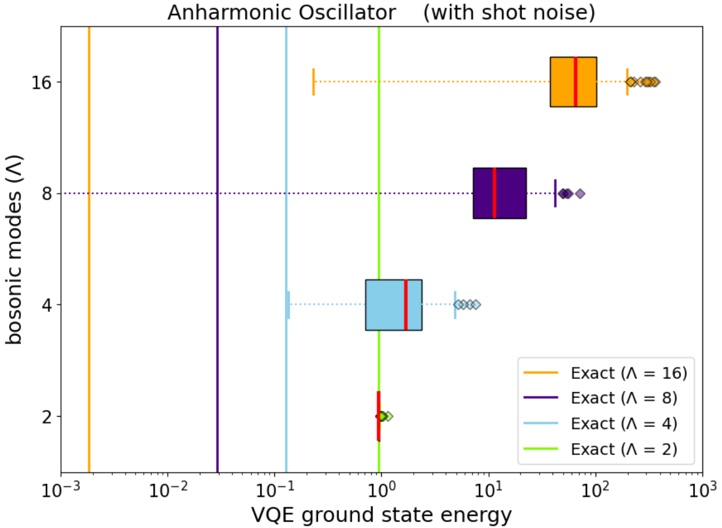




- Preserved supersymmetry, VQE disagrees!
- Supersymmetry breaking, VQE agrees!

VQE with/without shot noise





- Preserved supersymmetry, VQE disagrees!
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Conclusions

Take-home message

What we have seen:

- Supersymmetric lattice models can be encoded on quantum hardware using quantum resources that scale polynomially.
- Supersymmetry breaking/preserved can be checked with the VQE
- VQE agrees with classical results for small systems
- For larger systems the VQE classical optimizer can get stuck in local minima
- Shot noise negatively impacts VQE performance, especially for larger systems

Ongoing work:

- Explore different VQE ansatzes and optimizers
- Run on quantum hardware and mitigate the hardware noise

Thank you all for your time!

Time for Questions!

Backup Slides

VQE measurement protocol for checking supersymmetry breaking/preservation

- Choose a superpotential
- Fix the bosonic modes Λ
  - Choose an Ansatz (RealAmplitude)
  - Choose an optimizers (Constrained Optimization by Linear Approximation (COBYLA))
  - Perform N VQE runs
- Increase the bosonic modes Λ
- Conclude on breaking/preserved supersymmetry

Run specifications:

100 VQE runs, 10k shots, 10k max optimizer iterations

Number of Pauli strings $0+1$ Supersymmetric Quantum Mechanics

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Λ	H size	N. qubits	Harmonic Oscillator	Double Well	Anharmonic Oscillator
2	4×4	2	3	5	4
4	8×8	3	4	15	11
8	16×16	4	5	48	32
16	32×32	5	6	132	92
32	64×64	6	7	337	240