Doubly Charmed $\Lambda_c\Lambda_c$ Scattering from Lattice QCD

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Outline

- Motivation
- Lattice Setup
- Calculation Methodology
- Analyses and Result
- Summary

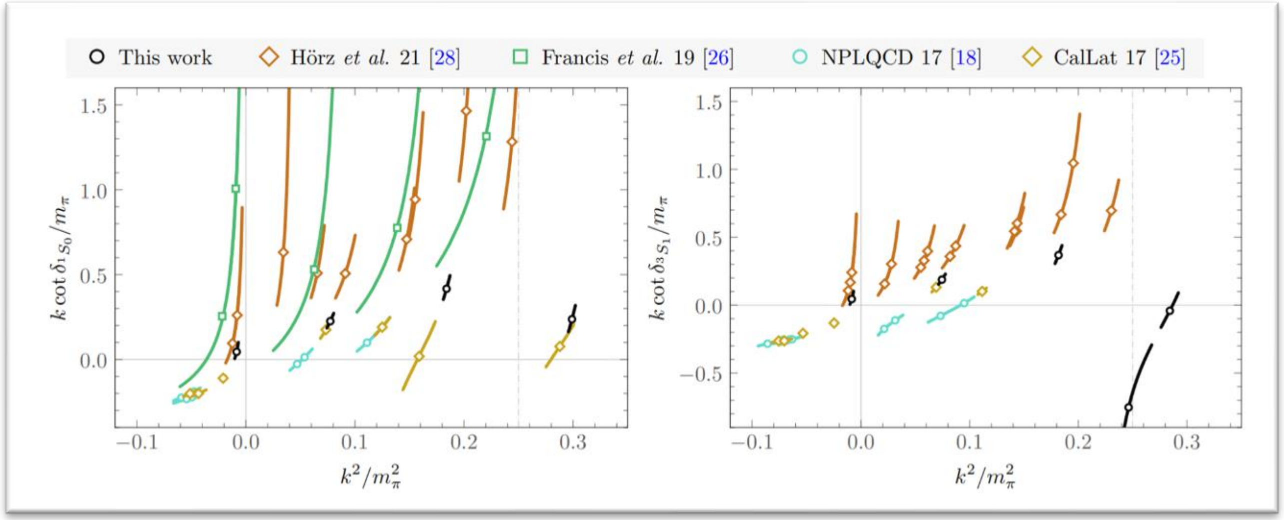
Motivation

- Recently, many charmed exotic states beyond conventional quark model were found in experiments. Such as T_{cc}^{+} which can be explained well on Lattice.

[see talks given at LT1 room, 29th afternoon]

- Study of Baryon-Baryon interaction have many challenges on Lattice.

- Deuteron bound state haven't been confirmed in the lattice calculation.



[Saman Amarasinghe et al. PRD 107 (2023) 9, 094508]

- The same with $\Lambda\Lambda$ system, which is also known as H dibaryon.

[Kenji Sasaki et al. Nucl.Phys.A 998 (2020) 121737]

[Jeremy R. Green et al. PRL. 127 (2021) 24, 242003]

What about $\Lambda_c\Lambda_c$?

Lattice Setup

[Z.C. Hu et al. PRD 109 (2024) 5, 054507]

CLQCD

- Two Wilson-Clover lattice ensembles are used in this work.
- They are space-time symmetrical and 2+1 flavor.
- The same pion mass and lattice spacing, but different volume.

ensemble	$(L/a)^3 \times T/a$	β	a(fm)	m_π(MeV)	m_K(MeV)	$m_\pi \times L$	N_{conf}
F32P30	$32^3 \times 96$	6.41	0.07746(18)	303.2(1.3)	524.6(1.8)	3.81	567
F48P30	$48^3 \times 96$	6.41	0.07746(18)	303.4(0.9)	523.6(1.4)	5.72	201

- Mass of particles/MeV

particle	$m_{latt.}$
π	303.2(1.2)
N	1069.7(4.8)
Λ_c	2411.5(2.9)
Σ_c	2571.8(2.9)
Ξ_{cc}	3747.7(1.0)

F32P30

particle	$m_{latt.}$
π	303.4(0.9)
N	1062.4(1.9)
Λ_c	2410.5(1.2)
Σ_c	2565.9(1.3)
Ξ_{cc}	3750.5(0.7)

F48P30

- Energy levels of thresholds /MeV

threshold	$E_{latt.}$
Ξ_{cc}N	4817.4(5.8)
$\Lambda_c\Lambda_c$	4822.9(5.8)
$\Xi_{cc}N\pi$	5120.6(6.0)
$\Sigma_c\Sigma_c$	5143.5(5.8)

F32P30

threshold	$E_{latt.}$
Ξ_{cc}N	4812.9(2.6)
$\Lambda_c\Lambda_c$	4820.9(2.5)
$\Xi_{cc}N\pi$	5116.3(3.5)
$\Sigma_c\Sigma_c$	5131.7(2.6)

F48P30

- **Lüscher's finite volume method** [M. Lüscher, Nucl. Phys. B. 354 (1991) 531-578]

$$\mathbf{det}[\mathbf{1} + \boldsymbol{i\rho T}(\mathbf{1} + \boldsymbol{i}\,\mathcal{M})] = \mathbf{0}$$

  - ρ ~ phase space
  - T ~ scattering amplitude
  - $\mathcal{M}$ ~ Lüscher matrix
  - For S wave spin-zero case: $$\mathcal{M}_{00}(\tilde{q}^2) = \frac{2}{\sqrt{\pi}L} Z_{00}(1; \tilde{q}^2)$$

  [see talk: LT1 room, 29th 11:35 given by Nelson Pitanga Lachini]

- **Distillation quark smearing method** [Michael Peardon et al, PRD 80 (2009) 054506]

  - Improve precision
  - Efficient for large numbers of operations

  [see talks given at LT1 room, 30th morning]

- **Operator construction**
  - Single baryon $\mathrm{B}(k, x^0) = \sum_{\vec{x}} P_+ \, \varepsilon_{abc} \, r_{ax} \left[s_{bx}^T \, (C\gamma_5) t_{cx} \right] e^{-i\vec{k}\cdot\vec{x}}$
  - Two baryons $\mathcal{O}_{B_1 B_2}^{\Lambda}(|\vec{k}|) = \sum_j c_j^{\Lambda} B_1^T(|\vec{k}|) \, C\gamma_5 B_2(-|\vec{k}|)$
- **Correlation function**

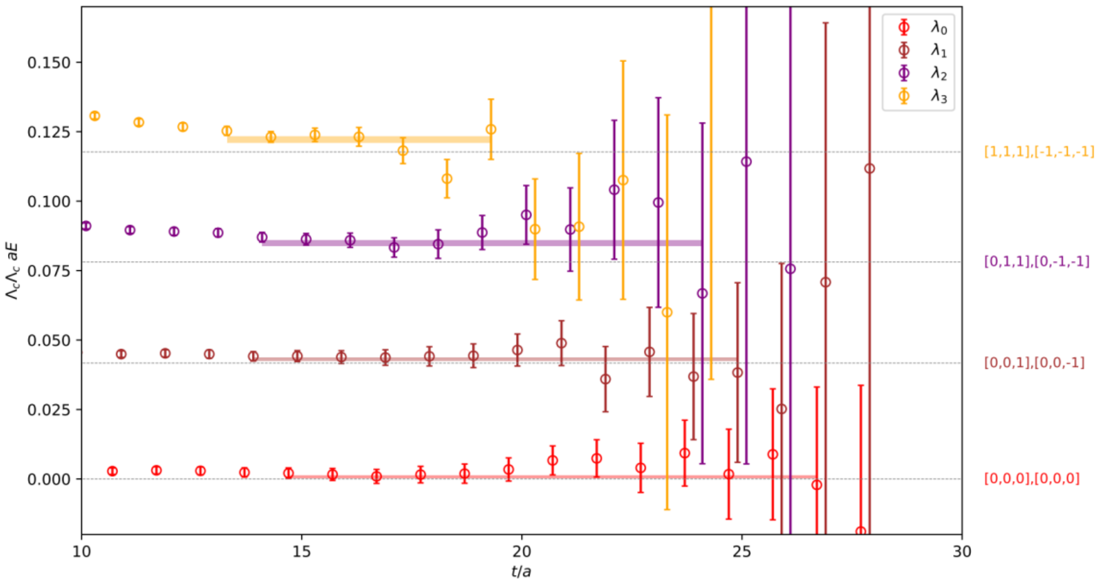
$$C(t) = \langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle \approx A e^{-E\,t}$$

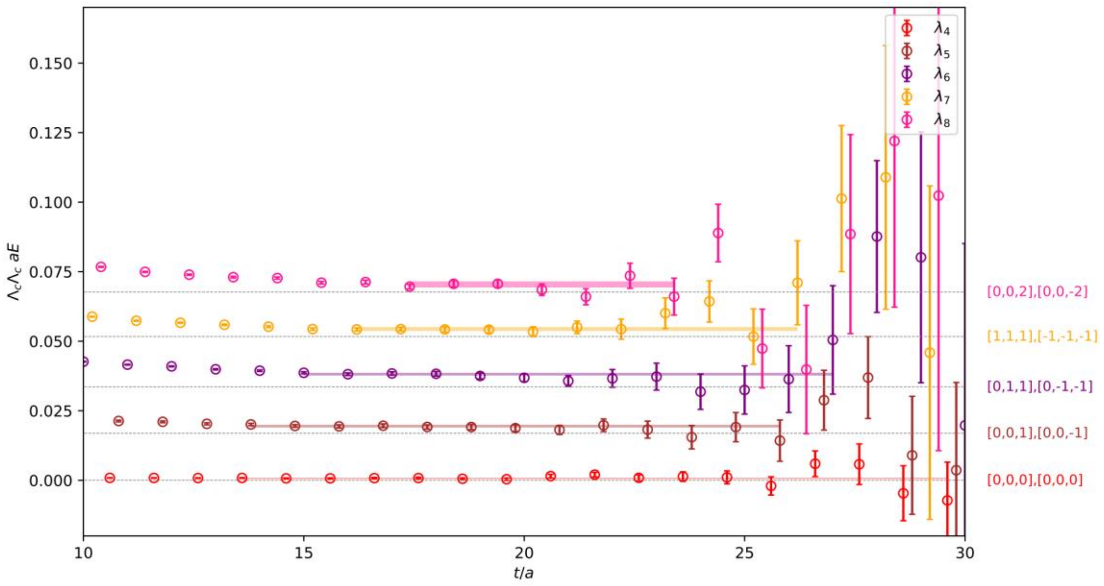
- **Generalized eigenvalue problem(GEVP)** $C(t) v_\alpha(t, t_0) = \lambda_\alpha(t, t_0) C(t_0) v_\alpha(t, t_0)$
  - Eigenvalue $\lambda_\alpha(t, t_0) \sim e^{-E_\alpha(t - t_0)}$

Energy Fitting Method

Since these energy levels are quite close to their corresponding free energies, a ratio form fitting could perform better

$$C_R(t) = \frac{C_{BB}(t)}{C_{thre.}(t)} \simeq Ae^{-(E_{BB}-2m_{\Lambda_c})t} = Ae^{-\Delta Et}$$





Couple channels

To explore the states near $\Lambda_c\Lambda_c$ $0(0^+)$ threshold, three channels are taken into consideration in A_1^+ irrep, i.e.$\Xi_{cc}N, \Lambda_c\Lambda_c$ and $\Sigma_c\Sigma_c$.

- single baryon $\boldsymbol{\Lambda_c}$ $\mathbf{0}\left(\frac{1}{2}^+\right)$, $\boldsymbol{\Sigma_c}$ $\mathbf{1}\left(\frac{1}{2}^+\right)$, $\boldsymbol{\Xi_{cc}}$ **and** $\boldsymbol{N}$ $\frac{1}{2}\left(\frac{1}{2}^+\right)$

- two baryons

$$\Lambda_c\Lambda_c^{I=0} = [\Lambda_c\Lambda_c]$$

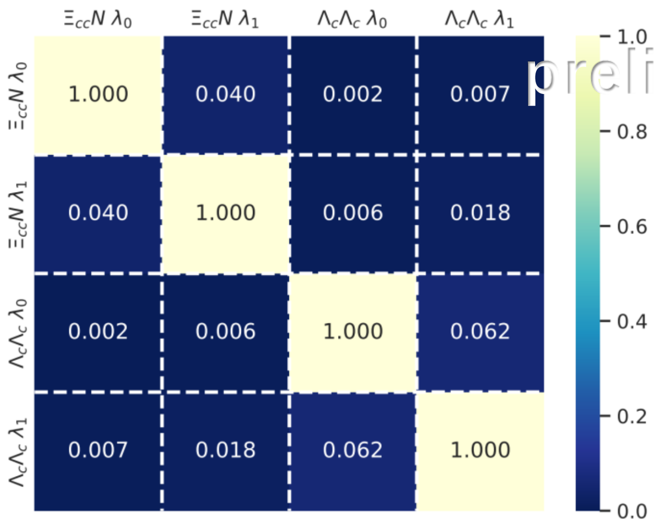
$$\Sigma_c\Sigma_c^{I=0} = \frac{1}{\sqrt{3}}([\Sigma_c^{++}\Sigma_c^0] - [\Sigma_c^+\Sigma_c^+] + [\Sigma_c^{++}\Sigma_c^0])$$

$$\Xi_{cc}N^{I=0} = \frac{1}{2}([p\Xi_{cc}^+] + [\Xi_{cc}^+p] - [n\Xi_{cc}^{++}] - [\Xi_{cc}^{++}n])$$

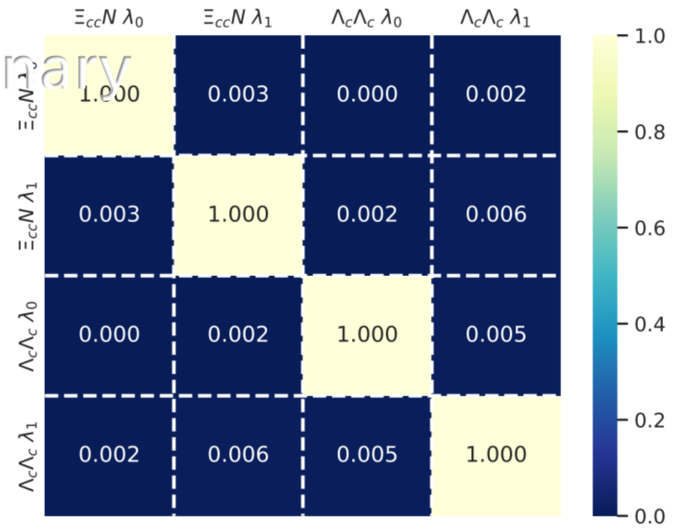
- Coupled with $\Xi_{cc}N$

Cross-correlation matrix

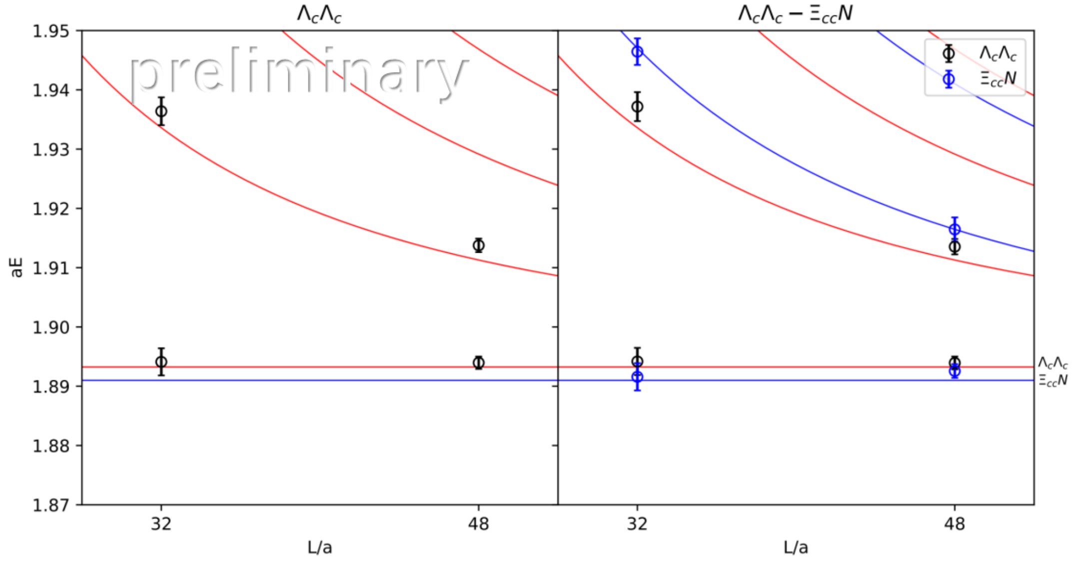
$$\widetilde{C}_{ij}(\vec{P},t) = C_{ij}(\vec{P},t)/\sqrt{|C_{ii}(\vec{P},t)C_{jj}(\vec{P},t)|}$$



(a)F32P30 $\Xi_{cc}N - \Lambda_c\Lambda_c$



(b)F48P30 $\Xi_{cc}N - \Lambda_c\Lambda_c$

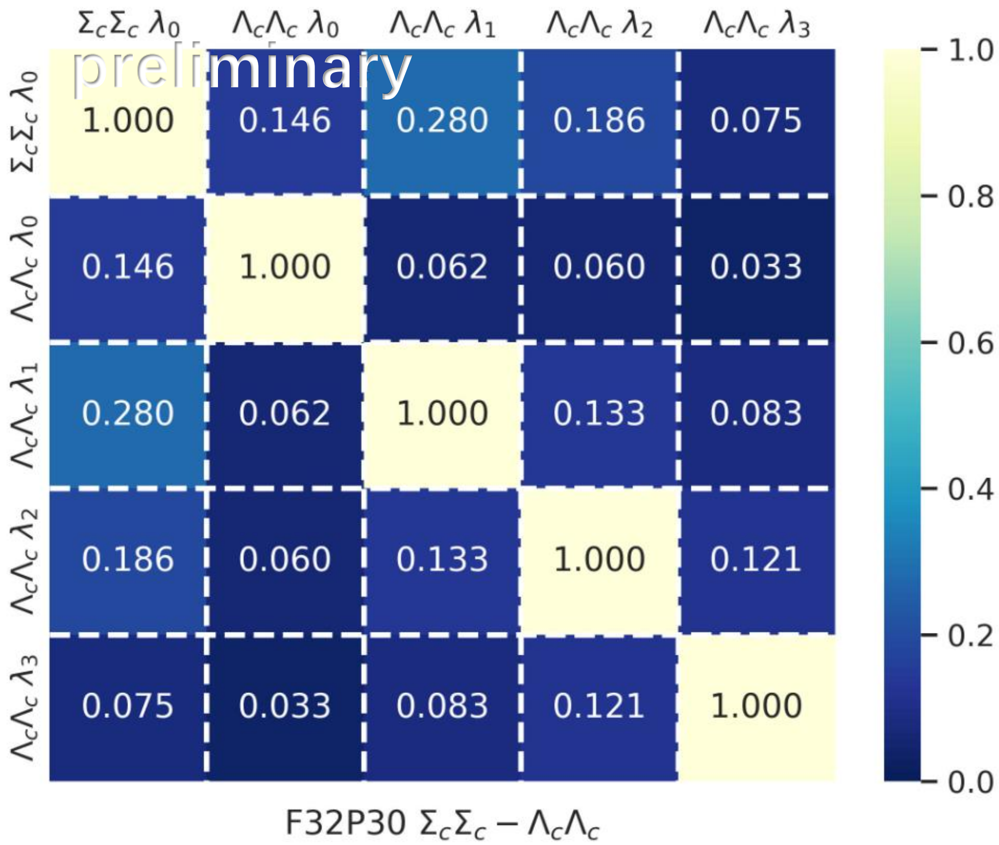


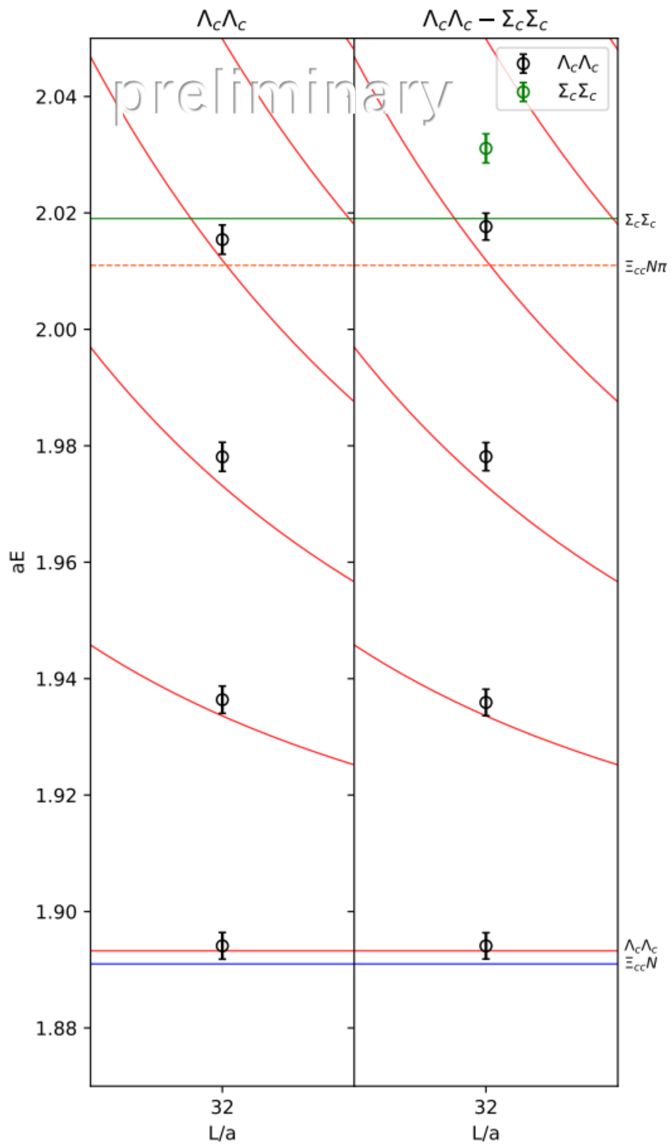
- The coupling between $\Xi_{cc}N$ and $\Lambda_c\Lambda_c$ is quite small.
- $\Lambda_c\Lambda_c$ energy levels haven't been shifted obviously.

Analyses and Result

- Coupled with $\Sigma_c\Sigma_c$

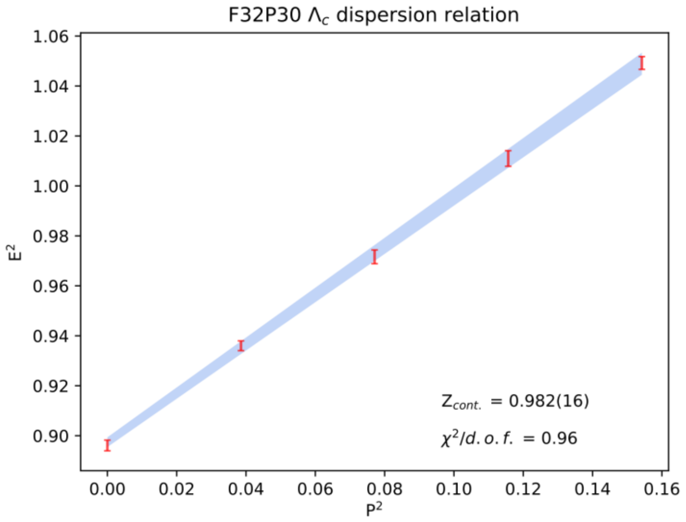
Shift is slight too, except the highest one.

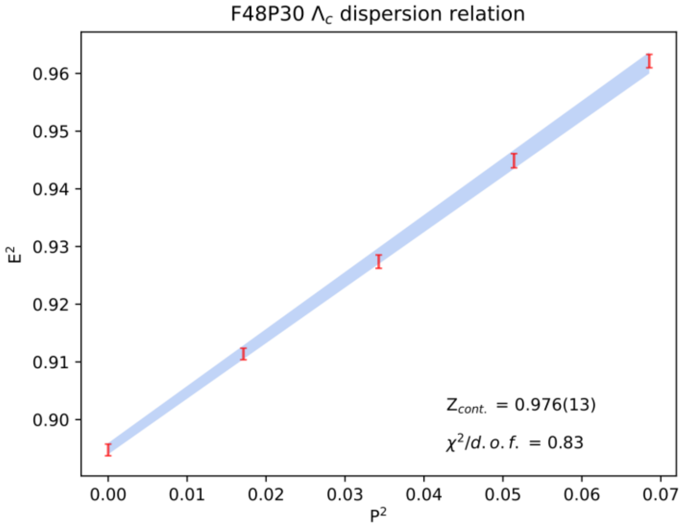




Single channel analysis

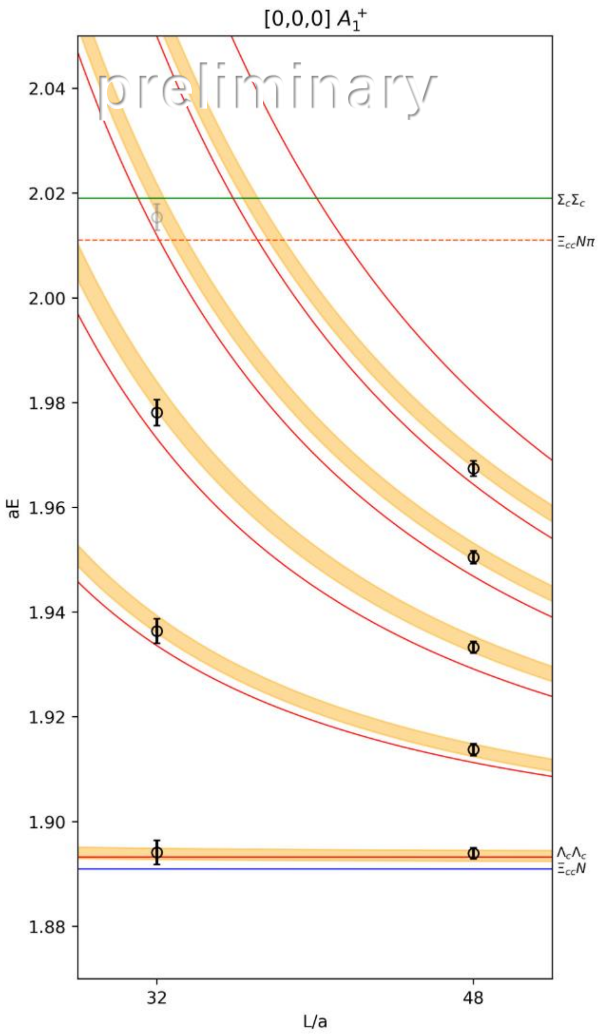
- Dispersion relation check $E^2(k) = m^2 + Z_{cont.} p^2$





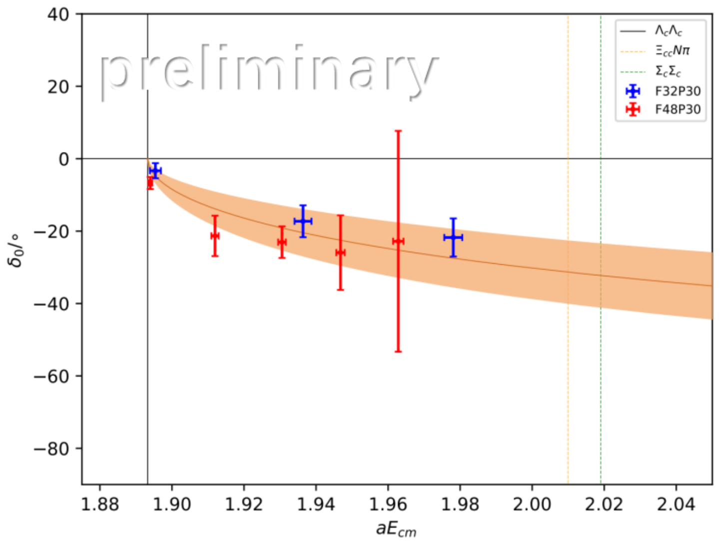
- Modified Lüscher formula

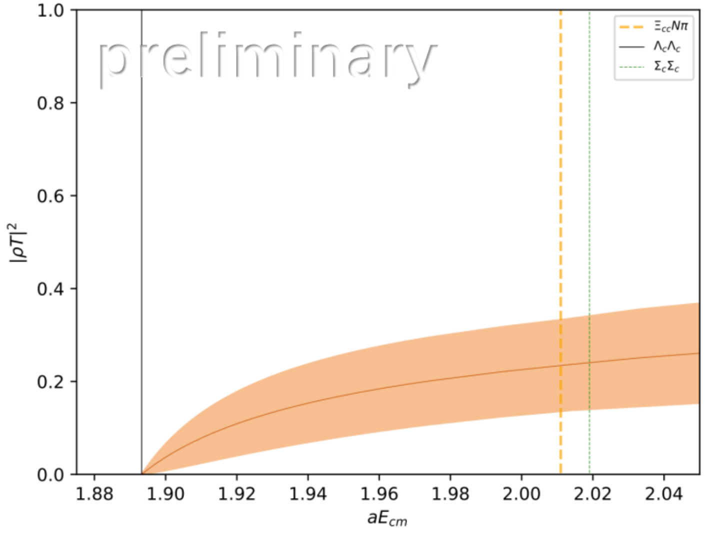
$$\mathcal{M}_{00}(\tilde{q}^2) = \frac{2}{\sqrt{\pi}L} \mathcal{Z}_{00}(1;\tilde{q}^2) \times \frac{E_1 + E_2}{Z_1 E_2 + Z_2 E_1}$$

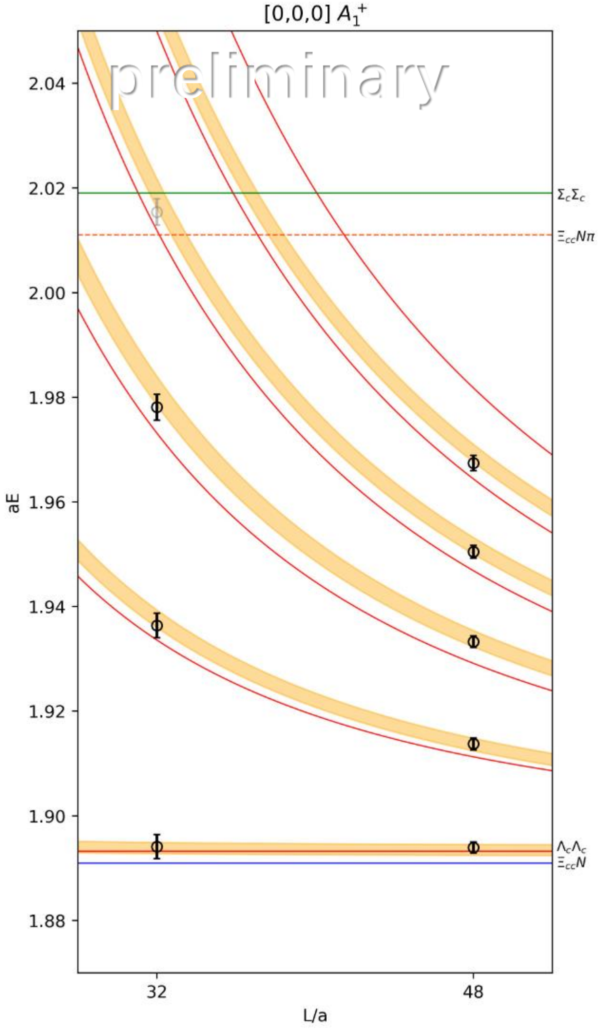


- Repulsive interaction

- Phase shift and scattering length







Parameterization: $k\cot\delta_0 = \frac{1}{a_0}$.

Fitting result: $a_0 = -0.143(49)$ fm, with $\frac{\chi^2}{dof} = 0.86$.

Conclusion: **No bound state** in S wave $\Lambda_c\Lambda_c$ system.

- Repulsive interaction

Summary

- Due to the **discretization** on lattice, a **modification on L$\ddot{u}$scher equation** is proposed in this work.

- Coupled with Ξ_{cc}N or $\Sigma_c\Sigma_c$, the energy levels of $\Lambda_c\Lambda_c$ haven't been shifted obviously. Therefore, **single channel** is contained.

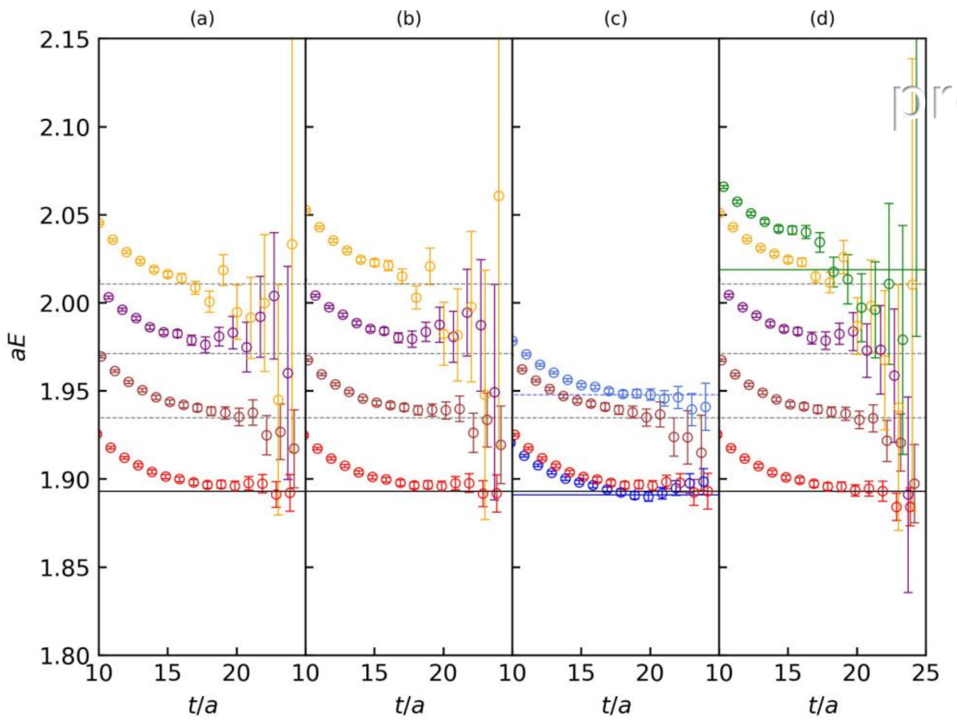
- Showing a **repulsive interaction**, there's **no bound state** in this system.

- Scattering length $\boldsymbol{a_0 = -0.143(49)}$ **fm.**

Thanks!

Back-up

- Effective mass in various cases



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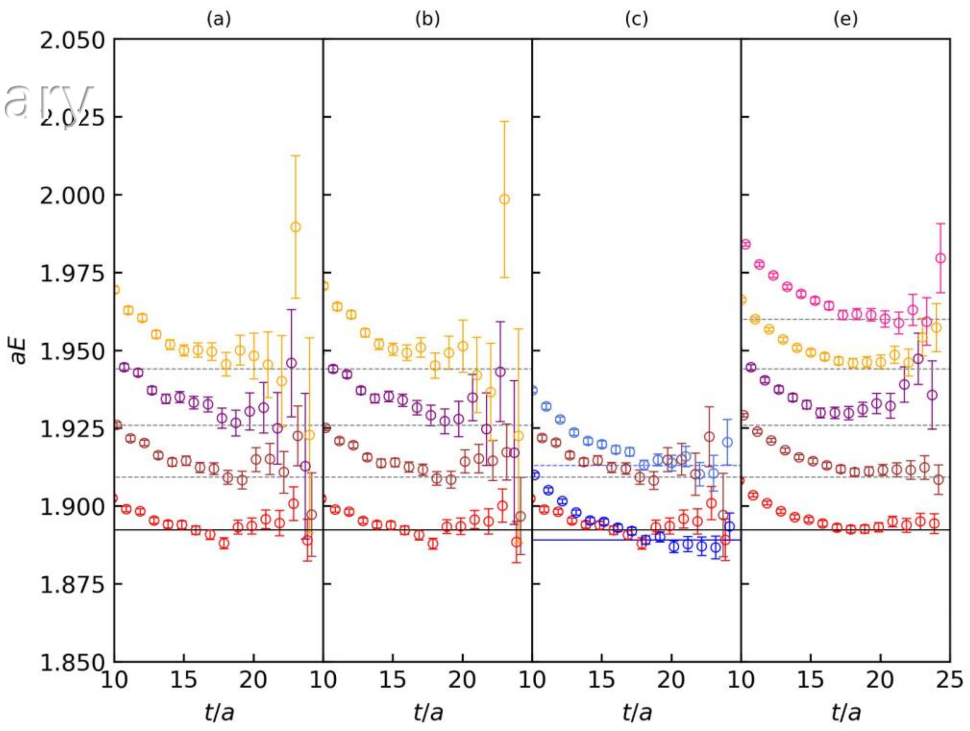

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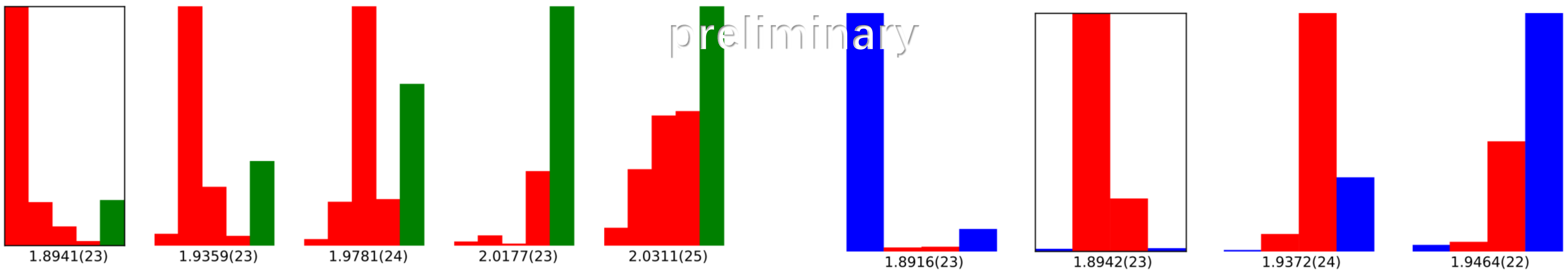
(a) Diagonal elements of $\Lambda_c\Lambda_c$ correlation matrix(red, orange, purple, brown)
(b) Single channel $\Lambda_c\Lambda_c$ with GEVP
(c) Coupled with $\Xi_{cc}N$(blue and blue) with GEVP
(d) Coupled with $\Sigma_c\Sigma_c$(green) with GEVP
(e) The same as (a) with Nev=200, added in higher momentum(pink).

- Overlap factors[J. J. Dudek et al, 1004.4930]

$$Z_i^\alpha \equiv \langle \alpha \,|\phi_i\,|\, 0 \,\rangle \;=\; \sqrt{2E_\alpha} e^{\frac{E_\alpha t_0}{2}} v_j^{\alpha *} C_{ji}(t_0)$$



Red ones belong to $\Lambda_c\Lambda_c$, while green one belongs to $\Sigma_c\Sigma_c$ and blue ones belong to $\Xi_{cc}N$

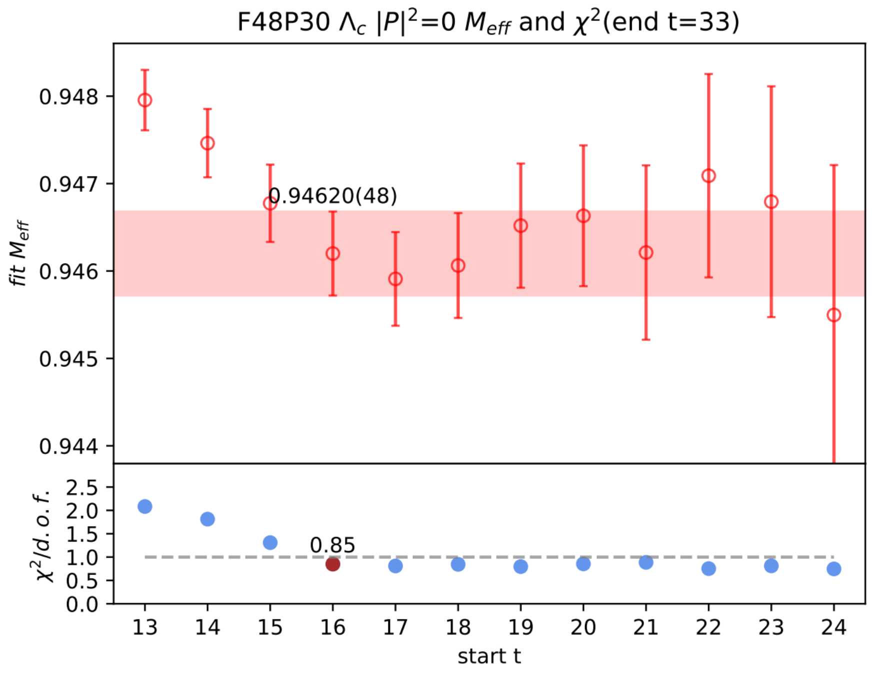
Black box represents the ground state of $\Lambda_c\Lambda_c$.

- Correlation function fitting details

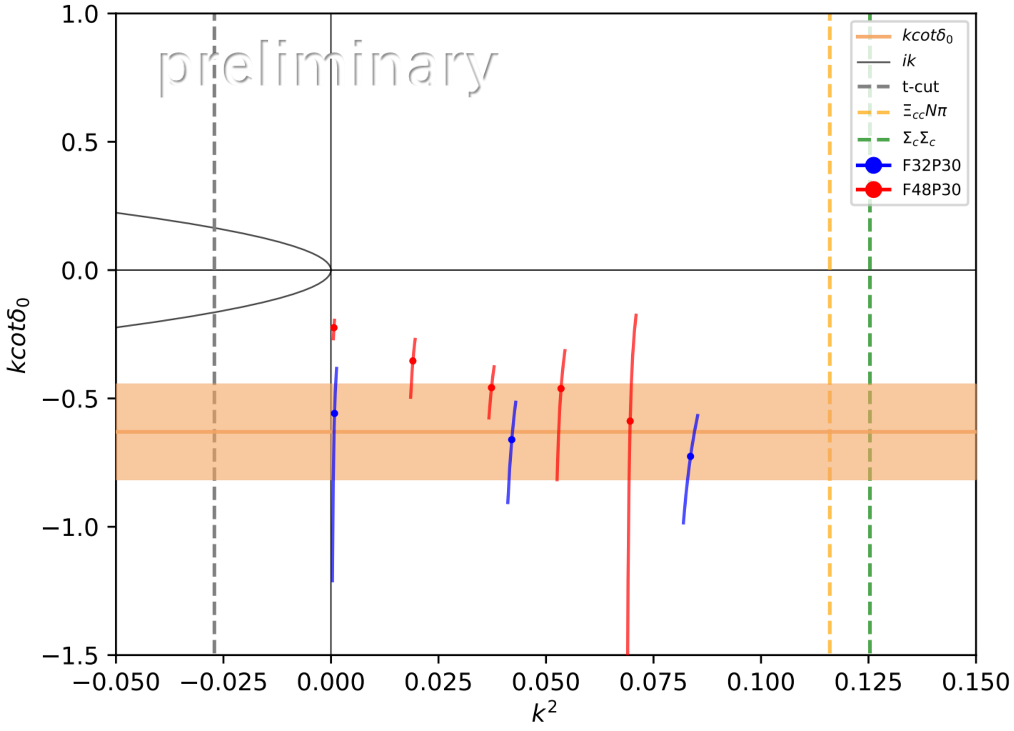
Correlation function of Single Λ_c then can be fitted as the parametrization

$$\mathcal{C}(t) = A\exp\left[-M_{eff}t\right]$$

- Fitting window is $[t_{start}, t_{end}]$ while t_{end} is fixed.
- Starting time-slice should given a stable fitting result.
- Fitting window should be as wide as possible.



- Left-hand cut



The left-hand cut gives the limit of pole extraction with $k^2 = -\frac{m_\omega^2}{4}$ from the exchanging of omega boson.

[X.-K. Dong et al, Phys.Rev.D 105 (2022) 7, 074506]
[Also see talk in Aug 1st 11:30 and 11:50 in LT1]

- Finite volume effect
  - The scattering length are consistent in the two ensembles.
  - However, there’s a little difference.