# Simulating an SO(3) Quantum Link Model with Dynamical Fermions in 2+1 Dimensions

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UK NATIONAL QUANTUM TECHNOLOGIES PROGRAMME

Quantum Technologies for Fundamental Physics

# Outline

- **Quantum Link Models: What and Why?**
  - Exact gauge symmetry with a finite-dimensional Hilbert space
- **SO(3) QLM and Nuclear Physics**
  - Imposing gauge-invariance on a per-site basis: 1+1d and 2+1d
  - Confinement and chiral symmetry-breaking in 1+1d
  - Phase diagram information from 1+1d
- **Exact Diagonalization for a Single Plaquette**
  - The plaquette observable as an order parameter
  - Explicit and spontaneous chiral symmetry-breaking
  - Exploiting discrete symmetries for variational quantum simulation

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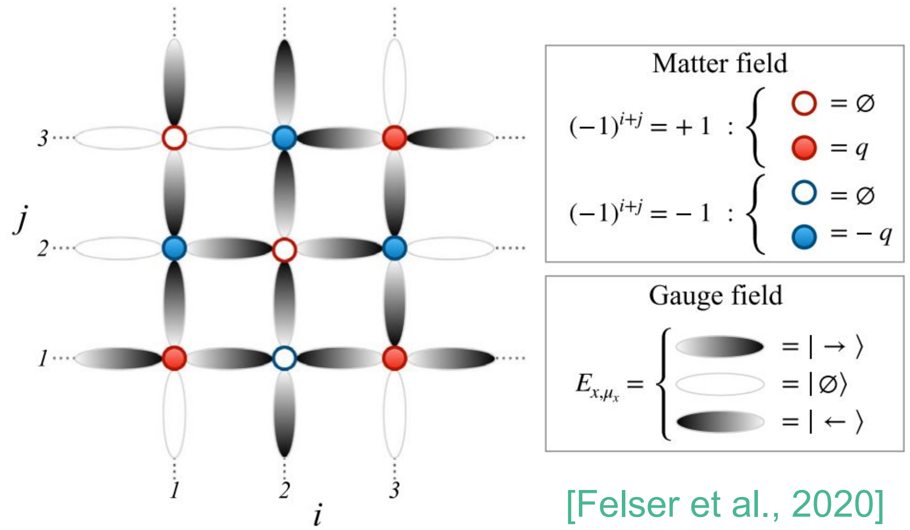
# Quantum Link Models (QLMs)

- Generalisations of Wilsonian LGTs, with link operators on finite-dimensional Hilbert spaces.
- Exact gauge symmetry
  - Choice of embedding algebra
- Well-posed continuum limits
  - Dim. reduction (D-theory)[1]
  - Large spin representations[2]




[Felser et al., 2020]

[1] Wiese, 2022, "From QLMs to D-theory…" [2] Zache et al., 2022, "Toward the continuum limit…"

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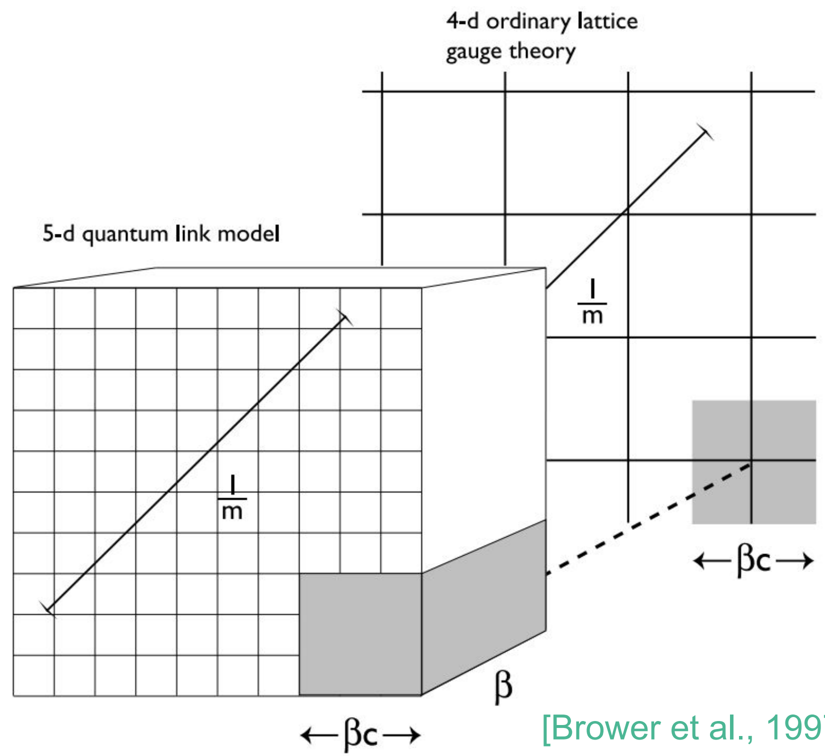

[Brower et al., 1997]

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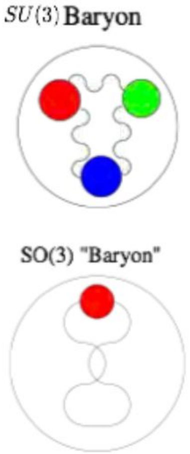
# Constructing an SO(3) QLM

- so(3) algebra realized on link operators $O^{ab}$, $L^a$, $R^b$
- Embed into so(6) bilinears:
  - $O^{ab} = \sigma^a \otimes \sigma^b$
  - $L^a = \sigma^a \otimes \mathrm{Id}$
  - $R^b = \mathrm{Id} \otimes \sigma^b$
- Adjoint fermions $\psi^a$
- H includes hopping, staggered mass, plaquette, and four-Fermi couplings

a)



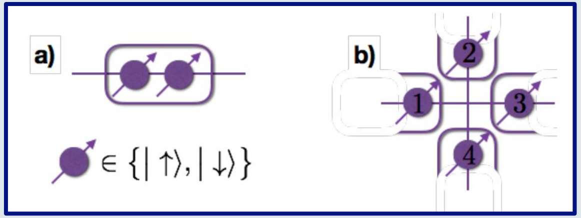

b) [adapted from Rico et al., 2018]

| | 3-d QCD | 1-d SO(3) | 2-d SO(3) |
|---|---|---|---|
| gauge symmetry | $SU(3)$ | $SO(3)$ | $SO(3)$ |
| chiral symmetry | $SU(2)_L \times SU(2)_R$ | $\mathbb{Z}_2$ | $\mathbb{Z}_2 \times \mathbb{Z}_2$ |
| flavor symmetry | $SU(2)_{L=R}$ | $\mathbb{I}$ | $\mathbb{Z}_2$ |
| baryon symmetry | $U(1)$ | $U(1)$ | $U(1)$ |
| charge conjugation | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ |
| parity | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ |

Hamiltonian:
$$H = -t \sum_{x,k} \left[ \eta_{x,k} B^\dagger_{x,\hat{k}} B_{x+k,-\hat{k}} + h.c. \right] + m \sum_x \eta_x M_x$$
$$- \frac{1}{4g^2} \sum_x \Phi_{x,34}\Phi_{x,23}\Phi_{x,12}\Phi_{x,41} + G \sum_x M_x^2 + V \sum_{x,k} M_x M_{x+k}$$

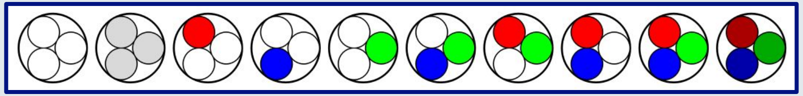
# Gauge-Invariant Basis

- Four spins per site: 2 singlets, 9 triplets
  - Fermions: singlets 0,3 ; triplets 1,2



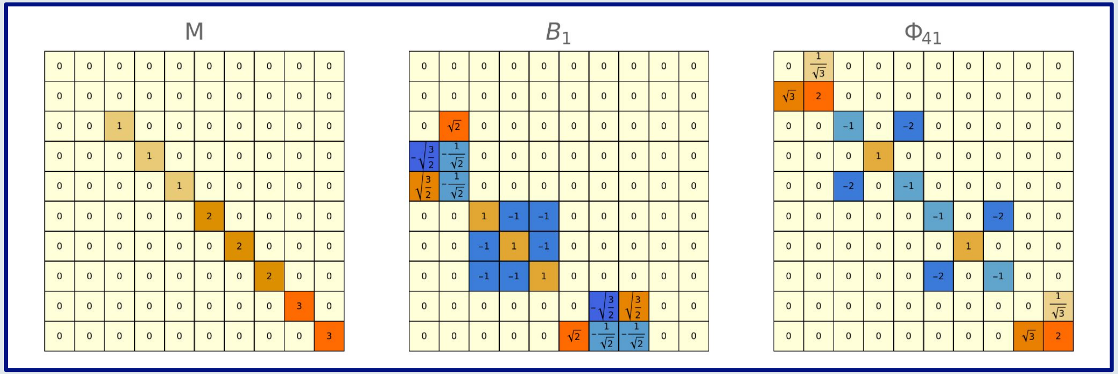

[Rico et al., 2018]

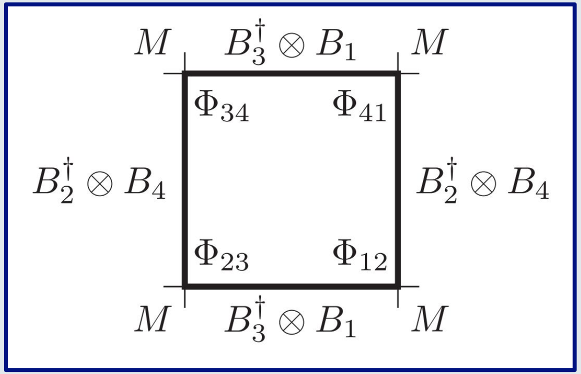
- Combining gives 10 gauge-singlets per site



- Gauge-invariant operators

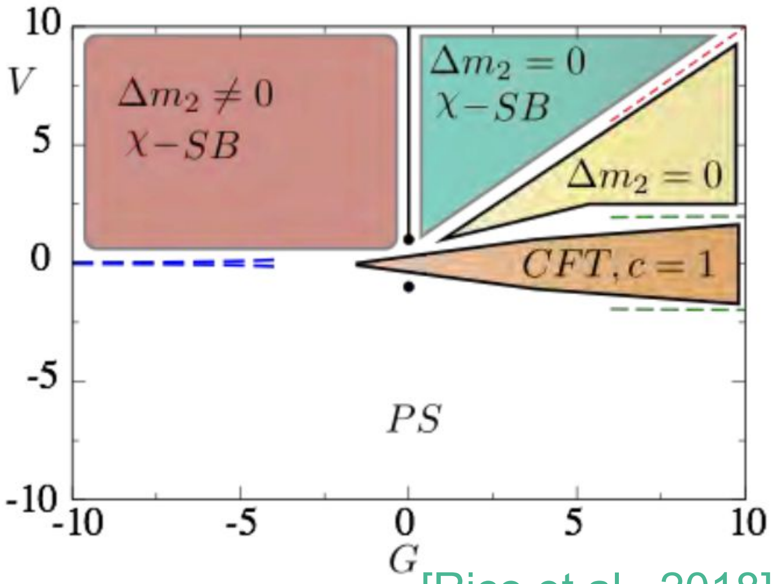
$$B_{x,k} = \psi^a_x \sigma^a_{x,k},\ M_x = \psi^{a,\dagger}_x \psi^a_x,\ \Phi_{x,k,l} = \sigma^a_{x,k}\sigma^a_{x,l}.$$



# SO(3) “Nuclear Physics” in (1+1)d

- For V>0 & G<V, chiral symmetry breaking – restored at finite B density
- Conformal phase for G >> V > 0, few-body bound states for G<0 or G << V




[Rico et al., 2018]

<u>(1+1)d Gauge-Invariant Basis</u>

- 4 gauge-singlet states
- Operators M, $B^+$, $B^-$
- Exact correspondence to spin-3/2 chain

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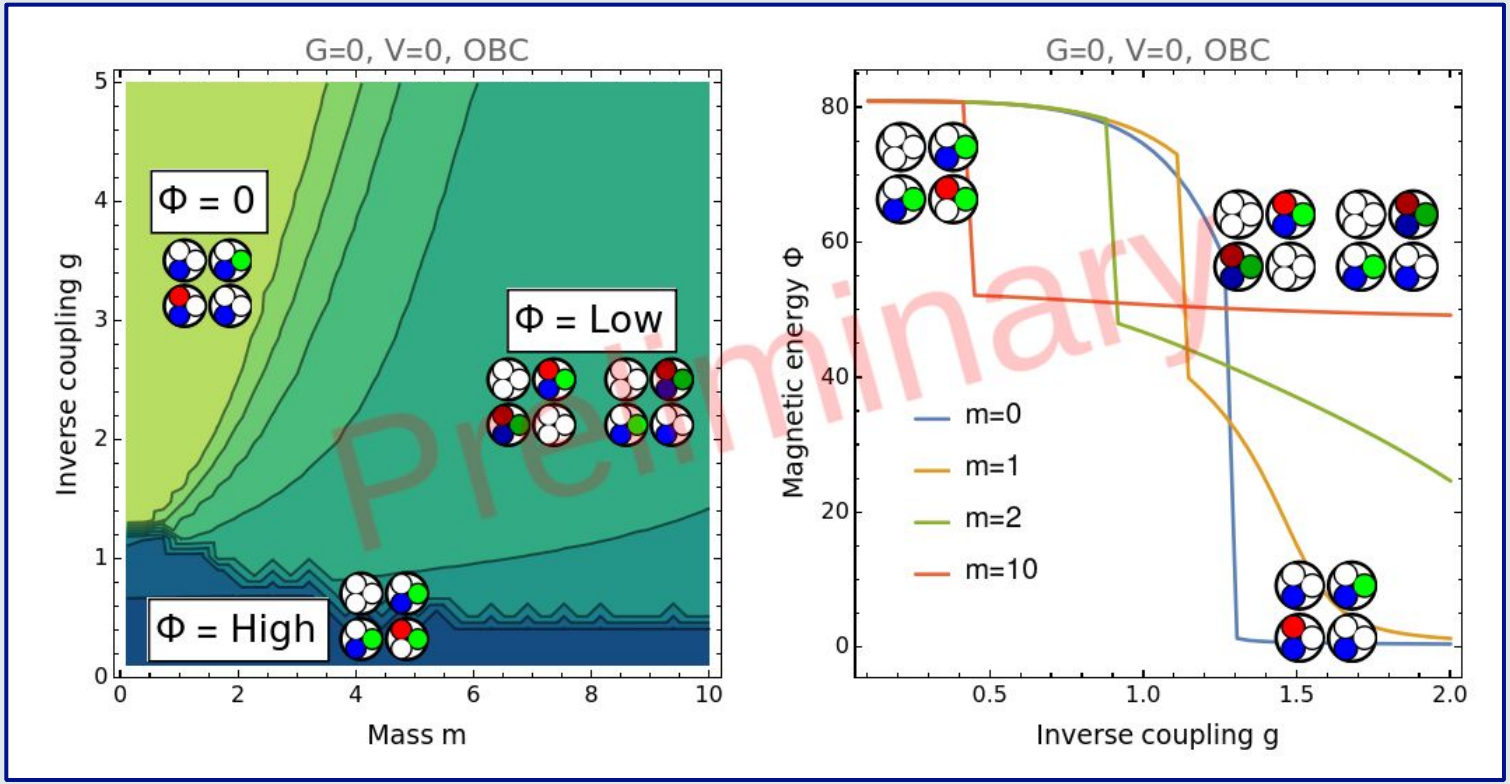
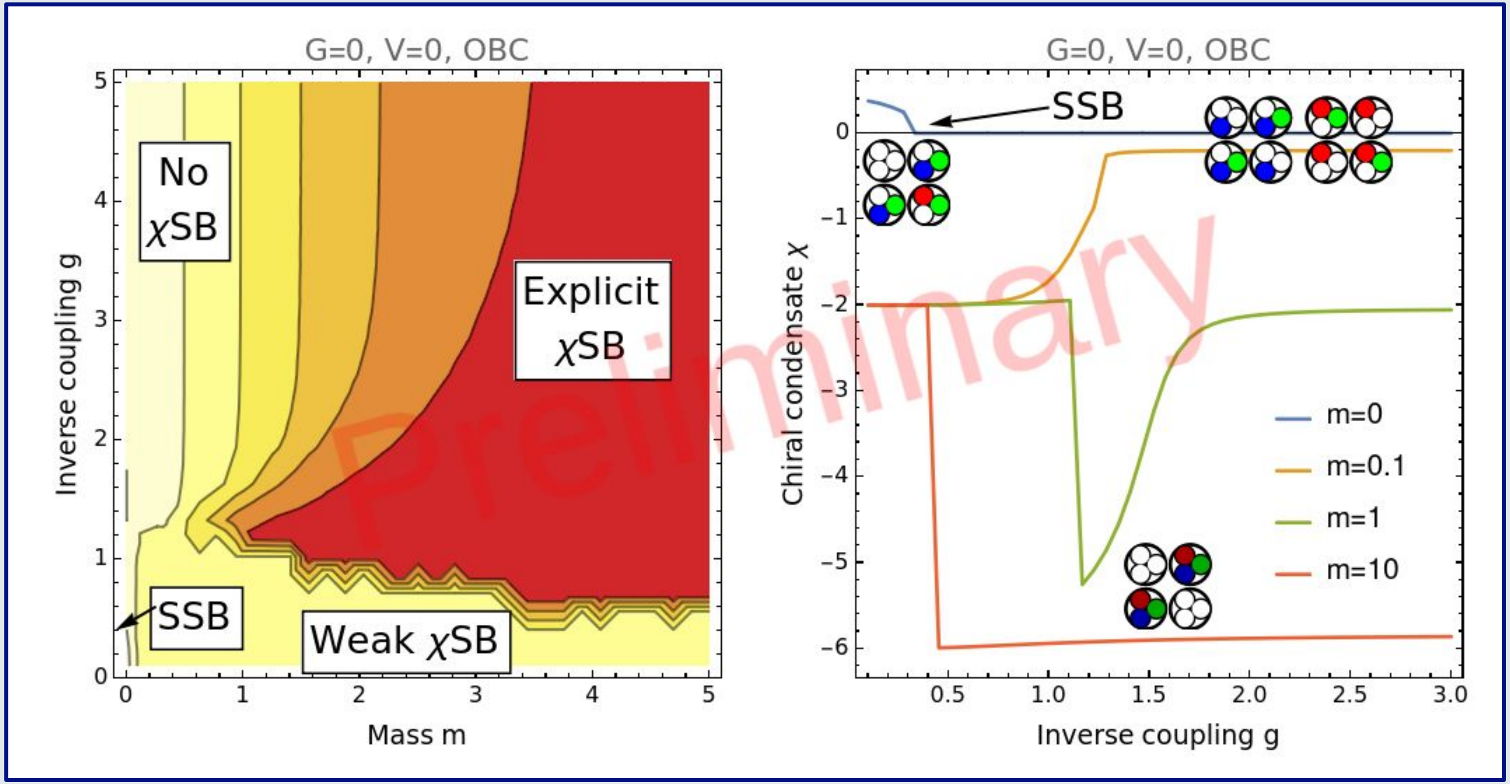
# Magnetic Observable (Plaquette)

- Three limiting phases – sharp massless transition to high-field, at $g_M \simeq 1.28$.



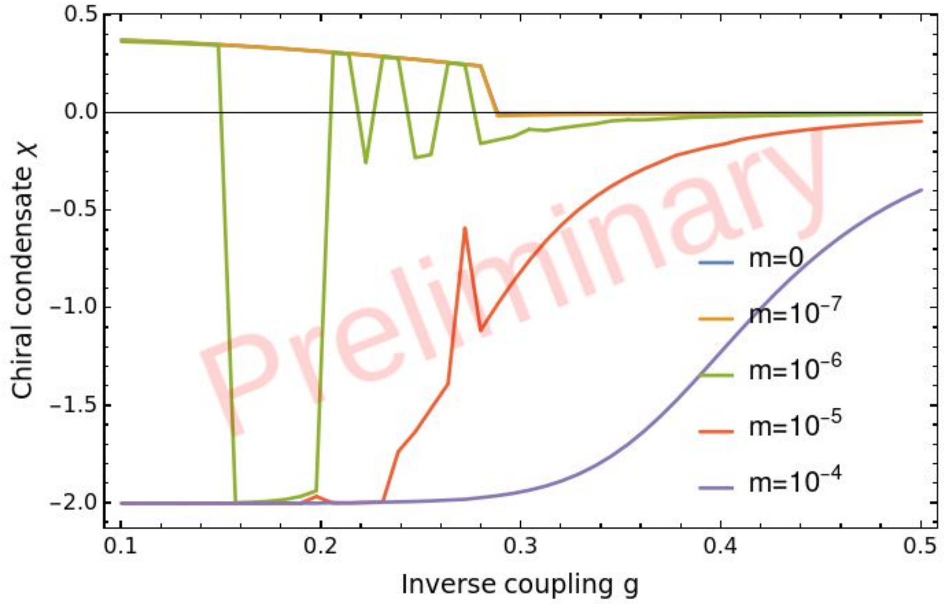
# Explicit Chiral Symmetry Breaking

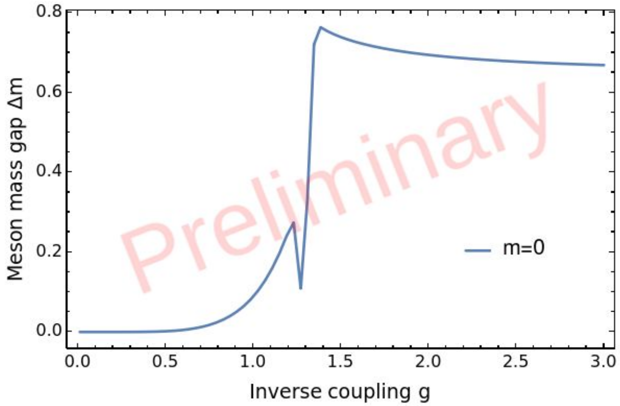
- m>0 explicitly breaks chiral symmetry – weakly within strong-field phase.



# Spontaneous Chiral Symmetry Breaking

- Closer look reveals SSB at $g_\chi \simeq 0.28$ for m = 0.
- Coincides with degenerate ground state.




(work in progress)

# Summary

- We have identified the gauge-invariant state space of an SO(3) QLM w/ fermions in (2+1)d.
- We have performed exact diagonalisation for a single plaquette, finding:
  - Distinct magnetic phases, discontinuous transition at $g_M \simeq 1.28$ ,
  - Explicit chiral symmetry breaking with two phases,
  - Spontaneous chiral symmetry breaking at strong-coupling, $g_\chi \simeq 0.28$ .

- *Caution*: we should take single-plaquette results as broad approximations.
- **Next:** study larger (2+1)d lattices, and simulate via variational QAs.

THANK YOU FOR LISTENING!

# Works Cited

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Thank you to Lattice 2024, and our hosts here in Liverpool!

# Extra: Variational QAs and Discrete Symmetries

- QLMs generically have numerous discrete symmetries (C, P, translation)
- Eliminating redundant basis states = smaller variational ansätze.