Handling challenges for robust and reliable quantum simulation of gauge theories on 1+1D and 2+1D

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Three of the key challenges for robust and reliable quantum simulation of gauge theories

- State preparation of gauge theory Hamiltonian – why so challenging? What are effective ways?

- Dealing with a large Hilbert space for gauge theories in contrast to quantum spins
  - We choose to work with a matter-free non-Abelian SO(3) lattice gauge theory in 2+1D
  - We impose the non-Abelian Gauss Law in the Rishon representation of the quantum link operator
  - Significantly reduces the degrees of freedom for gauge theories

- Sensitivity to errors and noise - an effective scheme for quantum error mitigation (QEM) using symmetry constraints and post-selection.

Challenge 1: State preparation for gauge theories

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Classically emulated digital quantum simulation of the Schwinger model with a topological term via adiabatic state preparation

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We designed a protocol for digital quantum computation of a gauge theory with a topological term in Minkowski spacetime, which is practically inaccessible by standard lattice Monte Carlo simulations. We focus on $1+1$ dimensional quantum electrodynamics with the θ term known as the Schwinger model and test our protocol for this on an IBM simulator. We construct the true vacuum state of a lattice Schwinger model using adiabatic state preparation which, in turn, allows us to compute an expectation value of the fermion mass operator with respect to the vacuum. Upon taking a continuum limit we find that our result in the massless case agrees with the known exact result. In the massive case, we find an agreement with mass perturbation theory in the small-mass regime and deviations in the large-mass regime. We estimate computational costs required to take a reasonable continuum limit. Our results imply that digital quantum simulation appears a promising tool to explore nonperturbative aspects of gauge theories with real time and topological terms.

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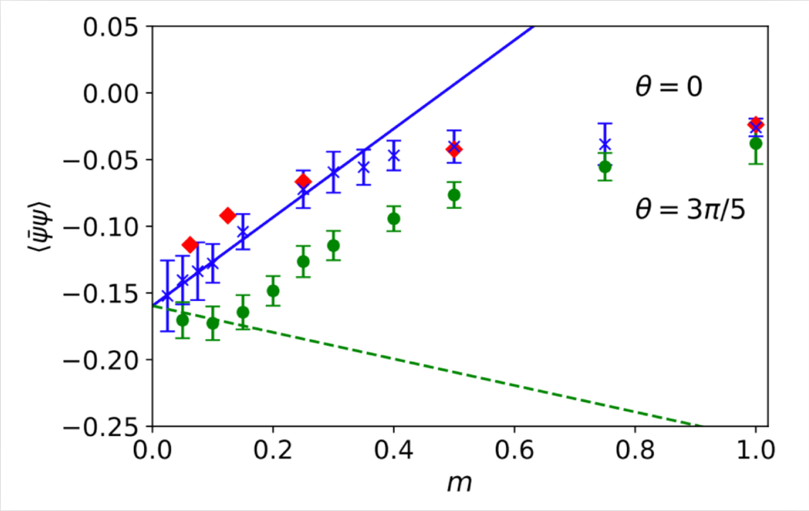
Adiabatic state preparation (ASP) for Schwinger Model Hamiltonian with theta term in 1+1D

$$H = -\mathrm{i}\sum_{n=1}^{N-1}\left(w-(-1)^n\frac{m}{2}\sin\theta\right)\left[\chi_n^\dagger e^{\mathrm{i}\phi_n}\chi_{n+1}-\mathrm{h.c.}\right] + m\cos\theta\sum_{n=1}^{N}(-1)^n\chi_n^\dagger\chi_n + J\sum_{n=1}^{N-1}L_n^2, \quad (5)$$

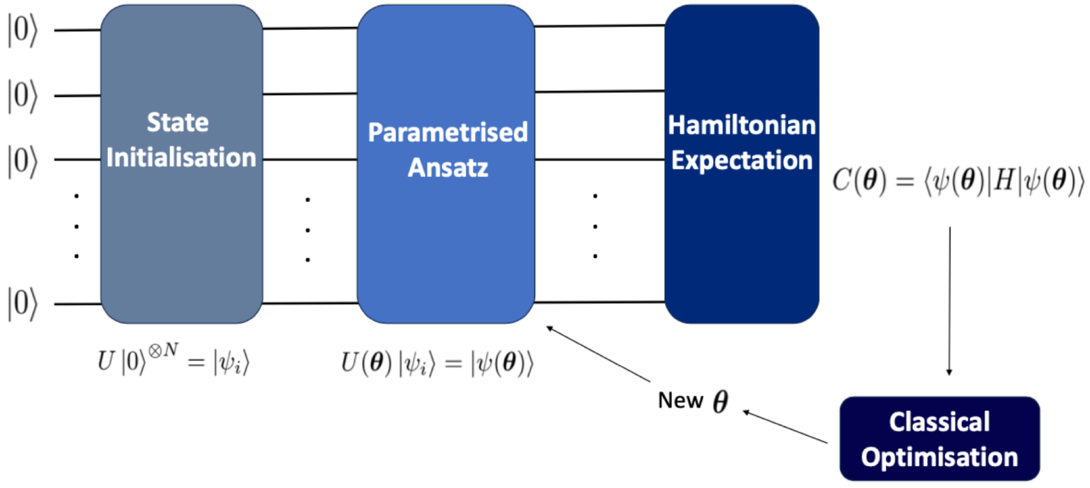
$$|\mathrm{vac}\rangle = \lim_{T\to\infty}\mathcal{T}\exp\left(-\mathrm{i}\int_0^T dt\ H_{\mathrm{A}}(t)\right)|\mathrm{vac}\rangle_0$$



However, ASP requires unfeasibly large circuit depth for near term Digital quantum computation

More feasible State preparation methods for NISQ era

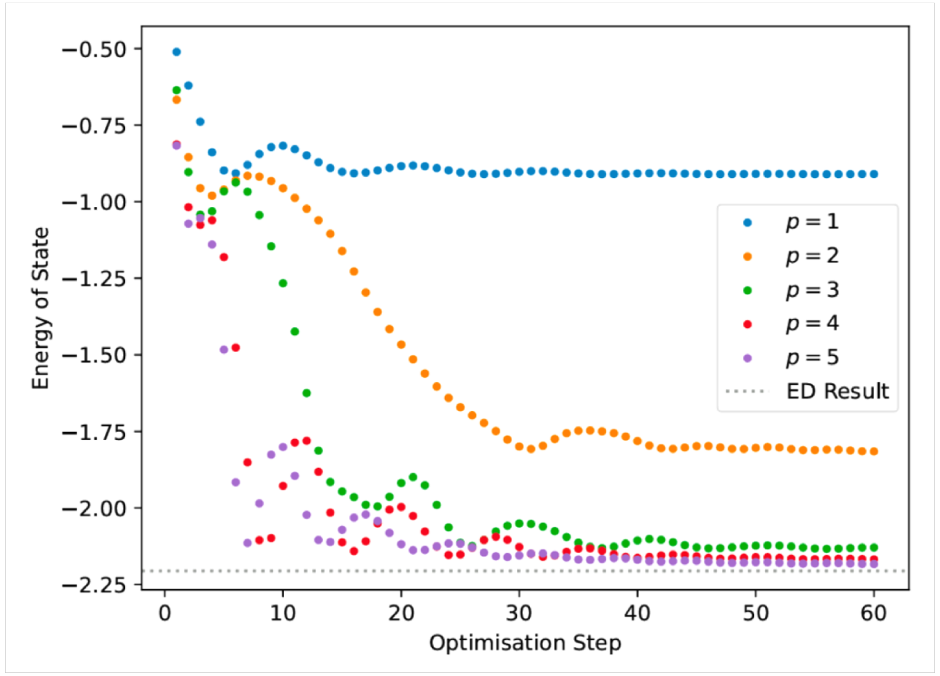
Variational quantum eigen solver (VQE)



Puruzzo et al. (2014) Nat Commun 5, 4213

Quantum Approximate Optimisation Algorithm (QAOA)

$$|\psi_p(\boldsymbol{\beta},\boldsymbol{\gamma})\rangle = e^{-i\beta_p \hat{H}_M} e^{-i\gamma_p \hat{H}_C} \ldots e^{-i\beta_1 \hat{H}_M} e^{-i\gamma_1 \hat{H}_C} |\psi_i\rangle$$



E. Farhi, J. Goldstone and S. Gutmann, A quantum approximate optimization algorithm, 2014

Further on this for U(1) 1+1D: Poster by Alex Tomlinson

Challenge 2: Large Hilbert space of gauge theories

We demonstrate the ability of Variational algorithms and QAOA to prepare ground states and excited states in a matter free non-Abelian SO(3) lattice gauge theories on 2+1 D

Additionally, we handle the exponentially decreasing mass gap due to the spontaneously broken global charge conjugation symmetry

Spontaneous symmetry breaking in an $SO(3)$ non-Abelian lattice gauge theory with quantum algorithms

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(Dated: July 27, 2024)

Simulation of various properties of quantum field theories are rapidly becoming a testing ground for demonstrating the prowess of quantum algorithms. Preparation of ground states, as well as various simple wave packets for demonstrating scattering phenomena are being extensively investigated. In this work, we study the ability of quantum algorithms for preparation of ground states in a matter-free non-Abelian $SO(3)$ lattice gauge theory in a phase where the global charge conjugation symmetry is spontaneously broken. This is challenging for two reasons: firstly, the necessity of dealing with a large Hilbert space for gauge theories in contrast to quantum spins, and secondly, the exponentially fast closing of the gap between the states which form the two ground states in an infinite volume. We demonstrate how the exact imposition of the non-Abelian Gauss Law in the rishon representation of the quantum link operator, significantly reduces the degrees of freedom, and alleviates the first problem. Further, in the Gauss-Law-resolved basis, symmetry-guided ansätze for trial states can be used as the starting point for the quantum algorithms to prepare the two lowest states, solving the second hurdle. We also provide experimental results from the quantum hardware, IonQ, when working on plaquettes with four qubits and before resolving the Gauss Law. Besides the two significant theoretical steps, the experimental results indicate the role of metrics, such as the energy and the infidelity, to assess the obtained results.

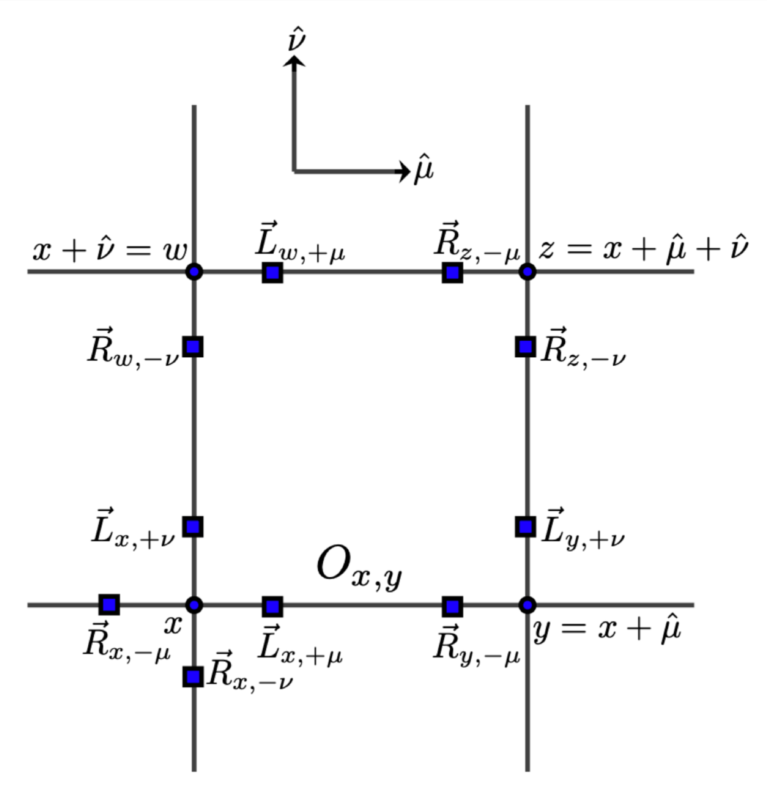
Soon to be arxived

SO(3) shares fundamental properties with QCD

Many groups have been working on various gauge theories in 2+1D and 3+1D

Rico et. al., Annals Phys. 393, 466-483 (2018)

An SO(3) Quantum Link Model



Model, symmetries and gauge invariant states:

$$\mathcal{H} = \mathcal{H}_E + \mathcal{H}_B$$

where

$$\mathcal{H}_E = \frac{g^2}{2}\sum_{x,\mu}\left(L^a_{x,+\mu}L^a_{x,+\mu} + R^a_{x+\mu,-\mu}R^a_{x+\mu,-\mu}\right)$$

$$\mathcal{H}_B = -\frac{1}{4g^2}\sum_{\Box}\mathrm{Tr}\mathcal{O}_{\Box}$$

Non-Abelian Gauss Law

$$G^a_x = \sum_{\mu}(L^a_{x,+\mu} + R^a_{x,-\mu}), \quad [G^a, G^b] = 2i\varepsilon^{abc}G^c$$

Choose a gauge invariant basis by directly projecting onto $\vec{G} = 0$

Rico et. al., Annals Phys. 393, 466-483 (2018)

D. Horn, Phys. Lett. 100B, 149 (1981)

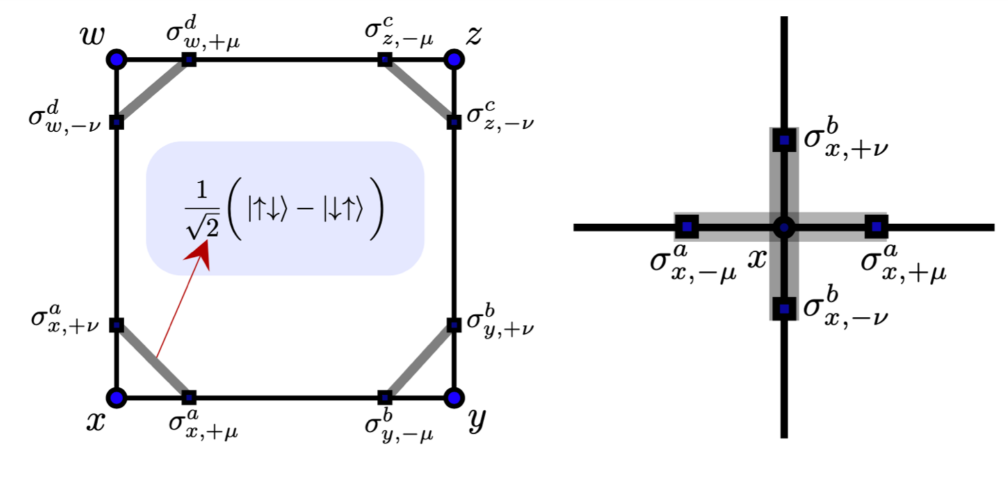
Brower, Chandrasekharan, Wiese, Phys. Rev. D 60, 094502 (1999)

An SO(3) Quantum Link Model (continued…)

$N^2 + 2N$ Hermitian operators needed to represent fields → SO(6) embedded algebra

→ Simplest representation in terms of spin 1/2 bilinear operators

$$O^{ab}_{xy} = \sigma^a_{x,+\mu} \otimes \sigma^b_{x+\mu,-\mu},$$
$$L^a_{x,+\mu} = \sigma^a_{x,+\mu} \otimes \mathbb{I}, \quad R^a_{x+\mu,-\mu} = \mathbb{I} \otimes \sigma^a_{x+\mu,-\mu}$$



Formation of gauge invariant states for four spins:

$$|\psi_{1s}\rangle_x = |\psi_s\rangle_{x,+\mu,-\mu} \otimes |\psi_s\rangle_{x,+\nu,-\nu}$$

$$\begin{aligned} |\psi_{2s}\rangle_x = &\ a\,|\psi_1\rangle_{x,+\mu,-\mu}\,|\psi_3\rangle_{x,+\nu,-\nu} \\ &+ b\,|\psi_2\rangle_{x,+\mu,-\mu}\,|\psi_2\rangle_{x,+\nu,-\nu} \\ &+ a\,|\psi_3\rangle_{x,+\mu,-\mu}\,|\psi_1\rangle_{x,+\nu,-\nu} \end{aligned}$$

An SO(3) Quantum Link Model (continued...)

$N^2 + 2N$ Hermitian operators needed to represent fields → SO(6) embedded algebra

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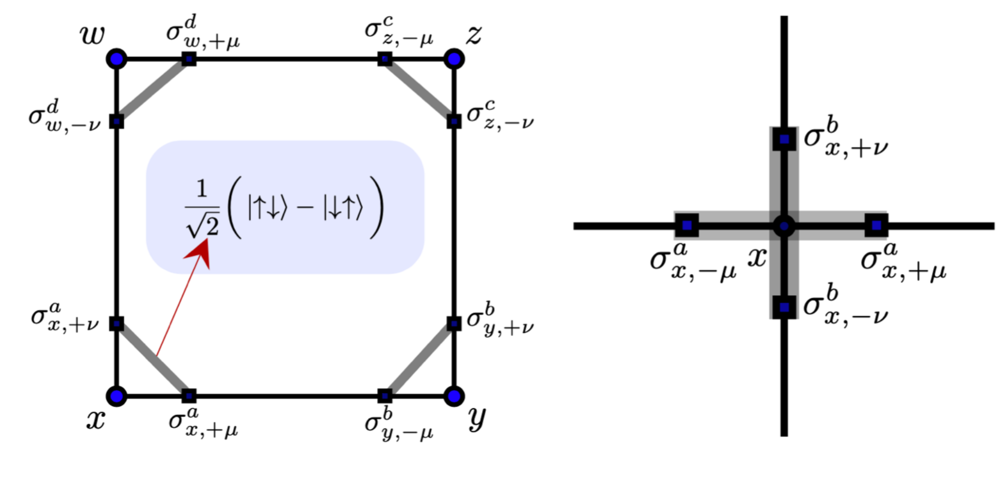


Hamiltonian in gauge invariant basis

$$H_{\text{inv}} = -\frac{1}{4g^2} \prod_{i=x,z} \left(\frac{1}{4}\left(\sigma^3_i - \mathbb{1}_i\right) + \frac{\sqrt{3}}{4}\sigma^1_i \right) \cdot \prod_{i=y,w} \left(\frac{1}{4}\left(\sigma^3_i - \mathbb{1}_i\right) - \frac{\sqrt{3}}{4}\sigma^1_i \right)$$

Charge conjugation symmetry is expected to break spontaneously

→ smallest energy gap decreases Exponentially with volume

→ to demonstrate with VQE and QAOA

Variational quantum algorithms: VQE and VQD

VQE: Minimize the cost functions $E(\vec{\theta}) = \langle\psi(\vec{\theta})|H|\psi(\vec{\theta})\rangle$ to obtain ground state energy

Similarly, use VQD by enforcing orthogonality to all previous states

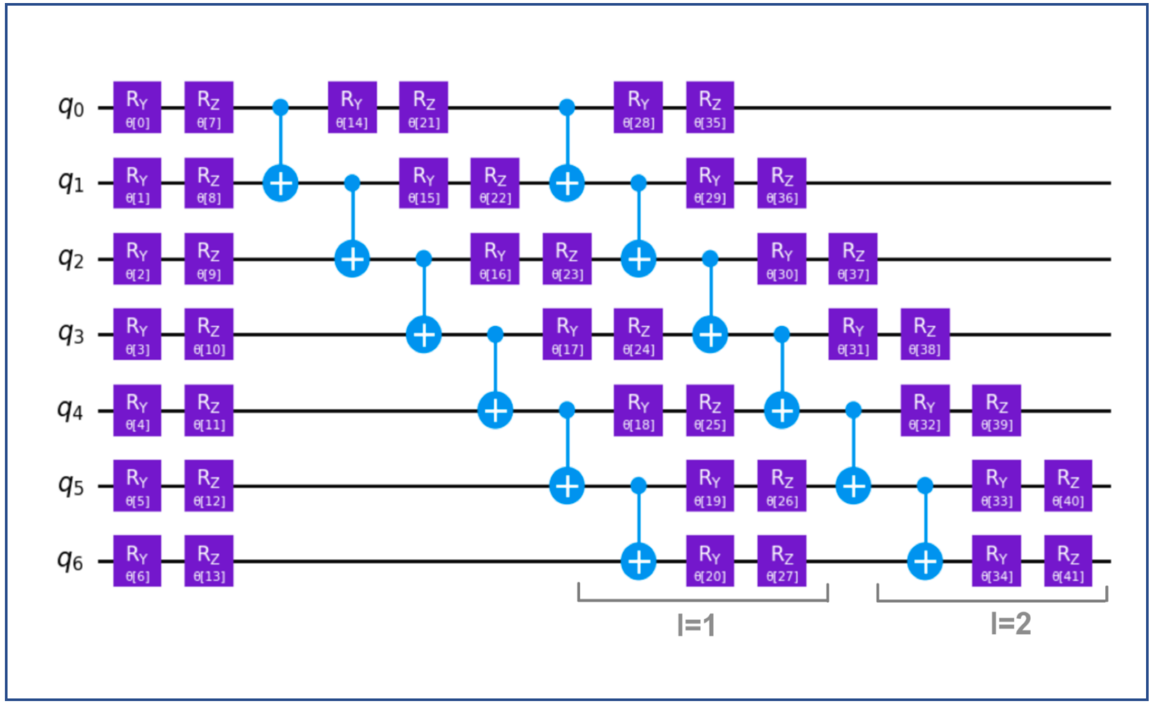
For *k*-th state:

$$F(\vec{\theta}_k) = \langle\psi(\vec{\theta}_k)|H|\psi(\vec{\theta}_k)\rangle + \sum_{i=0}^{k-1} \beta_i |\langle\psi(\vec{\theta}_k)|\psi(\vec{\theta}_i)\rangle|^2$$

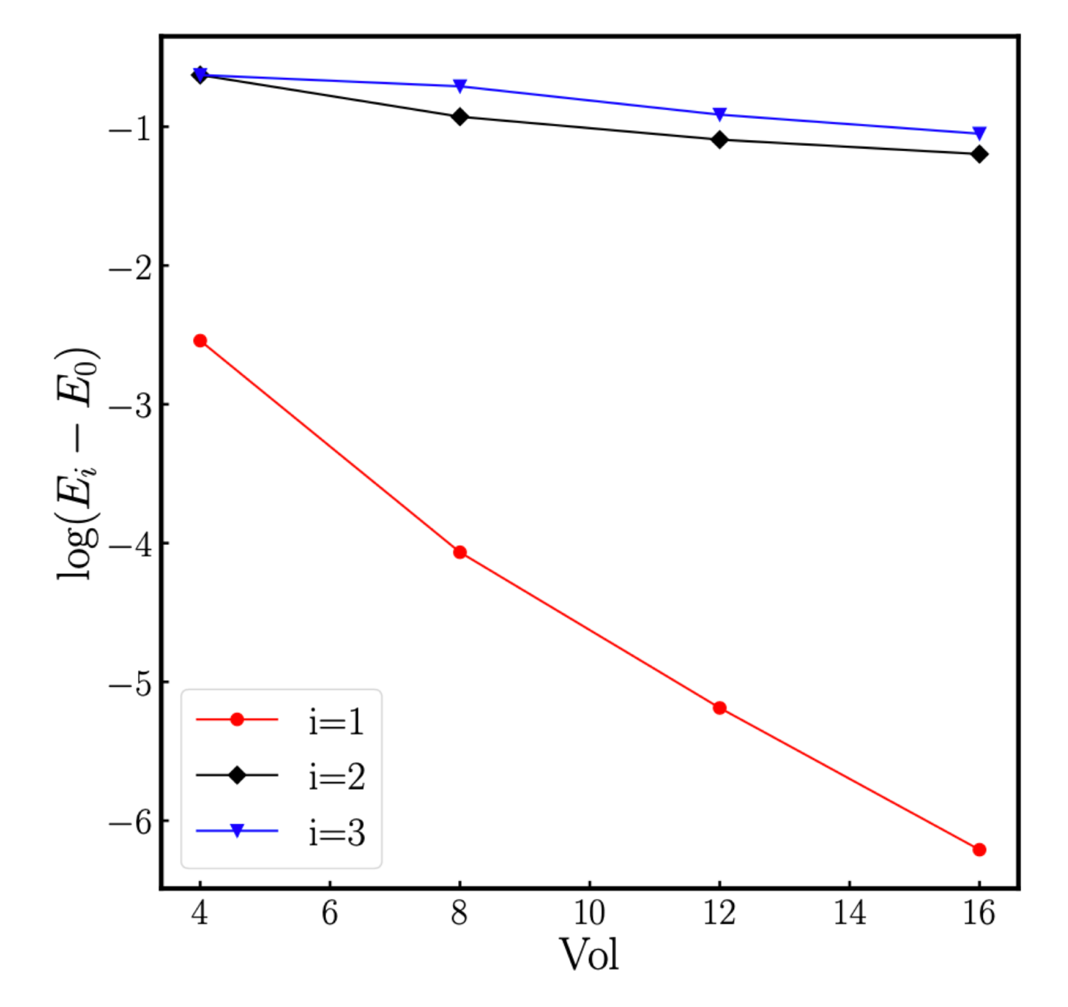
$$= E(\vec{\theta}_k) + \sum_{i=0}^{k-1} \beta_i |\langle\psi(\vec{\theta}_k)|\psi(\vec{\theta}_i)\rangle|^2,$$

Choose β for first excited state to be $\Delta E + \epsilon\Delta E$ and tune ϵ to get closer to the exact energy difference

Variational algorithms: VQE and VQD

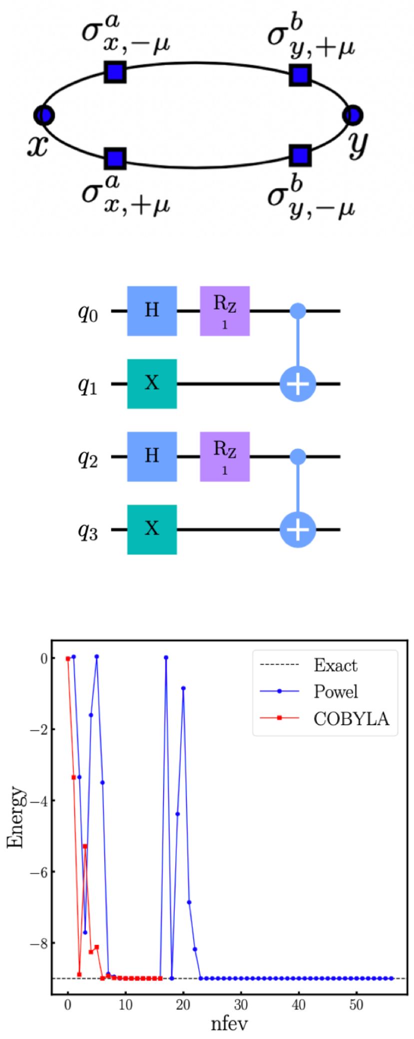


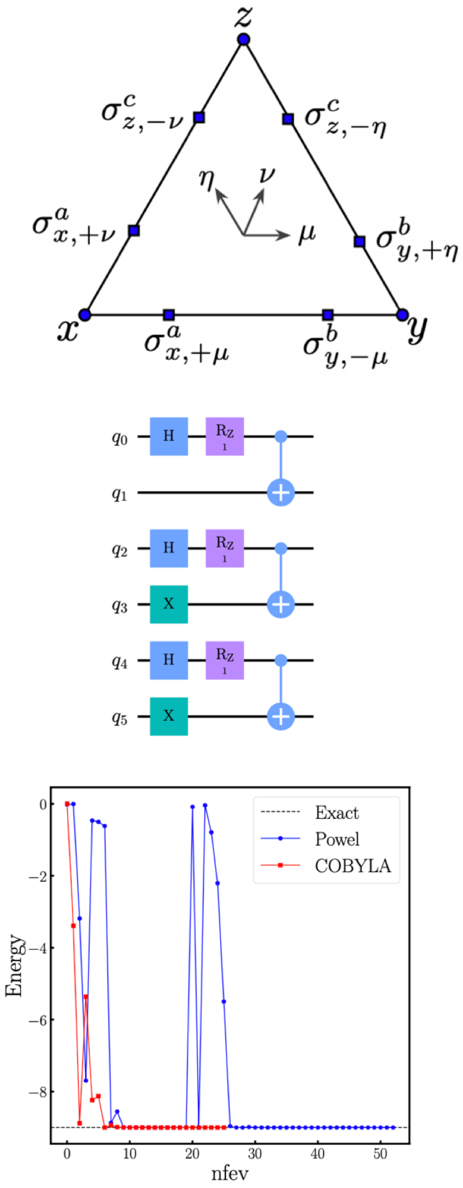
VQE ansatz with 6-qubits and 2 layers

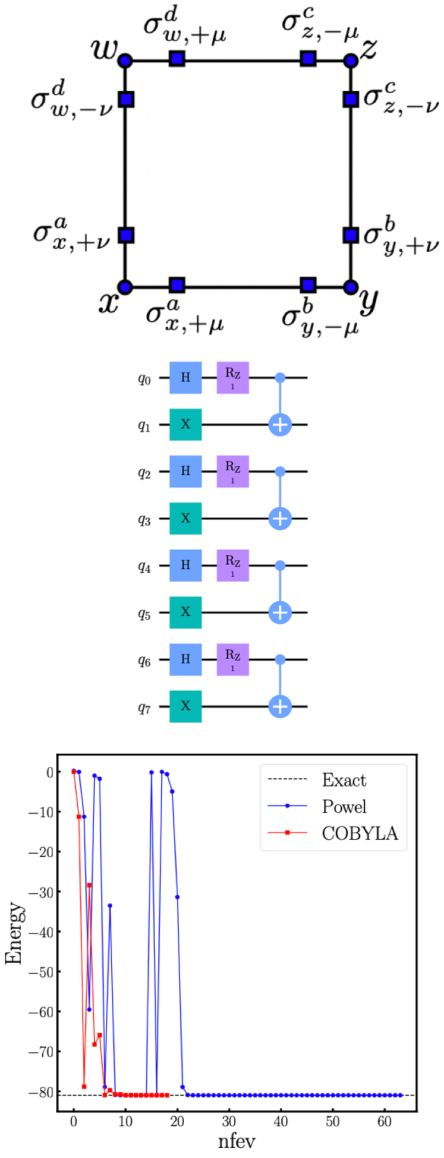


We witness the discrete symmetry breaking > exponential decrease of mass gap with volume

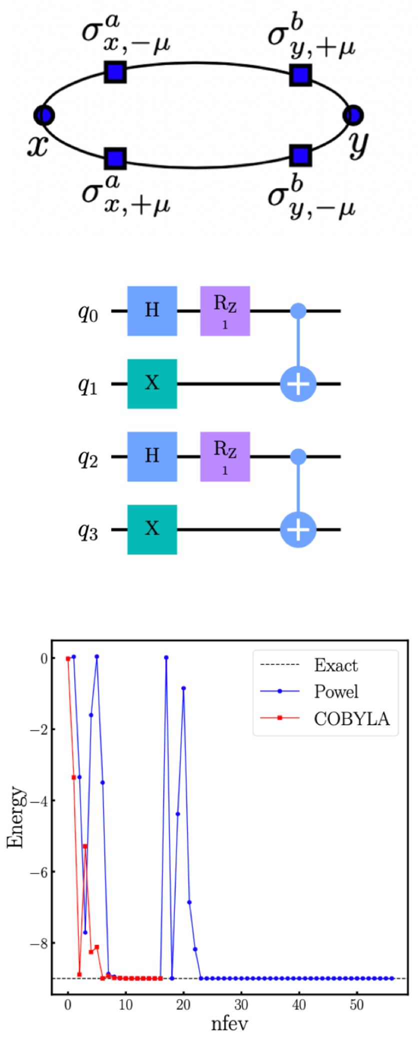
Variational algorithms: results

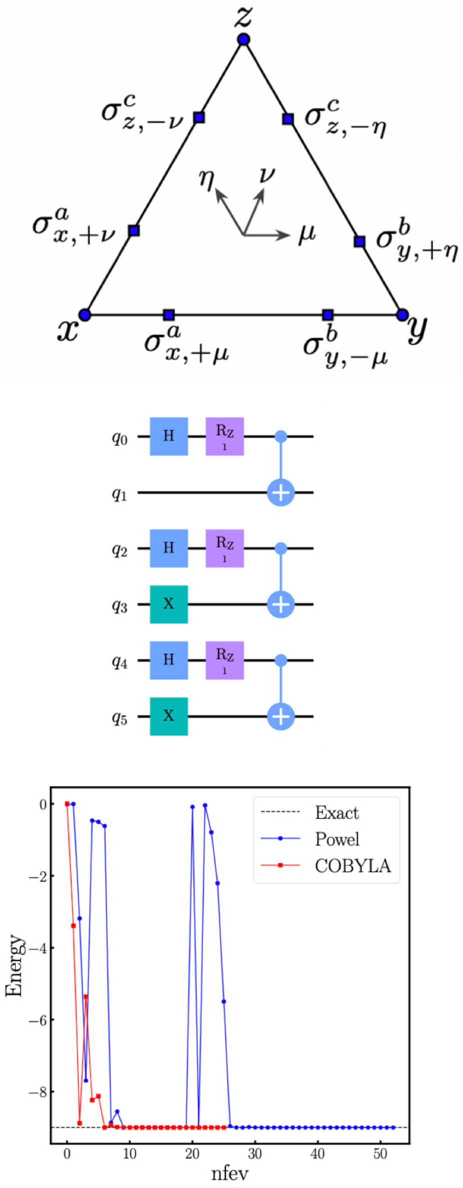


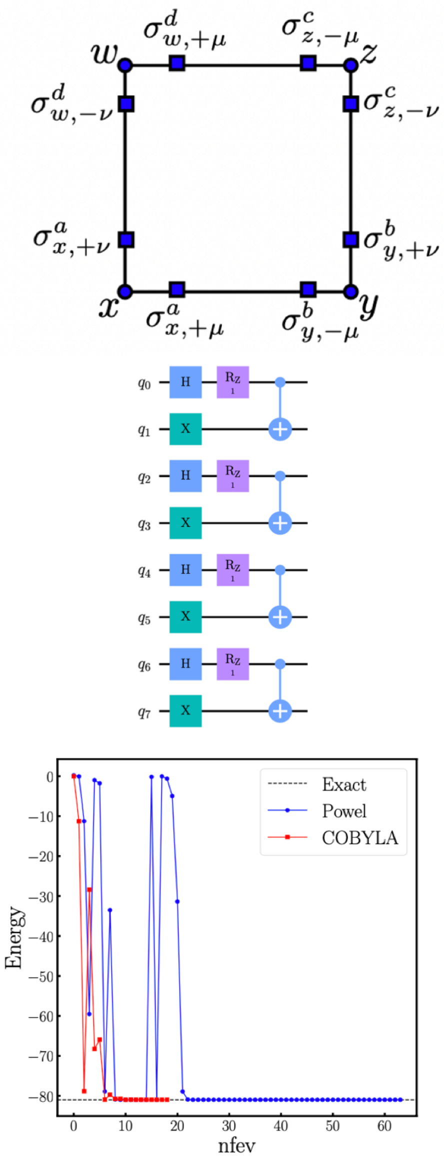


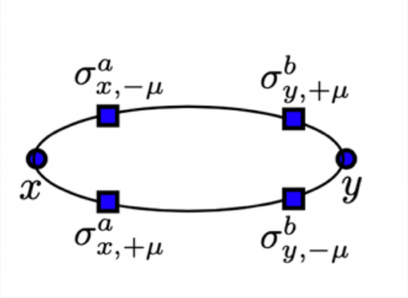


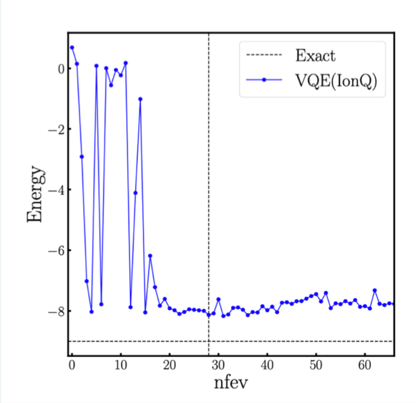
Variational algorithms: results

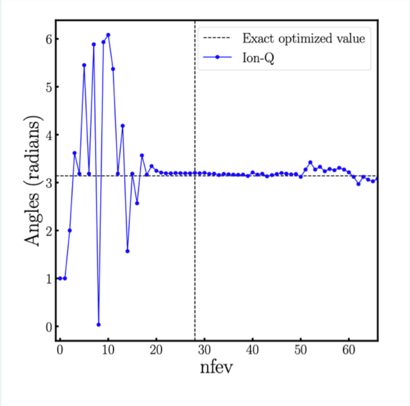




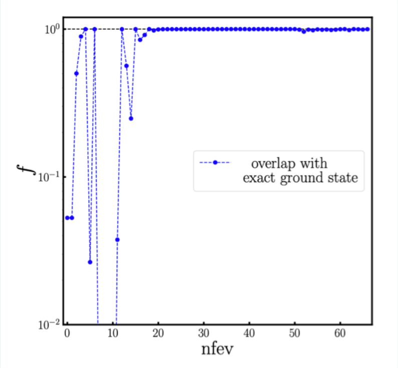








Results from real Hardware IonQ



Quantum Adiabatic Algorithm and QAOA

Gauge invariant states of 4-spins on 2 x 2 lattice:

$$H_1 = \frac{1}{4^3}\sum_{x=1}^{4}\sigma_x^3,$$

$$H_2 = -\frac{1}{4^3}\sum_{x\neq y}\sigma_x^3\sigma_y^3 + \frac{1}{4^3}\sum_{x\neq y\neq z}\sigma_x^3\sigma_y^3\sigma_z^3$$
$$-\frac{1}{4^3}\sigma_1^3\sigma_2^3\sigma_3^3\sigma_4^3,$$

$$H_3 = -\frac{3}{4^3}\sigma_1^1\sigma_2^1\left(-\sigma_3^3\sigma_4^3 + \sigma_3^3 + \sigma_4^3 - I\right),$$

$$H_4 = -\frac{3}{4^3}\sigma_3^1\sigma_4^1\left(-\sigma_1^3\sigma_2^3 + \sigma_1^3 + \sigma_2^3 - I\right),$$

$$H_5 = -\frac{3}{4^3}\sigma_2^1\sigma_3^1\left(\sigma_1^3\sigma_4^3 - \sigma_1^3 - \sigma_4^3 + I\right),$$

$$H_6 = -\frac{3}{4^3}\sigma_1^1\sigma_3^1\left(-\sigma_2^3\sigma_4^3 + \sigma_2^3 + \sigma_4^3 - I\right),$$

$$H_7 = -\frac{3}{4^3}\sigma_2^1\sigma_4^1\left(-\sigma_1^3\sigma_3^3 + \sigma_1^3 + \sigma_3^3 - I\right),$$

$$H_8 = -\frac{3}{4^3}\sigma_1^1\sigma_4^1\left(\sigma_2^3\sigma_3^3 - \sigma_2^3 - \sigma_3^3 + I\right),$$

$$H_9 = -\frac{3^2}{4^3}\sigma_1^1\sigma_2^1\sigma_3^1\sigma_4^1.$$

$$H_{\mathrm{inv}} = -\frac{1}{4g^2}\prod_{i=x,z}\left(\frac{1}{4}\left(\sigma_i^3 - \mathbb{1}_i\right) + \frac{\sqrt{3}}{4}\sigma_i^1\right)$$
$$\cdot\prod_{i=y,w}\left(\frac{1}{4}\left(\sigma_i^3 - \mathbb{1}_i\right) - \frac{\sqrt{3}}{4}\sigma_i^1\right)$$

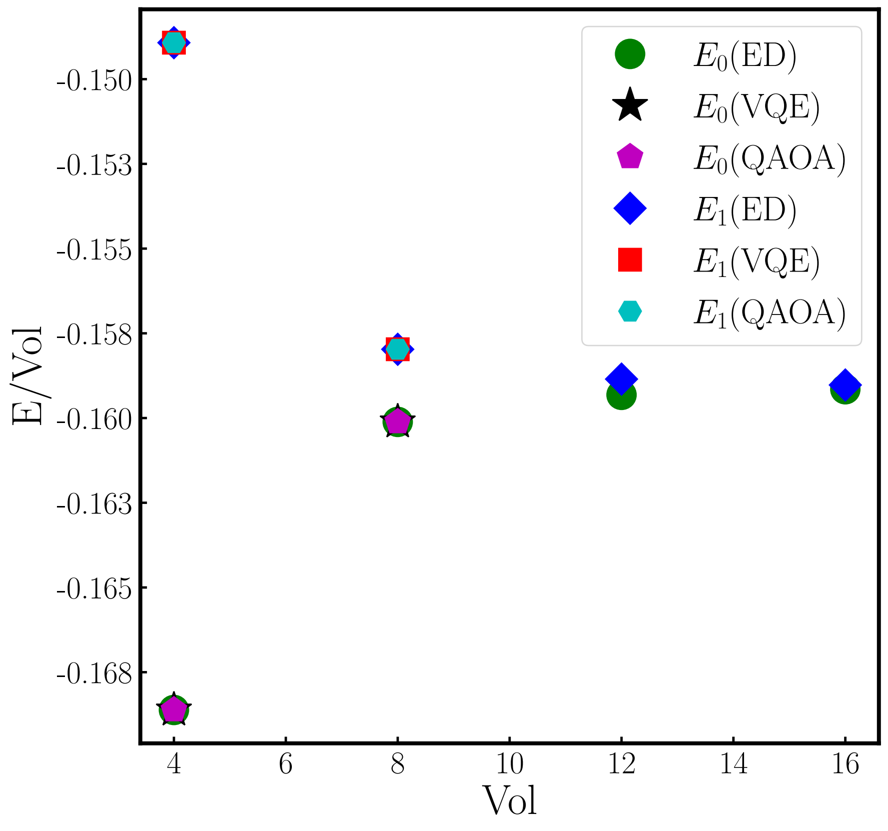
QAOA ansatz:

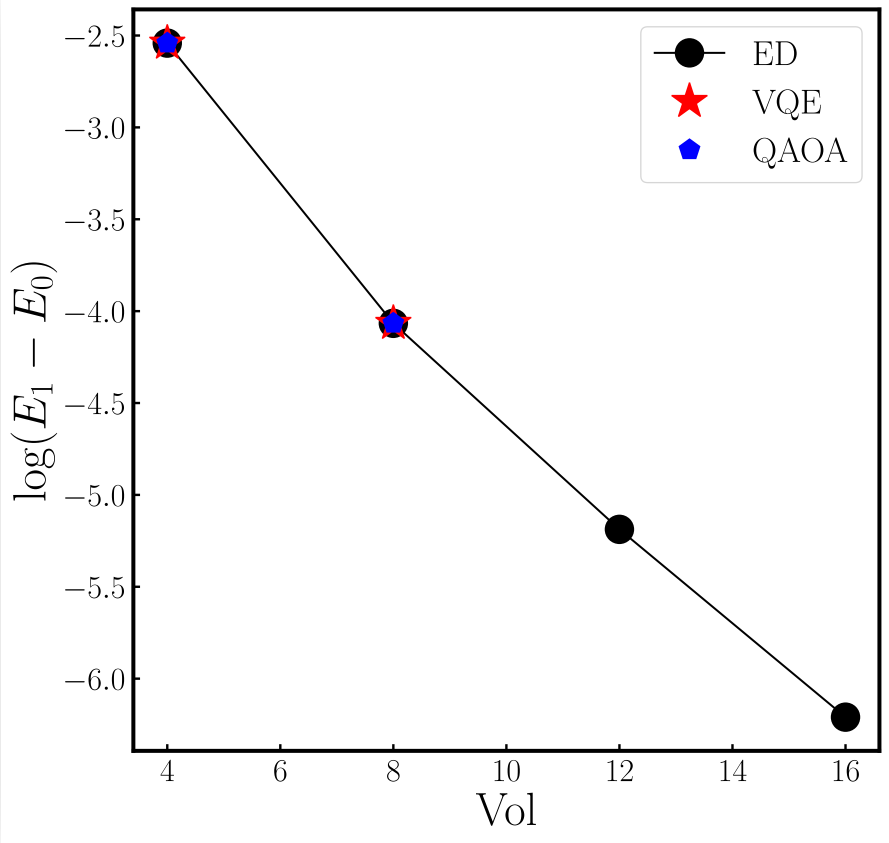
$$|GS\rangle_{\mathrm{QAOA}} = \prod_{k=1}^{p}\prod_{\alpha=1}^{N_\alpha} e^{iC_{\alpha,k}H_\alpha}\,|\psi_A\rangle$$

$$|\psi_{A=0}\rangle = |\uparrow\uparrow\ldots\uparrow\rangle$$

$$|\psi_{A=1}\rangle = |\uparrow\uparrow\ldots\downarrow\rangle$$

Spontaneous symmetry breaking with VQE and QAOA





We show that using VQE/VQD, and QAOA in a novel way spontaneous symmetry breaking in SO(3) in 2+1D has been achieved

Next talk by Dr. Graham Van Goffrier – SO(3) on 2+1D with fermions

Third challenge: QEM via symmetry constraints and post selection

- Imperfections in near-term quantum devices degrade the desired output information.
- A QEM protocol will aim to minimize this degradation.
- Although for scalability long term solution is QEC which needs fault tolerant qubits
- QEM is also feasible for NISQ era vs. QEC due to large overhead demand of QEC

One simple but effective QEM technique:
Use symmetry verification to identify errors that break the symmetries of the ideal quantum state and remove them via post-selection [Gottesman 1997, Tehral 2015]

QEM via symmetry constraints and post selection

Identify inherent symmetry of the circuit $[H, S] = HS - SH = 0$ and $S|\Psi_j\rangle = s|\Psi_j\rangle$

- We chose Schwinger Model Hamiltonian in 1+1D to work with
- We chose a common symmetry : Parity $\prod_i Z_i$
- We The input state, time evolution, output state – all should hold the symmetry ideally

Lets consider a Single Pauli symmetry operator S and where the ideal state lives within the +1 eigenspace of S defined by the projector $\Pi = (1/2)(1 + S)$

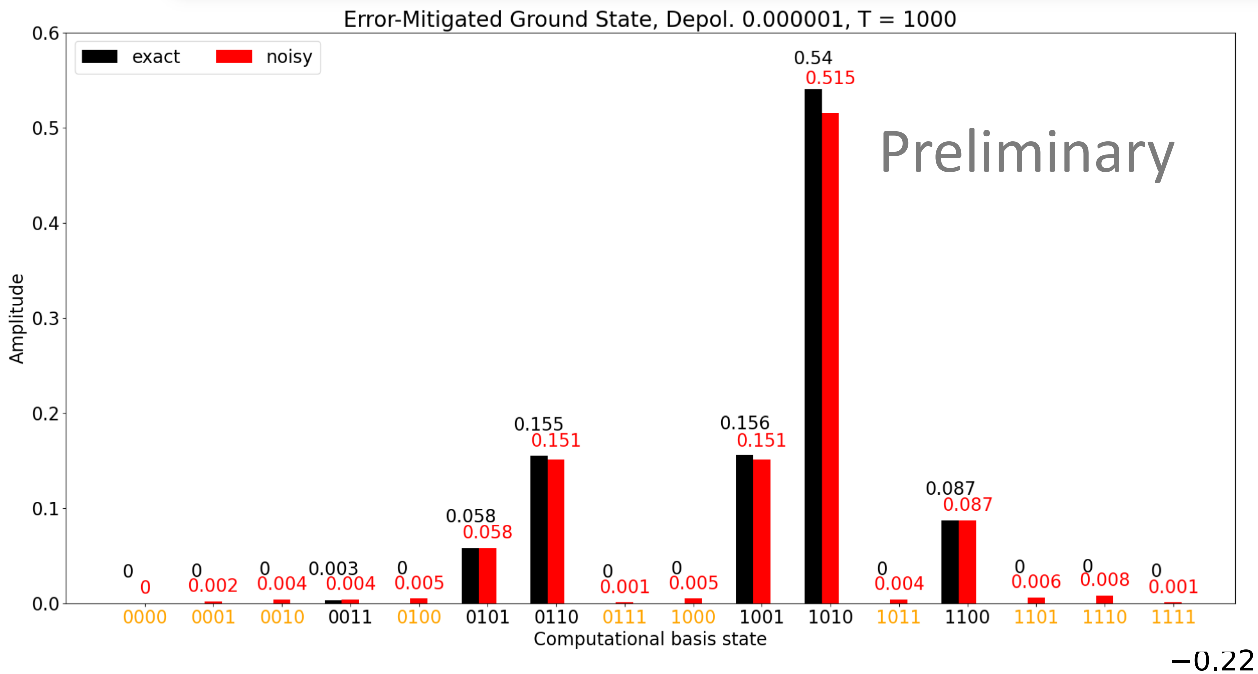
Post-selected state $\rho_{\text{sym}} = \dfrac{\Pi\rho\Pi}{\text{Tr}[\Pi\rho\Pi]}$

The symmetry-verified expectation value for the target observable *O* is given by

$$\text{Tr}[O\rho_{\text{sym}}] = \frac{\text{Tr}[O\Pi\rho\Pi]}{\text{Tr}[\Pi\rho\Pi]} = \frac{\text{Tr}[O_{\text{sym}}\rho]}{\text{Tr}[\Pi\rho]}$$

$$O_{\text{sym}} = \Pi O \Pi = (O + SO + OS + SOS)/4$$

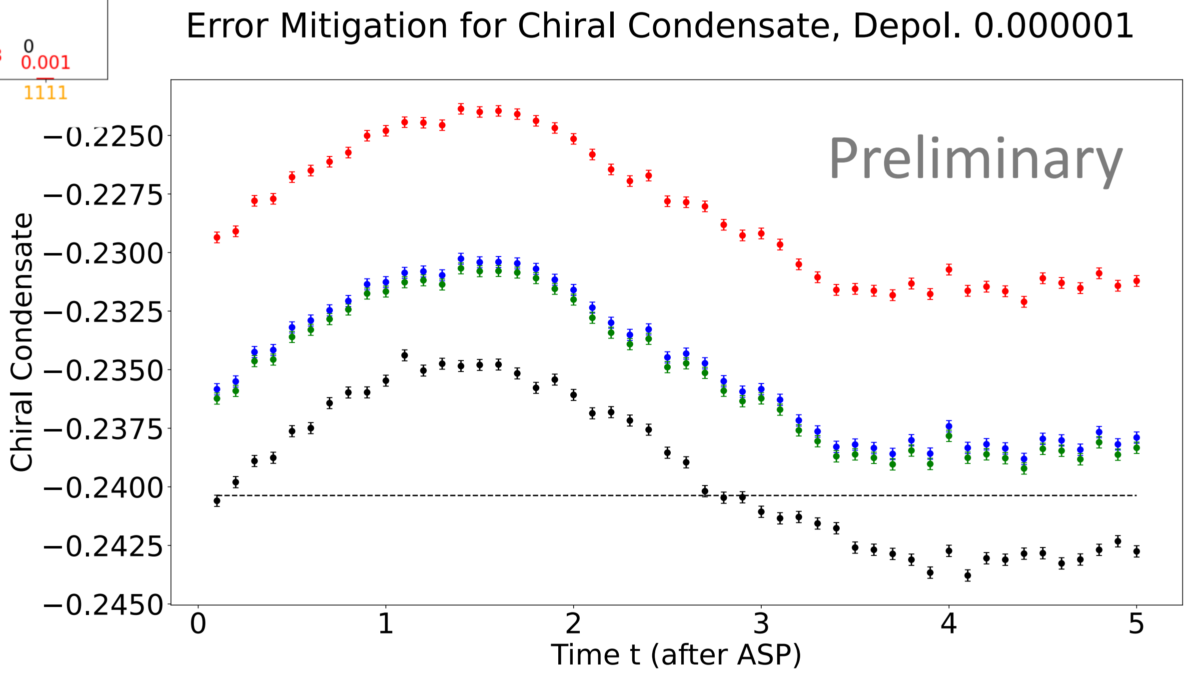
QEM implementation and outcome



$$H_{ZZ} = \frac{J}{2} \sum_{n=2}^{N-1} \sum_{1 \le k < \ell \le n} Z_k Z_\ell$$

$$H_{\pm} = \frac{1}{2} \sum_{n=1}^{N-1} \left(w - (-1)^n \frac{m}{2} \sin\theta \right) \left[X_n X_{n+1} + Y_n Y_{n+1} \right],$$

$$H_Z = \frac{m \cos\theta}{2} \sum_{n=1}^{N} (-1)^n Z_n - \frac{J}{2} \sum_{n=1}^{N-1} (n \bmod 2) \sum_{\ell=1}^{n} Z_\ell,$$



$$\langle \overline{\psi}(x)\psi(x) \rangle = \langle \mathrm{vac} | \overline{\psi}(x)\psi(x) | \mathrm{vac} \rangle$$

[Ongoing work]

Summary

- VQE and QAOA prepared ground state and excited state with novel implementation for SO(3) in 2+1D

- A matter-free SO(3) gauge theory Hamiltonian in 2+1D has be written in gauge invariant basis using Quantum Link Model, and the Hilbert space was heavily reduces

- The spontaneous discrete symmetry breaking in SO(3) in 2+1D was established via quantum simulation

- A simple but effective way of QEM is using symmetry constraints and post selection