Institute for Theoretical Physics
High Performance Computational Physics

Simulating (1+1)d Abelian Gauge Theories with Cluster Algorithms

Thea Budde, Marina Krstić Marinković, Joao C. Pinto Barros

Lattice 2024, July 30th

Outline

1. Hamiltonian Formalism
2. Recovering the continuum limit
3. Cluster Algorithm
4. Results

Outline

The Schwinger Model

Write a lattice Hamiltonian with staggered fermions

$$H = \sum_n \underbrace{-t\left(c_n^\dagger U_n c_{n+1} + h.c.\right)}_{\text{kinetic}} + \underbrace{m(-1)^n c_n^\dagger c_n}_{\text{mass}} + \underbrace{g\left(E_n + \frac{\theta}{2\pi}\right)^2}_{\text{gauge}} + \underbrace{u\left(\hat{n}_n - \frac{1}{2}\right)\left(\hat{n}_{n+1} - \frac{1}{2}\right)}_{\text{4-fermion interaction}}$$

where U_n is a raising operator for E_n.

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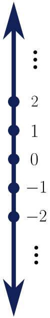
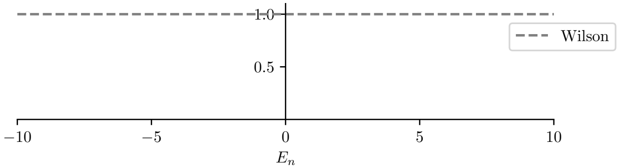
Local symmetry conserves $G_n = \underbrace{c_n^\dagger c_n + (1 - (-1)^n)/2}_{\text{charge } \rho_n} - \underbrace{E_n - E_{n-1}}_{\nabla E_n} = \rho_n - \nabla E_n$

Wilson formulation:

Infinite dimensional:

$$U_n \left| E_n \right\rangle = \left| E_n + 1 \right\rangle$$





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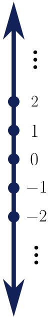
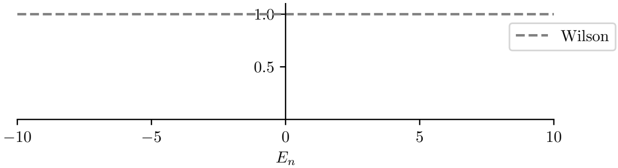


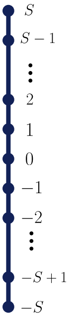


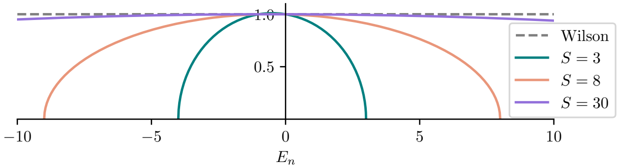
Simulations need finite-dimensional Hilbert space!

Quantum Link Models

Use a Spin S representation:

$$U_n \left| E_n \right\rangle \to \frac{S_n^+}{\sqrt{S(S+1)}} \left| E_n \right\rangle = \underbrace{\sqrt{\frac{S(S+1) - E_n(E_n+1)}{S(S+1)}}}_{t(E_n)} \left| E_n + 1 \right\rangle$$



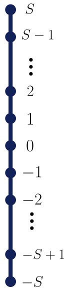


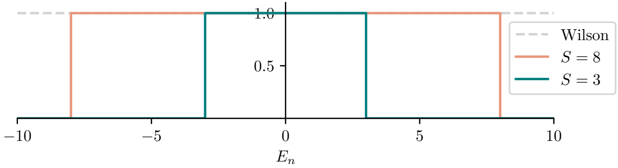
S. Chandrasekharan, U. J. Wiese, Nuclear Physics B 492 (1997) 455-471
V. Kasper et. al., Phys. Lett. B 760 (2016) 742 - 746

Truncated Link Models

Cut off large field values:

$$U_n |E_n\rangle \to \tau_n |E_n\rangle = \begin{cases} |E_n + 1\rangle & \text{if } -S \leq E_n < S, \\ 0 & \text{otherwise.} \end{cases}$$



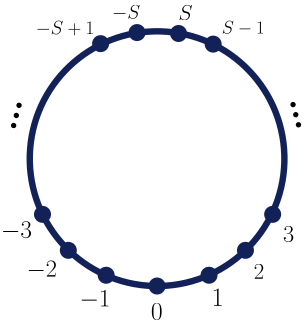


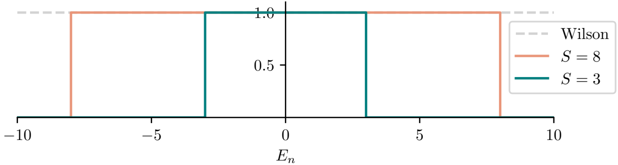
see e.g. J. Desaules et. al., Phys. Rev. B 107 (2023), 205112

$\mathbb{Z}_N$ gauge theories

Use periodic operator:

$$U_n |E_n\rangle \to \tilde{U}_n |E_n\rangle = |\text{mod}_{2S+1}(E_n + 1)\rangle$$





S. Notarnicola et. al., J. Phys. A: Math. Theor. 48 (2015) 30FT01

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2. Recovering the continuum limit

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Recovering the continuum limit

Multiple techniques exist, some don't require removing truncation[1]

Here:

1. Remove truncation: $S \to \infty$
2. Continuum limit: $a \to 0$

[1] R. Brower et. al., Nuclear Physics B 693 (2004) 149–175

[2] T. Zache et. al., PRD 106, L091502 (2022); J. Desaules et. al., PRB 107, 205112 (2023)

[3] P. Popov et. al., arXiv:2405.00745 (2024)

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- preserve $U(1)$ symmetry
- approach correct commutation relations

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Can we compare truncations directly?

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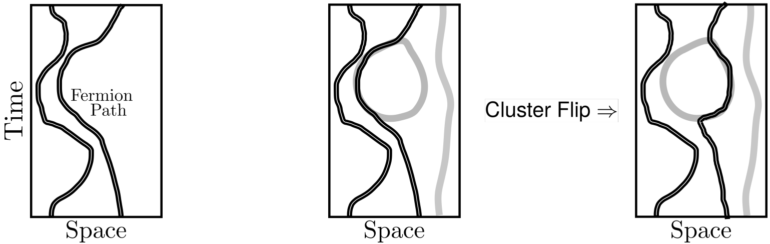
Use efficient cluster algorithm

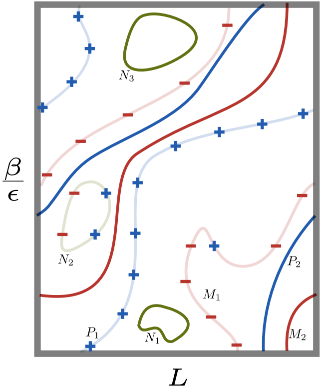
Sample spinless fermions like the Meron cluster algorithm does



S. Chandrasekharan, U.J. Wiese, Phys.Rev.Lett. 83 (1999) 3116-3119

Use efficient cluster algorithm

- Gauss' law prescribes description as charges
- Truncation and periodicity of gauge field not obeyed
- Flip clusters with conditional probabilities



J. Pinto Barros, **TB**, M. Kristc Marinkovic, PoS 453 LATTICE2023 024 (2024);
TB, M. Kristc Marinkovic, J. Pinto Barros, arXiv:2409.XXXXX

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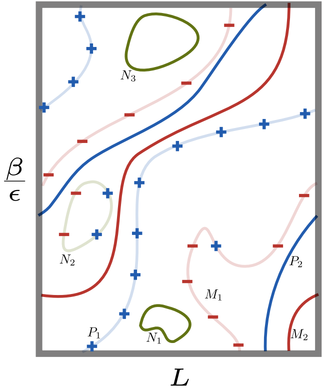
Generates TLM and $\mathbb{Z}_N$ configurations in $\mathcal{O}(L\beta S^2)$



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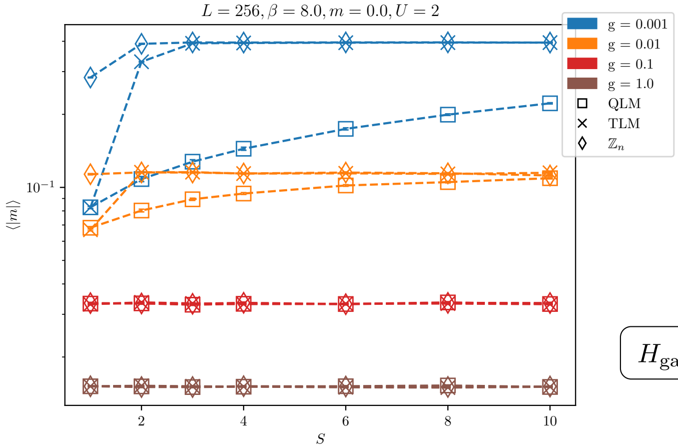
Comparisons of different truncations in the large S limit (preliminary)

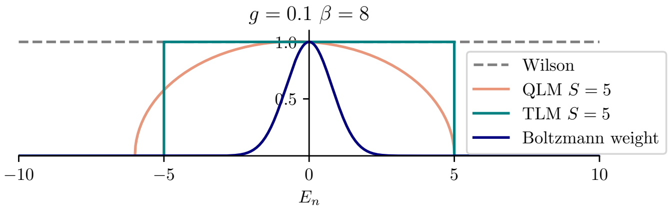


$$H_{\text{gauge}} = g \sum_n E_n^2$$

Comparisons of different truncations (preliminary)

What is the *relevant* Hilbert space?

$$w(E_n) \sim e^{-\beta g E_n^2}$$



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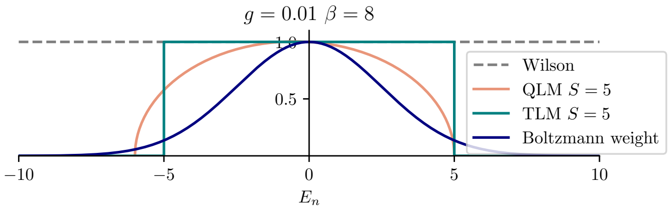

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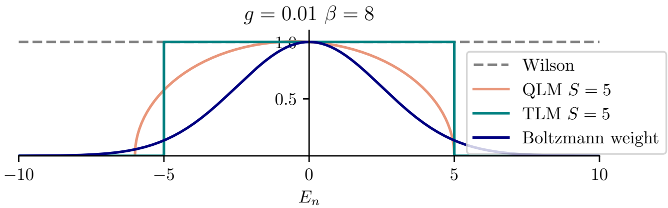
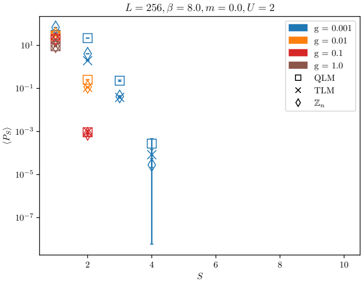
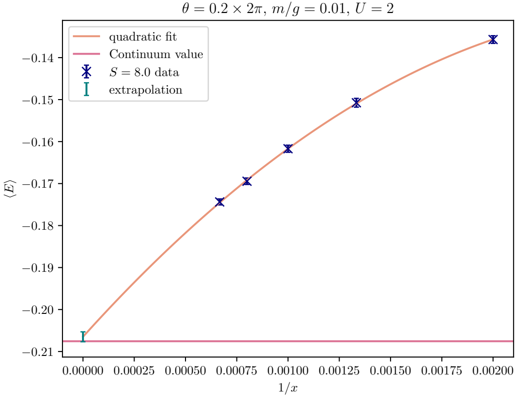




Taking continuum limits at finite theta with MCMC (preliminary)

$$H = \sum_n -\left(c_n^\dagger U_n c_{n+1} + h.c.\right) + \frac{2m}{g\sqrt{x}}(-1)^n c_n^\dagger c_n + \frac{1}{x}\left(E_n + \frac{\theta}{2\pi}\right)^2 + U\left(\hat{n}_n - \frac{1}{2}\right)\left(\hat{n}_{n+1} - \frac{1}{2}\right)$$



Conclusion

- Efficient cluster algorithm can simulate $U(1)$ gauge theories in $(1+1)$d
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- Low energy physics can be captured by small truncations $S \sim 3$

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Thank you