Update on the lattice calculation of $K \to \pi\pi$ decays with G-parity boundary conditions on a second lattice spacing

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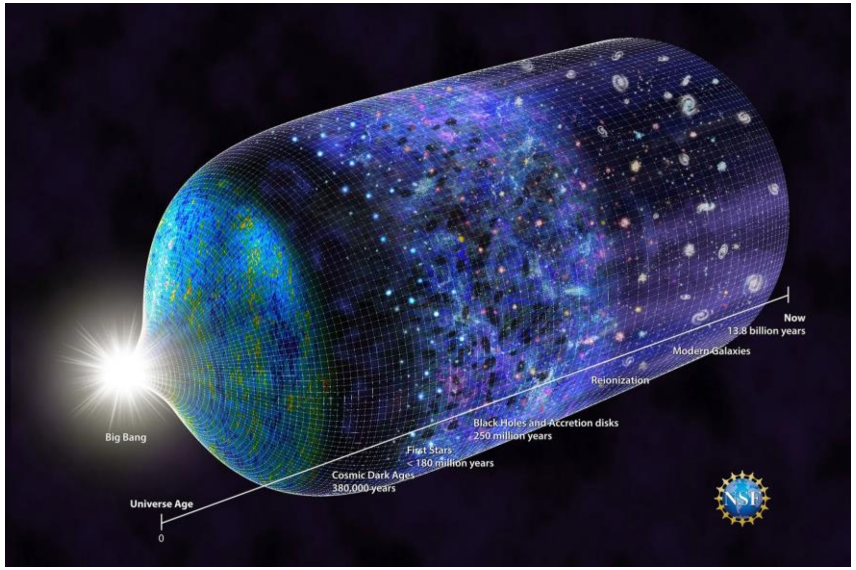
Motivation

- The RBC & UKQCD collaborations have a long-running series of lattice QCD calculations of **Direct CP-violation in $K \to \pi\pi$ decays**.
- Comparing theory to experiment allows us to probe for new physics.
- *Such new sources of CPV are needed to explain the dominance of matter over antimatter in the Universe.*
- CERN/FNAL (1990s) experimental result available with ~15% error.
- <u>Reliable, 1st principles calculation only possible in lattice QCD due to NP final-state interactions.</u>



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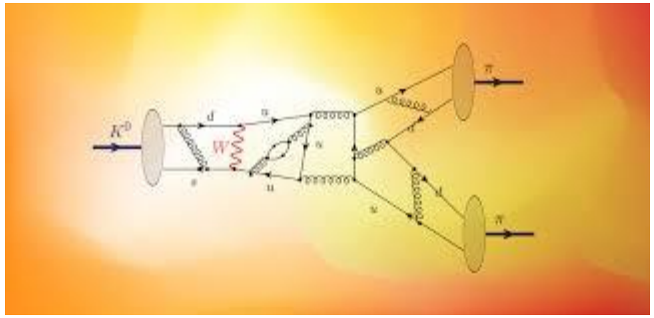
Strategy



Measure of direct CPV (*not including EM, isospin breaking)

$$\epsilon' \propto \frac{\mathrm{Re}(A_2)}{\mathrm{Re}(A_0)} \left(\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \right)$$

<u>lattice</u>

$$A_I = \langle (\pi\pi)_I | H_W | K^0 \rangle \Rightarrow H_W \propto \sum_{i=1}^{10} c_i(\mu) Q_i(\mu)$$

isospin

Perturbative 3f Wilson coeffs (high energy physics)

Effective 4-quark operators

NB: Renormalization in consistent scheme ($\overline{\mathrm{MS}}$) required

NB2: Lellouch-Lüscher finite-volume correction required!

- A_2 relatively straightforward with conventional methods, high precision (3% stat., 12% sys.)
- <u>A_0 much more challenging due to disconnected diagrams + nearby excited states.</u>

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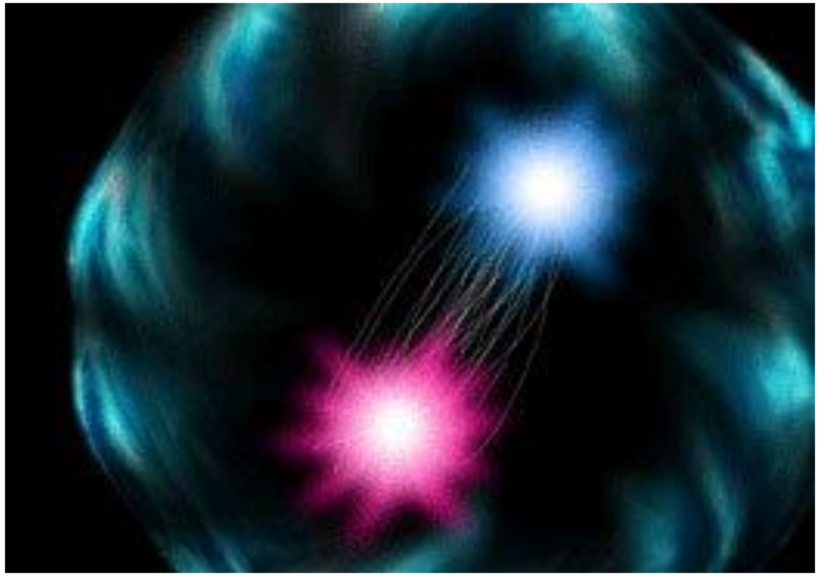
The Ground-state Conundrum

- $\pi\pi$ ground-state energy (~270 MeV) ≪ kaon mass (~500 MeV)
  - Ground-state matrix element is unphysical!
- For A_2 solve using antiperiodic BCs on down-quark in $n \leq 3$ spatial dirs.:

$$E^2_{\pi^\pm\ gnd} = m_\pi^2 + n(\pi/L)^2$$

  - Tune $\pi\pi$ energy via n, L
  - Only works for charged pions, breaks isospin (workaround only for I=2)
- For A_0 we might use periodic BCs and extract physical matrix elem as excited state.
  - Challenging due to large statistical errors *
- Or…



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G-parity Boundary Conditions

- Under the GPBC charged *and neutral* pions pick up a sign at the boundary.
  - Isospin is preserved.
- <u>But we cannot directly control the BCs of composite particles</u>, only those of their constituent *quarks*.
- *G-parity on quarks is more complicated!*
- *GPBC calculation much more expensive*
  - Requires explicit 2-flavor Dirac op <u>(x2 cost)</u>
  - Requires custom ensembles (x? cost)
  - Ensembles more expensive to generate as $\det(M^\dagger M)$ is 4-flavor: need sqrt *for light quarks (4th root for strange)* <u>(x4 cost)</u>

$$\hat{G}\pi^{\pm,0}\hat{G}^{-1} = -\pi^{\pm,0}$$

$$E^2_{\pi^{\pm,0}\ gnd} = m_\pi^2 + n(\pi/L)^2$$

$$\hat{G}\begin{pmatrix} u \\ d \end{pmatrix}\hat{G}^{-1} = \begin{pmatrix} -C\bar{d}^T \\ C\bar{u}^T \end{pmatrix}$$

spin matrix

The Past and the Present

[Phys.Rev.Lett. 115 (2015) 21, 212001]

- RBC & UKQCD performed first complete calculation of ϵ' in 2015.
- Improved result in 2020:
  - +3.5x statistics
  - multiple $\pi\pi$ operators to better control excited state systematics.
- Result:

[Phys.Rev.D 102 (2020) 5, 054509]

direct CPV, stat, sys, indirect CPV

$$\mathrm{Re}(\epsilon'/\epsilon) = 21.7(2.6)(8.0) \times 10^{-4} \quad \text{[Lattice]}$$

$$16.6(2.3) \times 10^{-4} \quad \text{[Experiment]}$$

- Agrees with experiment but with ~4x the total error.
- Systematics dominated: Wilson coeffs. (~12%), E&M (~23%), A_0 discretization (~12%)

unclear how reliable this estimate is!

The Edge (boundary) of Tomorrow?

- We are focusing on repeating calculation with finer lattices.
- Advent of multi-operator/variational techniques has opened the possibility of *reliably extracting the physical decay as an excited state with periodic BCs*.
  - Can reuse existing ensembles and eigenvectors where appropriate.
  - Measurements and ensemble generation much cheaper.
  - Possibility of including E&M effects in future (not so with GPBC?)

- RBC & UKQCD pilot calculation (led by M.Tomii) on coarse (1 GeV) lattice with ~1/3 of statistics of 2020 calc demonstrated method is effective:

[Phys.Rev.D 108 (2023) 9, 094517]

$$\mathrm{Re}(\epsilon'/\epsilon) = 29.4(5.2)(12.2) \times 10^{-4}$$ [periodic, a^{-1}=1.0 GeV]

$$\mathrm{Re}(\epsilon'/\epsilon) = 21.7(2.6)(8.0) \times 10^{-4}$$ [G-parity , a^{-1}=1.4 GeV]

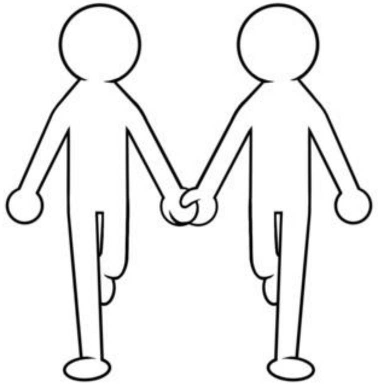
- Repeated calculation on finer lattice currently underway
  - For preliminary results including continuum limit, cf. M. Tomii talk 02/08 12:35pm (tomorrow)
- *Should we then abandon G-parity?*

X Reasons To Continue

(cf. my Lattice 2023 talk + proceedings)

- Under SciDAC-5 I have developed a series of **"X-conjugate"** algorithms exploiting a subtle symmetry of the GPBC Dirac Op.
  - 4x cost reduction in ensemble generation
  - 2x cost & memory reduction for eigenvectors
  - 2x cost reduction for inversions (for select sources)

- **Ensemble gen. now as cheap as periodic BCs.**
- **Measurement cost substantially reduced.**
- GPBC has different finite-volume systematics including reduced round-the-world pion propagation and differing $\pi\pi$ energy spectra.



- GPBC physical signal is dominant whereas periodic requires projecting out subdominant contribution.
- At this stage, a comprehensive cost-benefit analysis has not been performed of the two approaches.
- Our two analyses are somewhat independent (teams, lattices, techniques) which is beneficial given the lack of external competition for the calculation.
- Also: "general purpose" periodic lattices are trending towards larger physical volumes where the physical decay will be a higher excited state, much harder to extract.
  - *Periodic will likely also need custom ensembles in the future!*
    (or lots of operators and large statistics!)

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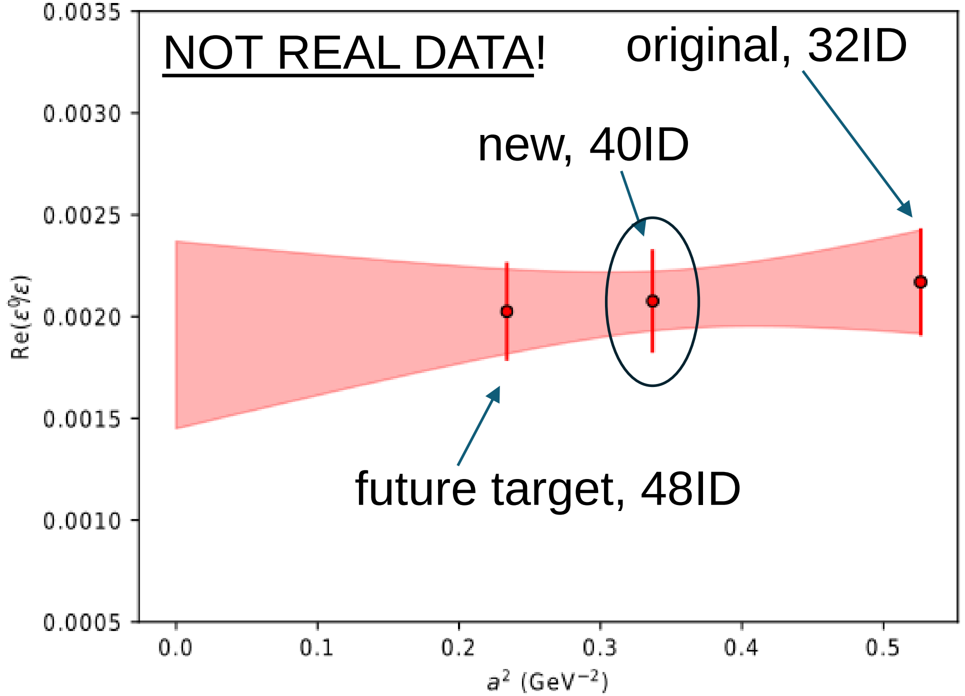
40ID GPBC ensemble

- $40^3 \times 64$ DWF+Iwasaki-DSDR ensemble
  - $a^{-1} = 1.73$ GeV vs 1.38 GeV previous
  - Same physical volume, physical masses
- Evolving on Perlmutter GPU



- Switched to X-conjugate action and retuned evolution:
  - Original: 4.36hrs (32 nodes) – 139.5 node-hrs
  - New : 1.12hrs (32 nodes) – 35.8 node-hrs
    : 1.61hrs (16 nodes) – 25.76 node-hrs

5.4x (or 3.9x) reduction in cost, 2.7x (or 3.9x) speedup

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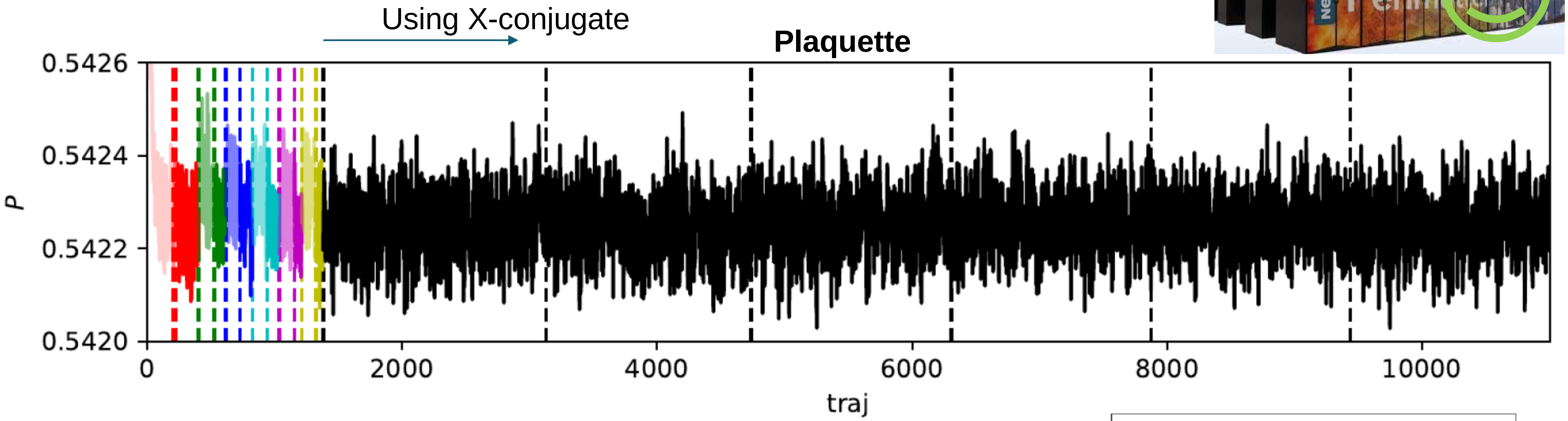
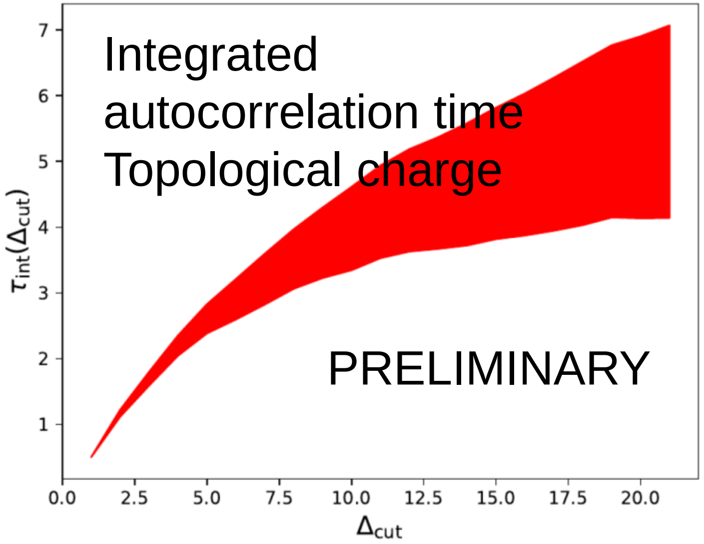
Ensemble status



- Excluding thermalization, ~9700 trajectories.
- $\tau_{int} \sim 5$ MDTU => 10 traj between independent.
- Sampling every 20th trajectory (conservative) this is <u>enough for 485 measurements</u>.
- *Very close to target ensemble size! (750 meas.)*



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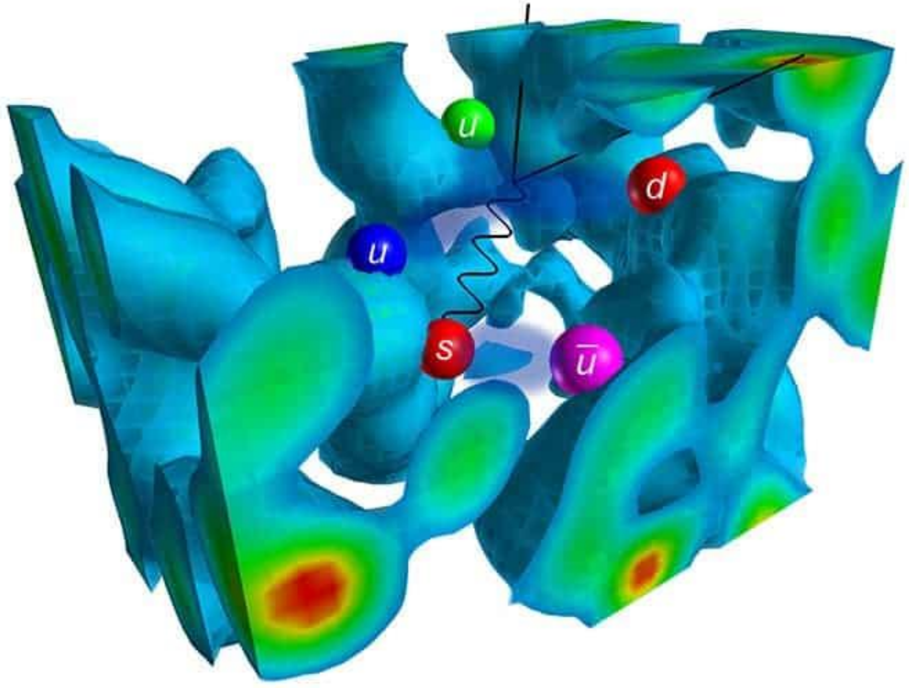
Measurements

- $K \to \pi\pi$ measurements performed on Polaris under ALCC allocation.
  ~6.2 hrs on 80 nodes Polaris
  (492 node-hours)
- Use all the latest performance tricks:
  - All-to-all propagators (disconn. diags.)
  - 2000 single-prec evecs w Block Lanczos
  - Deflated, split-Grid mixed-prec CG
  - Fourier-accelerated gauge fixing
  - 3 operators to control excited states
- Timing includes 2x reduction in Lanczos / inversion times due to X-conjugate algorithms.
- To-date 81 complete measurements (sep 40MDTU)



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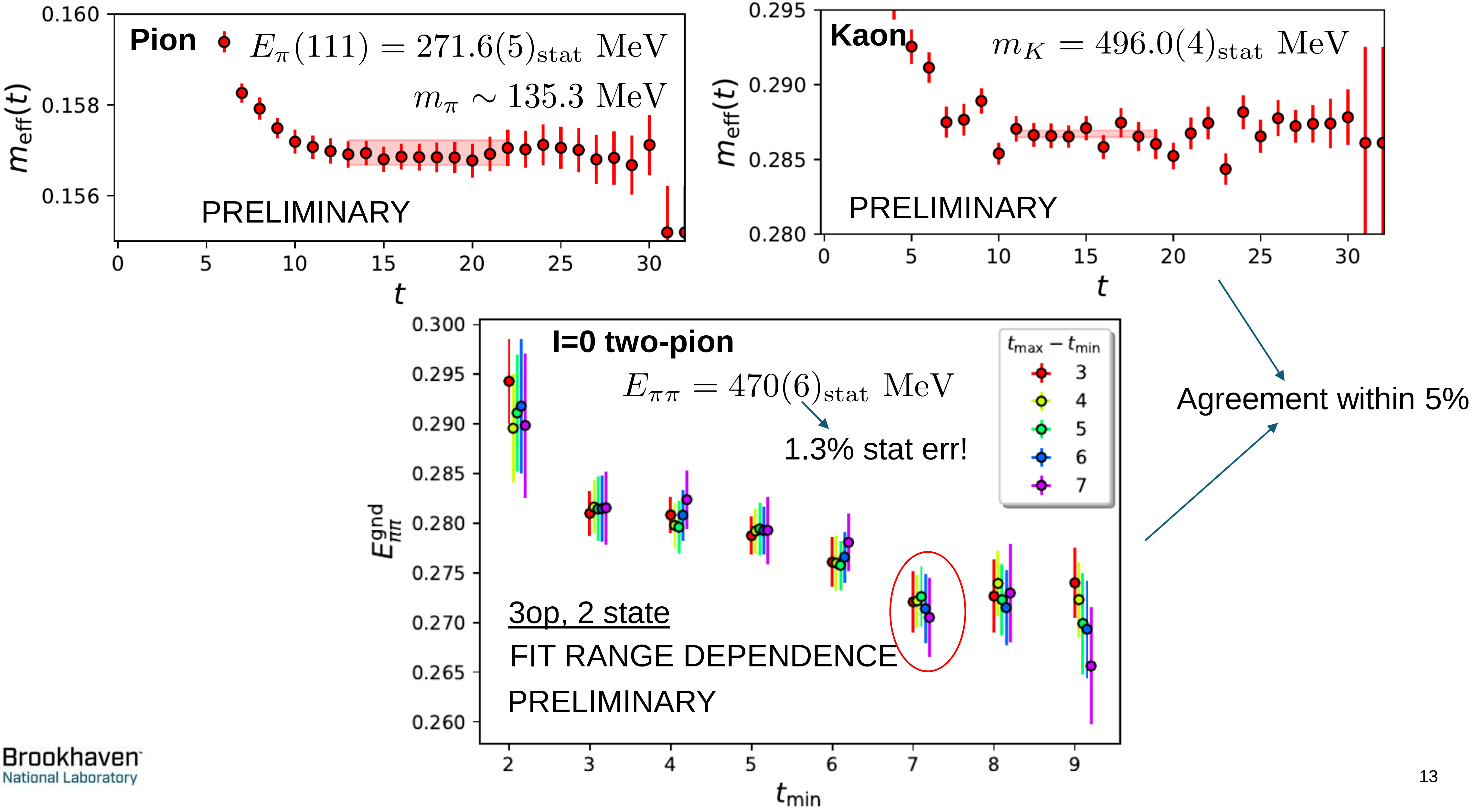
Results I



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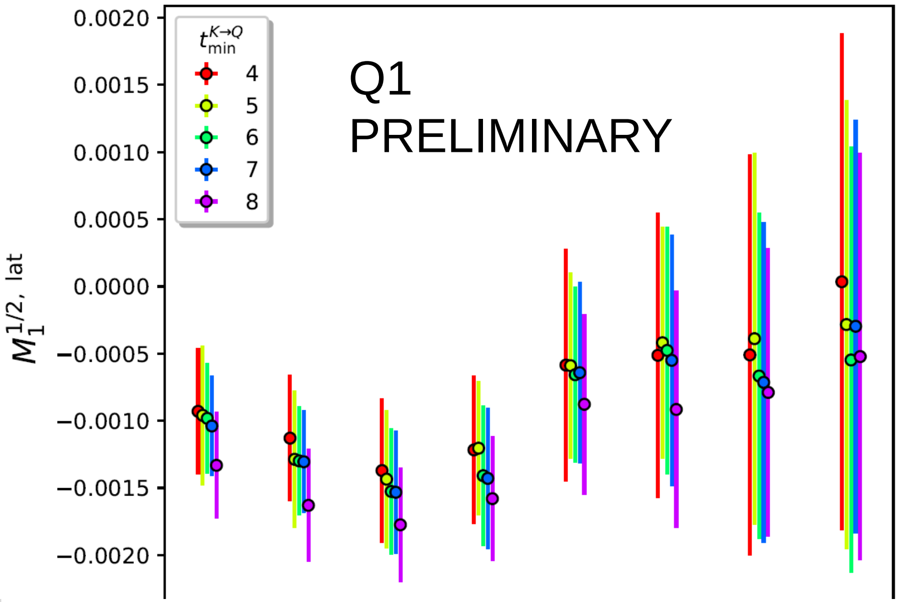
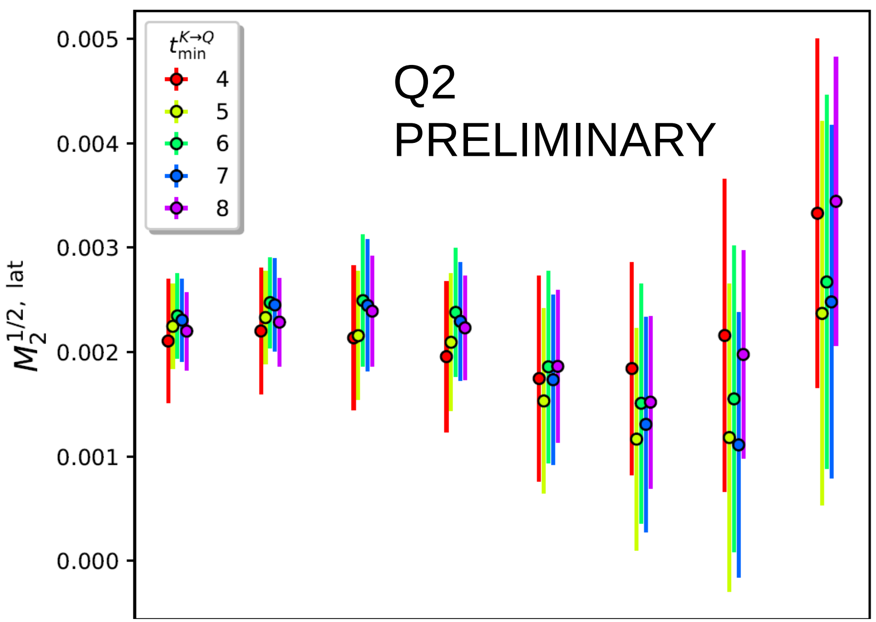
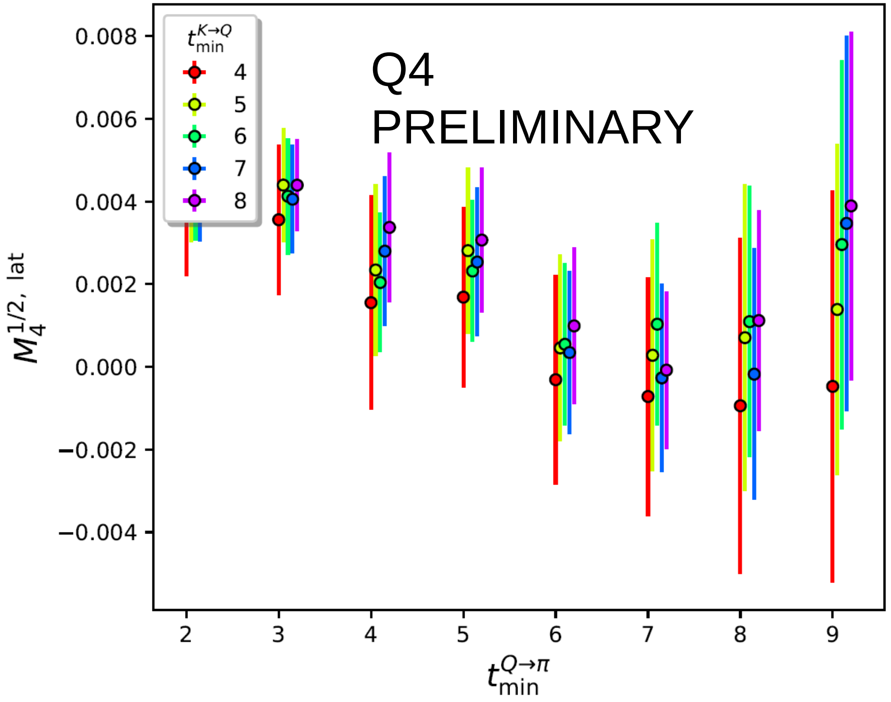
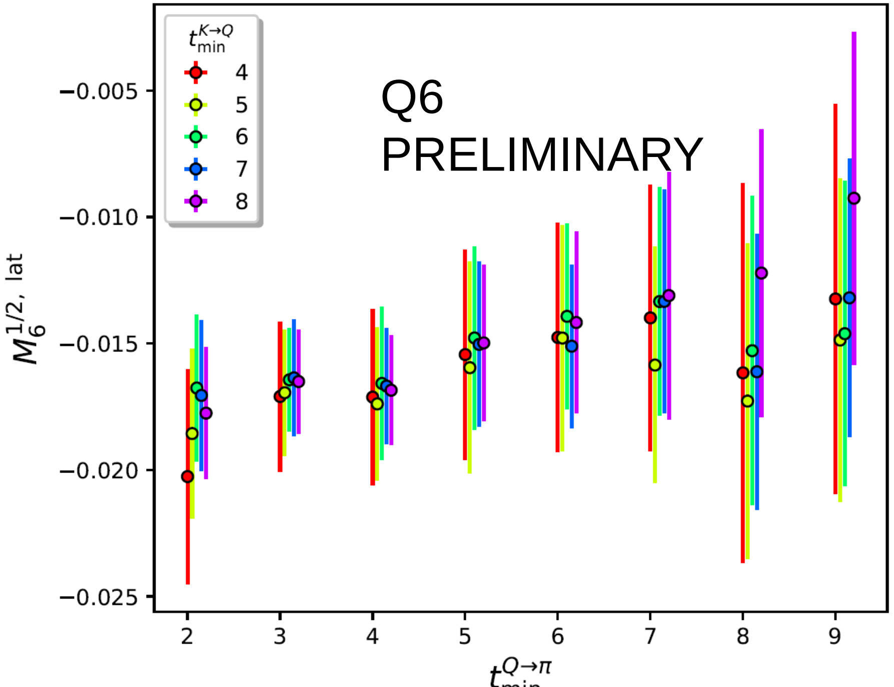
Results II


Q1
PRELIMINARY
$t_{min}^{K\to Q}$
4
5
6
7
8
$M_1^{1/2,\ lat}$
0.0020
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
−0.0020


Q2
PRELIMINARY
$t_{min}^{K\to Q}$
4
5
6
7
8
$M_2^{1/2,\ lat}$
0.005
0.004
0.003
0.002
0.001
0.000


Q4
PRELIMINARY
$t_{min}^{K\to Q}$
4
5
6
7
8
$M_4^{1/2,\ lat}$
0.008
0.006
0.004
0.002
0.000
−0.002
−0.004
2 3 4 5 6 7 8 9
$t_{min}^{Q\to\pi}$


Q6
PRELIMINARY
$t_{min}^{K\to Q}$
4
5
6
7
8
$M_6^{1/2,\ lat}$
−0.005
−0.010
−0.015
−0.020
−0.025
2 3 4 5 6 7 8 9

Conclusions

*custom lattice caveat!

- *Excellent progress towards repeating the GPBC calculation on a finer lattice.*
  - *Ensemble generation nearly complete*
  - *Measurements tuned and under production running. ~10% towards ultimate target.*
- New algorithms have reduced cost to be comparable to periodic calc*
- Todo:
  - Lots more statistics!
  - NPR required; either custom periodic lattice or RI-SMOM with GPBC
    - *Requires theory development*