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BERKELEY LAB

Subtleties and Systematics in achieving sub-percent uncertainty for g_A

Lattice 2024 Liverpool, UK, July 28 — August 3, 2024

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Non-monotonic FV corrections to *gA*

Subtleties and Systematics in achieving sub-percent uncertainty for *gA*

Why should we care about sub-percent uncertainty for g_A ?

QED corrections to g_A : estimates from $χ$ PT

The first row is showing robust tension , $\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 + |\dot{V}_{ub}|^2 - 1, \quad V_{ud}^{0^+ \to 0^+} = 0.97367(11)_{exp}(13)_{\Delta_v^R}(27)_{NS}[32]_{total}$ $V_{us}^{K_{e3}} = 0.22330(35)_{exp}(39)_{f_{+}}(8)_{IB}[53]_{total}$

Intense effort to test *heavy* flavor violation with charm/bottom quarks

become the frontier Original Sirlin & Marciano et al approach \square modern pheno and EFT treatments \Box lattice \angle CD + \angle ED

Cabibbo Angle Anomaly

At this level of precision, careful treatment of radiative QED corrections has

$$
\begin{array}{cc}\nV_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}\n\end{array}\n\begin{pmatrix}\nd \\
s \\
b\n\end{pmatrix}_{QCD}\n\end{array}
$$

CKM

In the absence of new physics, unitarity constrains the elements of CKM 2 $= 1$ for $i = u, c, t$

 \smile

First-row CKM Unitarity & Precision *β* **decays**

First-row CKM Unitarity & Precision *β* **decays FLAG 2023**

Exciting prospects for neutron *β*-decay to match precision from superallowed alleviating the need for modeling the nuclear structure (NS) corrections

 $\omega_d^{(0)} = 0.97367(11)_{exp}(13)_{\Delta_V^R}(27)_{NS}[32]_{total}$

 $\lambda = g_A/g_V$

$$
V_{ud}^{0^+\to0^+} = 0.97
$$

 $V^{n,\text{PDG}}_{\mu d} = 0.97441(3)_{f}(13)_{\Delta_N^R}(82)_{\lambda}(28)_{\tau_n}[88]_{\text{total}}$ $\mu_d^{m,\text{PDG}} = 0.97441(3)_{f}(13)_{\Delta_V^R}(82)_{\lambda}(28)_{\tau_n}$

 $V^{n, best}_{ud}$ $\mu_d^{n, \text{best}} = 0.97413(3) f(13) \Delta_V^R (35) \lambda^{(20)} \tau_n [43]_{\text{total}}$

Reaching target precision requires improving the uncertainty from radiative QED corrections, in particular, Δ^R_V

 Γ_n = G_F^2

 \Box The first row is showing robust tension — [some of the values in this estimate] , *Cabibbo Angle Anomaly* $\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 + |\dot{V}_{us}|^2 - 1$, $V_{ud}^{0^+ \to 0^+} = 0.97367(11)_{exp}(13)_{\Delta_N^R}(27)_{NS}[32]_{total}$ $V_{us}^{K_{e3}} = 0.22330(35)_{exp}(39)_{f_{+}}(8)_{IB}[53]_{total}$

First-row CKM Unitarity & Precision *β* **decays**

 ν

$$
= \frac{G_F^2 |V_{ud}|^2 m_e^5}{2\pi^3} (1 + 3\lambda_{\text{PDG}}^2) f_0 (1 + \Delta_f) (1 + \Delta_V^R)
$$

\n
$$
\lambda_{\text{PDG}} = \lambda_{\text{''exp}} - \Delta_A^{R, Sirlin, analytic} = \lambda_{\text{QCD-iso}} + \Delta_A^{R, other}
$$

\n
$$
\Delta_A^{R,other} \simeq O(2\%) \qquad \Delta_A^{R,other} = \text{QED correction to } g_A
$$

QED corrections to *gA*

 g_A^{PDG} is determined from an experimental measurement of $\lambda = g_A/g_V$ after some analytic long-distance QED effects are subtracted — see Hayen & Young, 2009.11364 for discussion *A*

 $g_A^{\text{PDG}} = g_A^{\text{QCD-iso}} + \Delta_A^{R,other}$

We compare our LQCD calculations of to *g*QCD−iso *^A ^g*PDG *A*

But it turns out - potentially significant low-energy nucleon structure corrections may spoil this comparison

 $\Delta_A^{R,other} \simeq O(2\%)$

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$m³$ for $n⁴$ for $n⁵$ for $n⁶$ rion-induced radiative correc Cirigliano, de Vries, Haven, Mereghetti & Walker-Loud, PRL 129 (2022) [2202,10439] $\mathcal{L}(\mathcal{B})$ at the multi-TeV scale, such as the comparison of the comparison $\mathcal{L}(\mathcal{B})$ In this Letter, we present a systematic electron and the systematic electron and the systematic electron and the
In this Letter, we present a systematic electron and the systematic electron and the systematic electron and ons to neutron beta-decay the neutron and proton, i.e. *q*ext ⇠ *mⁿ m^p* ⇠ 1 MeV, is tive corrections to neutrolative corrections to neutro. hand, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439] Ω eto Ω e Ω to the parameters of the Standard Model. In our analysis Pion-induced radiative corrections to neutron beta-decay
Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

Systematic, EFT treatment of neutron β-decay P_{deconv} P_{deconv} (Q_{tr}

If we want to connect them to Standard Model (SM) parameters we need to start from a Lagrangian with parameters related to SM parameters e want to connect them to Standard Model (SM) cision in the next few years of a new years with the new years of the new years of the second the second states. ϵ need to start from a Lagrangian with parameters. red to SM parameters $\frac{1}{2}$ tandard Model (SM) parameters an with parameters related to SM $_1$

 $\mathcal{L}_{\#}=-% {\displaystyle\sum\limits_{i=1}^{p}} \text{ }\left(-1\right) ^{i}\frac{1}{\left\vert i\right\vert }=\frac{1}{\left\vert j\right\vert }%$

pion-less low-energy EFT $i \bar{N}(\omega^{\mu} \omega^{\nu} - \alpha^{\mu} \omega^{\nu})$

The parameters can be measured probe of BSM right-handed charged currents.

$$
\mathcal{L}_{\neq} = -\sqrt{2}G_F V_{ud} \left[\bar{e}\gamma_{\mu} P_L \nu_e \left(\bar{N} \left(g_V v_{\mu} - 2g_A S_{\mu} \right) \tau^+ N \right. \right. \\ \left. + \frac{i}{2m_N} \bar{N} \left(v^{\mu} v^{\nu} - g^{\mu \nu} - 2g_A v^{\mu} S^{\nu} \right) \left(\overleftarrow{\partial} - \overrightarrow{\partial} \right)_{\nu} \tau^+ N \right) \\ \left. + \frac{i c_T m_e}{m_N} \bar{N} \left(S^{\mu} v^{\nu} - S^{\nu} v^{\mu} \right) \tau^+ N \left(\bar{e} \sigma_{\mu \nu} P_L \nu \right) \right. \\ \left. + \frac{i \mu_{\text{weak}}}{m_N} \bar{N} \left[S^{\mu}, S^{\nu} \right] \tau^+ N \partial_{\nu} \left(\bar{e} \gamma_{\mu} P_L \nu \right) \right] + \dots \qquad (2)
$$

Perform the calculation with SU(2) heavy-baryon χPT and match the results to this pion-less EFT whose parameters can be matched to experimentally measured quantities full Standard Model *n* ! *pe*⌫¯ decay amplitude includmaterial for the full expressions. I match the results to this pion-less EP I $\mathcal{P}(9)$ heavy beween $\mathcal{P}(\text{DT}$ and match the regults to α and the electric terms of α and the denotes by lowing standard practice, derivatives (\overline{P}) and the property of \overline{P} electroweak couplings *e*, *G^F* are assigned chiral dimen-

$$
\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{(G_F V_{ud})^2}{(2\pi)^5} (1 + 3\lambda^2) w(E_e)
$$
\n
$$
\times \left[1 + \bar{a}(\lambda) \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \bar{A}(\lambda) \frac{\vec{\sigma}_n \cdot \vec{p}_e}{E_e} + ...\right]
$$

$$
\lambda = \frac{g_A}{g_V}
$$

Pion-induced radiative corrections to neutron beta-decay
Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

\square Sub-set of $O(50)$ diagrams

including a new one needed to absorb divergence from \mathbf{r} lactual baryons, photons, photons, photons, photons, photons, photons, photons, photons, photons, and leads to **Cirigliano, de Vries, Hayen, Mereghetti & Wa** tributing to the matching between PT and */*⇡EFT at $\frac{1}{2}$ vanish to me internal beta-decay T T DDT 100 (0000) TO000 104901 $\text{Loud}_1 \text{KL}$ 120 (2022) [2202.10100] tributing to the matching between PT and */*⇡EFT at e corrections to neutron betalereghetti & Walker-Loud, PRL 129 (2 *Matching at O*(↵) *and O*(↵✏) *–* The diagrams conto the matrice of the matrice to the matrice of the matrice of the matrice of the matrice of the *[/]* $\overline{10}$ $$ Cinguano, de VIIes, Hayen, operte pour possible dopour tion (WFR) at zero momentum transfer (*q* = 0). As a r-Loud, PRL 129 (2022) [2202.10439] *^O*(✏⁰) and *O*(✏) are shown in Fig. 1. They imply for etions to neutron beta- \mathbb{C}^2 22) [2202.10459 X (*n*) $\frac{1}{2}$ diagrams (*b*1) and (*d*1) and (*c*1) and (*e*1) cancels, leav-**Fion-induced radiative corrections of a corrections of a corrections of** \mathbb{R}^2 $Cirigliano, de Vries, Hayen, Mereghetti & Walk$ $\sum_{i=1}^{n}$ 3 8*m^N* \overline{a} 9 16*m^N g* \overline{a} \overline{A} *i* \overline{A} where ✏*^R* ⇠ (246 GeV*/*⇤BSM)² is a BSM right-handed corrections to neutron heta-COTT CULTURE CONSIDER WE ARE WORKING TO THE RADIATION OF THE RADIATIO Pion-induced radiative corrections to neutron beta-decay
Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

Matching $\lambda - g_A^{\text{QCD}} \left(1 + \delta_{\text{RC}}^{(\lambda)} - 2 \text{Re}(\epsilon_R) \right) \qquad \delta_{\text{RC}}^{(\lambda)}$ **D** Matching the leading vector and axial operators were also the leading vector and axial operators of \mathbf{R} $\mu - g_A$ $\left(\frac{1 + \nu_{RC}}{RC} - 2\pi \nu(\epsilon_R)\right)$ σ_R *gV /A* = *g* λ - $-\mathcal{S}_A$ \overline{L} (1) *V /A, + /A* ↵ $\overline{\mathcal{L}}$ e (R') *V /A,*em $\overline{\mathbf{C}}$ **by corrections** $\lambda - g_A^{\prime\prime} - (1 + o_{RC}^{\prime\prime\prime}) - 2 \text{Re}(\epsilon_R)$ $\rho_{RC} = g_A^{\prime\prime\prime} - 2 \text{Re}(\epsilon_R)$ $M \cdot \text{chine}$ $\frac{1}{2} \cdot \frac{1}{2} \cdot \$ \mathbf{u} watching $\lambda - g_A^2 + 1 + o_{RC}^2 - 2\mathbf{I}$ the LECs of the pion-less Lagrangian. Specifically, the

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^O(✏⁰

 $\ddot{}$ $\overline{21}$ \mathbb{R} c thai ↵ ⇣ \cdot P. $vious estimates$ $P(A|V)$ *C* $P(A|V)$ **1 C** $P(A|V)$ DA to estimate $C_A(\mu)$ and $c_{3,4}$ from the lit NLO correction is somewhat larger than the LO one, we

$$
g_{V/A} = g_{V/A}^{(0)} \left[1 + \sum_{n=2}^{\infty} \Delta_{V/A,\chi}^{(n)} + \frac{\alpha}{2\pi} \sum_{n=0}^{\infty} \Delta_{V/A,\text{em}}^{(n)} + \left(\frac{m_u - m_d}{\Lambda_{\chi}} \right)^{n_{V/A}} \sum_{n=0}^{\infty} \Delta_{V/A,\delta m}^{(n)} \right]
$$

\n
$$
n_V = 2 \qquad n_A = 1
$$

\n
$$
\Delta_{\chi,\text{em},\delta m}^{(n)} \sim O(\epsilon_{\chi}^{n})
$$

\nCVC explicit calculation: $\Delta_{A,\delta m}^{(0),(1)} = 0$
\n
$$
\Delta_{V,\delta m}^{(0)} = 0
$$

$$
\Delta_{A,\text{em}}^{(0)} = Z_{\pi} \left[\frac{1 + 3g_A^{(0)2}}{2} \left(\log \frac{\mu^2}{m_{\pi}^2} - 1 \right) - g_A^{(0)2} \right] + \hat{C}_A(\mu) \qquad \text{Low-Energy-Constants (LEC)}
$$

$$
\Delta^{(1)}_{A,\text{em}} = Z_\pi \, 4\pi m_\pi \left[c_4 - c_3 + \frac{3}{8 m_N} + \frac{9}{16 m_N} g_A^{(0)2} \right]
$$

 $\overline{\text{I}}$ cing N. **Vaive Dimensional Analysis (NDA) to estin** $S(\lambda)$ – 11 ˆ 261.10^{-2} aporter of mag α_{RC} = (1.1,2.0) To an order of mag dominated by the NLO ⇡*N* LECs *c*3*,*⁴ via topology (a2). Using Naive Dimensional Analysis (NDA) to estin

c(4) – (1,4,0,6) – 10–² $\delta_{\rm RC}^{(4)} \in \{1.4, 2.6\} \cdot 10^{-2}$ an order of mag Using Naive Dimensional Analysis $s^{(\lambda)} = 114261.10^{-2}$ $\delta_{\text{RC}}^{(\lambda)} \in \{1.4, 2.6\} \cdot 10^{-2}$ a $\delta_{\rm RC}^{(\lambda)} \in \{1.4, 2.6\} \cdot 10^{-2}$ an order of magnitude larger than previous estimates Using Naive Dimensional Analysis (NDA) to estimate $C_A(\mu)$ and $c_{3,4}$ from the literature

V,m, Low-Energy-Constants (LECs) $\bigcap_{i=1}^n$ $CA(\mu)$ - C ˆ *V,A* on the LECs of *^O*(*e*²*p*). Here we note C₃ & C₄ are esti $\overline{\mathbf{u}}$ mated from literature $\hat{c}_n(0)$ 2 $\hat{c}_n(0)$ *V,* vanish due to conservation of that as written, *C* the vector current, while (*n*) *A,* have been calculated up ˆ *C* constants (LECs) distribution physics and in particular large logarithms con- (μ) V, vanish due to conservation of the conse the vector current vector c λ \tilde{h}_1 the vector current, while (*n*) $+$ *C* $\hat{\bigcirc}$ $A(\mu)$ $\overline{1}$ $\overline{\Omega}$ \overline{A} (Γ, Γ, C_s) $C_A(\mu)$ - completely unknown c₃ & c₄ are estimated from literature Low-Energy-Constants (LECs) $\hat{c}(0)$ \hat{c} data is \hat{c} data is \hat{c} data is \hat{c} at \hat{c} and \hat{c} and \hat{c} is \hat{c} and \hat{c} a $A \begin{bmatrix} +\mathbf{C}A(\mu) \\ \mathbf{I} \end{bmatrix}$ stabilizing between N2CM-EHCTgy-GOIISLO. $P_A^{(0)2}$ **dia**_{α} β *x* β *x*₁*.* 1*c* 1*.*¹ where the range in (0)

$$
\delta_{\mathrm{RC}}^{(\lambda)} = \frac{\alpha}{2\pi} \left(\Delta_{A,\mathrm{em}}^{(0)} + \Delta_{A,\mathrm{em}}^{(1)} - \Delta_{V\mathrm{em}}^{(0)} \right)
$$

LO

with the LEC \mathcal{L} fixed by the relation \mathcal{L} contributions arise from tree-level graphs with one $\delta_{\rm RC}^{(\prime)} \in \{1.4, 2.6\} \cdot 10^{-2}$ with one vertex from higher order Lagrangians *^L^p*² Lagrangians are presented in the Supplemental Material, including a new one needed to absorb divergences from $\det(\mathbf{r})$ at $\det(\mathbf{r})$ and $\det(\mathbf{r})$ and $\det(\mathbf{r})$ and $\det(\mathbf{r})$ are $\det(\mathbf{r})$ and $\det(\mathbf{r})$ and $\det(\mathbf{r})$ are $\det(\mathbf{r})$ and $\det(\mathbf{r})$ are $\det(\mathbf{r})$ and $\det(\mathbf{r})$ are $\det(\mathbf{r})$ and $\det(\mathbf{r})$ are $\det(\mathbf{r})$ a \Box need LQCD+QED calculation to determine $\delta_{\rm RC}^{(\lambda)}$ the leading vector and axial operators $\varrho^{\text{PDG}} = \varrho^{\text{QCD} - \text{iso}} + \delta^{(\lambda)}(\alpha, \hat{C}_{\cdot}(\mu))$ $_{\text{RC}}^{(\lambda)} \in \{1.4, 2.6\} \cdot 10^{-2}$ \Box seems to move g_A^2 towards g_A^2 . $g_A^{\text{PDG}} = g_A^{\text{QCD-iso}} + \delta_{\text{RC}}^{(\lambda)}(\alpha_{fs}, \hat{C}_A(\mu), \dots)$ ̂ seems to move g_A^{QCD} towards g_A^{exp}

$\overline{ }$ Pion-induced radiative corrections to neutron beta-decay
Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439] *^AV,*em ² *{*2*.*4*,* ⁵*.*7*} ,* (1) *A,*em = *{*10*.*0*,* 14*.*5*,* 15*.*9*},* (13) range de la communicación de V e Vries, Hayen, Mereghetti & Walker-Lou

pion electromagnetic mass splitting $\frac{1}{\sqrt{2\pi}}$ $m_{\pi^{\pm}}^2 - m_{\pi^0}^2 = 2e^2F_{\pi}^2Z_{\pi}$ $m^2 - m^2_0 = 2e^2F^2Z$ $\frac{1}{a}$ addition, in an EFT with $\frac{1}{a}$ and $\frac{1}{b}$ with $\frac{1}{b$

RC

QED corrections to Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439] *gA* measured nucleon axial charge with first-principles computations using lattice QCD and on the potential of -decay experiments to constrain beyond-the-Standard-Model interactions.

calculations [9–11]. This test is a unique and sensitive $dE_{e}d\Omega_{e}d\Omega_{\nu}$ $\frac{(r_F v_{ud})}{(2\pi)^5}(1+3\lambda^2)w(E_e)$ ⇥ $\overline{1}$ $1+\bar{a}(\lambda)$ $\vec{p}_e \cdot \vec{p}_\nu$ $E_e E_\nu$ $+ \bar{A}(\lambda)$

- An O(2%) QED correction to g_A was estimated with $χ$ PT
	- Assume χ PT is at least qualitatively correct (if not accurate) (no significant cancellation between analytic terms and LECs) $e^{i\pi}$ and $n = 1$ is at idast quantum very correct at me cient precision. Currently, lattice QCD calculations are $\frac{\text{c} \cdot \text{c} \cdot \text{c}}{\text{c} \cdot \text{c}}$
- In order to compare LQCD results of g_A to experiment, this QED correction MUST be $determined - LOCD + QED$ is the only way Γ \cap \cap Γ \cap \cap Γ er to compare $LQCD$ results of g_A to experim $\mathbf{u} \cdot \mathbf{u}$ requisited correction model be
	- It is a scheme (and possibly QED-gauge) dependent quantity
- This correction does NOT impact extraction of V_{ud} it is a "right handed" correction The λ in Γ is the same as in beta-assymetry (A) $s = \frac{1}{2}$ have highlighted and additional and and average for $\alpha = \frac{1}{2}$ orrection does iv $\bm{\cup}$ i impact extraction of $\bm{\mathit{V}}_{\mathit{uc}}$ \mathfrak{g} at the multi-TeV scale, such as the comparison of \mathfrak{g} the extracted weak as the extracted weak assignment of \mathcal{A} , t_{max} study of radiative corrections to the neu-attention to the neu-attentions to the neu-attentions to the neu-attention of \mathcal{L} tron decay die rate given by the divided divided by α $d\Gamma$ = $(G_F V_{ud})^2$
- It does prevent us from using LQCD to constrain BSM right-handed currents better than a few percent prevent us from using LQCD to co. cision in the next few years α next few years α next few years α next few years α

 E_e

QED corrections to Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439] *gA* measured nucleon axial charge with first-principles computations using lattice QCD and on the potential of -decay experiments to constrain beyond-the-Standard-Model interactions.

will be some years before these calculations reach sub-

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- In order to compare LQCD results of g_A to experiment, $\overline{}$ $determined - LOCD + QED$ is the only way Γ \cap \cap Γ \cap \cap Γ er to compare $LQCD$ results of g_A to experim
	-
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- It does prevent us from using LQCD to constrain BSM right-handed currents better than a few percent calculations [9–11]. This test is a unique and sensitive cision in the next few years α next few years α next few years α next few years α

QED corrections to Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439] *gA* measured nucleon axial charge with first-principles computations using lattice QCD and on the potential of -decay experiments to constrain beyond-the-Standard-Model interactions.

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	-
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Non-monotonic FV corrections to *gA*

Non-monotonic FV corrections to *gA*

- What is the issue?
- We (the LQCD community) think of FV corrections in the asymptotic scaling regime We have numerical evidence that the sign of the FV correction depends upon *mπ* We have qualitative evidence that the sign of FV corrections at $m_\pi \approx 300 \text{ MeV}$ is not the
-
- same as at m_π^phys
- We have qualitative evidence that the sign of the FV corrections can change
	- at fixed as one varies *mπL m^π*
	- at fixed as one varies *m^π mπL*
- We should not find this surprising, after all, for nucleon quantities

0.000 0.005 0.010 0.015 0.020 0.025 *^e*−*m*π*L*/(*m*π*L*) 1/2 1.23 1.25 1.27 1.29 *gA* NNLO+ct χ PT NLO χ PT prediction 1*.*20 $\sum_{i=1}^{\infty} 1.22$ 1*.*24 1*.*26 *A* $m_{\pi} \approx 220 \text{ MeV}$ $m_{\pi} \approx 310 \text{ MeV}$

Numerical Evidence:

At $m_{\pi} \approx 220$ MeV, results are consistent with leading prediction from χ PT (and also consistent with no correction or opposite sign)

At $m_{\pi} \approx 300$ MeV, results constrain the sign of the volume correction opposite of χPT prediction

Non-monotonic FV corrections to *gA*

- Expectations from PT *χ*
	-
	- therefore, higher order corrections are relatively more important The nucleon has a much richer spectrum of virtual excited states $(N\pi, \Delta\pi, \dots)$ In the large N_c limit, there is an exact cancellation of most NLO corrections to g_A α at fixed m_{π} vs $m_{\pi}L$

SU(2) HB_XPT(Δ) at NNLO also predicts change in sign of FV corrections

The chiral expansion for nucleons is a series in $\epsilon_{\pi} = \frac{m_{\pi}}{4\pi F}$, while for pions, it is in *mπ* $4\pi F_\pi$ ϵ_π^2

The finite volume corrections also respect this cancellation and lead to a sign change

Non-monotonic FV corrections to *gA*

Non-monotonic FV corrections to *gA* \blacksquare **for-monotonic FV com** Non-monotonic FV co PT are derived by the time extent as infinite the time extent as in the time extent as in the time extent as i \mathbf{r} \blacksquare \blacks <u>илс т</u> у ditions (typically periodic) \bf{r} v corrections to g_A

Z. Hall, D. Pefkou, A.S. Meyer, R. Briceño, M.A. Clark, M. Hoferichter, E. Mereghetti, H. Monge-Camacho, C. Morningstar, A. Nicholson, P. Vranas, A. Walker-Loud — In preparation H. Monge-Camacho, C. Morningstar, A. Nicholson, P. Vran *H. Monge-Camacho, C. Morningstar, A. Nicholson, P. Vranas, A. Walker-Loud — In preparation* H. Monge-Camacho, C. Mornin ceño, M.A. Clark, M. Hoferichter, E. Me ereghetti, icholson, P. Vranas, A. Walker-Loud — In pi

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Expectations from PT *χ* \Box SU(2) HB_XPT(Δ) at NNLO also predicts change in sign of FV corrections \blacksquare \Box Expectations from χ P \Box $g_A = g_0 + \Delta^{(2)} + \delta_{\rm FV}^{(2)} + \Delta^{(3)} + \delta_{\rm FV}^{(3)}$ FV *,* (2.2) $\mathcal{L}(\alpha_0 + \Lambda(2) + \mathcal{S}(2) + \Lambda(3) + \mathcal{S}(3)$ g_0 are more complex than σ_{FV} than σ_{FV} $SU(2) HByFI(\Delta)$ at infinition also predicts change in sign of FV corrections F<u>ronce</u>tations from vPT productions non χ ¹ $\overline{\mathbf{1}}$ $\mathbf{B}\chi\mathbf{P} \mathbf{1}(\mathbf{X})$ at NNLO also \Box SU(2) HB χ PT(Δ) at NNLO also pr $g_A = g_0$ \pm λ $\Delta^{(2)} + 0$ $\frac{2}{\sqrt{2}}$ *M*⁰ $-\Delta$ ² $+o$ ^r $\frac{1}{2}$ \Box Expectations from χ PT α are more complex than α are α or other α nucleon quantities is a series in ✏⇡, defined as g *,* ⇤ ⌘ 4⇡*F*⇡ *,* (2.1)

$$
\delta_{\rm FV}^{(2)} = \frac{8}{3} \epsilon_{\pi}^2 \left[g_0^3 F_1^{(2)}(m_{\pi} L) + g_0 F_3^{(2)}(m_{\pi} L) \right] \qquad \delta_{\rm FV}^{(3)} = \epsilon_{\pi}^3
$$

$$
F_1^{(2)}(x) = \sum_{\vec{n} \neq 0} \left[K_0(x|\vec{n}|) - \frac{K_1(x|\vec{n}|)}{x|\vec{n}|} \right]
$$

\n
$$
F_3^{(2)}(x) = -\frac{3}{2} \sum_{\vec{n} \neq 0} \frac{K_1(x|\vec{n}|)}{x|\vec{n}|},
$$

\n
$$
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$$

$$
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\n
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\n
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$$
\n
$$
\delta_{\rm FV}^{(2)} = \frac{8}{3} \epsilon_{\pi}^{2} \left[K_0(x|\vec{n}|) - \frac{K_1(x|\vec{n}|)}{x|\vec{n}|} \right]
$$
\n
$$
F_4^{(3)}(x) = -\frac{3}{2} \sum_{\vec{n} \neq 0} \frac{K_1(x|\vec{n}|)}{x|\vec{n}|},
$$
\n
$$
\delta_{\rm FV}^{(3)}(x) = \frac{8}{3} \epsilon_{\pi}^{3} \left[\frac{K_1(x|\vec{n}|)}{M_0} \right]
$$
\n
$$
F_4^{(3)}(x) = \sum_{\vec{n} \neq 0} \frac{K_1(x|\vec{n}|)}{x|\vec{n}|},
$$
\n
$$
F_3^{(3)}(x) = \sum_{\vec{n} \neq 0} \frac{K_2(x|\vec{n}|)}{\sqrt{\frac{\pi}{2}x|\vec{n}|}} x|\vec{n}|
$$
\n
$$
F_5^{(3)}(x) = \sum_{\vec{n} \neq 0} \frac{K_2(x|\vec{n}|)}{\sqrt{\frac{\pi}{2}x|\vec{n}|}} \frac{1}{\sqrt{\pi}} \sum_{\vec{n} \neq 0} \frac{K_2(x|\vec{n}|)}{(L - \vec{n}\vec{n})} \frac{1}{\sqrt{\frac{\pi}{2}x|\vec{n}|}} = \sum_{\vec{n} \neq 0} \frac{e^{L}x|\vec{n}|}{x|\vec{n}|}
$$

the order in ✏*ⁿ* ⇡ at which they contribute. change in l sig 11 OI T **l** 4⇡*F* + 4(2˜*c*⁴ *c*˜3)

$$
\delta_{\text{FV}}^{(3)} \qquad \Delta^{(2)} = \epsilon_{\pi}^{2} \left[-g_{0} (1 + 2g_{0}^{2}) \ln \epsilon_{\pi}^{2} + 4 \tilde{d}_{16}^{r} - g_{0}^{3} \right] \n\Delta^{(3)} = \epsilon_{\pi}^{3} g_{0} \frac{2\pi}{3} \left[3(1 + g_{0}^{2}) \frac{4\pi F}{M_{0}} + 4(2\tilde{c}_{4} - 4\tilde{c}_{4}^{2}) \right]
$$

g

Expectations from PT *χ* **Ω** SU(2) HB_XPT(Δ) at NNLO also predicts change in sign of FV corrections \blacksquare \Box Expectations from χ P \Box $g_A = g_0 + \Delta^{(2)} + \delta_{\rm FV}^{(2)} + \Delta^{(3)} + \delta_{\rm FV}^{(3)}$ FV *,* (2.2) $\delta_{\rm FV}^{(2)} = \frac{8}{2} \epsilon_{\pi}^2 \left[g_0^3 F_1^{(2)}(m_{\pi}L) + g_0 F_3^{(2)}(m_{\pi}L) \right]$ *g^A* are more complex than for other quantities, particu- $\frac{1}{2}$ $(2) H R v P T$ \bigvee $\overline{\mathbf{C}}$ at **NNI** *M*⁰ $T(\mathbf{X})$ at NNLO also predicts change and λ (2), ζ (2), λ (3), ζ (3) $A - y_0 + \Delta$ $+ 0$ δ (2) $F_V^{(2)} =$ 8 3 ϵ_π^2 π h $g_0^3 F_1^{(2)}(m_\pi L) + g_0 F_3^{(2)}(m_\pi L)$ $\overline{1}$ $\mathcal{L}(\alpha_0 + \Lambda(2) + \mathcal{S}(2) + \Lambda(3) + \mathcal{S}(3)$ g_0 are more complex than σ_{FV} than σ_{FV} $\begin{pmatrix} 0 \end{pmatrix}$ $\begin{pmatrix} 0 \end{pmatrix}$ for $\begin{pmatrix} 0 \end{pmatrix}$ for $\begin{pmatrix} 1 \end{pmatrix}$ for $\begin{pmatrix} 1 \end{pmatrix}$ $\delta_{\text{FV}}^{(2)} = \frac{1}{3} \epsilon_{\pi}^2 \left[g_0^3 F_1^{(2)}(m_{\pi}L) + g_0 F_3^{(2)}(m_{\pi}L) \right]$

NOTE: the leading FV correction is a \mathbf{g}_0 is determined in the chiral extractions are given by a set \mathbf{g}_0 for heavy mesons involving a charm or bottom quark. $t \cdot \frac{1}{2}$ $\overline{2}$ $\frac{d}{dV}$ co *F*(3) ¹ (*m*⇡*L*) \overline{a} $leter$ mined in the chiral ϵ $ext{rapolat}$ \cdot
 \cdot *.* NOTE: the leading FV correction is a prediction g_0 is determined in the chiral extrapolation $^{\bullet}$ no **,** , (2.1), (2.

 $\int_{\Omega} \rho$ for $g_0 \sim 1.2$, $\delta_{\text{FV}}^{(2)} > 0$ the state of \int In these expressions, *F* = lim*^m*⇡!⁰ *F*⇡, $m \in (2)$. The same holds for a given precision. The same holds for a given precision $\mathcal{L}(\mathcal{D})$. \sim 1.2, $o_{\text{FV}} > 0$

Non-monotonic FV corrections to *gA* \blacksquare **for-monotonic FV com** where we denote the infinite volume corrections by (*n*) and the finite volume corrections by the finite volume correction of the finite volume corrections by the finite volume corrections of the finite volume corrections of the finite volume corrections of the finite volume cor (*n*) phic FV cc Non-monotonic FV co PT are derived by the time extent as infinite the time extent as in the time extent as in the time extent as i \mathbf{r} sion for \mathcal{S}_A

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> the order in ✏*ⁿ* ⇡ at which they contribute. Ign of L A corrections

 $\overline{1}$

$$
F_1^{(2)}(x) = \sum_{\vec{n} \neq 0} \left[K_0(x|\vec{n}|) - \frac{K_1(x|\vec{n}|)}{x|\vec{n}|} \right]
$$

$$
F_3^{(2)}(x) = -\frac{3}{2} \sum_{\vec{n} \neq 0} \frac{K_1(x|\vec{n}|)}{x|\vec{n}|},
$$

$$
\Delta^{(2)} = \epsilon_{\pi}^{2} \left[-g_{0} (1 + 2g_{0}^{2}) \ln \epsilon_{\pi}^{2} + 4 \tilde{d}_{16}^{r} - g_{0}^{3} \right]
$$

$$
\Delta^{(3)} = \epsilon_{\pi}^{3} g_{0} \frac{2\pi}{3} \left[3(1 + g_{0}^{2}) \frac{4\pi F}{M_{0}} + 4(2\tilde{c}_{4} -
$$

Expectations from PT *χ* \Box SU(2) HB_XPT(Δ) at NNLO also predicts change in sign of FV corrections \blacksquare \Box Expectations from χ P \Box $g_A = g_0 + \Delta^{(2)} + \delta_{\rm FV}^{(2)} + \Delta^{(3)} + \delta_{\rm FV}^{(3)}$ FV *,* (2.2) $\delta_{\rm FV}^{(3)} = \epsilon_{\pi}^3 g_0 \frac{2\pi}{r^2} \left\{ g_0^2 \frac{4\pi F}{r^2} F_1^{(3)}(m_{\pi}L) \right\}$ $\left[4\pi F\right]_{\alpha=2}^{1/40}$ $t_0 = \left[\frac{\pm nT}{M_0}(3+2g_0^2)+4(2\tilde{c}_4-\tilde{c}_3)\right]F_3^{(3)}$ $\tilde{c}_i = (4\pi F) c_i$ $\mathcal{L}(\alpha_0 + \Lambda(2) + \mathcal{S}(2) + \Lambda(3) + \mathcal{S}(3)$ $g_A - g_0 + \Delta$ \rightarrow \rightarrow $v_{\text{FV}} + \Delta$ \rightarrow \rightarrow v_{FV} Ω_{π} for $\Lambda_{\pi}F$ and the chiral expansion for Ω $\delta_{\rm FV}^{(3)} = \epsilon_{\pi}^{3} g_0 \frac{2\pi}{\rho} \left\{ g_0^2 \frac{1}{\rho} \frac{1}{\rho} F_1^{(3)}(m_{\pi}L) \right\}$ πF (: *m*⇡ $+$ 2 g_0^2 + 4(2 \tilde{c}_4 - \tilde{c}_2) $F_s^{(3)}(m_{\pi}L)$ \mathcal{T}_{max} therefore, corrections from higher orders are relatively are relatively are relatively are relatively as the order in ✏*ⁿ* ⇡ at which they contribute. Ign of L A corrections $\Delta^{(2)}=\epsilon$ 2 π $\sqrt{ }$ $-g_0(1+2g_0^2)$ $\Delta^{(3)} = \epsilon$ 3 $\frac{3}{\pi}g_0$ 2π 3 $\sqrt{ }$ $3(1+g_0^2)$ $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $F_1(x) = \sum \frac{\pi}{\sqrt{\pi} \pi |\vec{n}|}$ $\overline{1}$ $\frac{1}{2}$ $\overline{}$ ⇡ h \overline{K} $\overline{Y}_{\frac{1}{2}}(x)$ $\sqrt{2}$ \ddagger $m \nu PT$ $\overline{\text{S}}$ e^{λ} each order are given by λ \overline{A} $A =$ $\overline{6}$ \int_0^1 ⇡ $\overline{}$ $(x + \Delta^{(2)} + \delta_{\text{FV}}^{(2)} + \Delta^{(3)} + \delta_{\text{FV}}^{(3)} + \delta_{\text{FV$ i (3) $\epsilon_{\rm FV}^{(3)}=\epsilon_{\pi}^3 g_0$ 2π 3 $\sqrt{ }$ g_0^2 $4\pi F$ M_0 $F_1^{(3)}(m_\pi L)$ $\overline{}$ $\lceil 4\pi F \rceil$ *M*⁰ $(3 + 2g_0^2) + 4(2\tilde{c}_4 - \tilde{c}_3)$ $\overline{1}$ $F_3^{(3)}(m_\pi L)$ \mathcal{L} $\int f \cdot \nabla V$ ~ *n*6=0 K $\ddot{\theta}$ *ⁿ|*) *^K*1(*x|*[~] $\frac{f(2)}{2}$ = ϵ_{π}^{2} $\left[-g_{0}(\mathbf{r})\right]$ \overline{a} $+\frac{6}{4}$ $\sim \pi$ 90 3 \sim $(1 + 1)$ $F_1^{(3)}$ $J_1^{(3)}(x) = \sum$ \bar{n} $\vec{n}\neq$ $\overline{0}$ 0 $K_{\frac{1}{2}}$ $\frac{1}{2} (x | \bar{n})$ $|\vec{n}|)$ $\sqrt{\frac{\pi}{2}}$ $\frac{\pi}{2}x|\bar{n}$ $|\vec{n}|$ $x|\bar{n}$ $K_{\frac{1}{2}}$

 \int_{Ω} in SU(2) HB χ י⊐
∍t $\frac{P T(\boldsymbol{\alpha})}{1610.08978}$ phase shift analysis $c_4 =$ 6 $c_4 = 4.26(4) \text{ GeV}^{-1}$ 2⇡ $c_3 = -5.60(6) \text{ GeV}^{-1}$ $\text{H}\text{B}\chi\text{PT}(\mathbf{\mathbf{\measuredangle}}),$ with N^3LO $N\pi$ phase shift analysis Siemens et al, 1610.08978 matrix elements. Therefore, the delta-resonance makes significant contributions to many quantities, adding an- $I(2) H BvPT(X)$, with N^3I O $N\pi$ phase shift $\frac{1}{2}$ in SU(2) $\text{HB}_{\chi} \text{PT}(\cancel{\Delta})$, with N³LO $N\pi$ phase shift analysis

> Fitting $2c_4 - c_3$ to our LQCD results yields a value $\sim 10 \times$ smaller — leads to change in sign of δ_{FV} as function of m_π This leads to LARGE, negative FV correction

Non-monotonic FV corrections to *gA* \blacksquare **for-monotonic FV com** Non-monotonic FV co PT are derived by the time extent as infinite the time extent as in the time extent as in the time extent as i \mathbf{r} sion for \mathcal{S}_A Non-monate The infinite volume corrections are given by

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$$
\Delta^{(2)} = \epsilon_{\pi}^{2} \left[-g_{0}(1 + 2g_{0}^{2}) \ln \epsilon_{\pi}^{2} + 4 \tilde{d}_{16}^{r} - g_{0}^{3} \right]
$$

$$
\Delta^{(3)} = \epsilon_{\pi}^{3} g_{0} \frac{2\pi}{3} \left[3(1 + g_{0}^{2}) \frac{4\pi F}{M_{0}} + 4(2\tilde{c}_{4} - \tilde{c}_{3}) \right]
$$

$$
F_{1}^{(3)}(x) = \sum_{\vec{n} \neq \vec{0}} \frac{K_{\frac{1}{2}}(x|\vec{n}|)}{\sqrt{\frac{\pi}{2}x|\vec{n}|}} x|\vec{n}| = \sum_{\vec{n} \neq \vec{0}} e^{-x|\vec{n}|}
$$

$$
F_{3}^{(3)}(x) = \sum_{\vec{n} \neq \vec{0}} \frac{K_{\frac{1}{2}}(x|\vec{n}|)}{\sqrt{\frac{\pi}{2}x|\vec{n}|}} = \sum_{\vec{n} \neq \vec{0}} \frac{e^{-x|\vec{n}|}}{x|\vec{n}|}
$$

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- Current strategy (of most groups)
	- and only leading volume correction

- Fit c_2 essentially to heavy m_π results
- Use this m_{π} -independent value of c_2 to extrapolate to infinite volume at all *mπ*
- If the volume corrections do change sign (to agree with χ PT prediction close to m_{π}^{phys}) the current strategy will lead to an error
- At what precision will this occur?

take asymptotic form of Bessel functions and leading "wrap around the world" mode

$$
\mathsf{L}\left[g_{A}(L)\right] = g_{A} + c_{2} \frac{m_{\pi}^{2}}{(4\pi F_{\pi})^{2}} \frac{e^{-m_{\pi}L}}{\sqrt{m_{\pi}L}}
$$

 0.010

- One needs to perform a volume study at multiple pion masses with sufficient precision to constrain the sign of the volume correction as a function of m_{π} $g_A(L) = g_A + c_2$ m_π^2 (4*πFπ*)2 $e^{-m_\pi L}$ $m_\pi L$ $+c_3$ m_π^3 (4*πFπ*)3 $e^{-m_\pi L}$ $m_\pi L$ $+ \cdots$
- Or we need to rely only upon $m_{\pi} \approx m_{\pi}^{\text{phys}}$ with sufficient precision to control the final uncertainty of g_A as well as the volume correction
- Or determine quantitatively that some variant of HBxPT provides an accurate description of both the m_{π} dependence as well as $m_{\pi}L$ dependence

What should we do?

Non-monotonic FV corrections to *gA*

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Non-monotonic FV corrections to *gA*

ing fraction [84–87] are taken from Ref. [12]), since in this ratio

Z. Hall, D. Pefkou, A.S. Meyer, R. Briceño, M.A. Clark, M. Hoferichter, E. Mereghetti,

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Non-monotonic FV corrections to *gA*

oosed measurement of the ratio to π_{ℓ_2} decays [27] (pion lifetime [62, 79–83] and branching fraction [84–87] are taken from Ref. [12]), since in this ratio

Subtleties and Systematics in achieving sub-percent uncertainty for g_A

- There is tension in the first-row CKM unitarity, BSM right-handed currents offer a favored solution to the tension LQCD calculation of g_A , plus radiative QED corrections, provides such a constraint
- estimates from χ PT suggests $\Delta_A^{R,other} = O(2\%)$, *A*
- seems to exhibit non-monotonic FV corrections *gA* As the precision of results improves, the current strategy of most groups $g_A(L) = g_A + c_2$ m_{π}^2 $(4πF_π)²$ $e^{-m_\pi L}$ $m_\pi L$

will lead to an error

At what precision of results will this become important?

$$
= O(2\%), g_A^{PDG} = g_A^{QCD-iso} + \Delta_A^{R,other}
$$

