Subtleties and Systematics in achieving sub-percent uncertainty for g_A

Lattice 2024

Liverpool, UK, July 28 — August 3, 2024

André Walker-Loud

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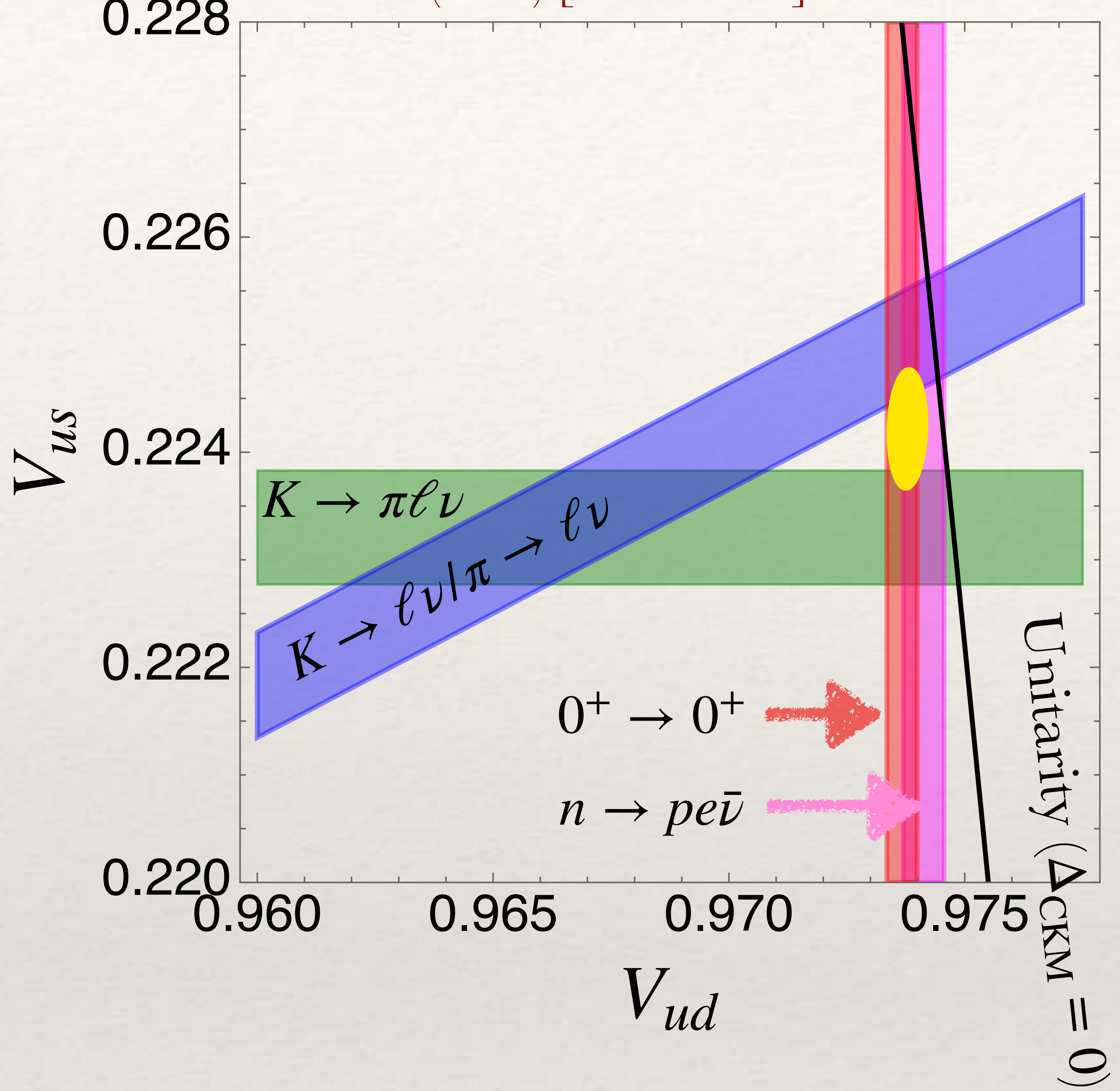
Subtleties and Systematics in achieving sub-percent uncertainty for g_A

- ❑ Why should we care about sub-percent uncertainty for g_A?
- ❑ QED corrections to g_A: estimates from χPT
- ❑ Non-monotonic FV corrections to g_A

First-row CKM Unitarity & Precision β decays

Cirigliano, Crivellin, Hoferichter, Moulson
PLB 838 (2023) [2208.11707]



$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm Weak} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{QCD}$$

- In the absence of new physics, unitarity constrains the elements of CKM e.g. $\sum_{j=d,s,b} |V_{ij}|^2 = 1$ for $i = u, c, t$

- Intense effort to test *heavy* flavor violation with charm/bottom quarks

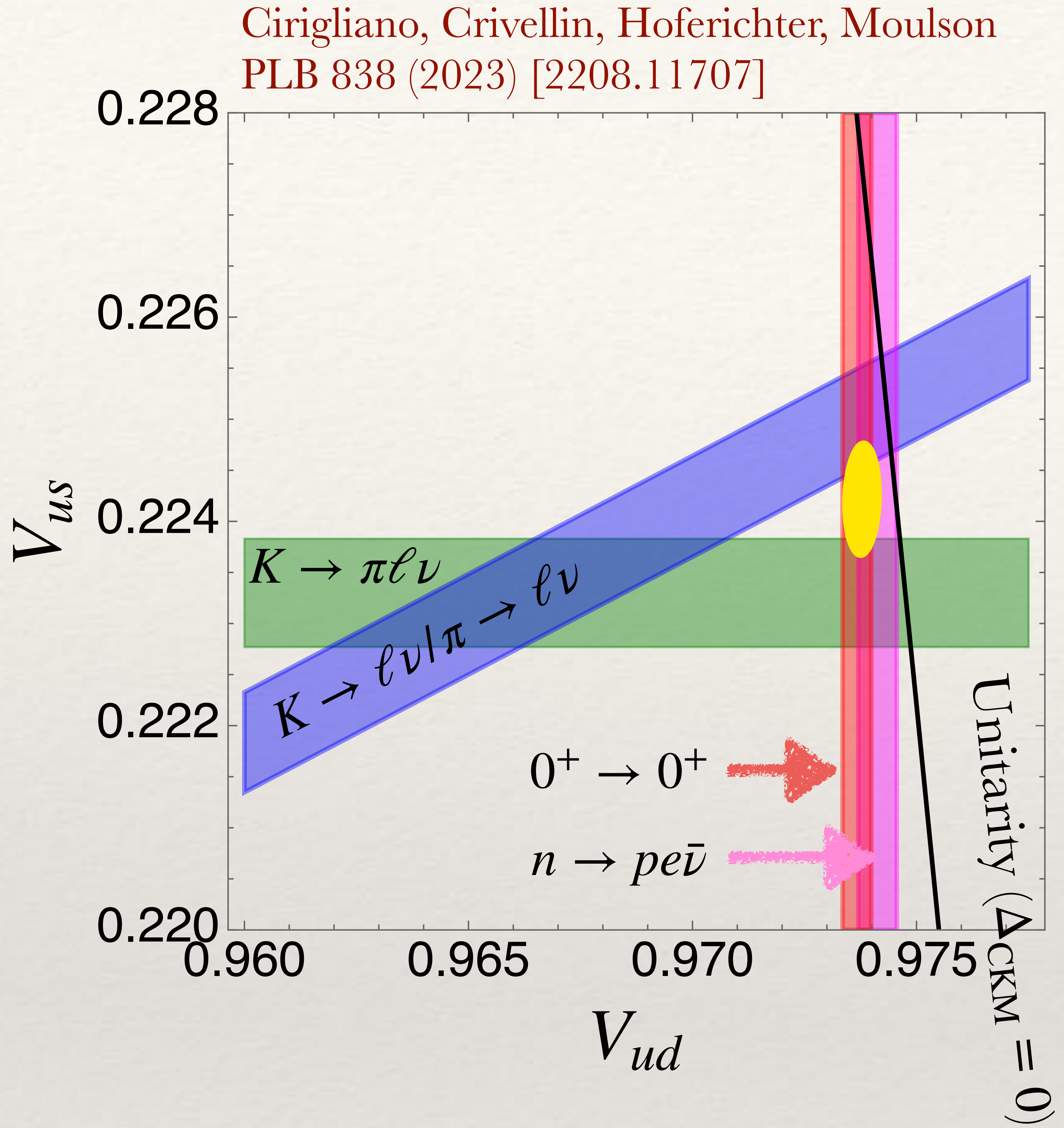
- The first row is showing robust tension

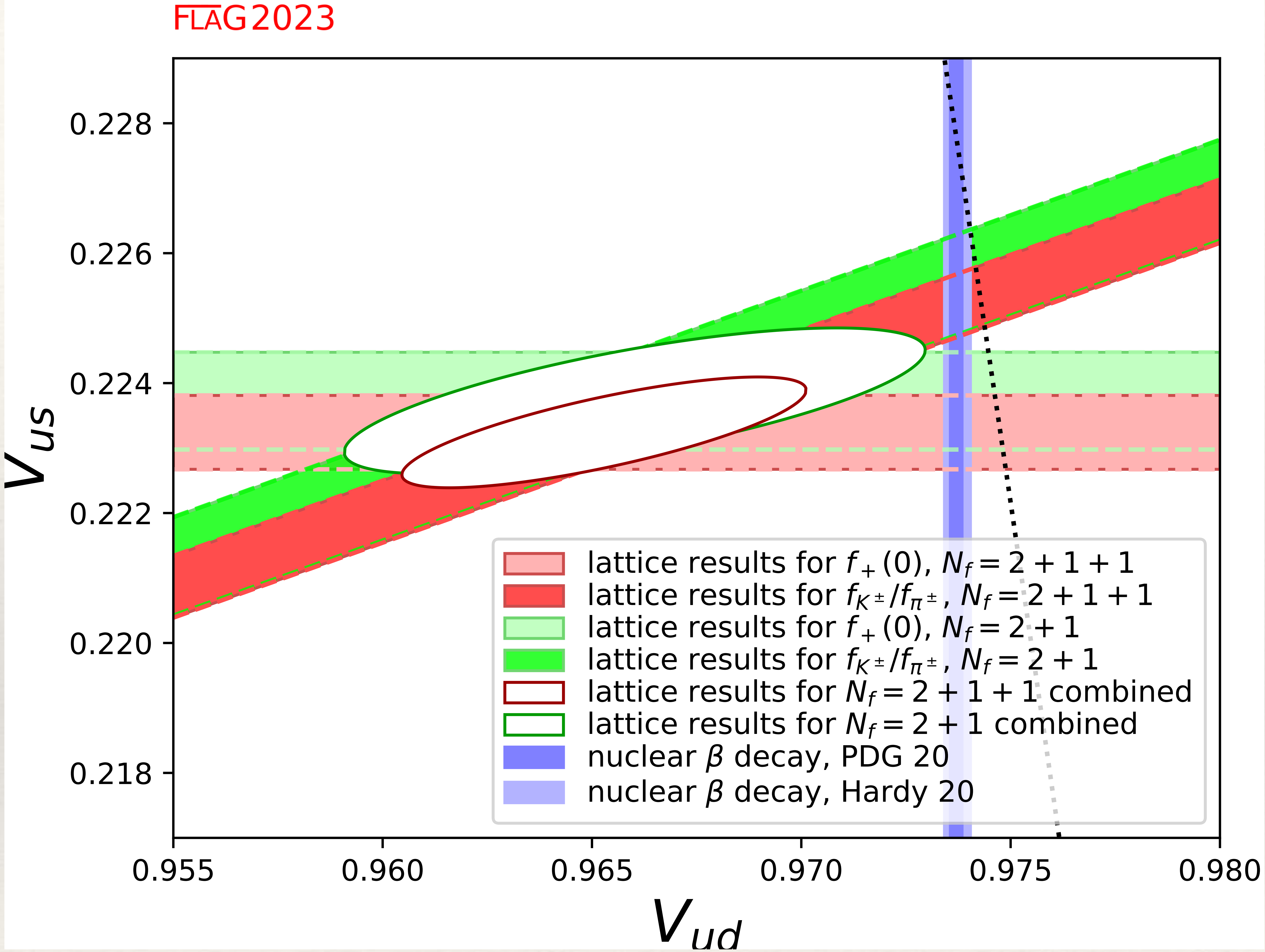
$$\Delta_{\rm CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1, \qquad V_{ud}^{0^+\to 0^+} = 0.97367(11)_{\rm exp}(13)_{\Delta_V^R}(27)_{\rm NS}[32]_{\rm total}$$
$$= -0.00176(56) \qquad V_{us}^{K_{\ell 3}} = 0.22330(35)_{\rm exp}(39)_{f_+}(8)_{\rm IB}[53]_{\rm total}$$

Cabibbo Angle Anomaly

- At this level of precision, careful treatment of radiative QED corrections has become the frontier
  - Original Sirlin & Marciano et al approach
  - modern pheno and EFT treatments
  - lattice QCD + QED

First-row CKM Unitarity & Precision β decays





- $> 3\sigma$ tension is seen with $N_f = 2+1+1$
- less tension with $N_f = 2+1$

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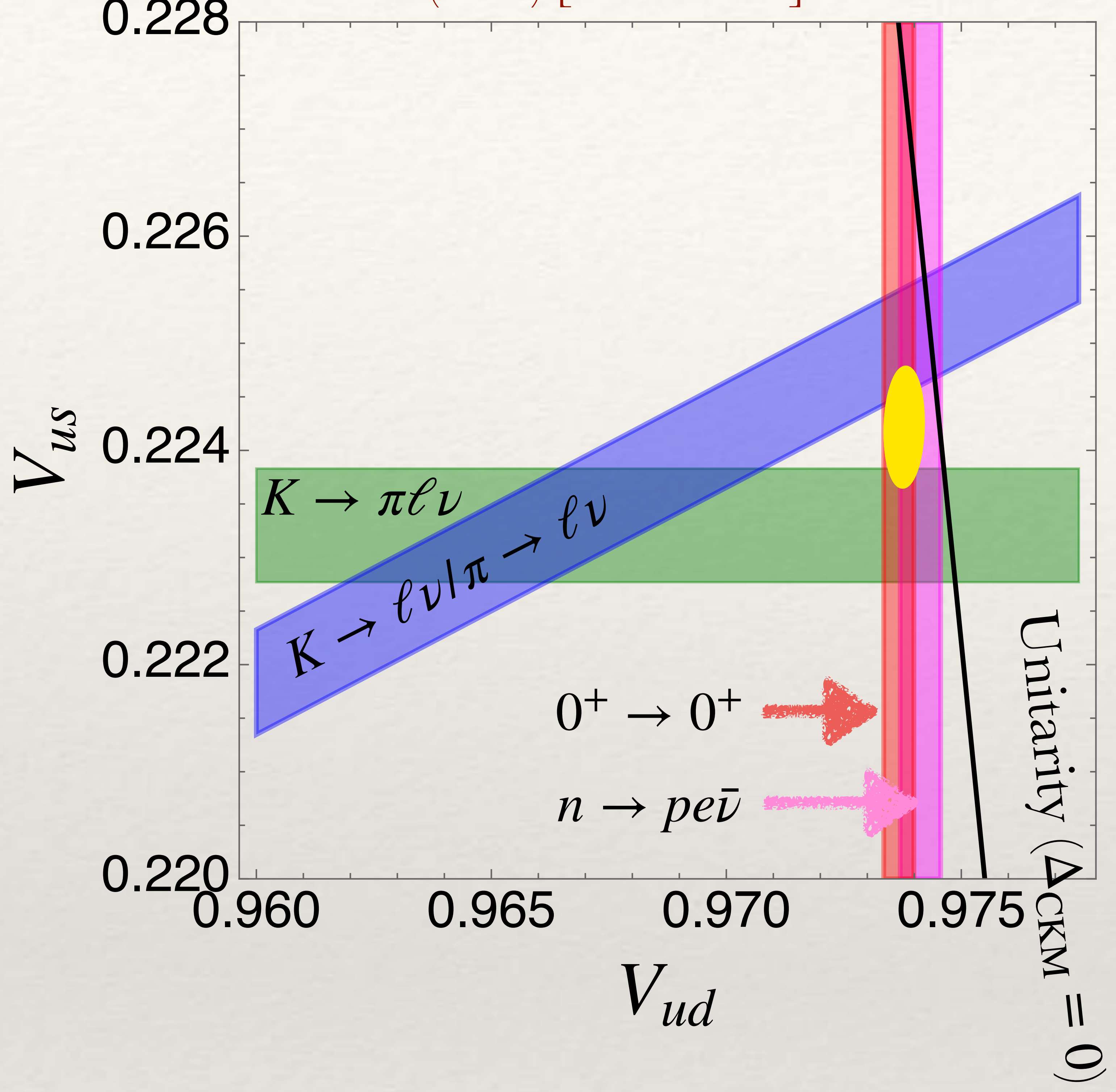


- The first row is showing robust tension — [some of the values in this estimate]

$$\Delta_{\rm CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1, \quad V_{ud}^{0^+\to 0^+} = 0.97367(11)_{\rm exp}(13)_{\Delta_V^R}(27)_{\rm NS}[32]_{\rm total}$$

$$= -0.00176(56) \quad V_{us}^{K_{\ell 3}} = 0.22330(35)_{\rm exp}(39)_{f_+}(8)_{\rm IB}[53]_{\rm total}$$

Cabibbo Angle Anomaly

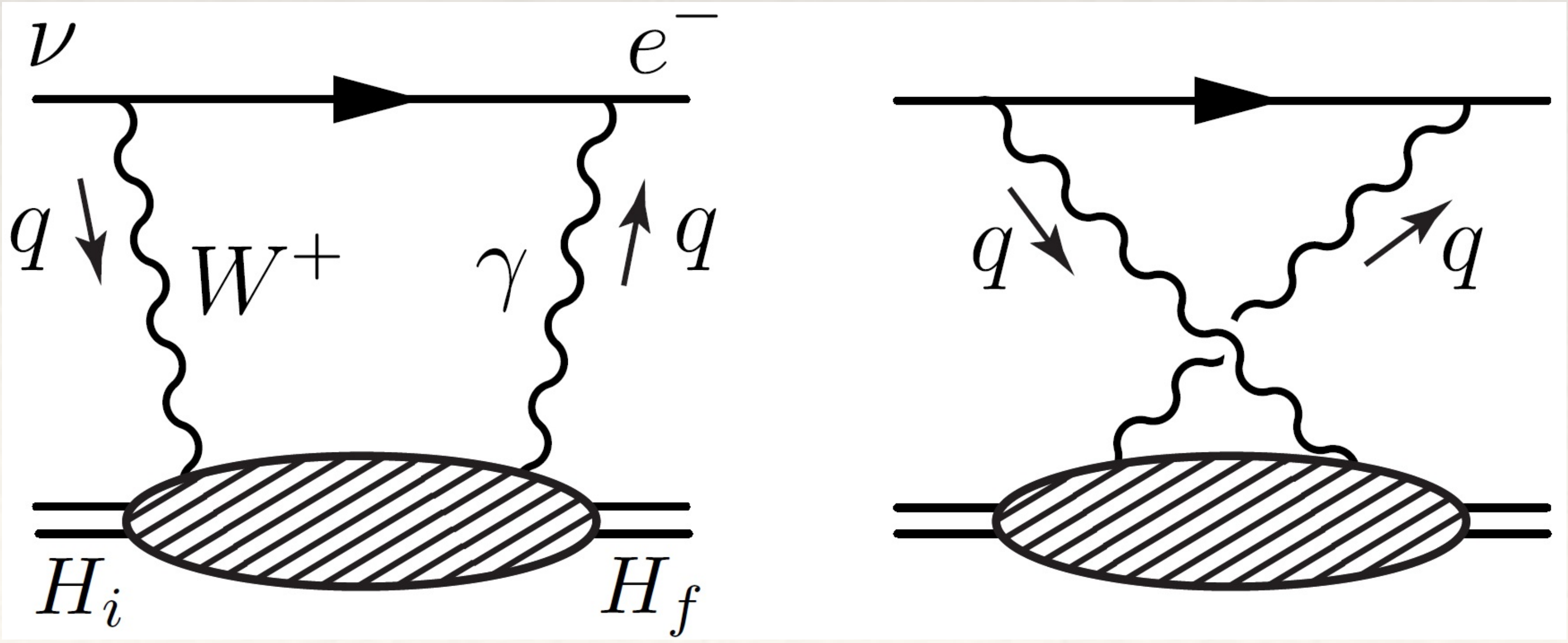
- Exciting prospects for neutron β-decay to match precision from superallowed alleviating the need for modeling the nuclear structure (NS) corrections

$$V_{ud}^{0^+\to 0^+} = 0.97367(11)_{\rm exp}(13)_{\Delta_V^R}(27)_{\rm NS}[32]_{\rm total}$$

$$V_{ud}^{n,{\rm PDG}} = 0.97441(3)_f(13)_{\Delta_V^R}(82)_\lambda(28)_{\tau_n}[88]_{\rm total} \qquad \lambda = g_A/g_V$$

$$V_{ud}^{n,{\rm best}} = 0.97413(3)_f(13)_{\Delta_V^R}(35)_\lambda(20)_{\tau_n}[43]_{\rm total}$$

- Reaching target precision requires improving the uncertainty from radiative QED corrections, in particular, Δ_V^R



$$\Gamma_n = \frac{G_F^2 |V_{ud}|^2 m_e^5}{2\pi^3}(1+3\lambda_{\rm PDG}^2) f_0 (1+\Delta_f)(1+\Delta_V^R)$$

$$\lambda_{\rm PDG} = \lambda_{\rm "exp"} - \Delta_A^{R,Sirlin,analytic} = \lambda_{\rm QCD-iso} + \Delta_A^{R,other}$$

$\Delta_A^{R,other} \simeq O(2\%)$ $\qquad$ $\Delta_A^{R,other}$ = QED correction to g_A

QED corrections to g_A

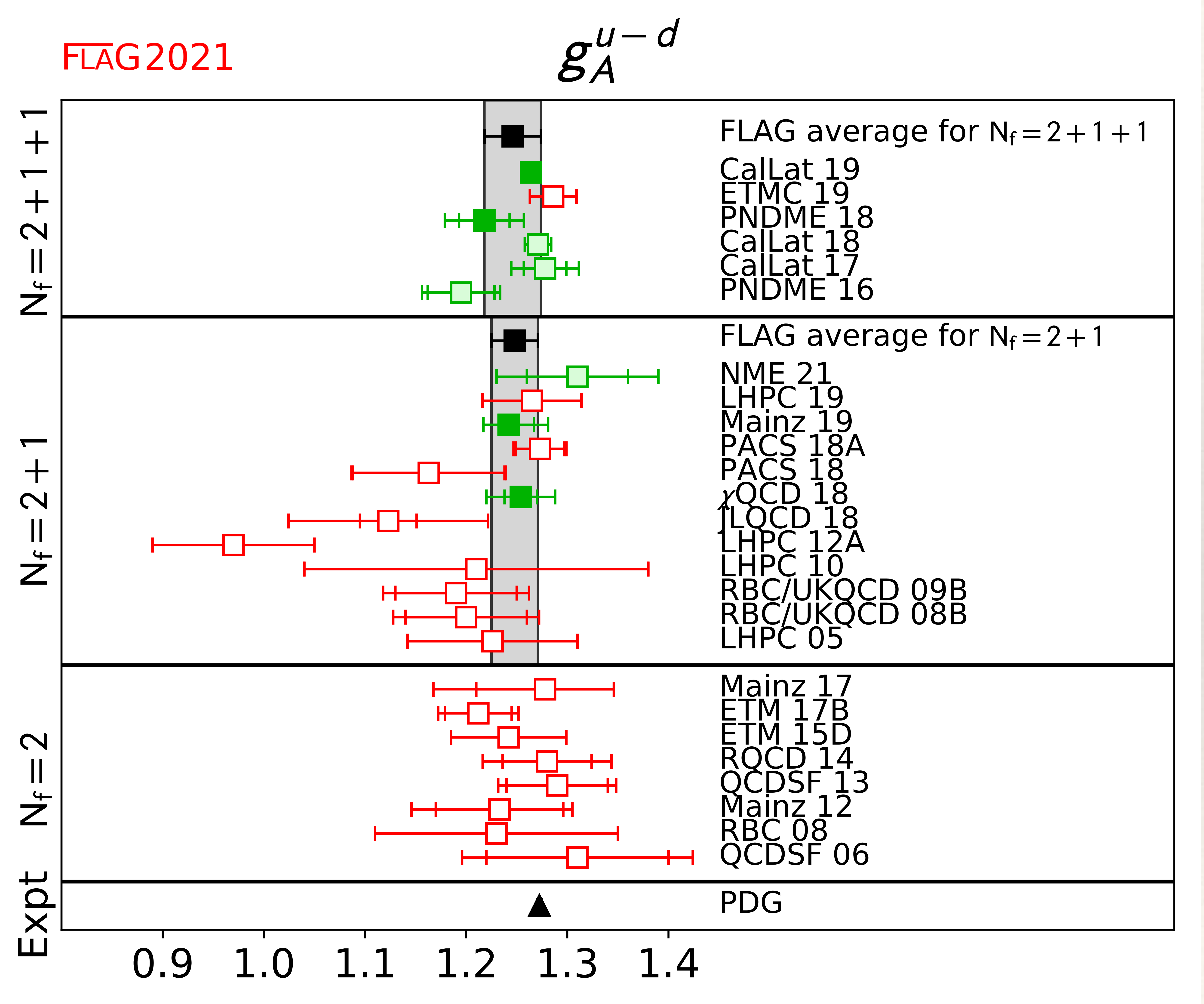
- We compare our LQCD calculations of $g_A^{\text{QCD-iso}}$ to g_A^{PDG}

- g_A^{PDG} is determined from an experimental measurement of $\lambda = g_A/g_V$ after some analytic long-distance QED effects are subtracted — see Hayen & Young, 2009.11364 for discussion

$$g_A^{\text{PDG}} = g_A^{\text{QCD-iso}} + \Delta_A^{R,other}$$

- But it turns out - potentially significant low-energy nucleon structure corrections may spoil this comparison

$$\Delta_A^{R,other} \simeq O(2\%)$$



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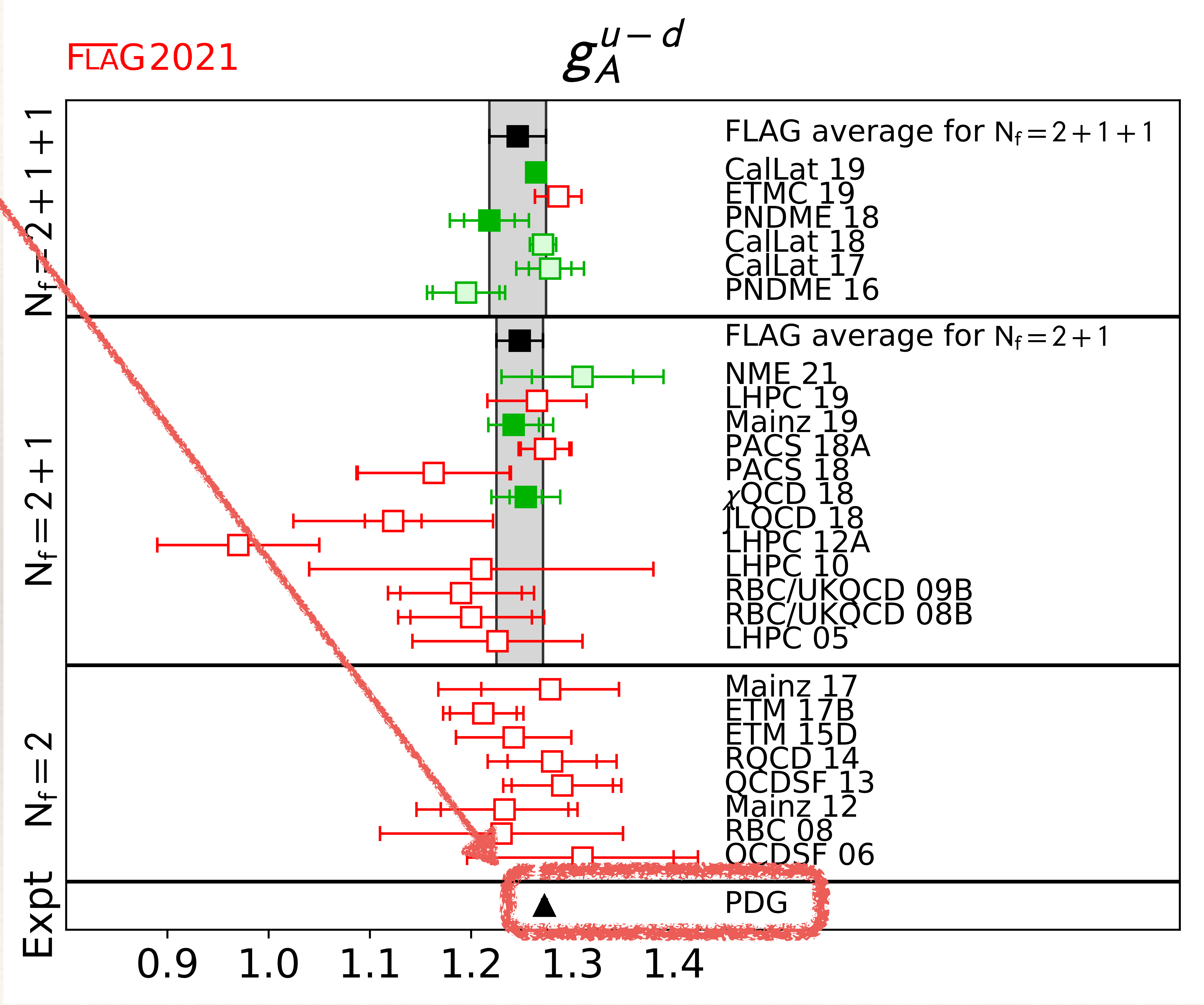

Pion-induced radiative corrections to neutron beta-decay

Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

- Systematic, EFT treatment of neutron β-decay

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{(G_F V_{ud})^2}{(2\pi)^5}(1+3\lambda^2)w(E_e) \times \left[1 + \bar{a}(\lambda)\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu} + \bar{A}(\lambda)\frac{\vec{\sigma}_n\cdot\vec{p}_e}{E_e} + \ldots\right]$$

The parameters can be measured

If we want to connect them to Standard Model (SM) parameters
we need to start from a Lagrangian with parameters related to SM parameters

pion-less low-energy EFT

$$\lambda = \frac{g_A}{g_V}$$

$$\begin{aligned}
\mathcal{L}_{\not\pi} = -\sqrt{2}G_F V_{ud}\Big[&\bar{e}\gamma_\mu P_L \nu_e\Big(\bar{N}\left(g_V v_\mu - 2g_A S_\mu\right)\tau^+ N \\
&+ \frac{i}{2m_N}\bar{N}(v^\mu v^\nu - g^{\mu\nu} - 2g_A v^\mu S^\nu)(\overleftarrow{\partial} - \overrightarrow{\partial})_\nu \tau^+ N\Big) \\
&+ \frac{i c_T m_e}{m_N}\bar{N}\left(S^\mu v^\nu - S^\nu v^\mu\right)\tau^+ N\ \left(\bar{e}\sigma_{\mu\nu}P_L \nu\right) \\
&+ \frac{i\mu_{\text{weak}}}{m_N}\bar{N}[S^\mu, S^\nu]\tau^+ N\,\partial_\nu\left(\bar{e}\gamma_\mu P_L \nu\right)\Big] + \ldots \qquad (2)
\end{aligned}$$

Perform the calculation with SU(2) heavy-baryon χPT and match the results to this pion-less EFT whose parameters can be matched to experimentally measured quantities

Pion-induced radiative corrections to neutron beta-decay

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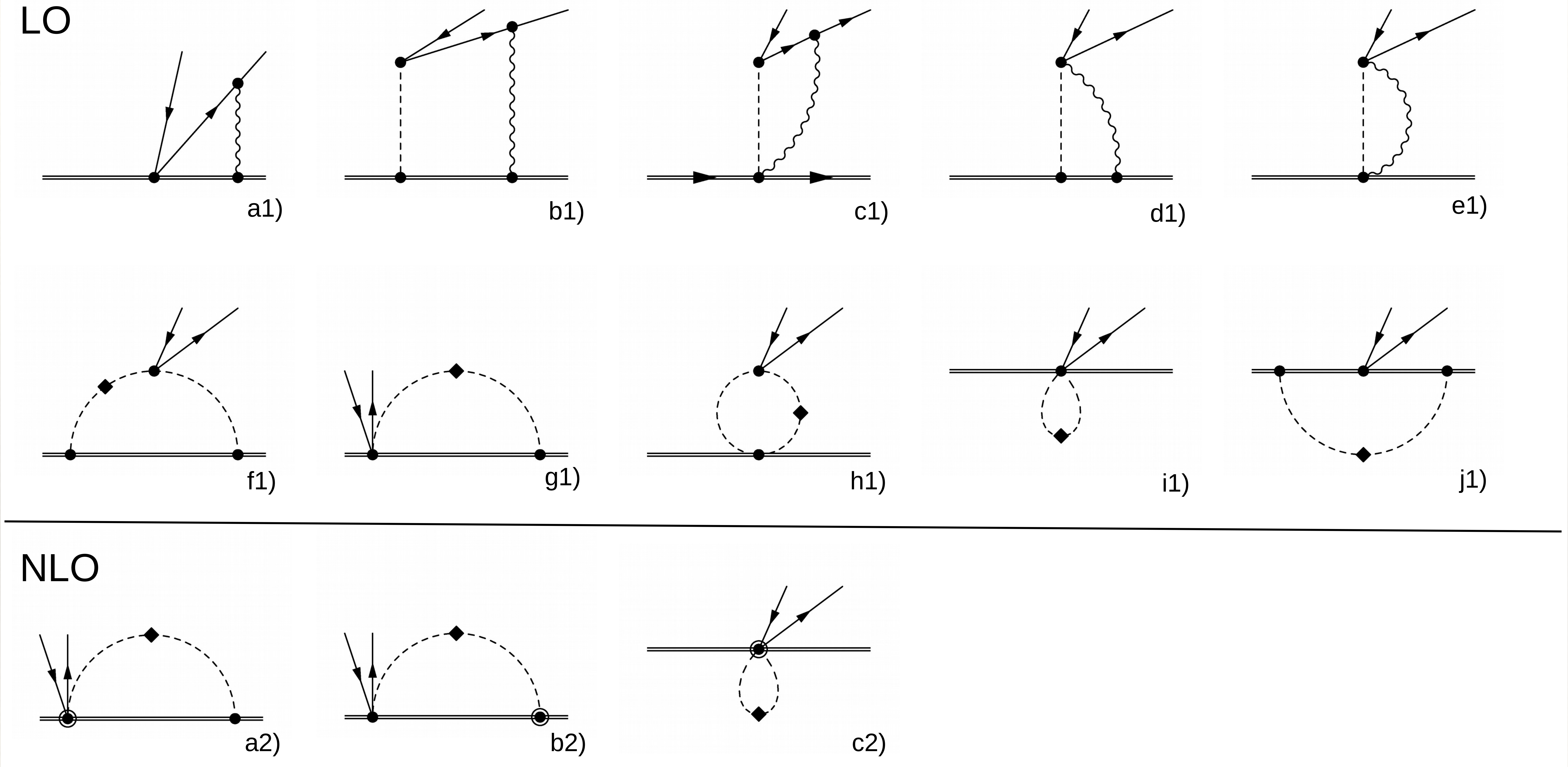
- Sub-set of O(50) diagrams



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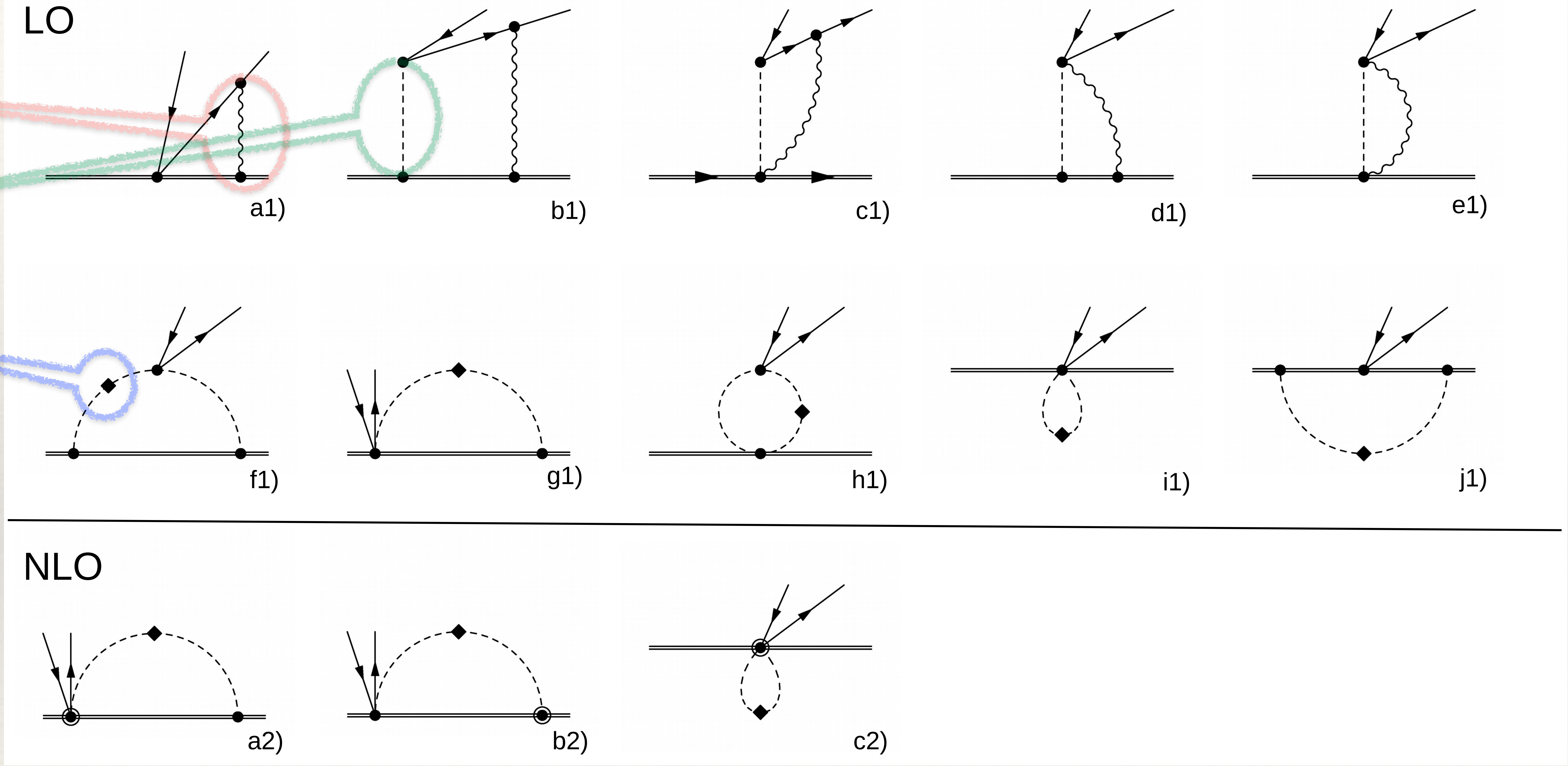
photons

pions

pion electromagnetic mass splitting

$$m^2_{\pi^\pm} - m^2_{\pi^0} = 2e^2 F_\pi^2 Z_\pi$$



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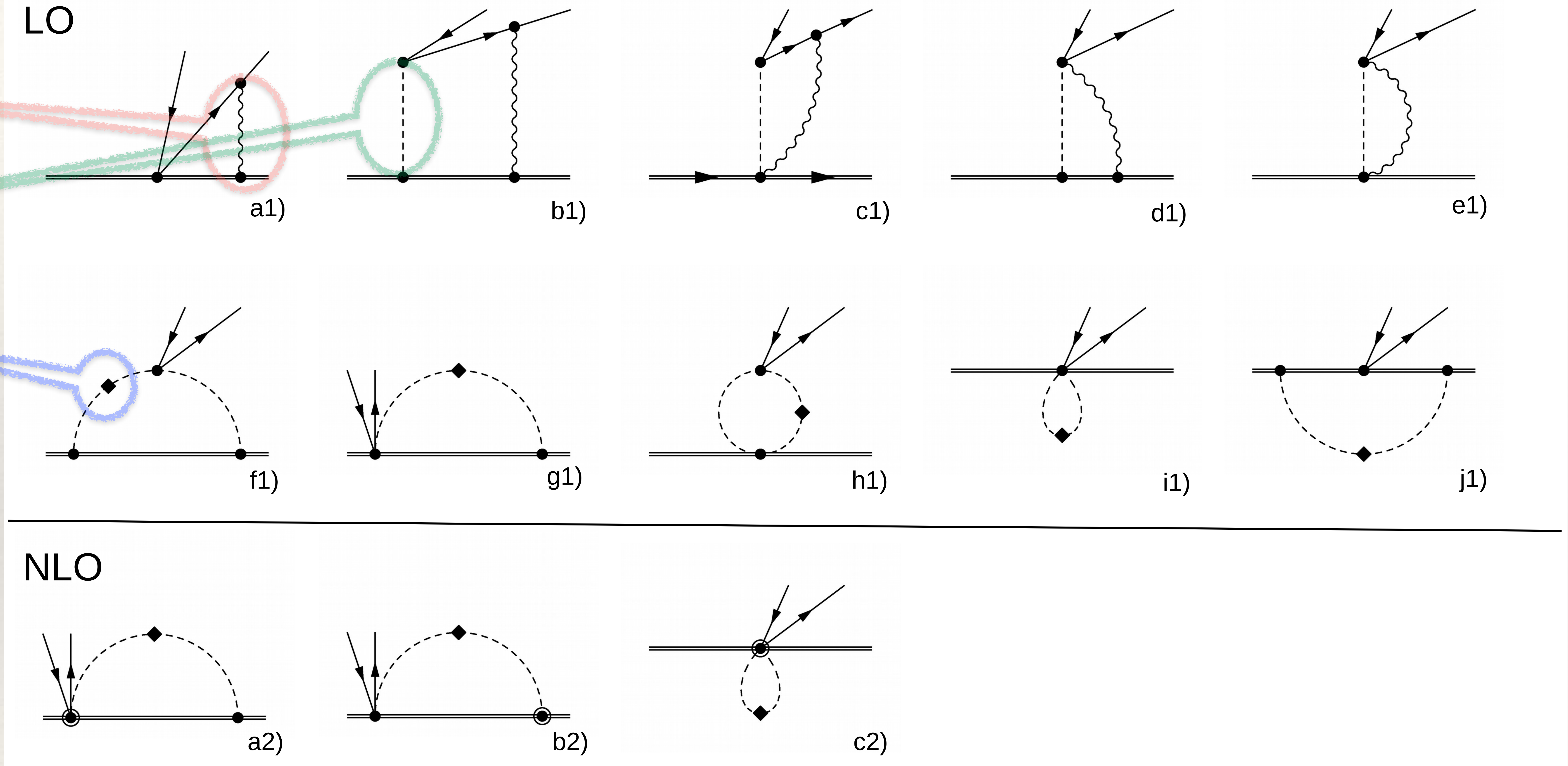

NOTE: at this order, we also include QED, m_d-m_u corrections to M_n-M_p

- iso-vector contributions to M_n-M_p vanish from symmetry constraints for τ^+ current
- iso-scalar contributions do not vanish - but the sum of all of them does vanish through NLO

Pion-induced radiative corrections to neutron beta-decay

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- Matching $\lambda - g_A^{\mathrm{QCD}}\left(1+\delta_{\mathrm{RC}}^{(\lambda)} - 2\mathrm{Re}(\epsilon_R)\right)$ $\quad \delta_{\mathrm{RC}}^{(\lambda)} = \frac{\alpha}{2\pi}\left(\Delta_{A,\mathrm{em}}^{(0)} + \Delta_{A,\mathrm{em}}^{(1)} - \Delta_{V\mathrm{em}}^{(0)}\right)$

$$g_{V/A} = g_{V/A}^{(0)}\left[1 + \sum_{n=2}^{\infty}\Delta_{V/A,\chi}^{(n)} + \frac{\alpha}{2\pi}\sum_{n=0}^{\infty}\Delta_{V/A,\mathrm{em}}^{(n)} + \left(\frac{m_u - m_d}{\Lambda_\chi}\right)^{n_{V/A}}\sum_{n=0}^{\infty}\Delta_{V/A,\delta m}^{(n)}\right]$$

$\Delta_{\chi,\mathrm{em},\delta m}^{(n)} \sim O(\epsilon_\chi^n)$

$n_V = 2$ CVC

$n_A = 1$ explicit calculation: $\Delta_{A,\delta m}^{(0),(1)} = 0$, $\Delta_{V,\delta m}^{(0)} = 0$

$$\Delta_{A,\mathrm{em}}^{(0)} = Z_\pi\left[\frac{1+3g_A^{(0)2}}{2}\left(\log\frac{\mu^2}{m_\pi^2} - 1\right) - g_A^{(0)2}\right] + \hat{C}_A(\mu)$$

$$\Delta_{A,\mathrm{em}}^{(1)} = Z_\pi\, 4\pi m_\pi\left[c_4 - c_3 + \frac{3}{8m_N} + \frac{9}{16m_N}g_A^{(0)2}\right]$$

Low-Energy-Constants (LECs)

$C_A(\mu)$ - completely unknown

c_3 & c_4 are estimated from literature

Using Naive Dimensional Analysis (NDA) to estimate $C_A(\mu)$ and $c_{3,4}$ from the literature

$\delta_{\mathrm{RC}}^{(\lambda)} \in \{1.4, 2.6\}\cdot 10^{-2}$ an order of magnitude larger than previous estimates

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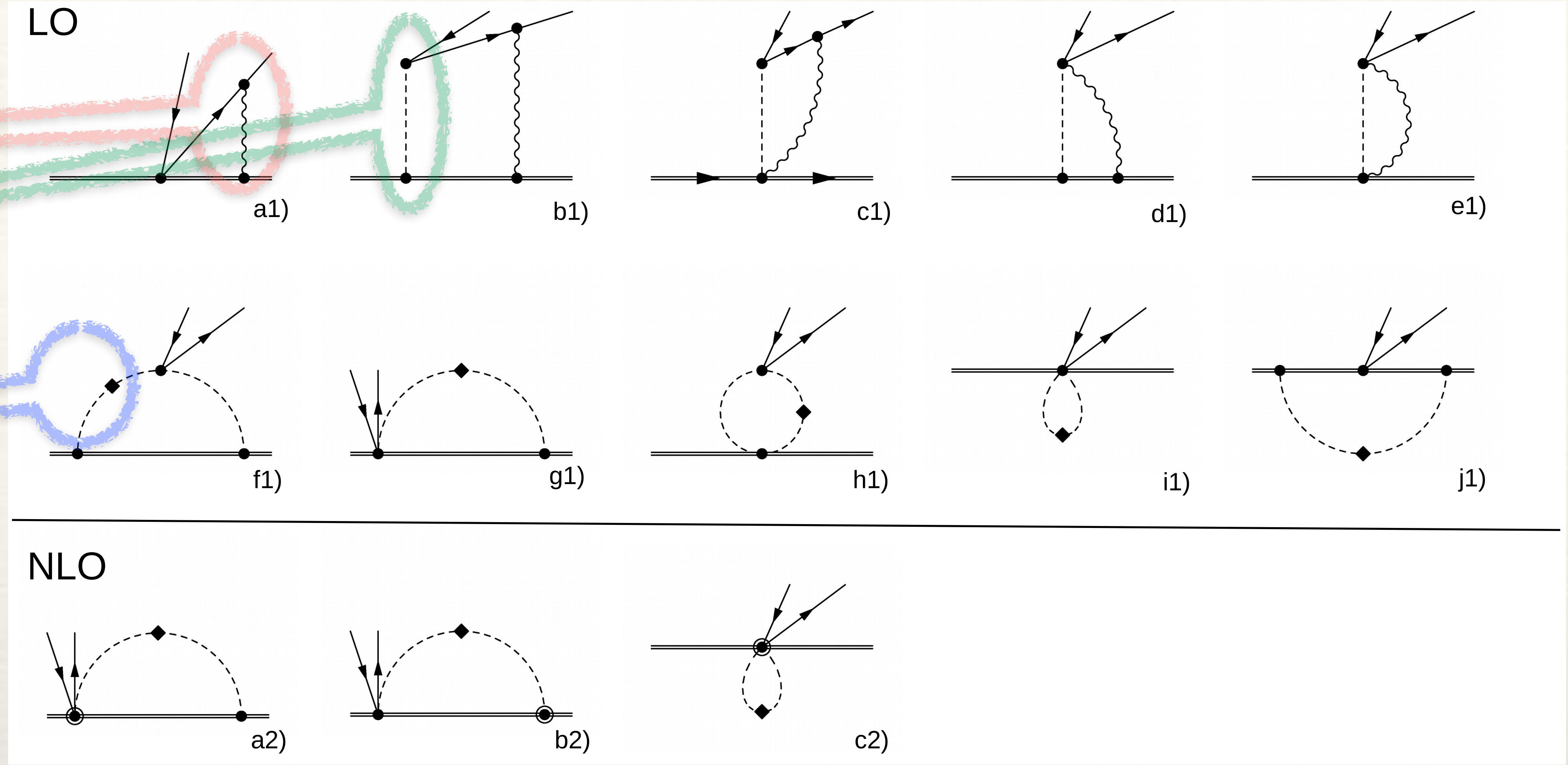
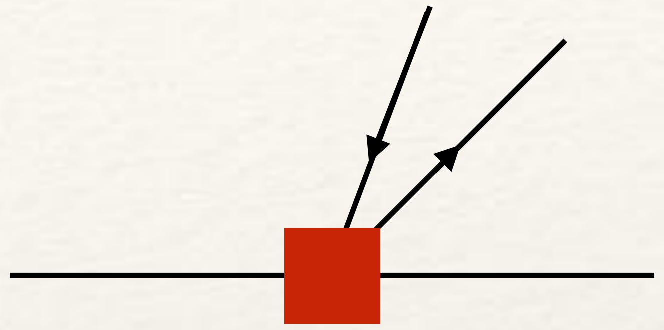


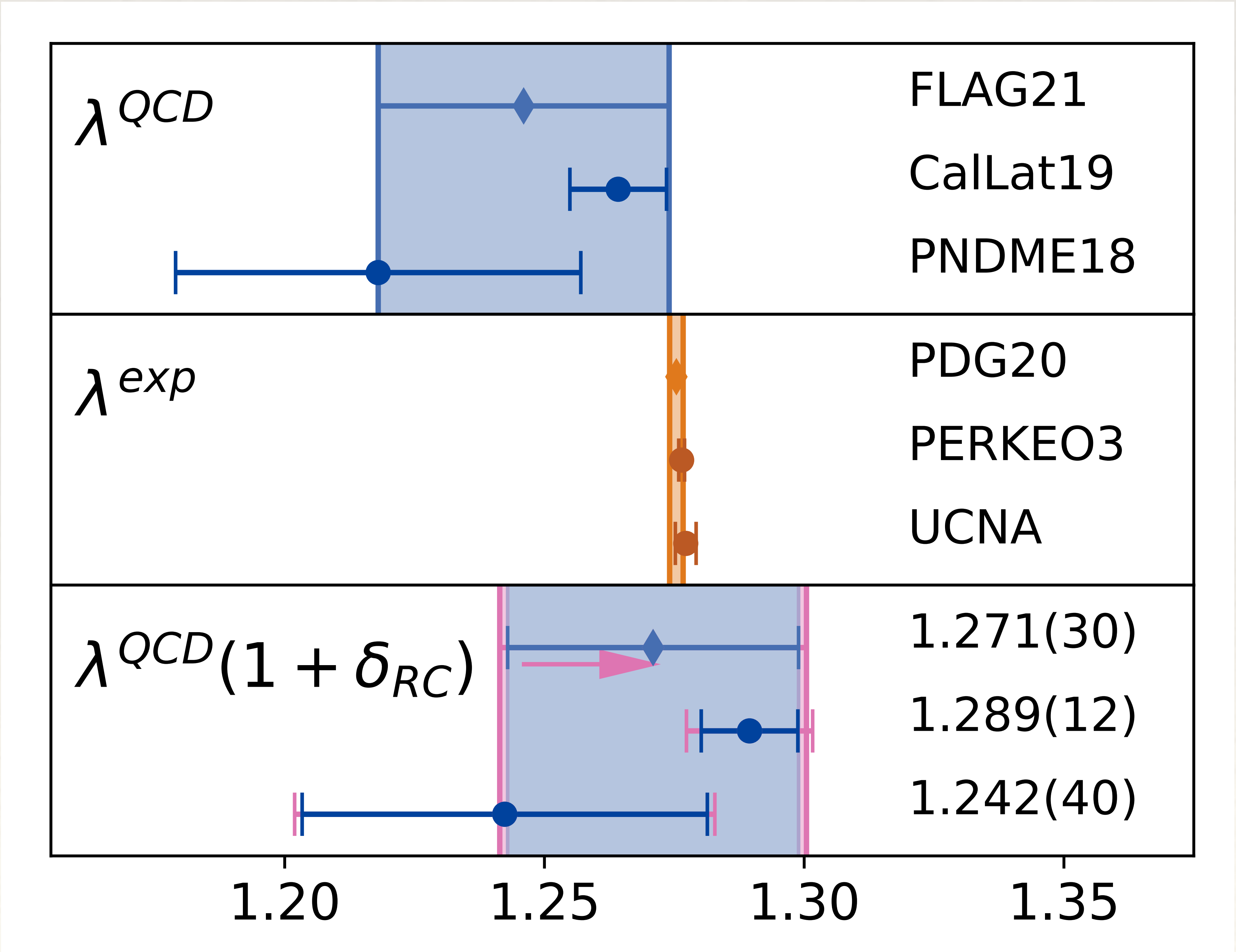
+ 

Low-Energy-Constants (LECs)

$$g_A^{\rm PDG} = g_A^{\rm QCD-iso} + \delta_{\rm RC}^{(\lambda)}(\alpha_{fs}, \hat{C}_A(\mu), \dots)$$

$$\delta_{\rm RC}^{(\lambda)} \in \{1.4,\ 2.6\} \cdot 10^{-2}$$

- seems to move g_A^{QCD} towards $g_A^{\rm exp}$
- need LQCD+QED calculation to determine $\delta_{\rm RC}^{(\lambda)}$



QED corrections to g_A

Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

- An O(2%) QED correction to g_A was estimated with χPT
  - Assume χPT is at least qualitatively correct (if not accurate) (no significant cancellation between analytic terms and LECs)
- In order to compare LQCD results of g_A to experiment, this QED correction MUST be determined — LQCD + QED is the only way
  - It is a scheme (and possibly QED-gauge) dependent quantity
- This correction does NOT impact extraction of V_{ud} — it is a "right handed" correction
  - The λ in Γ is the same as in beta-assymetry (A)
- It does prevent us from using LQCD to constrain BSM right-handed currents better than a few percent

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{(G_F V_{ud})^2}{(2\pi)^5}(1+3\lambda^2)w(E_e) \times \left[1 + \bar{a}(\lambda)\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \bar{A}(\lambda)\frac{\vec{\sigma}_n \cdot \vec{p}_e}{E_e} + \dots\right]$$

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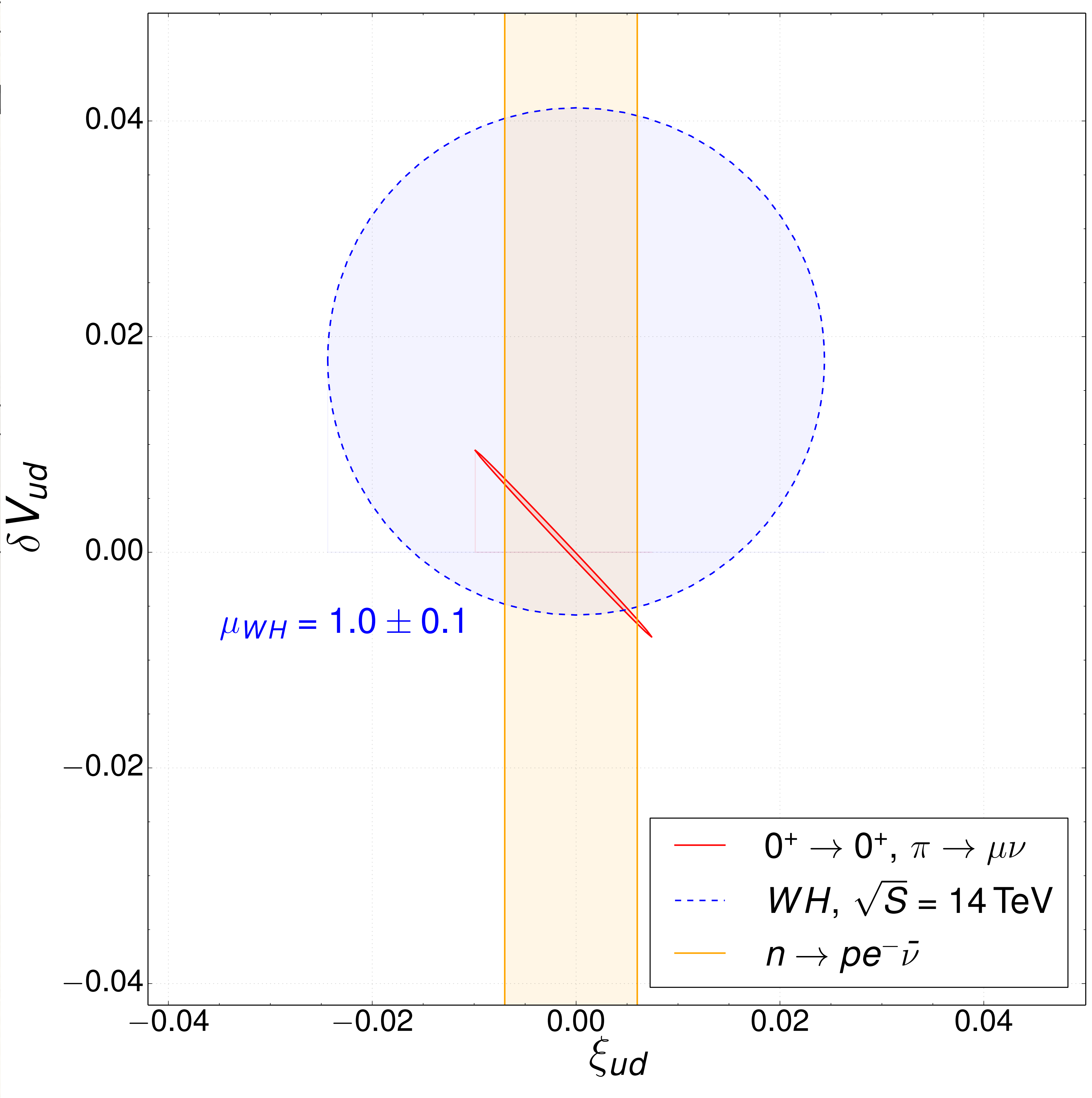

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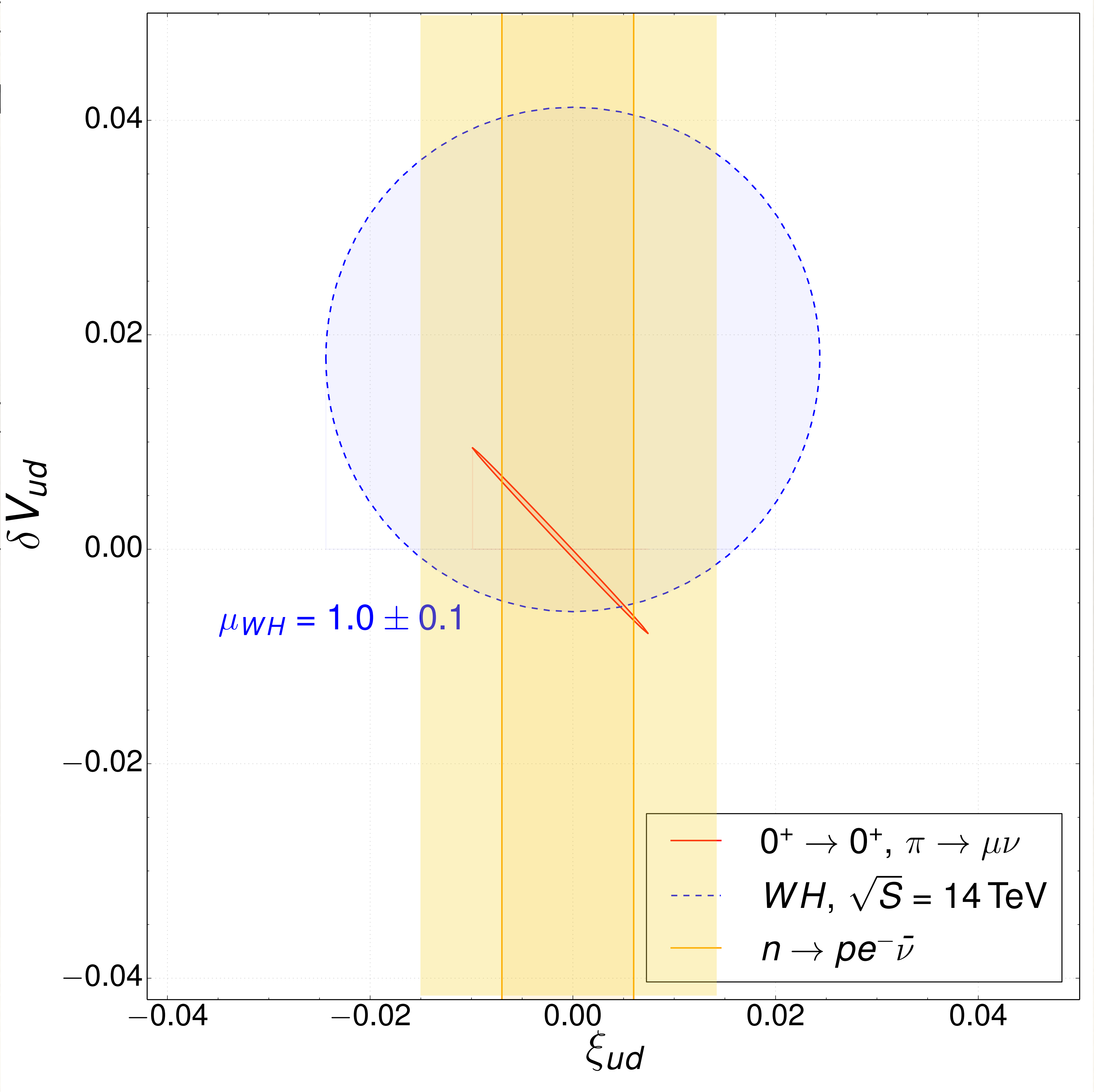

Non-monotonic FV corrections to g_A

Z. Hall, D. Pefkou, A.S. Meyer, R. Briceño, M.A. Clark, M. Hoferichter, E. Mereghetti,
H. Monge-Camacho, C. Morningstar, A. Nicholson, P. Vranas, A. Walker-Loud — In preparation

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- What is the issue?
- We (the LQCD community) think of FV corrections in the asymptotic scaling regime
- We have numerical evidence that the sign of the FV correction depends upon m_π 😱
- We have qualitative evidence that the sign of FV corrections at $m_\pi \approx 300$ MeV is not the same as at m_π^{phys}
- We have qualitative evidence that the sign of the FV corrections can change
  - at fixed $m_\pi L$ as one varies m_π
  - at fixed m_π as one varies $m_\pi L$
- We should not find this surprising, after all, for nucleon quantities

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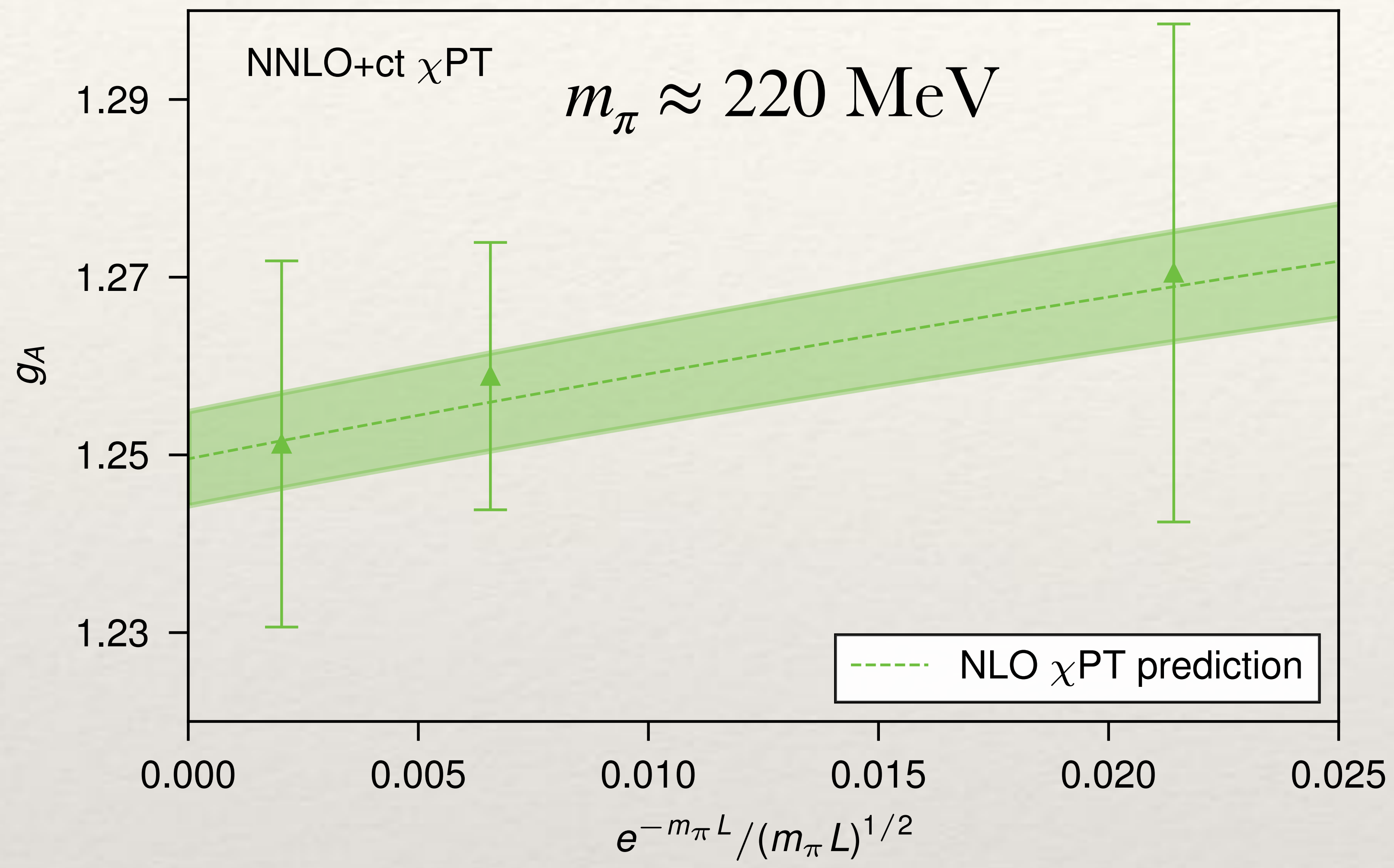
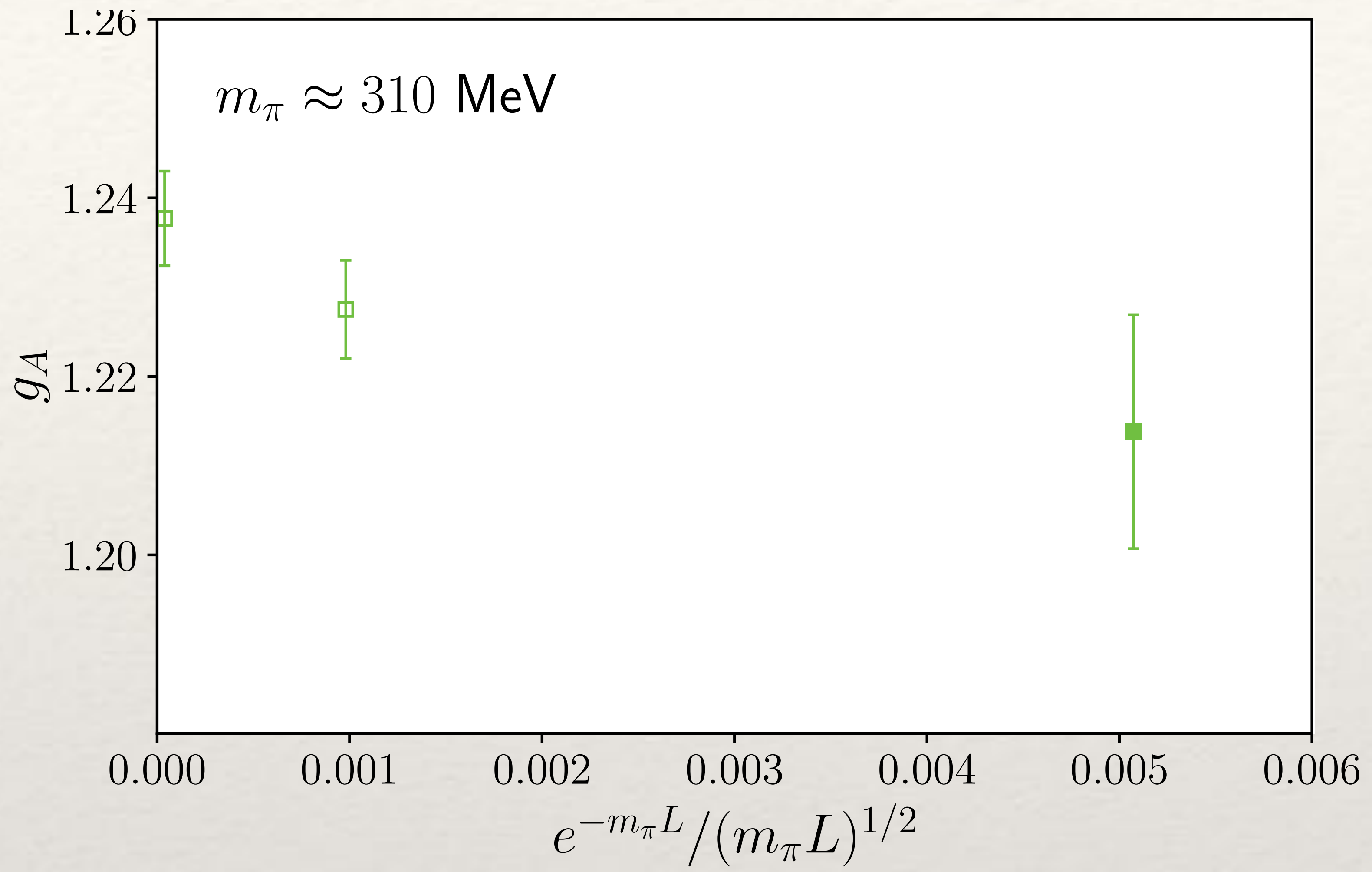
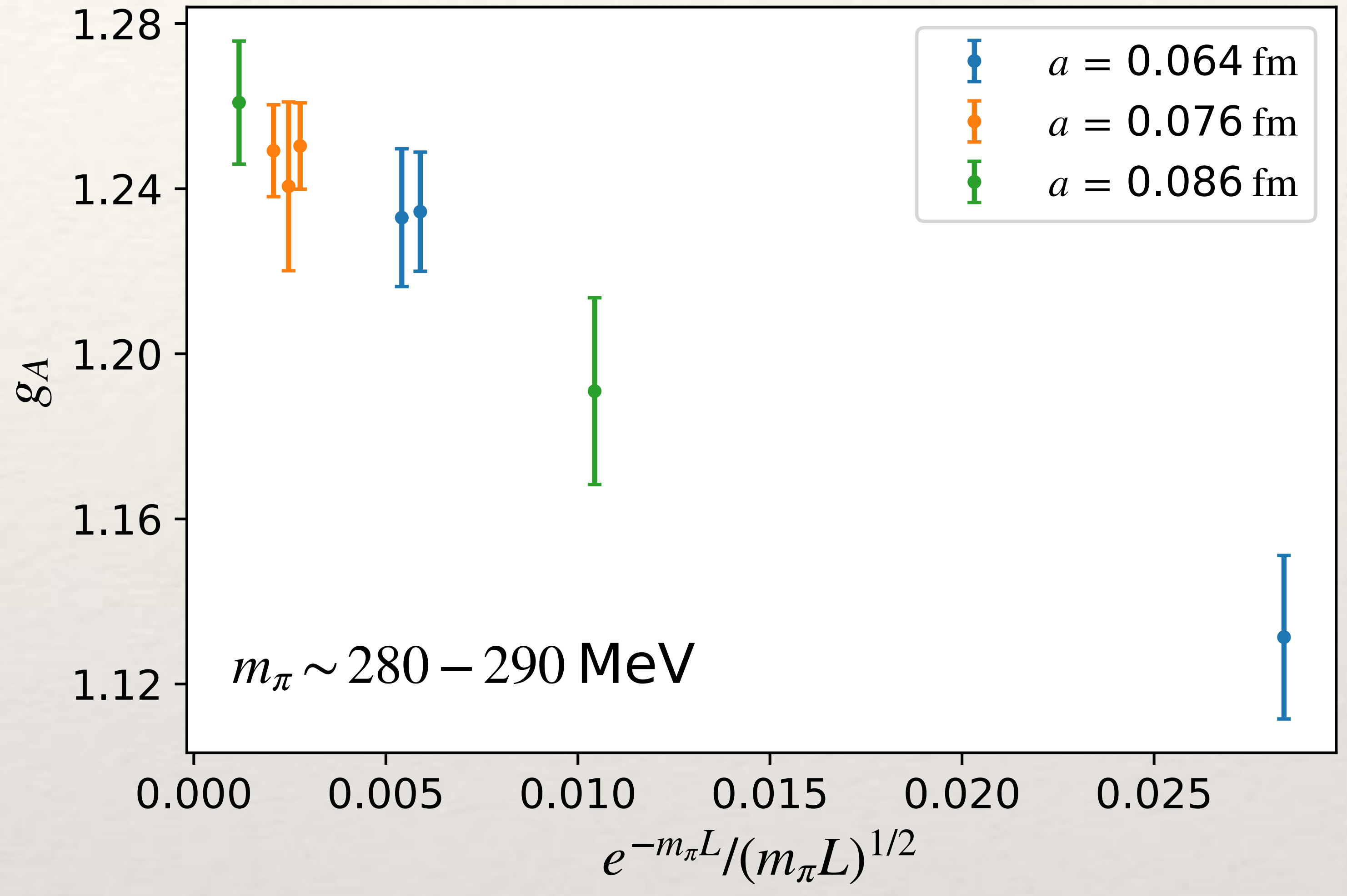
- Numerical Evidence:



CalLat [1805.12130]



CalLat - unpublished



RQCD - 2305.04717

- At $m_\pi \approx 220$ MeV, results are consistent with leading prediction from χPT (and also consistent with no correction or opposite sign)
- At $m_\pi \approx 300$ MeV, results constrain the sign of the volume correction opposite of χPT prediction

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- Expectations from χPT
  - The chiral expansion for nucleons is a series in $\epsilon_\pi = \dfrac{m_\pi}{4\pi F_\pi}$, while for pions, it is in ϵ_π^2
    - therefore, higher order corrections are relatively more important
  - The nucleon has a much richer spectrum of virtual excited states $(N\pi, \Delta\pi, \ldots)$
  - In the large N_c limit, there is an exact cancellation of most NLO corrections to g_A
    - The finite volume corrections also respect this cancellation and lead to a sign change at fixed m_π vs $m_\pi L$
  - SU(2) HBχPT($\not\Delta$) at NNLO also predicts change in sign of FV corrections

Non-monotonic FV corrections to g_A

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  - SU(2) HBχPT($\not\Delta$) at NNLO also predicts change in sign of FV corrections

$$g_A = g_0 + \Delta^{(2)} + \delta^{(2)}_{\rm FV} + \Delta^{(3)} + \delta^{(3)}_{\rm FV}$$

$$\Delta^{(2)} = \epsilon_\pi^2 \left[-g_0(1+2g_0^2)\ln\epsilon_\pi^2 + 4\tilde{d}_{16}^r - g_0^3 \right]$$

$$\Delta^{(3)} = \epsilon_\pi^3 g_0 \frac{2\pi}{3} \left[3(1+g_0^2)\frac{4\pi F}{M_0} + 4(2\tilde{c}_4 - \tilde{c}_3) \right]$$

$$\delta^{(2)}_{\rm FV} = \frac{8}{3}\epsilon_\pi^2 \left[g_0^3 F_1^{(2)}(m_\pi L) + g_0 F_3^{(2)}(m_\pi L) \right]$$

$$F_1^{(2)}(x) = \sum_{\vec{n}\neq 0} \left[K_0(x|\vec{n}|) - \frac{K_1(x|\vec{n}|)}{x|\vec{n}|} \right]$$

$$F_3^{(2)}(x) = -\frac{3}{2} \sum_{\vec{n}\neq 0} \frac{K_1(x|\vec{n}|)}{x|\vec{n}|} \,,$$

$$\delta^{(3)}_{\rm FV} = \epsilon_\pi^3 g_0 \frac{2\pi}{3} \left\{ g_0^2 \frac{4\pi F}{M_0} F_1^{(3)}(m_\pi L) - \left[\frac{4\pi F}{M_0}(3+2g_0^2) + 4(2\tilde{c}_4 - \tilde{c}_3) \right] F_3^{(3)}(m_\pi L) \right\}$$

$$F_1^{(3)}(x) = \sum_{\vec{n}\neq\vec{0}} \frac{K_{\frac{1}{2}}(x|\vec{n}|)}{\sqrt{\frac{\pi}{2}x|\vec{n}|}} x|\vec{n}| \qquad = \sum_{\vec{n}\neq\vec{0}} e^{-x|\vec{n}|}$$

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- Expectations from χPT
  - SU(2) HBχPT($\not\Delta$) at NNLO also predicts change in sign of FV corrections

$$g_A = g_0 + \Delta^{(2)} + \delta^{(2)}_{\rm FV} + \Delta^{(3)} + \delta^{(3)}_{\rm FV}$$

$$\delta^{(2)}_{\rm FV} = \frac{8}{3}\epsilon_\pi^2\left[g_0^3 F_1^{(2)}(m_\pi L) + g_0 F_3^{(2)}(m_\pi L)\right]$$

$$\Delta^{(2)} = \epsilon_\pi^2\left[-g_0(1+2g_0^2)\ln\epsilon_\pi^2 + 4\tilde{d}^r_{16} - g_0^3\right]$$

$$\Delta^{(3)} = \epsilon_\pi^3 g_0 \frac{2\pi}{3}\left[3(1+g_0^2)\frac{4\pi F}{M_0} + 4(2\tilde{c}_4 - \tilde{c}_3)\right]$$

NOTE: the leading FV correction is a prediction
g_0 is determined in the chiral extrapolation

$$F_1^{(2)}(x) = \sum_{\vec{n}\neq 0}\left[K_0(x|\vec{n}|) - \frac{K_1(x|\vec{n}|)}{x|\vec{n}|}\right]$$

$$F_3^{(2)}(x) = -\frac{3}{2}\sum_{\vec{n}\neq 0}\frac{K_1(x|\vec{n}|)}{x|\vec{n}|},$$

for $g_0 \sim 1.2$, $\delta^{(2)}_{\rm FV} > 0$

Non-monotonic FV corrections to g_A

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- Expectations from χPT
  - SU(2) HBχPT($\not\Delta$) at NNLO also predicts change in sign of FV corrections

$$g_A = g_0 + \Delta^{(2)} + \delta^{(2)}_{\rm FV} + \Delta^{(3)} + \delta^{(3)}_{\rm FV}$$

$$\delta^{(3)}_{\rm FV} = \epsilon_\pi^3 g_0 \frac{2\pi}{3}\left\{ g_0^2 \frac{4\pi F}{M_0} F_1^{(3)}(m_\pi L) - \left[\frac{4\pi F}{M_0}(3+2g_0^2) + 4(2\tilde{c}_4 - \tilde{c}_3)\right] F_3^{(3)}(m_\pi L)\right\}$$

$$\tilde{c}_i = (4\pi F)\, c_i$$

$$\Delta^{(2)} = \epsilon_\pi^2\left[-g_0(1+2g_0^2)\ln\epsilon_\pi^2 + 4\tilde{d}^r_{16} - g_0^3\right]$$

$$\Delta^{(3)} = \epsilon_\pi^3 g_0 \frac{2\pi}{3}\left[3(1+g_0^2)\frac{4\pi F}{M_0} + 4(2\tilde{c}_4 - \tilde{c}_3)\right]$$

$$F_1^{(3)}(x) = \sum_{\vec{n}\neq\vec{0}} \frac{K_{\frac{1}{2}}(x|\vec{n}|)}{\sqrt{\frac{\pi}{2}x|\vec{n}|}}\, x|\vec{n}| = \sum_{\vec{n}\neq\vec{0}} e^{-x|\vec{n}|}$$

$$F_3^{(3)}(x) = \sum_{\vec{n}\neq\vec{0}} \frac{K_{\frac{1}{2}}(x|\vec{n}|)}{\sqrt{\frac{\pi}{2}x|\vec{n}|}} = \sum_{\vec{n}\neq\vec{0}} \frac{e^{-x|\vec{n}|}}{x|\vec{n}|}$$

in SU(2) HBχPT($\not\Delta$), with N^{3}LO $N\pi$ phase shift analysis
Siemens et al, 1610.08978
$c_3 = -5.60(6)$ GeV^{-1}
$c_4 = \quad 4.26(4)$ GeV^{-1}

This leads to LARGE, negative FV correction
Fitting $2c_4 - c_3$ to our LQCD results yields a value $\sim 10\times$ smaller — leads to change in sign of $\delta_{\rm FV}$ as function of m_π

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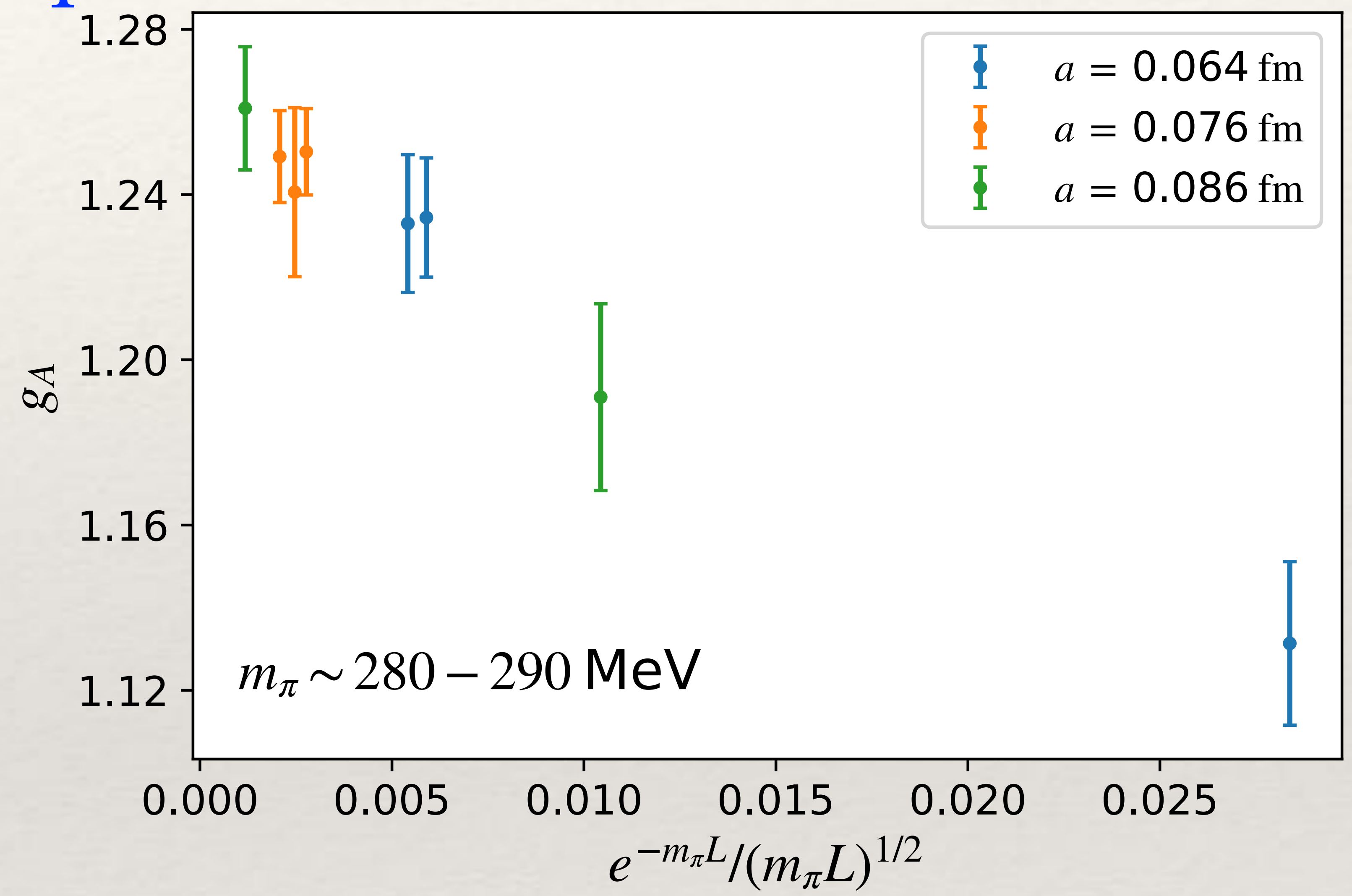
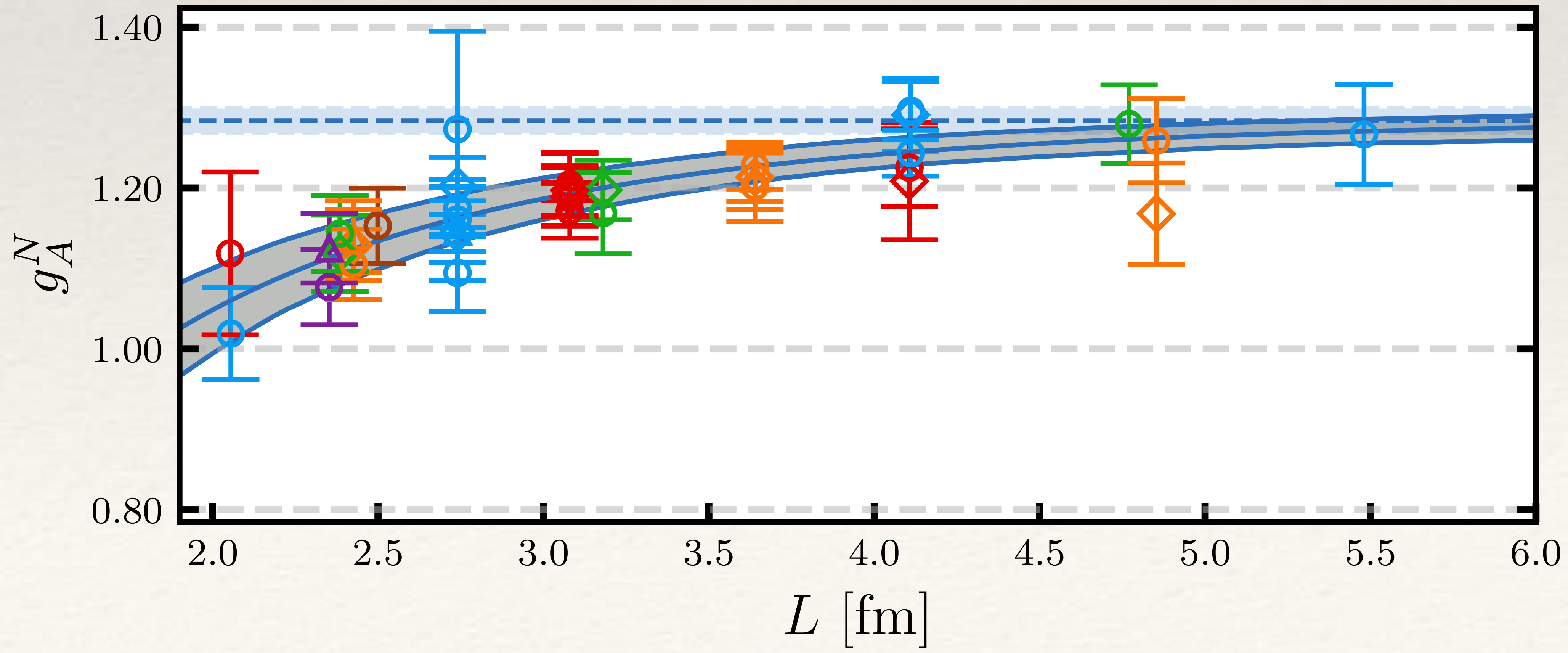
- Current strategy (of most groups)
  - take asymptotic form of Bessel functions and leading "wrap around the world" mode and only leading volume correction

$$g_A(L) = g_A + c_2 \frac{m_\pi^2}{(4\pi F_\pi)^2} \frac{e^{-m_\pi L}}{\sqrt{m_\pi L}}$$



  - Fit c_2 essentially to heavy m_π results
  - Use this m_π-independent value of c_2 to extrapolate to infinite volume at all m_π
  - If the volume corrections do change sign (to agree with χPT prediction close to m_π^{phys}) the current strategy will lead to an error
  - At what precision will this occur?



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- What should we do?
  - One needs to perform a volume study at multiple pion masses with sufficient precision to constrain the sign of the volume correction as a function of m_π

$$\boxed{g_A(L)} = g_A + c_2 \frac{m_\pi^2}{(4\pi F_\pi)^2} \frac{e^{-m_\pi L}}{\sqrt{m_\pi L}} + c_3 \frac{m_\pi^3}{(4\pi F_\pi)^3} \frac{e^{-m_\pi L}}{m_\pi L} + \cdots$$

  - Or - we need to rely only upon $m_\pi \approx m_\pi^{\rm phys}$ with sufficient precision to control the final uncertainty of g_A as well as the volume correction
  - Or - determine quantitatively that some variant of HBχPT provides an accurate description of both the m_π dependence as well as $m_\pi L$ dependence

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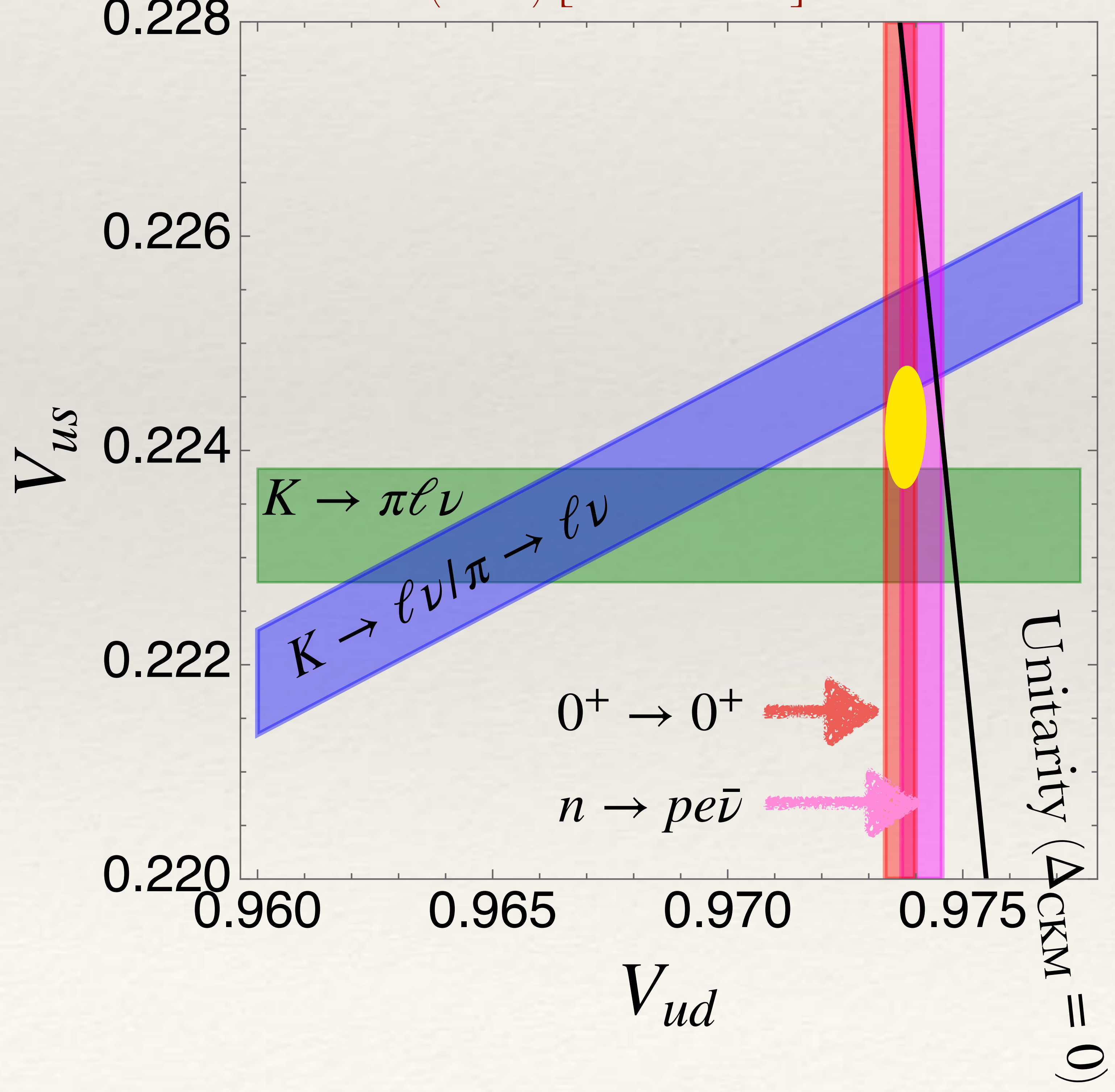
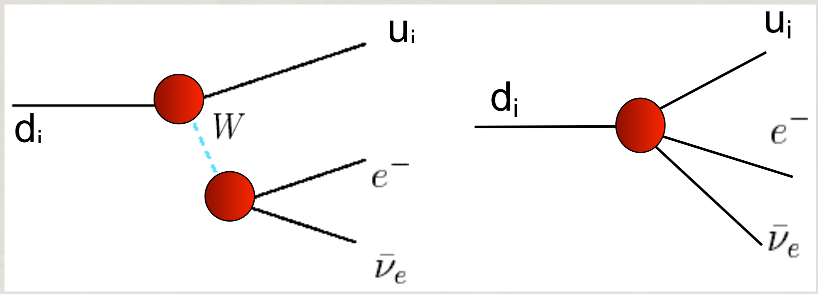
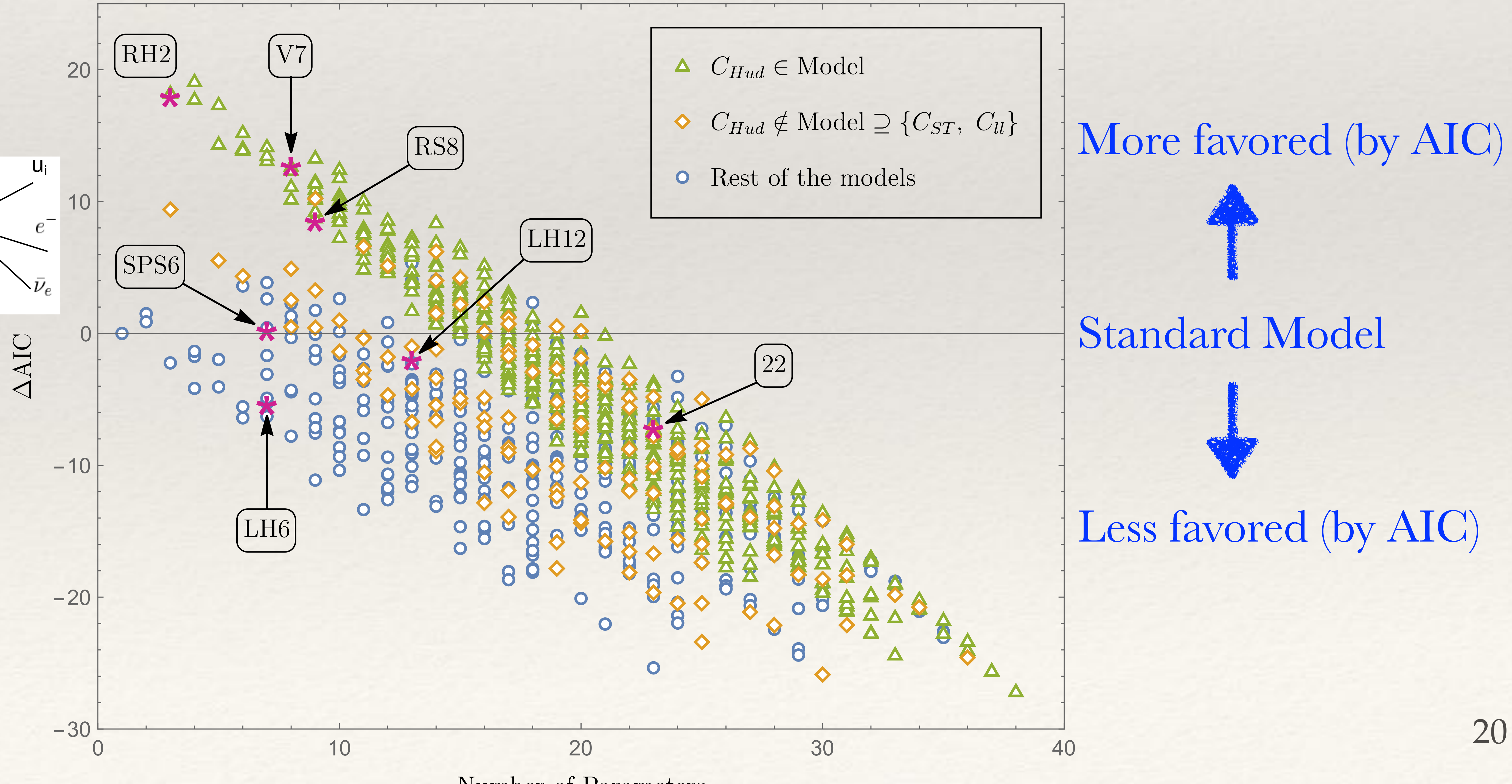
- "But you just told me there is an unknown O(2%) QED correction to g_A, so why should I care?"
  - Presumably, we will figure out how to determine this QED correction, which will allow us to utilize our high-precision iso-symmetric LQCD determination of g_A by applying the QED correction in a correlated way

Cirigliano, Crivellin, Hoferichter, Moulson
PLB 838 (2023) [2208.11707]





- Global analysis of first-row CKM constraints, including collider constraints, favors BSM Right-handed currents
  Cirigliano, Dekens, de Vries, Mereghetti, Tong, JHEP 03 (2024) [2311.00021]



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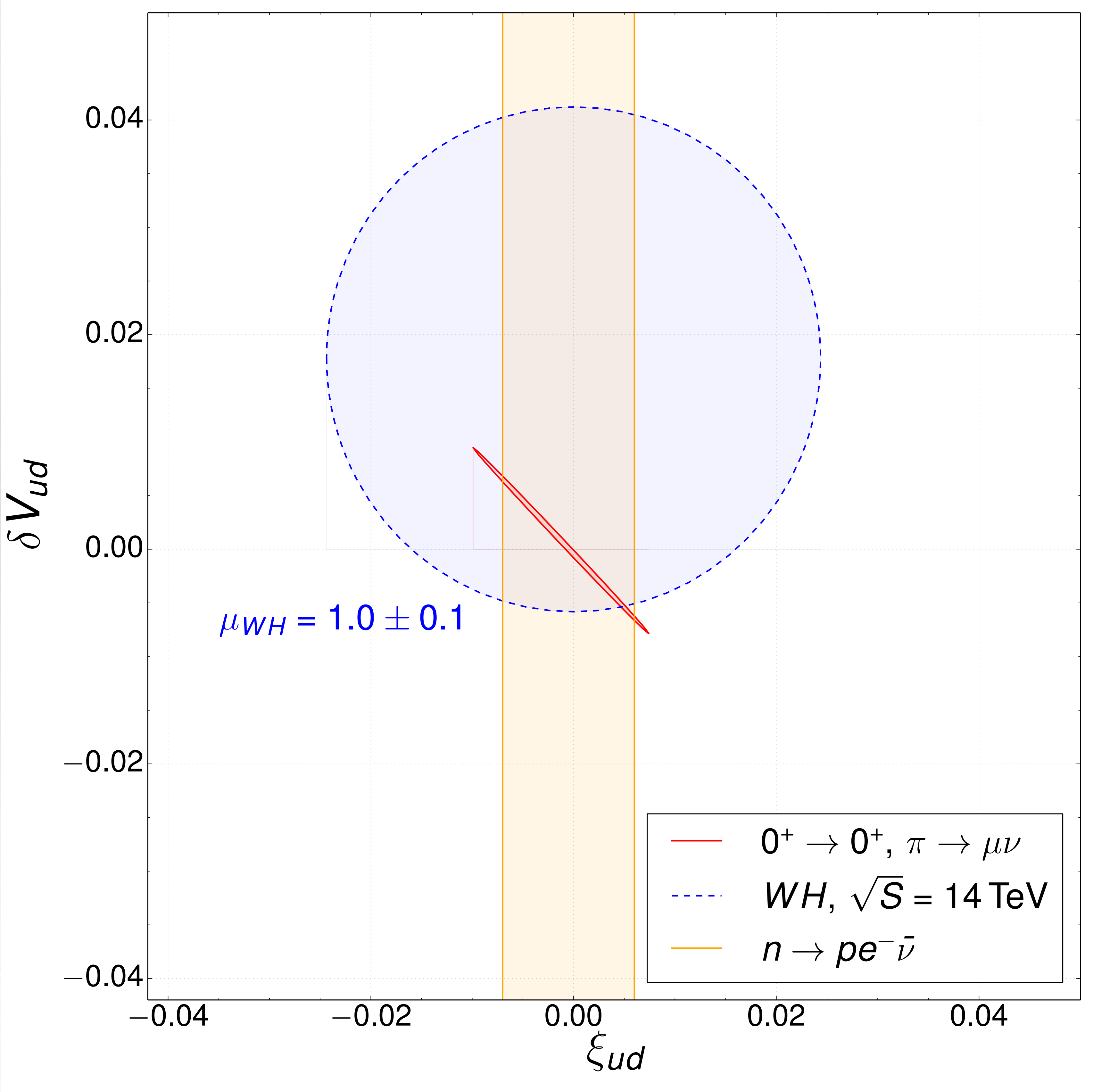
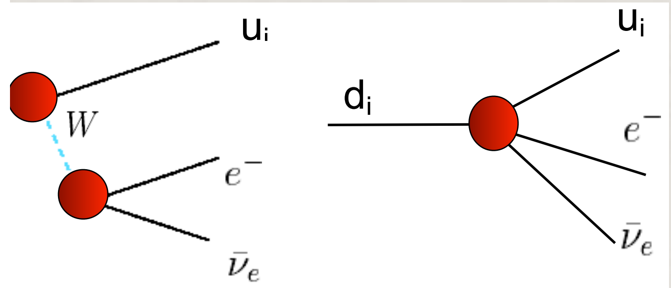
  - Presumably, we will figure out how to determine this QED correction, which will allow us to utilize our high-precision iso-symmetric LQCD determination of g_A by applying the QED correction in a correlated way
  - Comparing g_A^{QCD} to g_A^{PDG} including control of $\Delta_A^{R,other}$, allows us to constrain BSM right-handed currents





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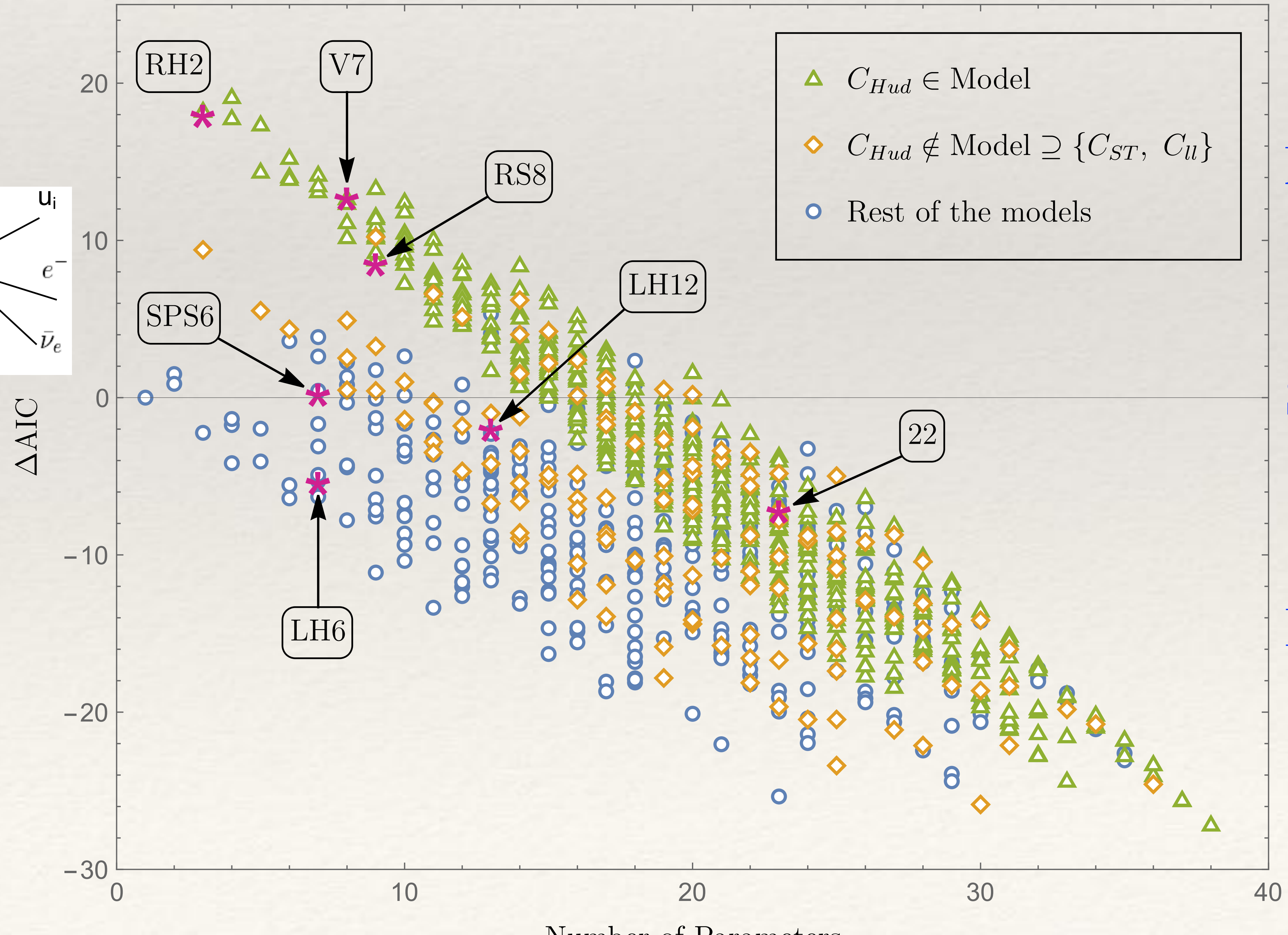

More favored (by AIC)

Standard Model

Less favored (by AIC)

Subtleties and Systematics in achieving sub-percent uncertainty for g_A

- There is tension in the first-row CKM unitarity,
  - BSM right-handed currents offer a favored solution to the tension
  - LQCD calculation of g_A, plus radiative QED corrections, provides such a constraint

- estimates from χPT suggests $\Delta_A^{R,other} = \mathrm{O}(2\%)$, $g_A^{\mathrm{PDG}} = g_A^{\mathrm{QCD-iso}} + \Delta_A^{R,other}$

- g_A seems to exhibit non-monotonic FV corrections
  - As the precision of results improves, the current strategy of most groups

$$g_A(L) = g_A + c_2 \frac{m_\pi^2}{(4\pi F_\pi)^2} \frac{e^{-m_\pi L}}{\sqrt{m_\pi L}}$$

    will lead to an error
  - At what precision of results will this become important?

Thank You