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Gradient Flow of the Weinberg Operator

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Abstract

We will present latest results on the gradient flow of our Weinberg three-gluon operator and its mixing with the topological term. This will allow us to combine our calculation of neutron electric dipole moments (nEDM) due to the lattice Weinberg operator with our previous results on the nEDM due to the topological term to constrain beyond-the-standard model contributions to the Weinberg operator.

Outline

Motivation

Power divergence

Gluonic CP Violation

Lattice Data and Fits

Future

Motivation
BSM physics exists: we need to find it

Two incompatible 'standard models':

Standard Model of Particle Physics X Standard Model of Cosmology

- Universe has gravity, is too big and too old.
- Universe too isotropic, homogeneous: Needs high-pressure 'dark energy'
- Universe too clumpy: Needs particulate 'dark matter'
- Has too much matter: Needs baryogenesis that needs 'CP violation'

Of these, CP-violation touches most particle physics.

Motivation
EDMs clean BSM signals

Electric Dipole Moments of non-degenerate systems prohibited by CP symmetry.
CP-violation effecting baryogenesis now within experimental reach:

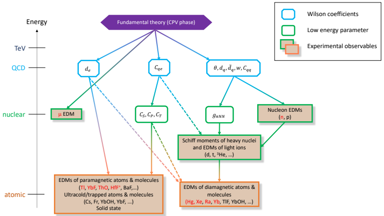
electron	$4.1 \times 10^{-30} e \cdot \mathrm{cm}$
neutron	$1.8 \times 10^{-26} e \cdot \mathrm{cm}$
proton	$2.1 \times 10^{-25} e \cdot \mathrm{cm}$

Improvements by a factor of 2–3 orders of magnitude may be within reach.
Standard model contribution about a million times smaller!

Motivation

Low-dimension CP violating operators

- Dimension-3 or 4
  - CP-violating lepton mass
  - CP-violating quark mass = Theta-term
- Dimension-5
  - Lepton electric dipole moment
  - Quark electric dipole moment
  - Quark chromo-electric dipole moment
- Dimension-6
  - Four-lepton operators
  - Lepton-quark operators
  - Four-quark operators
  - CP-violating three-gluon Weinberg operator



- A single detectable EDM would prove BSM.
- Need multiple EDM measurements to probe BSM.

Power divergence
Effective Field Theory: Separation of scales

Higher-dimensional operators strongly scheme dependent.
In EFT approach, we integrate out degrees heavier than the BSM scale.

$$\begin{aligned}\text{BSM MatrixElement} = \text{WilsonCoeff}_1 &\times \text{MatrixElement}(\text{O}_1) + \\ \frac{\text{WilsonCoeff}_2}{\text{BSM scale}^n} &\times \text{MatrixElement}(\text{O}_2) + \ldots\end{aligned}$$

Wilson coefficients are a function of $\alpha_s(\text{BSM scale})$. Usually calculated perturbatively.

Ambiguous up to non-perturbative effects like

$$\exp(-n/2\beta_0\alpha_s^2(\text{BSM scale})) \sim (\Lambda_{\text{QCD}}/\text{BSM scale})^n$$

Ambiguity absorbed in redefining $\text{O}_2 - \text{MixingCoeff}_{21}\Lambda^n_{\text{QCD}}\ O_1$, etc.

Power divergence
Nonperturbative definition: renormalization scheme

Usual tradeoff between Wilson Coefficients and Operator Renormalization.

$\mathrm{MatrixElement}(\mathrm{O}_2)$ is power-divergent.
Divergence proportional to $\mathrm{MatrixElement}(\mathrm{Cutoff}^{\mathrm{n}}\mathrm{O}_1)$.
Put a condition on $\mathrm{MatrixElement}(\mathrm{O}_2)$; this fixes coefficient of $\mathrm{Cutoff}^{\mathrm{n}}\mathrm{O}_1$.
But: want $\mathrm{MatrixElement}(\mathrm{O}_2)/\mathrm{MatrixElement}(\mathrm{O}_1) \sim \Lambda^{\mathrm{n}}_{\mathrm{QCD}}$, but $(\Lambda_{\mathrm{QCD}}/\mathrm{Cutoff})^n$ is a nonperturbatively small change.
For example, for $\mathrm{Cutoff} \sim 3\mathrm{GeV}$, $\alpha_s \approx 0.5$, $(\Lambda_{\mathrm{QCD}}/\mathrm{Cutoff})^2 \approx 0.01$.

So, calculating O_2 is usually not useful, unless either

- O_1 matrix element and Wilson coefficients known very accurately, or
- Lower-dimension operator has a suppressed matrix-element.

Power divergence
Gradient Flow scheme

In Gradient Flow Renormalization Scheme, there is a 'hard' cutoff $\tau_{\mathrm{gf}} = \sqrt{8t_{gf}}a$.

$$\frac{\partial U(t_{\mathrm{gf}})}{\partial t_{\mathrm{gf}}} = \nabla S[U(t_{\mathrm{gf}})] \cdot U(t_{\mathrm{gf}})$$

with $a \to 0$ holding $\tau_{\mathrm{gf}} \neq 0$ fixed.

The scheme has

- Good chiral symmetry when $\tau_{\mathrm{gf}} \neq 0$.
- No renormalization for composite operators.
- Only one multiplicative 'wavefunction renormalization' for fermions

Gluonic CP Violation
Weinberg and topological charge

Two gluonic CP-violating operators up to dimension 6: topological charge, $G \cdot G$, and Weinberg, $G \cdot \tilde{G} \cdot G$, operators.

With good chiral symmetry and massless quarks, gradient flow and $\overline{\mathrm{MS}}$ schemes related as:

$$\begin{pmatrix} G \cdot \tilde{G} \\ G \cdot \tilde{G} \cdot G \\ \cdots \end{pmatrix}_{\overline{\mathrm{MS}}} = \begin{pmatrix} Z_{\mathrm{Top}} & 0 & 0 \\ Z_{\mathrm{Mixed}} \frac{1}{(\tau_{\mathrm{gf}} a)^2} & Z_{\mathrm{Wein}} & O((\tau_{\mathrm{gf}} a)^2) \\ O(\frac{1}{(\tau_{\mathrm{gf}} a)^4}) & O(\frac{1}{(\tau_{\mathrm{gf}} a)^2}) & \cdots \end{pmatrix} \begin{pmatrix} G \cdot \tilde{G} \\ G \cdot \tilde{G} \cdot G \\ \cdots \end{pmatrix}_{\mathrm{gf}}$$

Gluonic CP Violation
Susceptibilities: Weinberg, Topological and Mixed

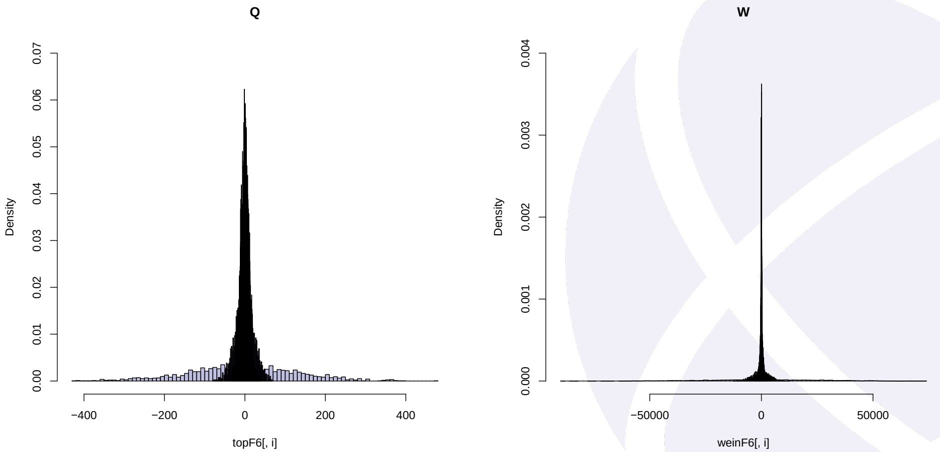
Susceptibility is the quadratic coefficient in the effective action:

$$\chi_{\text{Top}} = \partial^2 \ln Z / \partial\Theta^2$$

Variance of the topological charge: $\chi_{\text{Top}} = \langle Q^2 \rangle - \langle Q \rangle^2$, where $Q = \int d^4x G \cdot \tilde{G}$.
Important quantities in theories with Peccei-Quinn symmetry:
$\Theta_{\text{induced}} = -w(\chi_{\text{Mixed}}/\chi_{\text{Top}})$
Under gradient flow,

- Topological charge becomes an integer.
- Topological charge distribution almost stabilizes.
- Finite volume effects makes the topological charge distribution ‘flow’.

Gluonic CP Violation
Charge Distribution Under Gradient Flow



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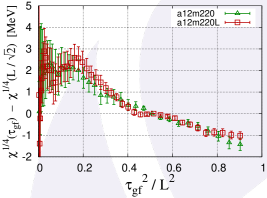
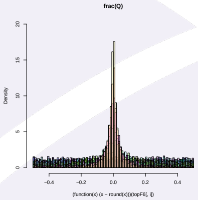
Gluonic CP Violation
Topological susceptibility and quark mass

Topological charge is topological (though takes long flow!)
Does not mix with Weinberg.
Fermion determinant suppresses topology.

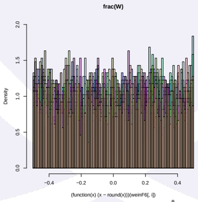
$$\frac{1}{\chi} \approx \frac{1}{\chi^q} + \frac{4}{M_\pi^2 F_\pi^2 \left(1 - \frac{M_\pi^2}{3M_\eta^2}\right)}$$

So, Weinberg has no power-divergence (as small flow-time) in the chiral limit. No reason to suspect large chiral corrections.





Basis of the preliminary nEDM data presented in LATTICE2021 (arXiv:2203.03746 [hep-lat])



Lattice Data and Fits

Lattice Setup and Parameters

Tadpole-improved clover fermions; errors $O(\alpha_s a)$ and $O(a^2)$.

Name	a (fm)	M_π (MeV)	M_K (MeV)	$L^3 \times T$
C13	0.127(2)	285(5)	476(5)	$32^3 \times 96$
D220	0.094(1)	214(3)	543(6)	$48^3 \times 128$
D5L	0.094(1)	268(3)	512(5)	$48^3 \times 96$
D6	0.0914(9)	175(2)		$64^3 \times 128$
D7	0.091(1)	170(2)	491(5)	$64^3 \times 128$
E5	0.0728(8)	272(3)	575(6)	$48^3 \times 128$
E6	0.0707(8)	223(3)	539(6)	$64^3 \times 192$
E7	0.0706(7)3	167(2)	538(6)	$72^3 \times 192$
E9	0.0700(7)	128(2)	521(5)	$96^3 \times 192$
F5	0.056(1)	280(5)	526(6)	$64^3 \times 192$
F5	0.0555(6)	216(2)	527(5)	$72^3 \times 192$

Lattice Data and Fits
Chiral-Continuum-Finite-Volume extrapolation: strategy

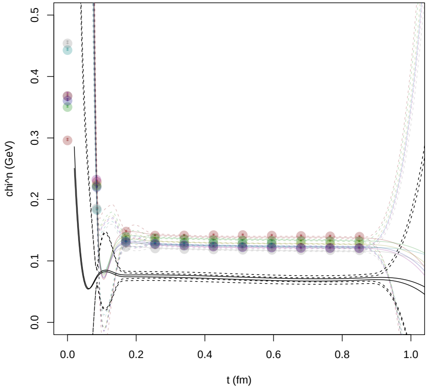
Fit form:

$$\chi^{1/n} = s_0(t_{\mathrm{gf}}) + s_1(t_{\mathrm{gf}})a^p + s_2(t_{\mathrm{gf}})M_\pi^2$$

where

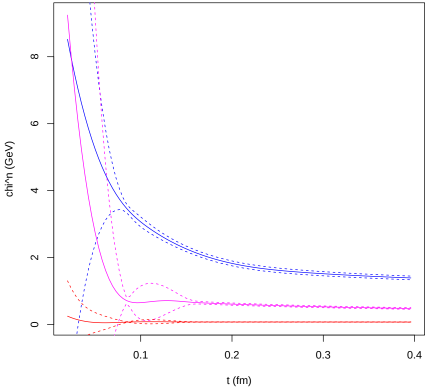
- n chosen to get dimensions of MeV.
- Gradient-flow-time dependence modeled as a spline.
- Linear extrapolation in a (or a^2) and M_π^2
- Neglect dependence on M_K^2 and finite volume.
- Neglect cross-term between a and quark-mass.
- Number of knots and knot positions chosen (globally) using AIC.

Lattice Data and Fits
Fit results



- Uncorrelated fits.
- Omit very small τ_{gf}.
- All the fits have reasonable quality by eye.
- Fit doesn't constrain $\chi = 0$ at chiral limit for Q.
- Fit doesn't go through $\chi = 0$ at chiral limit for Q.

Lattice Data and Fits
Study of mixing



At $M_\pi = 140$ MeV, $a = 0$:

- Topological susceptibility $(71\mathrm{MeV})^4$ close to previous determination.
- Weinberg shows a large t dependence, Mixed does not.
- Related to nonzero Q at chiral limit?

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Future
Ongoing Calculation: extraction of nEDM

- Topological charge behaves nicely:
  - Is an integer.
  - Is zero in the chiral limit.
- Since gradient-flow is chirally improved, in the chiral limit:
  - Weinberg can only have ‘divergent’ mixing with topological charge
  - Topological charge can be rotated away in the chiral limit.
  - Weinberg operator ‘safe’ in the chiral theory.
- Numerical extrapolations see residual effects.
  - Problems with lowest order fits?
- Will un-flowed vector current spoil the picture?