Numerical evidence for a CP broken deconfined phase at $\theta = \pi$ in 4D SU(2) Yang-Mills theory through simulations at imaginary θ

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θ term in SU(N) Yang-Mills theories

partition function $$Z = \int \mathcal{D}A_\mu e^{-S_g + i\theta Q}$$

topological charge $$Q = \frac{1}{32\pi^2}\int d^4x \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left[F_{\mu\nu}F_{\rho\sigma}\right]$$

($Q \in \mathbb{Z}$ on T^4)

- periodicity : $\theta \to \theta + 2\pi n \ \ (n \in \mathbb{Z})$
- CP symmetric at $\theta = 0 \text{ and } \pi$
- phase structure at $\theta = \pi$ predicted by 't Hooft anomaly matching condition

[D. Gaiotto, A.Kapustin, Z. Komargodski, N.Seiberg (2017)]

Phase structure at $\theta = \pi$ in SU(2) Yang-Mills theory

- previous studies
  - SSB of CP at zero temperature（first principle calculations on lattice）
    [R. Kitano, R. Matsudo, N. Yamada, M. Yamazaki (2021)]
  - CP symmetric deconfined phase at high temperature（1 loop analysis）
    [D.J. Gross, R.D. Pisarski, L.G. Yaffe (1981)], [N. Weiss (1981)]

- anomaly matching condition [D. Gaiotto, A.Kapustin, Z. Komargodski, N.Seiberg (2017)]
  - mixed 't Hooft anomaly between CP and $Z_N^{(1)}$ symmetries
    - At least, one of them should be broken.

↓

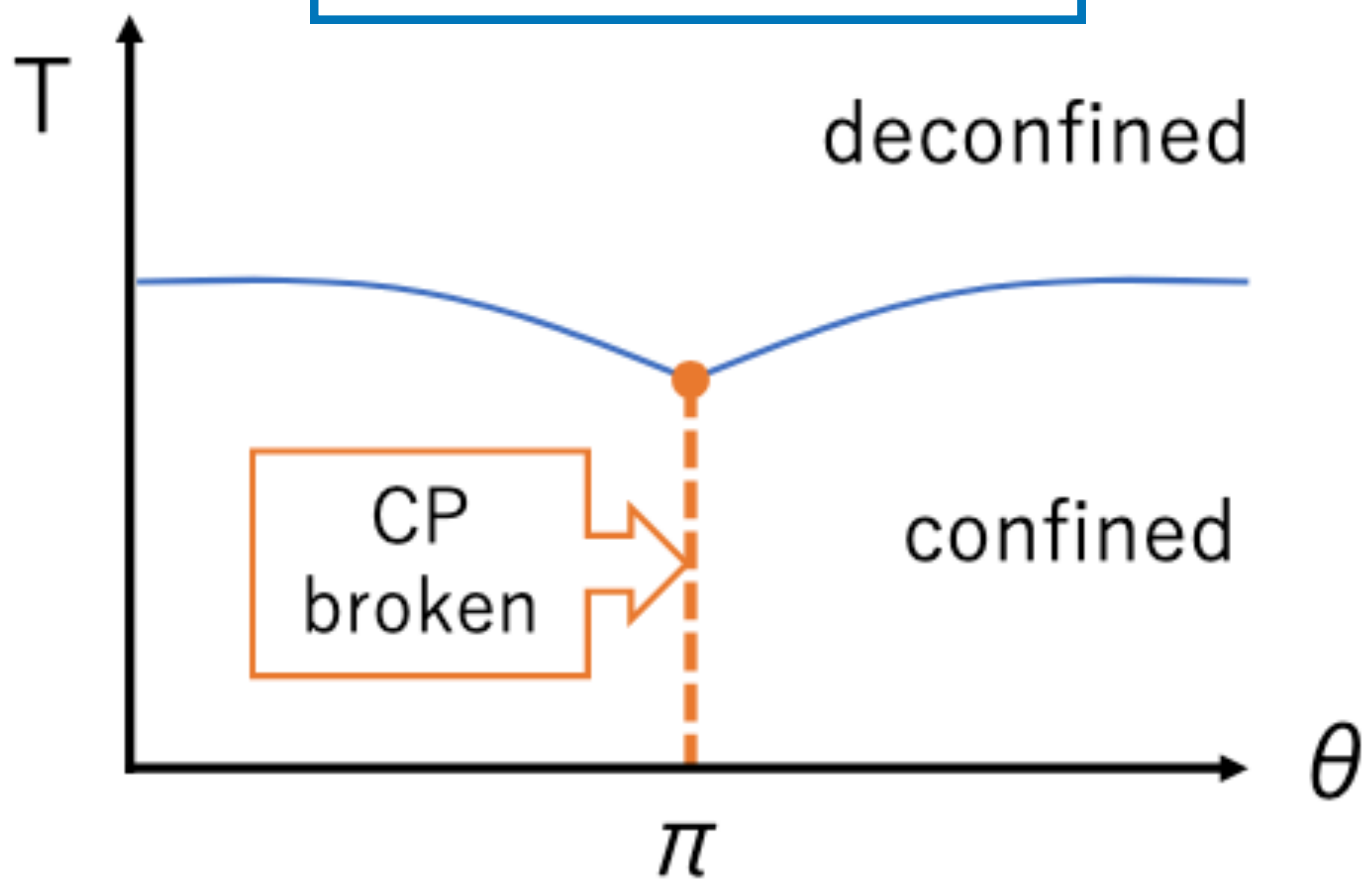
relation between two phase transition : $T_{\rm CP} \geq T_{\rm dec}$

- large N : $T_{\rm CP} = T_{\rm dec}$ [F. Bigazzi, A. L. Cotrone, R. Sisca (2015)]
- N = 2 : ?

Predicted phase diagrams in SU(2) Yang-Mills theory

anomaly matching condition : $T_{\rm CP} \geq T_{\rm dec}$ at $\theta = \pi$

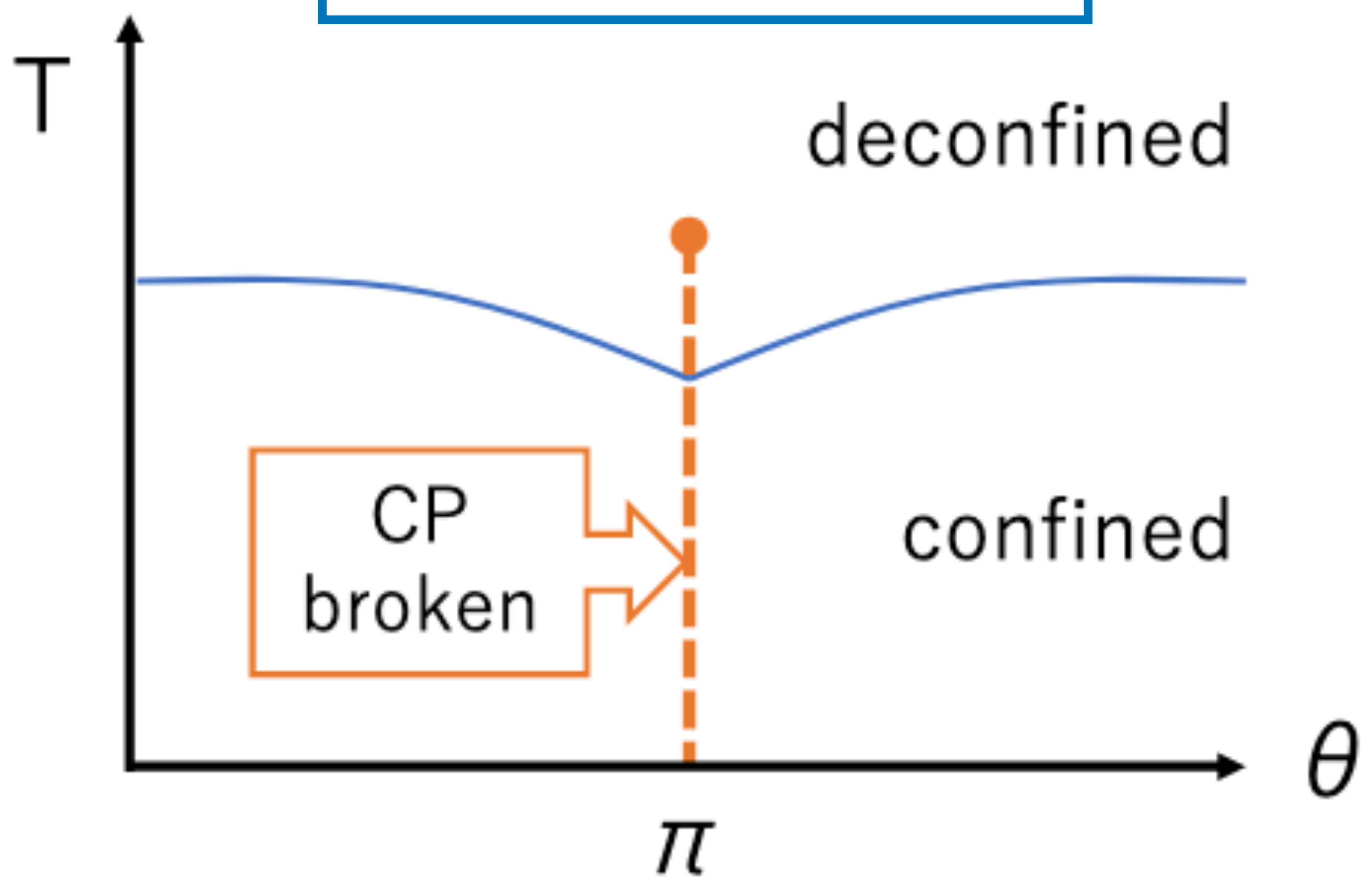
$T_{\rm CP} = T_{\rm dec}(\theta = \pi)$



Holographic analysis at large N

[F. Bigazzi, A. L. Cotrone, R. Sisca (2015)]

$T_{\rm CP} > T_{\rm dec}(\theta = \pi)$



Softly broken SUSY SU(2) case

[S. Chen, K. Fukushima, H. Nishimura, Y. Tanizaki (2020)]

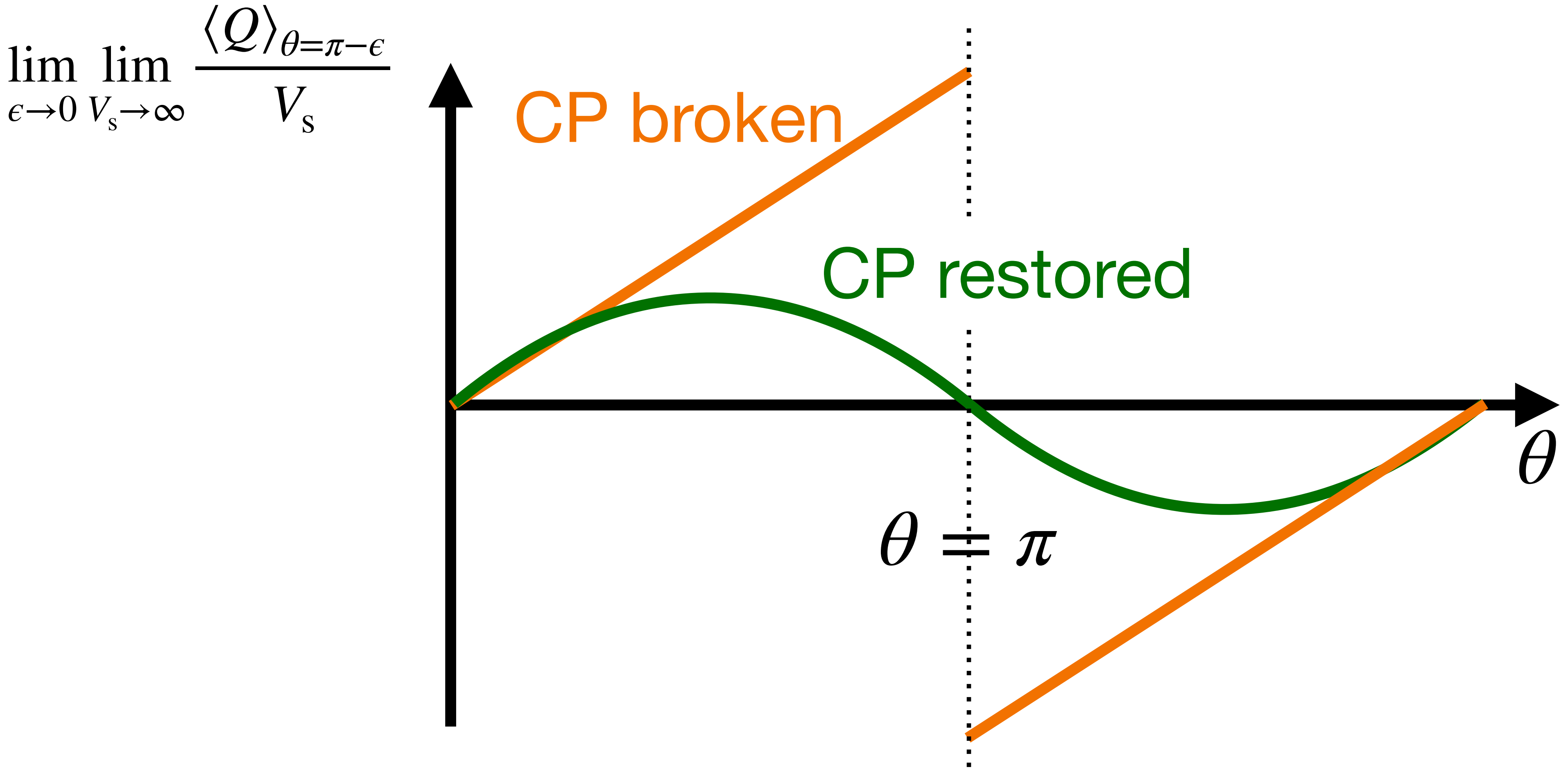
Which phase structure appears in pure SU(2) Yang-Mills theory?

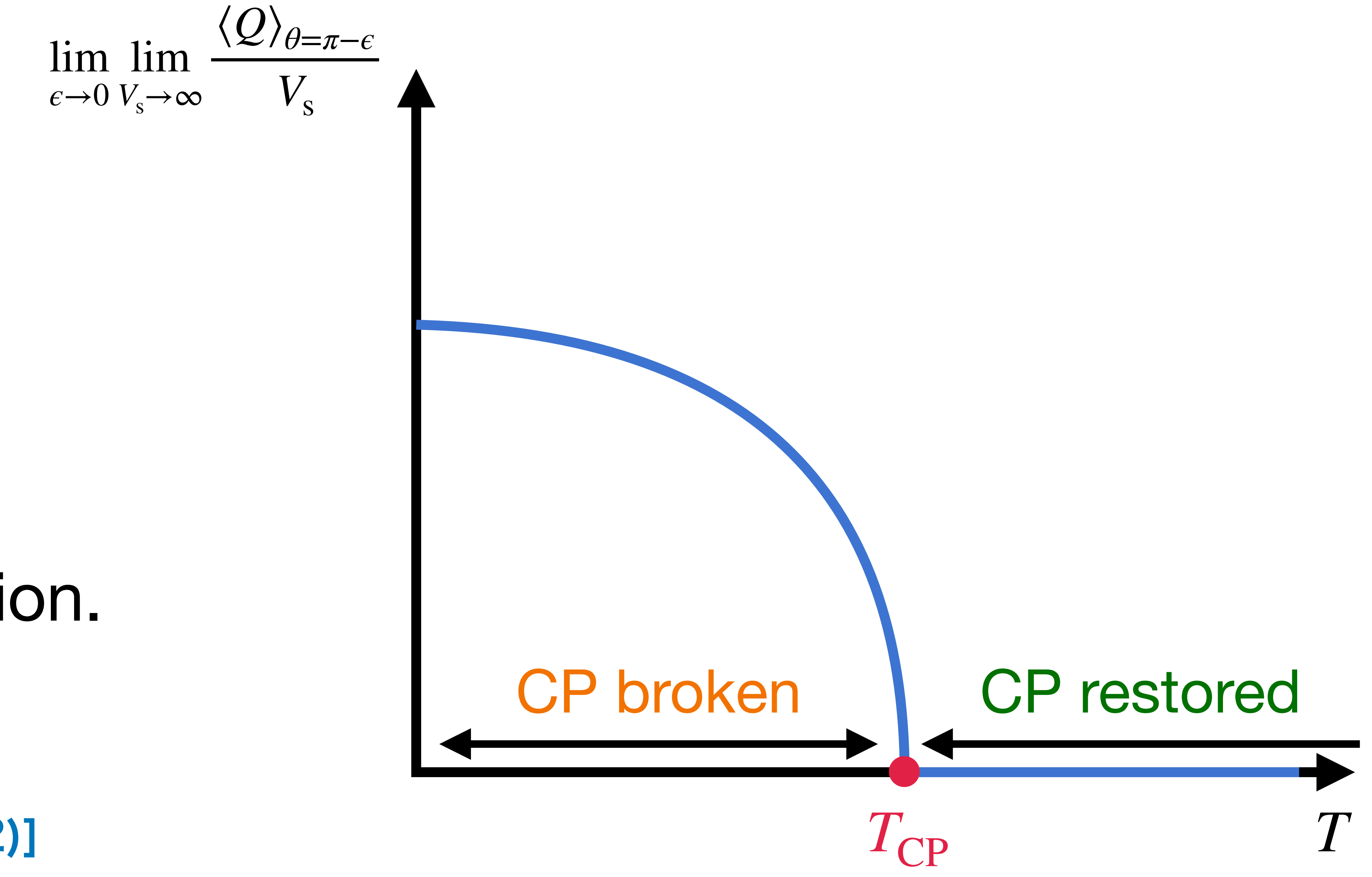
Spontaneous CP breaking at θ=π

- order parameter : topological charge density

$$\lim_{\epsilon\to 0}\lim_{V_s\to\infty}\frac{\langle Q\rangle_{\theta=\pi-\epsilon}}{V_s}=\begin{cases}0 & \text{CP restored}\\ \not 0 & \text{CP broken}\end{cases}$$

V_s : spatial volume





It is difficult to measure it directly due to the sign problem.

We study the behaviour of $\langle Q\rangle$ at $\theta=\pi$ using analytic continuation.

c.f. *θ* dependencce of $\langle Q\rangle_\theta$ at $T=0$ in CP^3 model

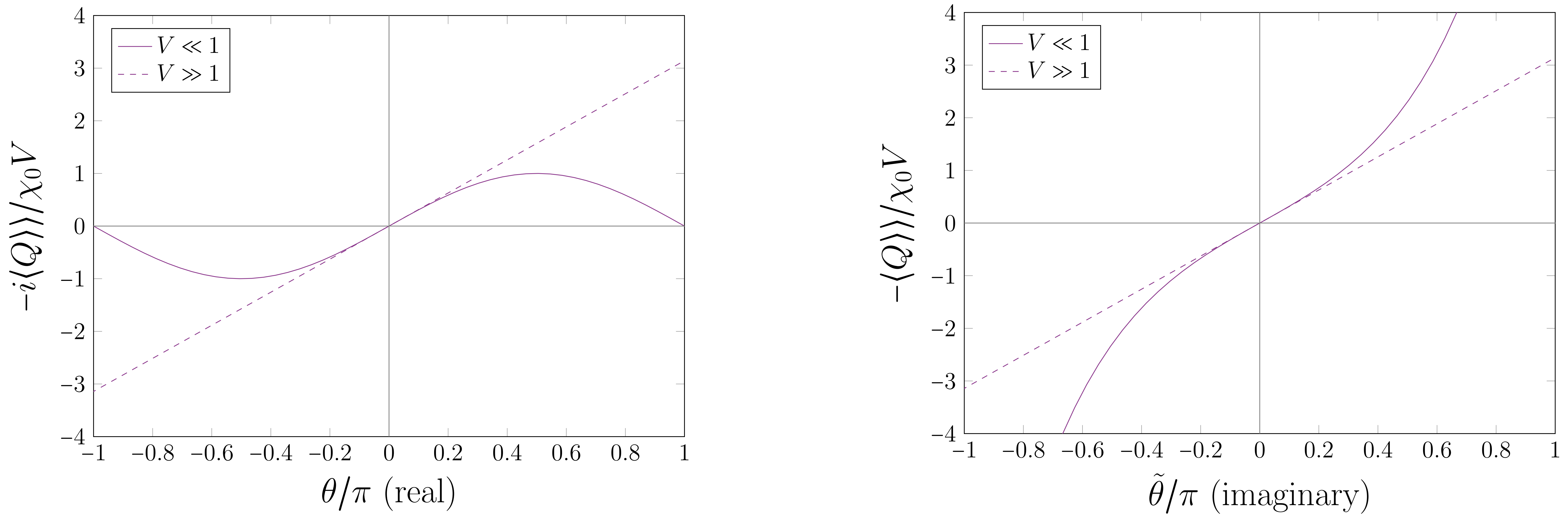
[V. Azcoiti, G. D. Carlo, A. Galante, V. Laliena (2002)]

θ dependence of $\langle Q \rangle_\theta$ — analytic results for 2d U(1) case —

limits	$\theta \in \mathbb{R}$	$\theta = i\tilde{\theta} \quad (\tilde{\theta} \in \mathbb{R})$	CP at $\theta = \pi$
$\frac{V}{\beta} \ll 1$	$i \sin\theta$	$\sinh\tilde{\theta}$	restored
$\frac{V}{\beta} \gg 1$	$i\theta$	$\tilde{\theta}$	broken

$(-\pi < \theta \leq \pi)$

c.f.) dilute instanton gas approximation in SU(N) Yang-Mills theory $\langle Q \rangle_\theta \propto i \sin\theta$ (CP restored)



$$\lim_{\epsilon \to 0} \lim_{V_s \to \infty} \frac{\langle Q \rangle_{\theta = \pi - \epsilon}}{V_s} = \begin{cases} 0 & \text{CP restored} \\ \not{0} & \text{CP broken} \end{cases}$$

The SSB of CP at $\theta = \pi$ can be judged by $\tilde{\theta}$ dependence of $\langle Q \rangle_{\tilde{\theta}}$.

Phase structure at $\theta = \pi$ by analytic continuation

- ansatz for θ dependence of $\langle Q \rangle$
  - low temperature (CP broken) : $g(\theta) = b_1\theta + b_3\theta^3 + b_5\theta^5$ $\quad (-\pi < \theta \le \pi)$ — discontinuity at $\theta = \pi$

    only odd poweres due to symmetry
  - high temperature (CP restored) : $h(\theta) = a_1\sin(\theta) + a_2\sin(2\theta) + a_3\sin(3\theta)$ — continuity at $\theta = \pi$

⬇

- our analysis
  - fit $\langle Q \rangle_{\tilde{\theta}}$ obtained at pure imaginary θ to two types of fitting function
    - $g(\tilde{\theta}) = b_1\tilde{\theta} - b_3\tilde{\theta}^3 + b_5\tilde{\theta}^5$
    - $h(\tilde{\theta}) = a_1\sinh(\tilde{\theta}) + a_2\sinh(2\tilde{\theta}) + a_3\sinh(3\tilde{\theta})$
  - estimate $\langle Q \rangle_\theta$ at $\theta = \pi$ by analytic continuation ($\tilde{\theta} = i\theta$)

Because of $\left.\frac{\partial \langle Q \rangle}{\partial \tilde{\theta}}\right|_{\tilde{\theta}=0} = \chi_0$,
- $b_1 = \chi_0$
- $a_1 = \chi_0 - 2a_2 - 3a_3$

Lattice setup

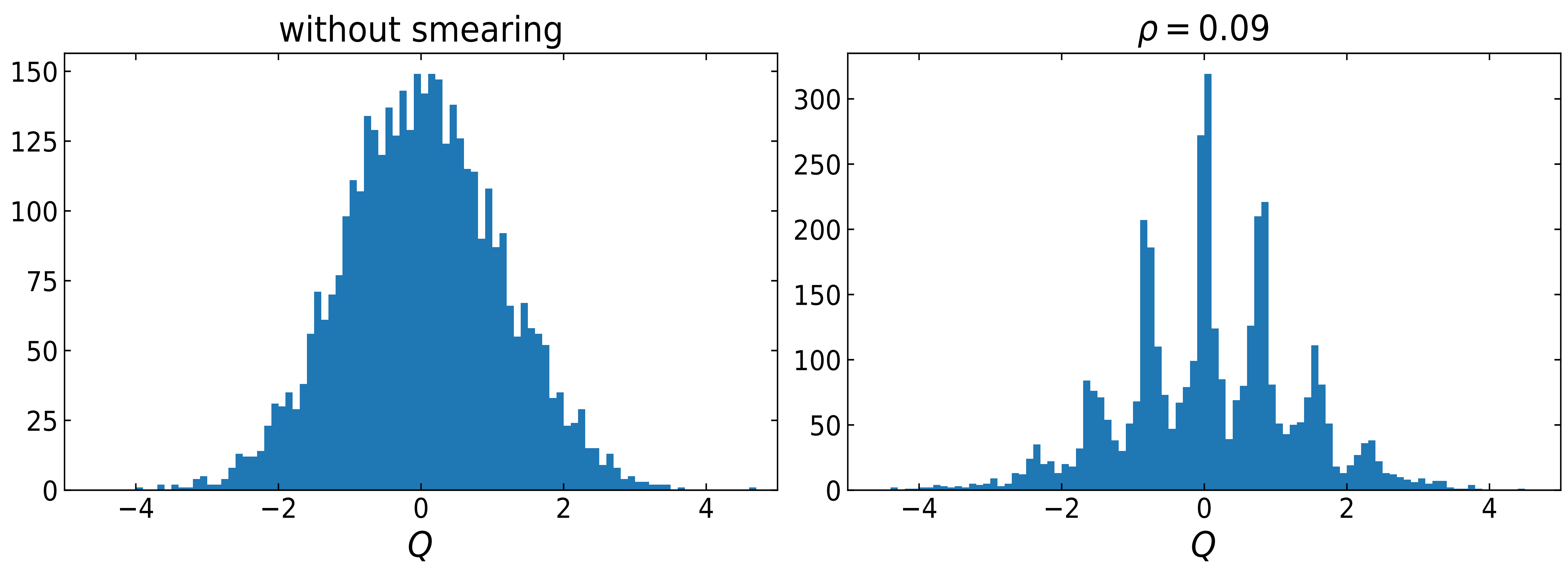
- gauge action : Wilson plaquette action
- definition of the topological charge : clover leaf + stout smearing

[P. Di Vecchia, K. Fabricius, G. C. Rossi, G. Veneziano (1981)]
[C. Morningstar, M. Peardon (2004)]

$$S = S_g(U) + \tilde{\theta}\, Q[U_{\text{smeared}}] \qquad U_{\text{smeared}} \ : \text{smeared link}$$

step size of the stout smearing : $\rho = 0.09$

number of steps of the stout smearing : $N_\rho = 40$



Peak positions of Q deviate slightly from integer values due to finite a.

→ This is taken care by renormalizing theta.

- algorithm for updates : HMC

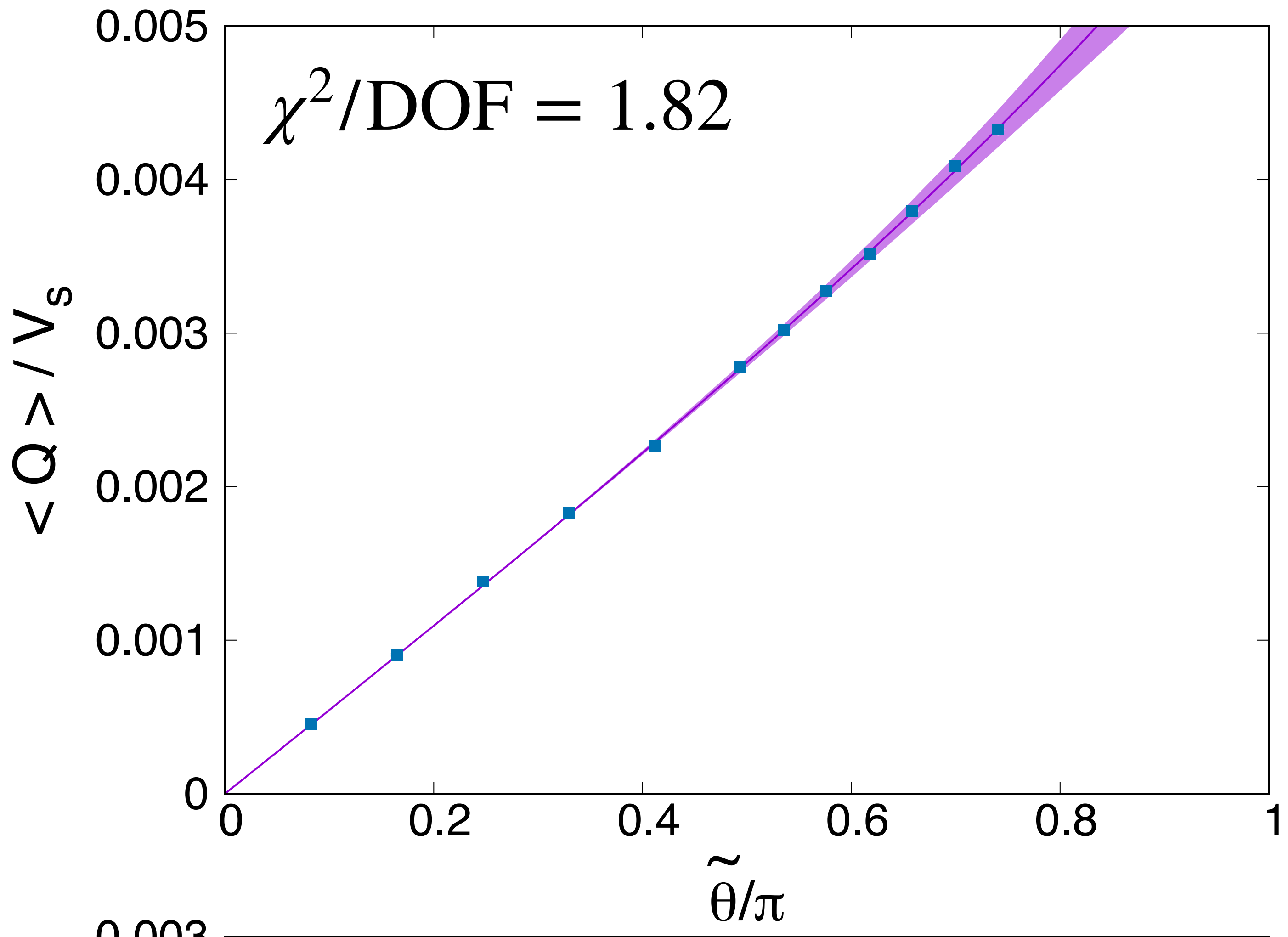
Configurations are generated using the above action.
($Q[U_{\text{smeared}}]$ is used also for updates.)

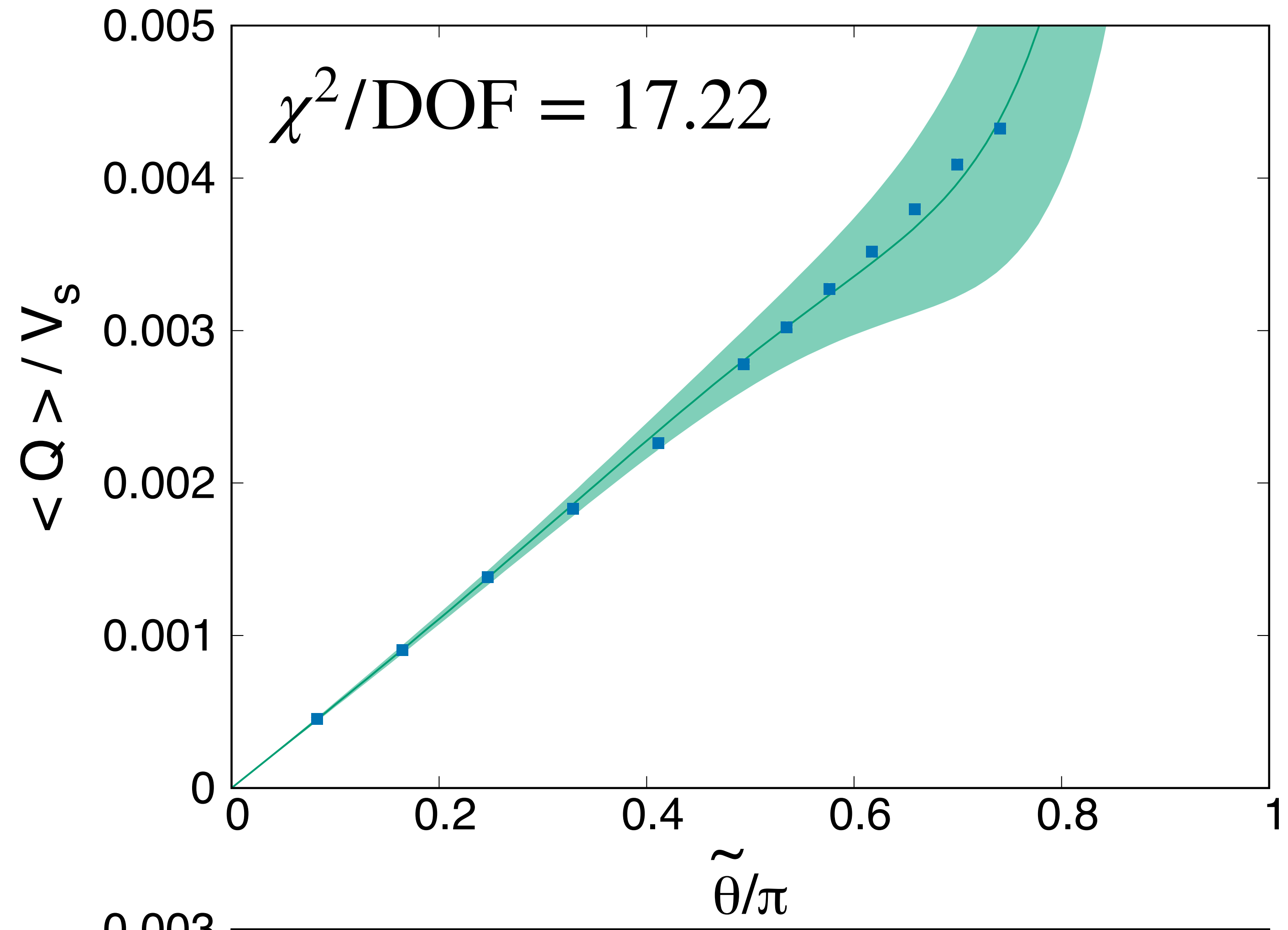
$\tilde{\theta}$ dependence of $\langle Q \rangle_{\tilde{\theta}}$ after the $V \to \infty$ limit

$g(\tilde{\theta}) = b_1\tilde{\theta} - b_3\tilde{\theta}^3 + b_5\tilde{\theta}^5$

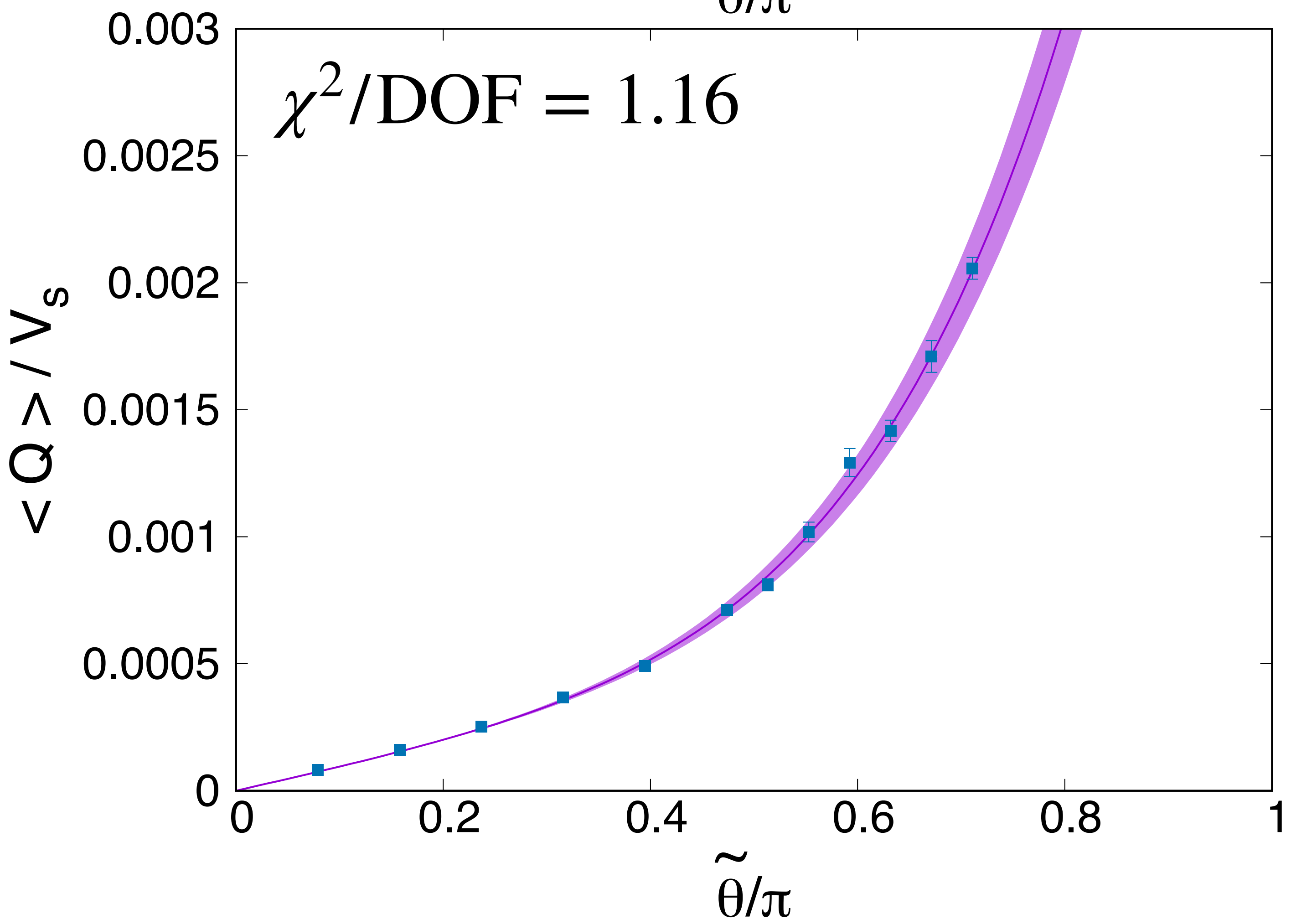
$h(\tilde{\theta}) = a_1\sinh(\tilde{\theta}) + a_2\sinh(2\tilde{\theta}) + a_3\sinh(3\tilde{\theta})$

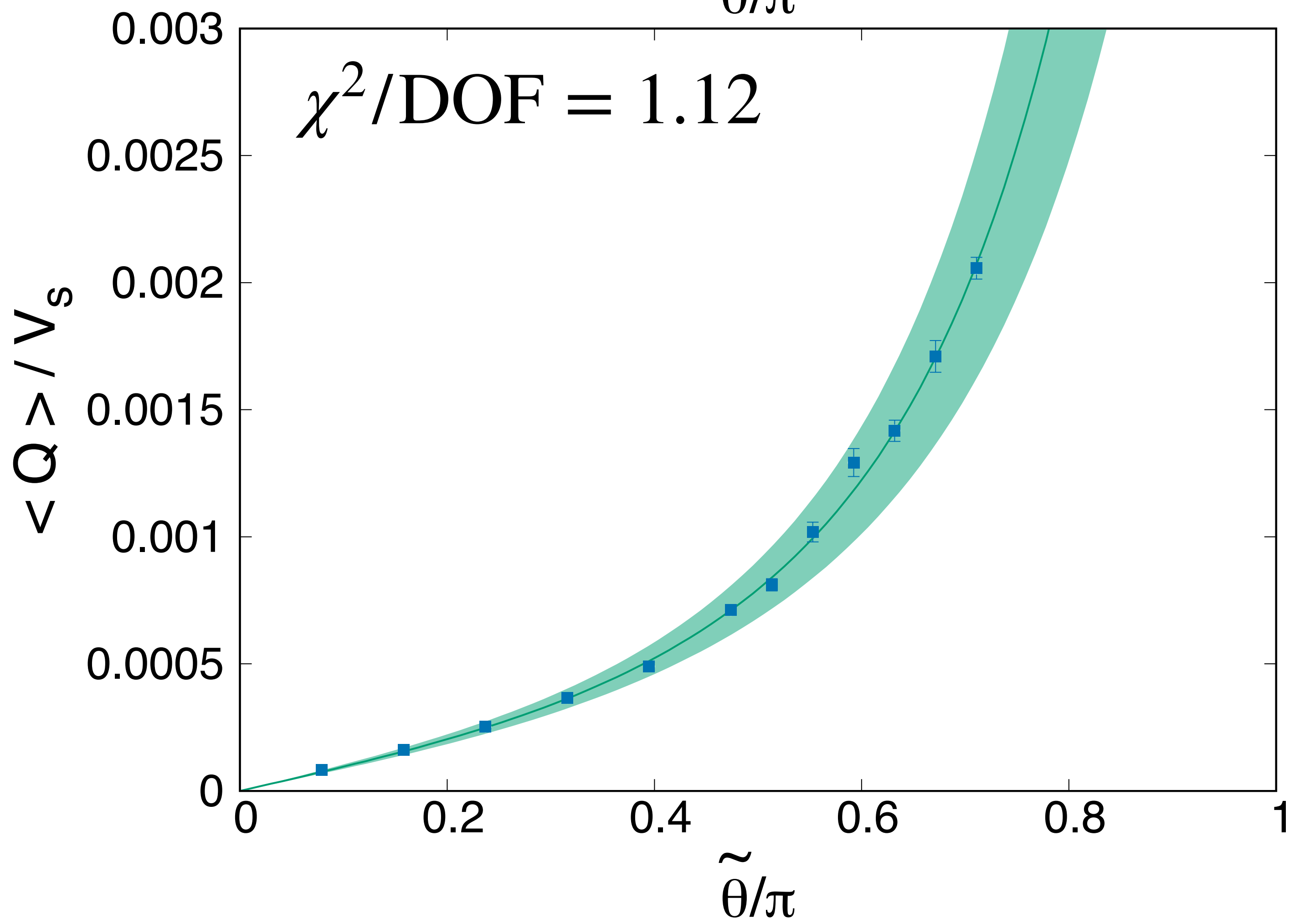
lower temperature
$(T = 0.9T_c)$





higher temperature
$(T = 1.2T_c)$





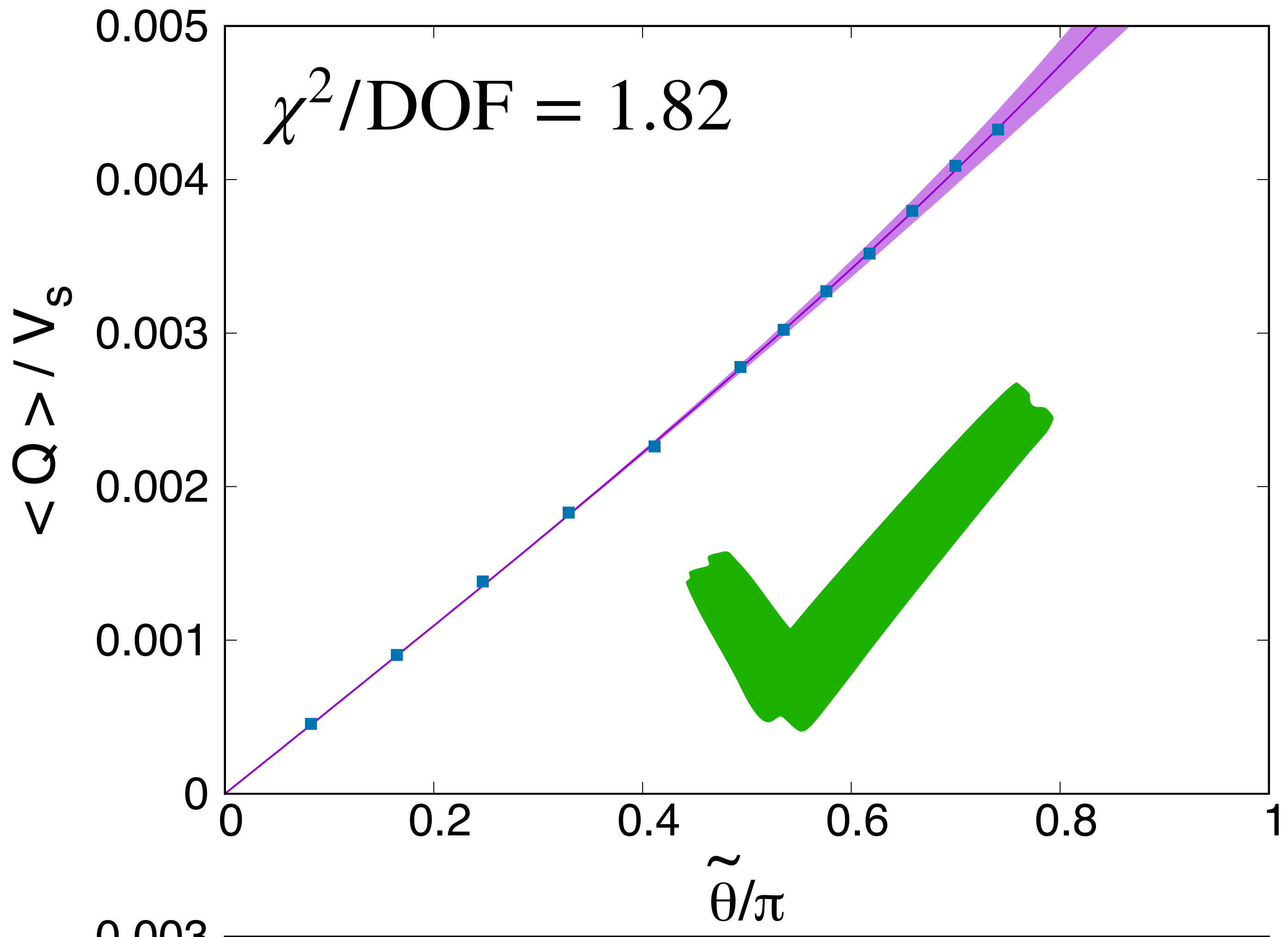
(T_c : deconfining temperature at $\theta = 0$ in the continuum limit)

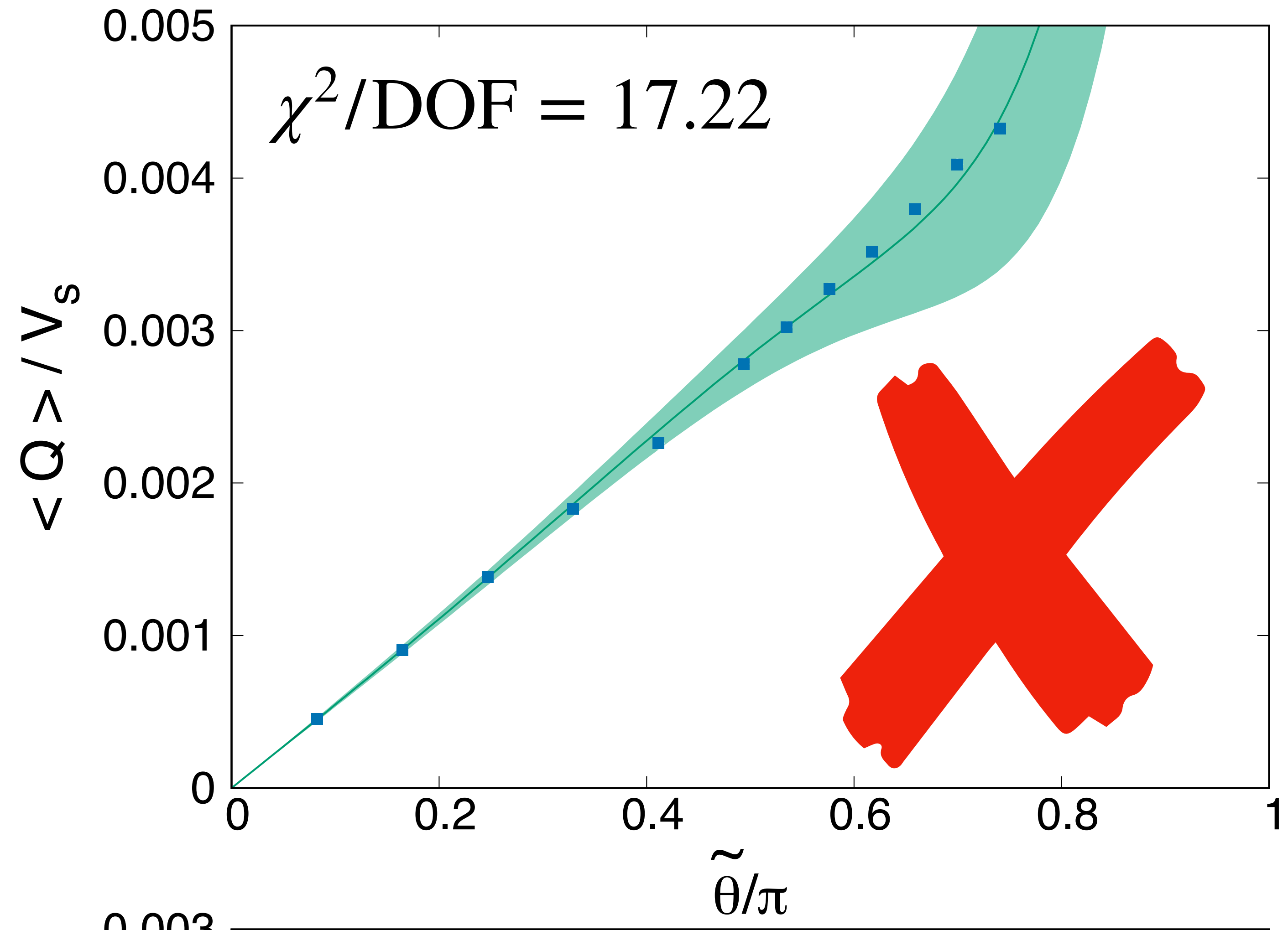
$\tilde{\theta}$ dependence of $\langle Q \rangle_{\tilde{\theta}}$ after the $V \to \infty$ limit

$g(\tilde{\theta}) = b_1\tilde{\theta} - b_3\tilde{\theta}^3 + b_5\tilde{\theta}^5$

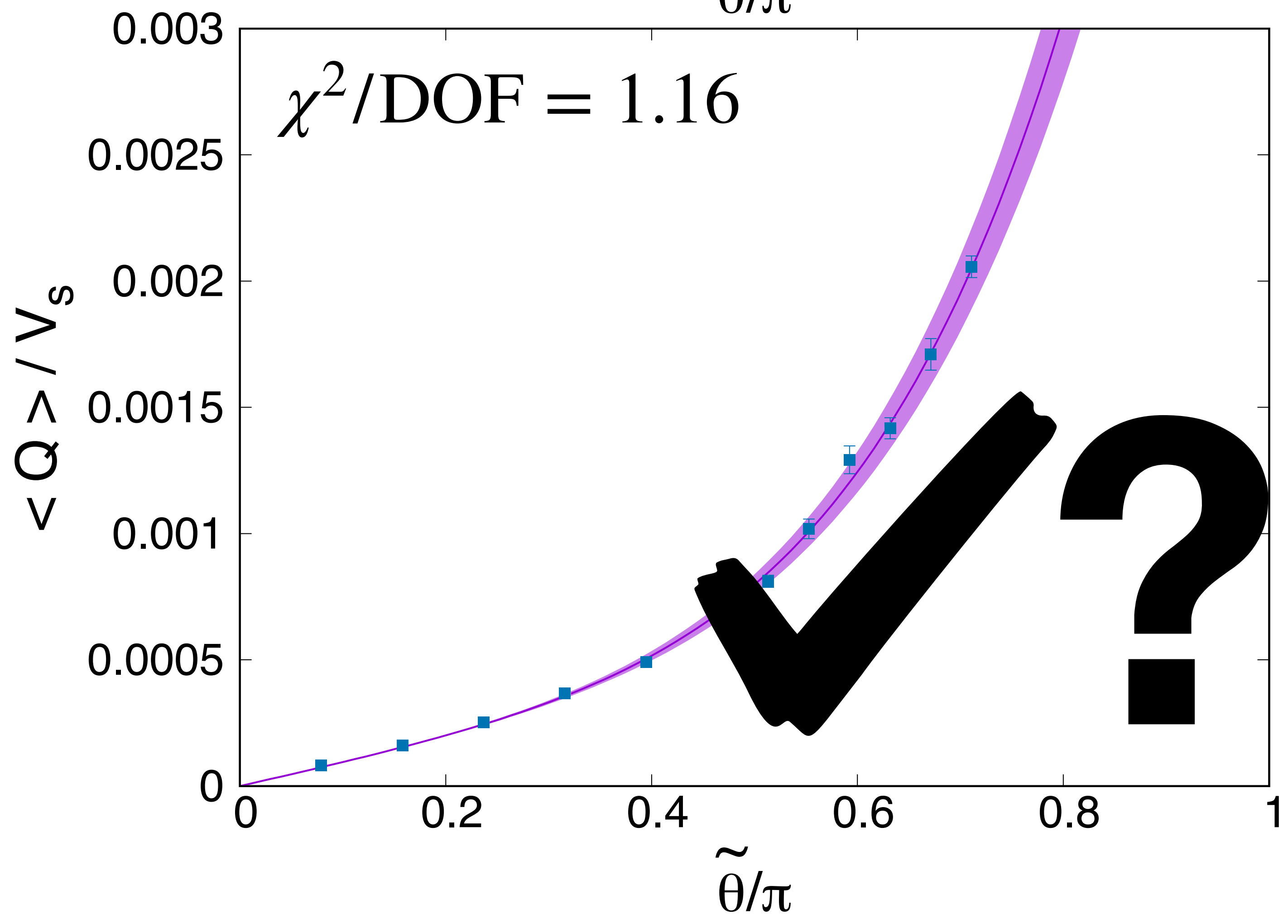
$h(\tilde{\theta}) = a_1\sinh(\tilde{\theta}) + a_2\sinh(2\tilde{\theta}) + a_3\sinh(3\tilde{\theta})$

lower temperature ($T = 0.9T_c$)

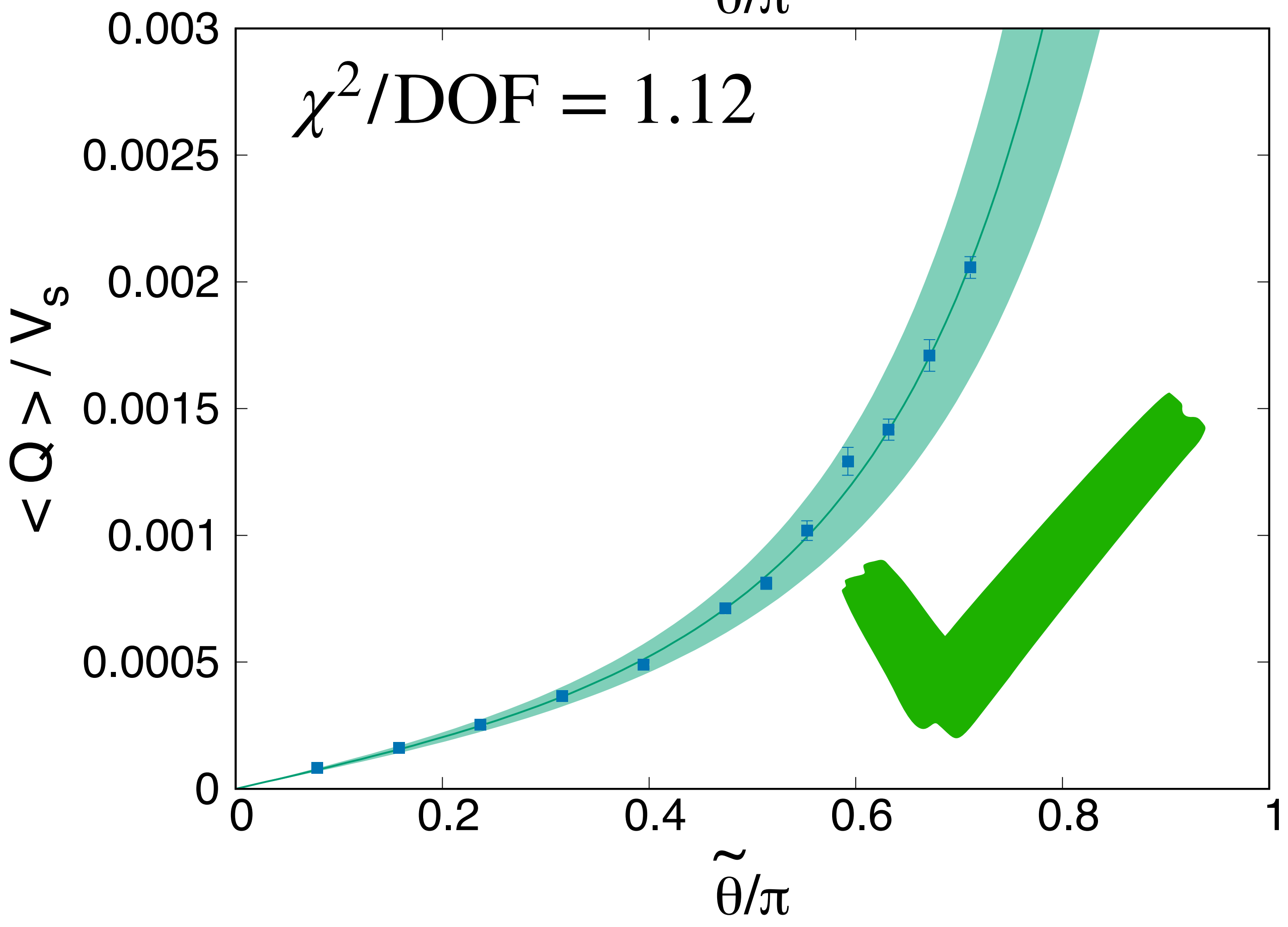




higher temperature ($T = 1.2T_c$)



$|b_5| > |b_3|$ (unnatural)



(T_c : deconfining temperature at $\theta = 0$ in the continuum limit)

It is clear that fitting by polynomial is better at lower temperature.

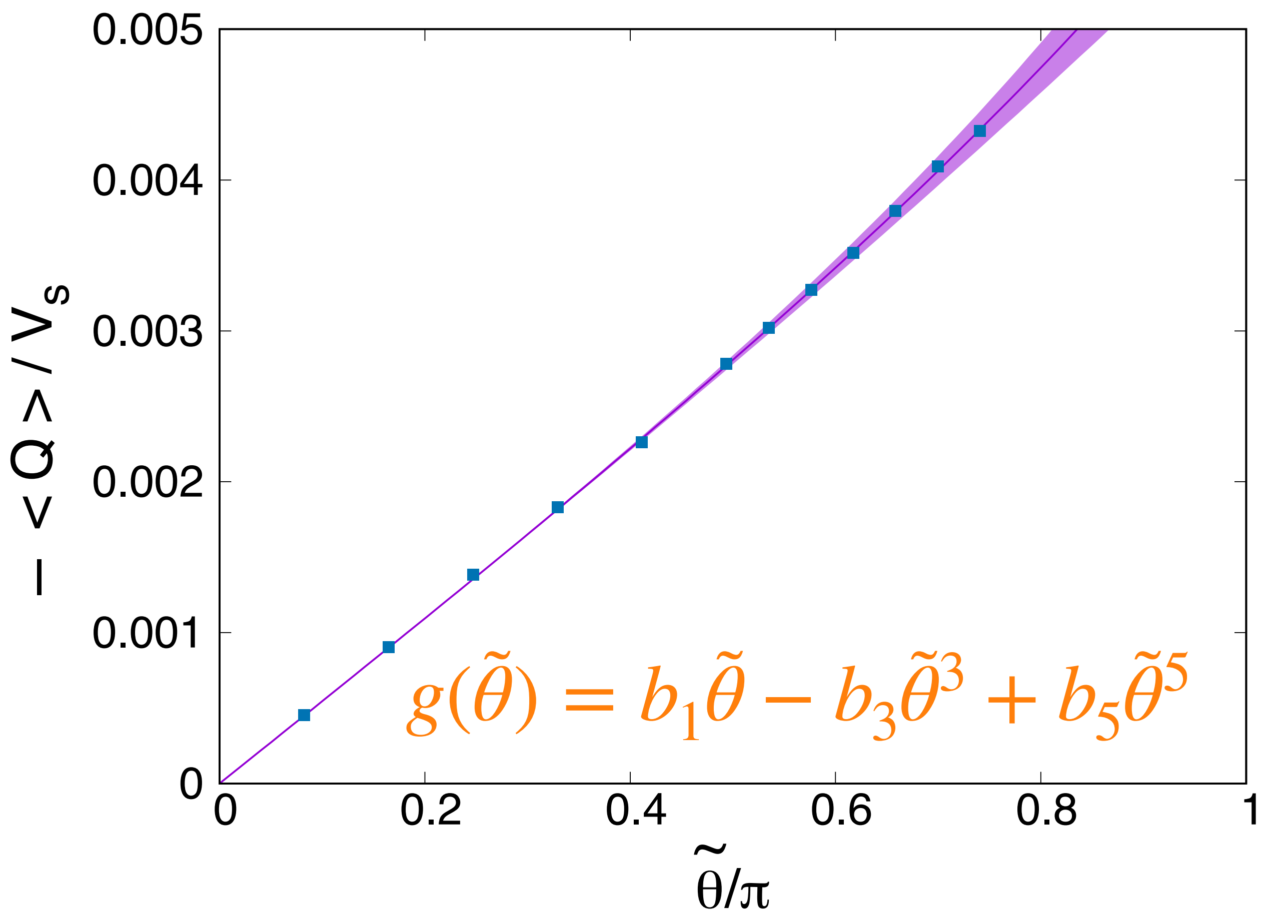
Analytic continuation

$\tilde{\theta}$ dependence of $\langle Q \rangle_{\tilde{\theta}}$

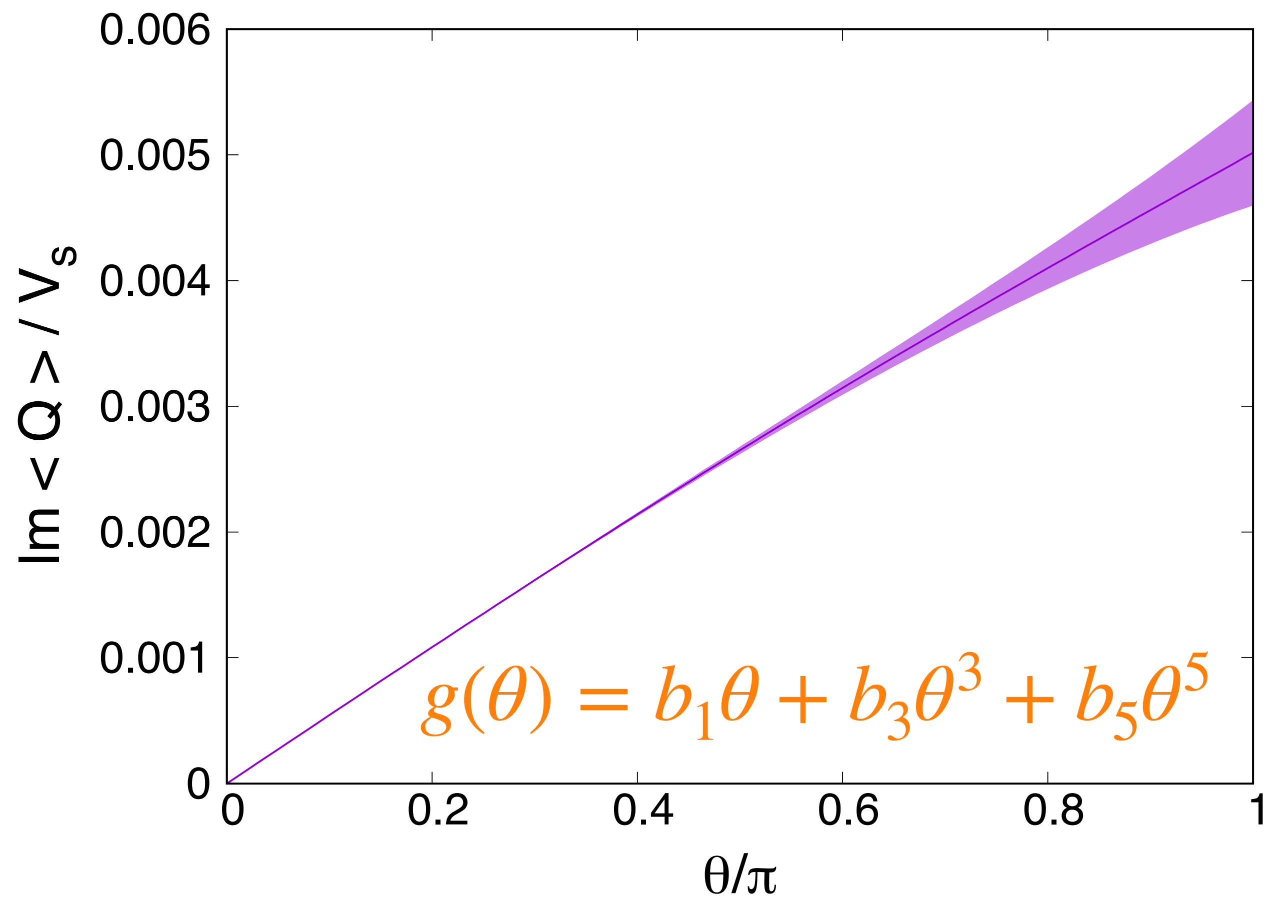
$\langle Q \rangle_{\theta}$ after the analytic continuation

lower temperature

$(T = 0.9T_c)$

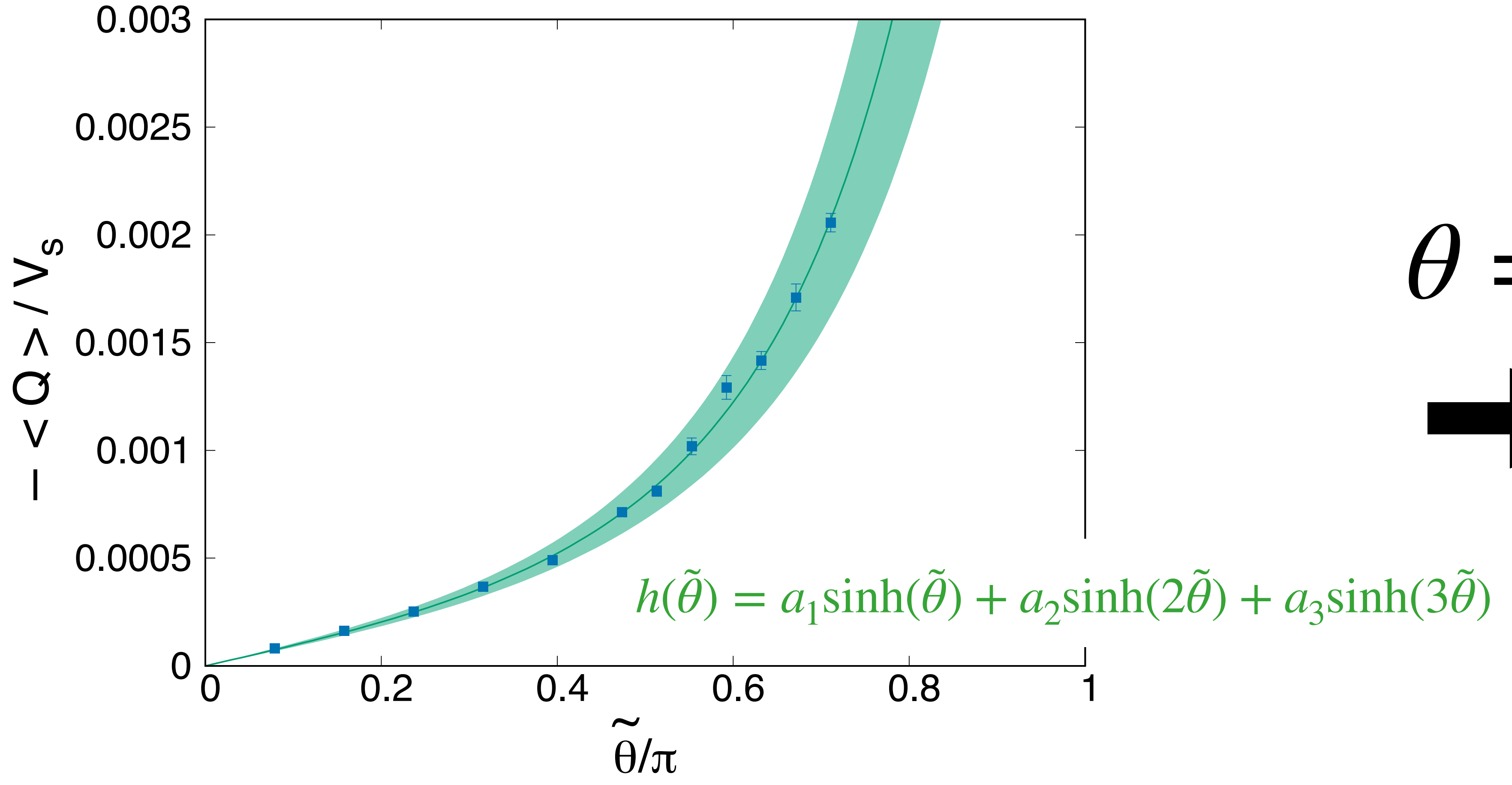


$\theta = i\tilde{\theta}$

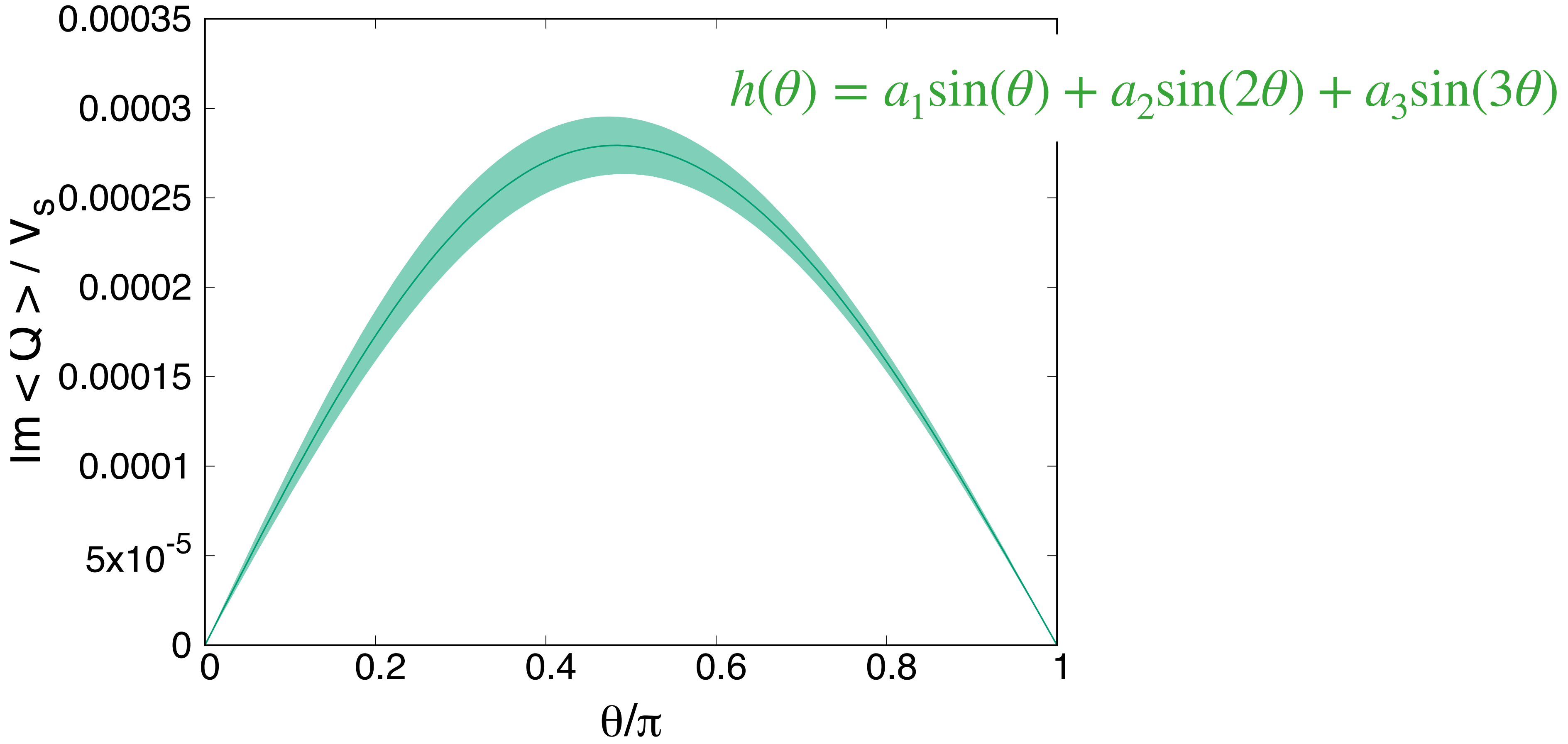


higher temperature

$(T = 1.2T_c)$



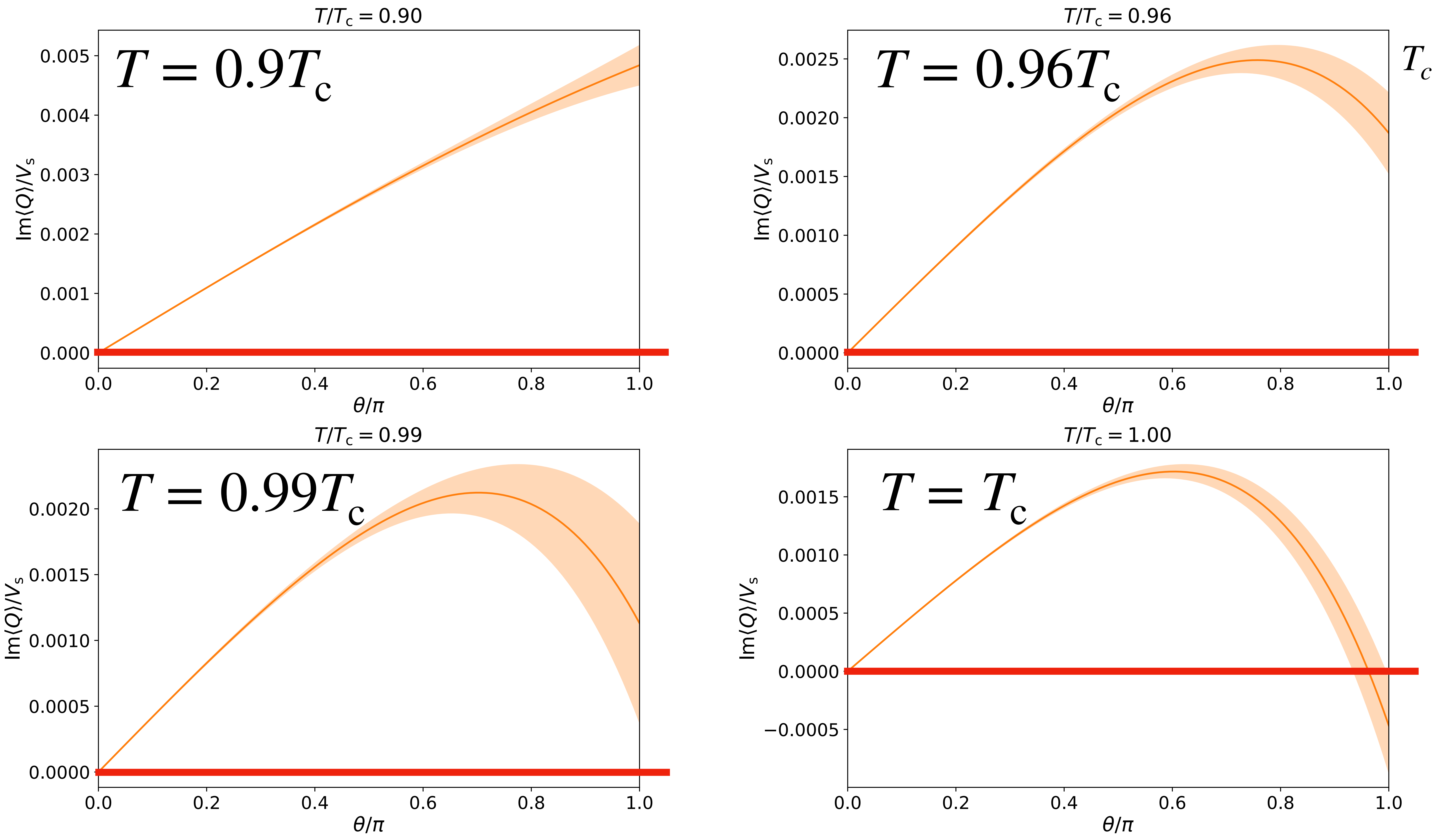
$\theta = i\tilde{\theta}$



We can translate the obtained $\tilde{\theta}$ dependence into θ dependence of $\langle Q \rangle_{\theta}$ by analytic continuation

θ dependence of $\langle Q \rangle_\theta$ near CP restoration temperature

We focus on the polynomial fitting in the low temperature region. $g(\theta) = b_1\theta + b_3\theta^3 + b_5\theta^5$



T_c : deconfining temperature at $\theta = 0$ in the continuum limit

$g(\theta = \pi) = 0 \leftrightarrow$ CP restoration $\rightarrow$ $0.99 < T_{\mathrm{CP}}/T_c \lesssim 1.00$

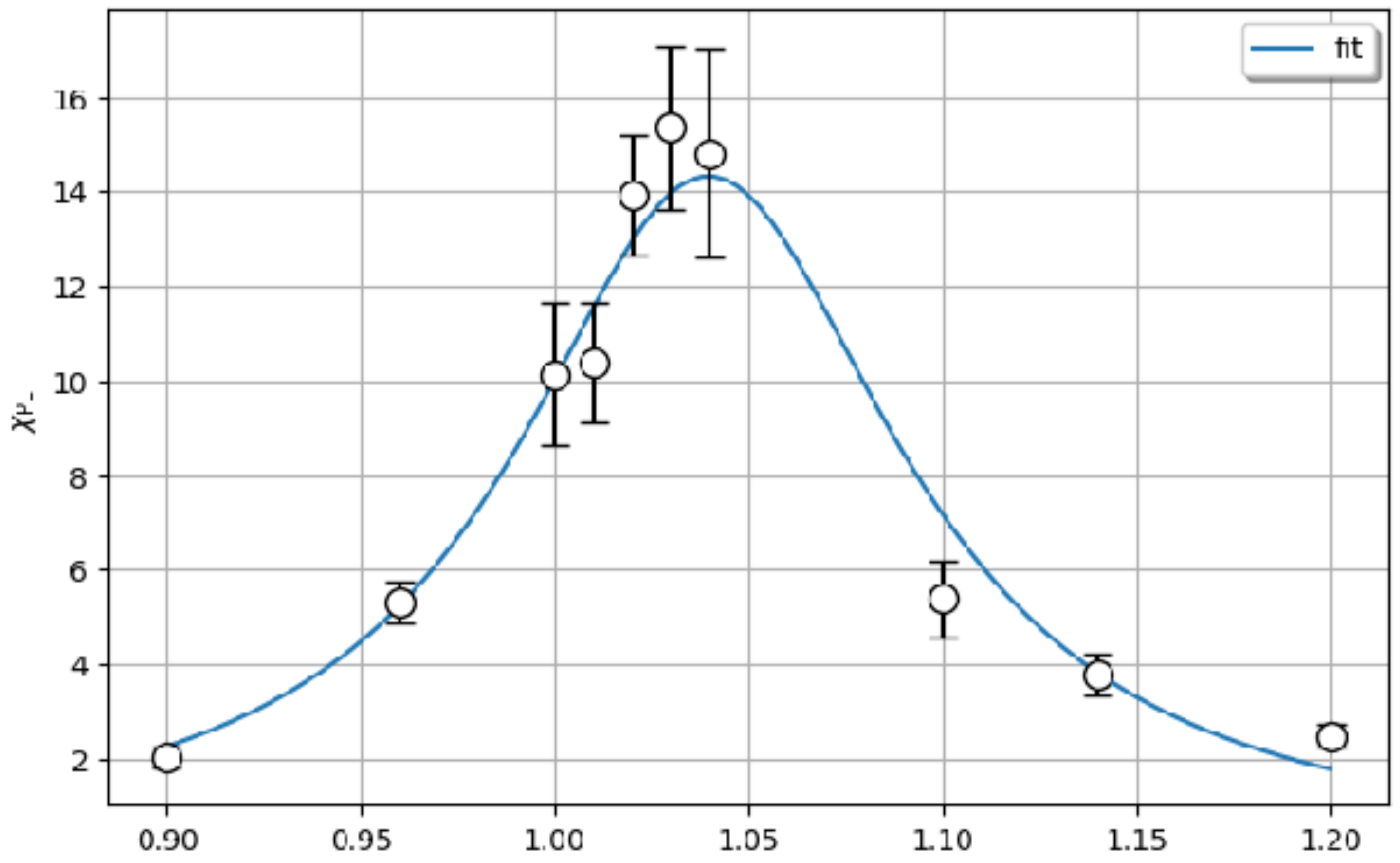
In order to determine the phase diagram, we still need to obtain $T_{\mathrm{dec}}(\theta = \pi)$.

θ dependence of the deconfining temperature

- order parameter : Polyakov loop susceptibility

A peak appears near the critical temperature.

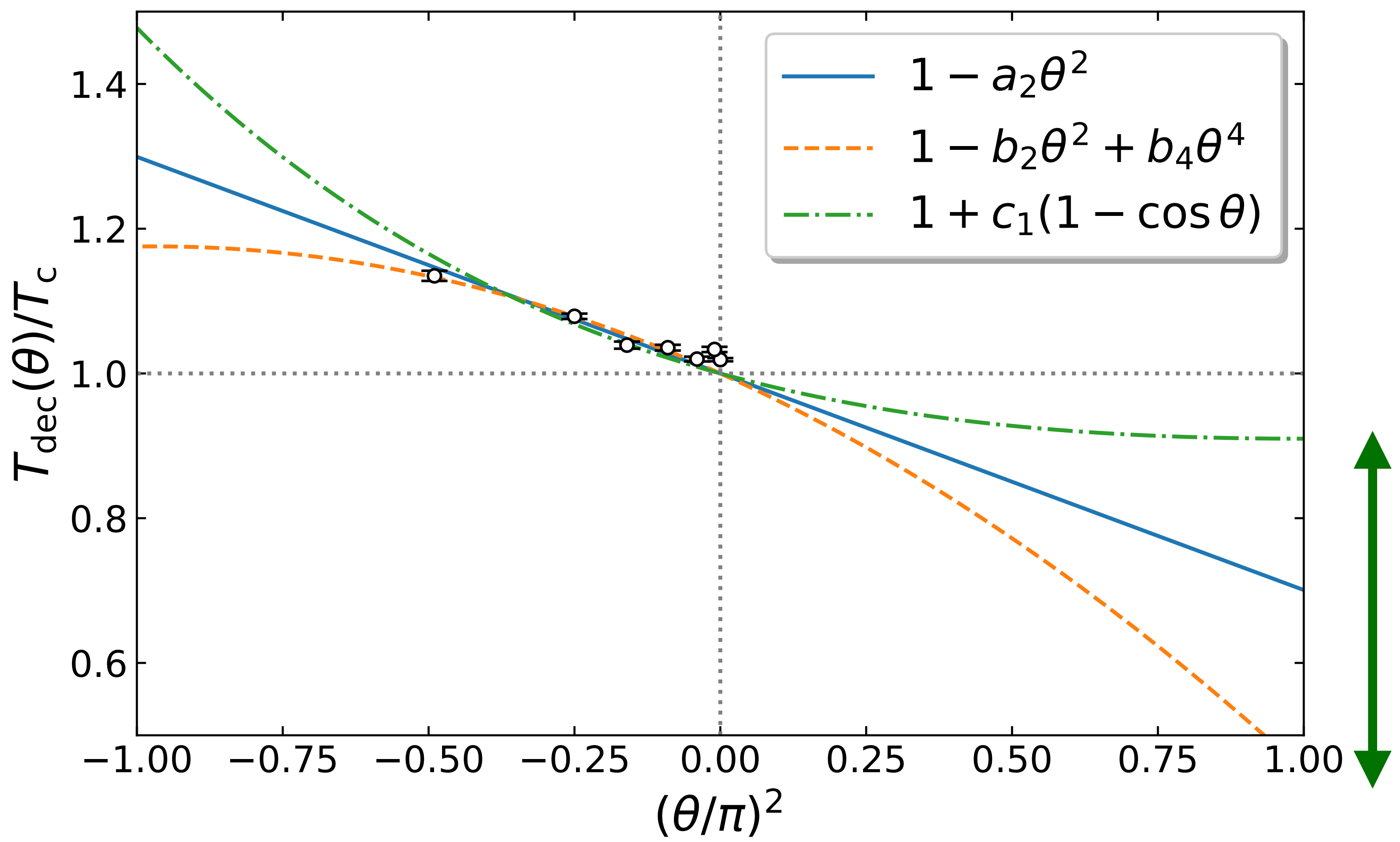
Polyakov loop susceptibility



T/T_c

($L_s = 24,\ L_t = 5,\ \tilde{\theta} = 0.3\pi$)

θ dependence of the deconfining temperature at $V = \infty$



$a_2 = 0.0303$
$b_2 = 0.0367,\ b_4 = -0.0019$
$c_1 = -0.045$

$T_{dec}(\theta = \pi) \lesssim 0.91\ T_c$

pure imaginary θ → real θ

analytic continuation

SU(3) case

$$\frac{T_{dec}(\theta)}{T_{dec}(\theta = 0)} \sim 1 - 0.0178\ \theta^2 + \dots$$

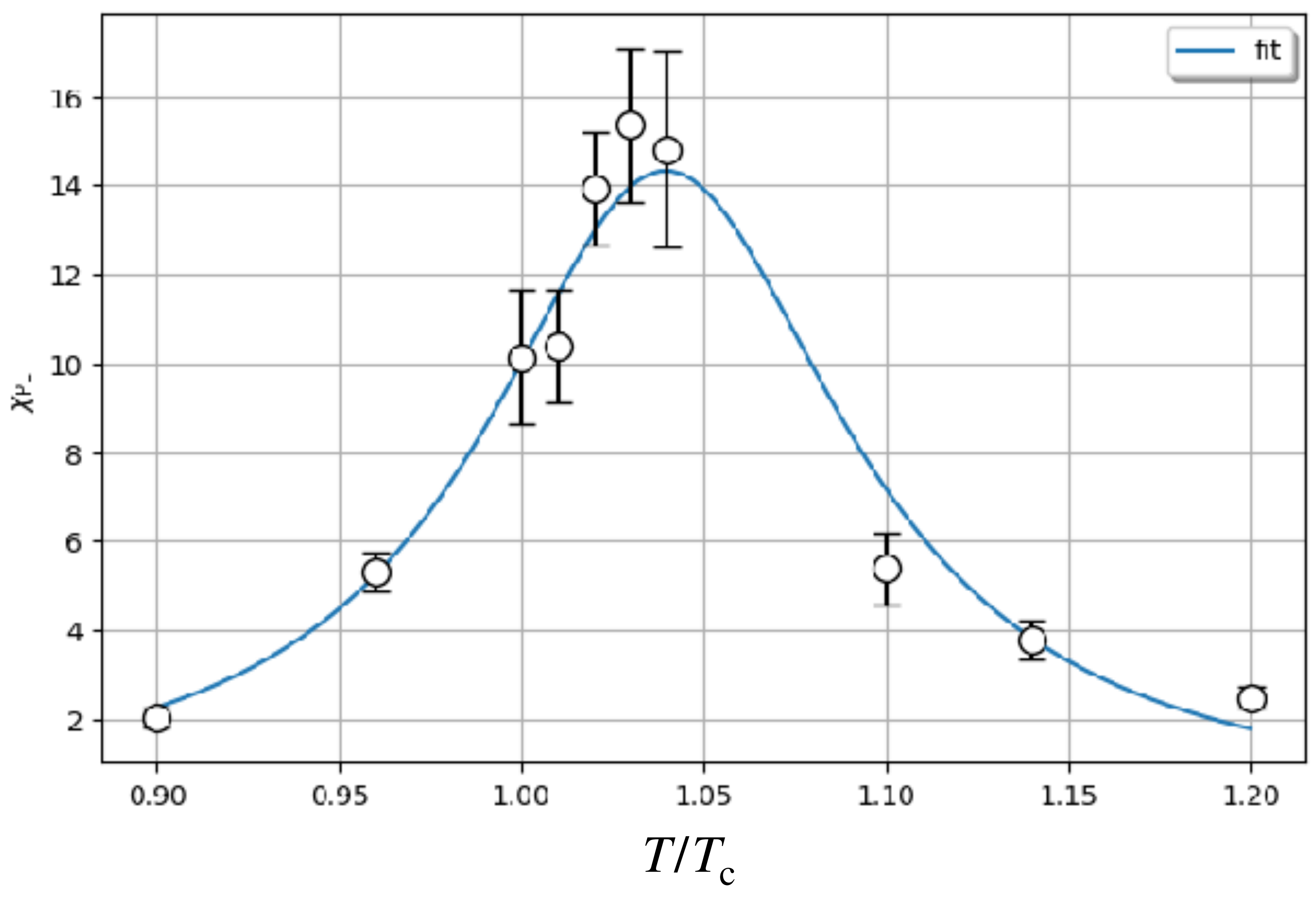
[M. D'Elia, F. Negro(2013)]
[N. Otake, N. Yamada(2022)]

θ dependence of the deconfining temperature

- order parameter : Polyakov loop susceptibility

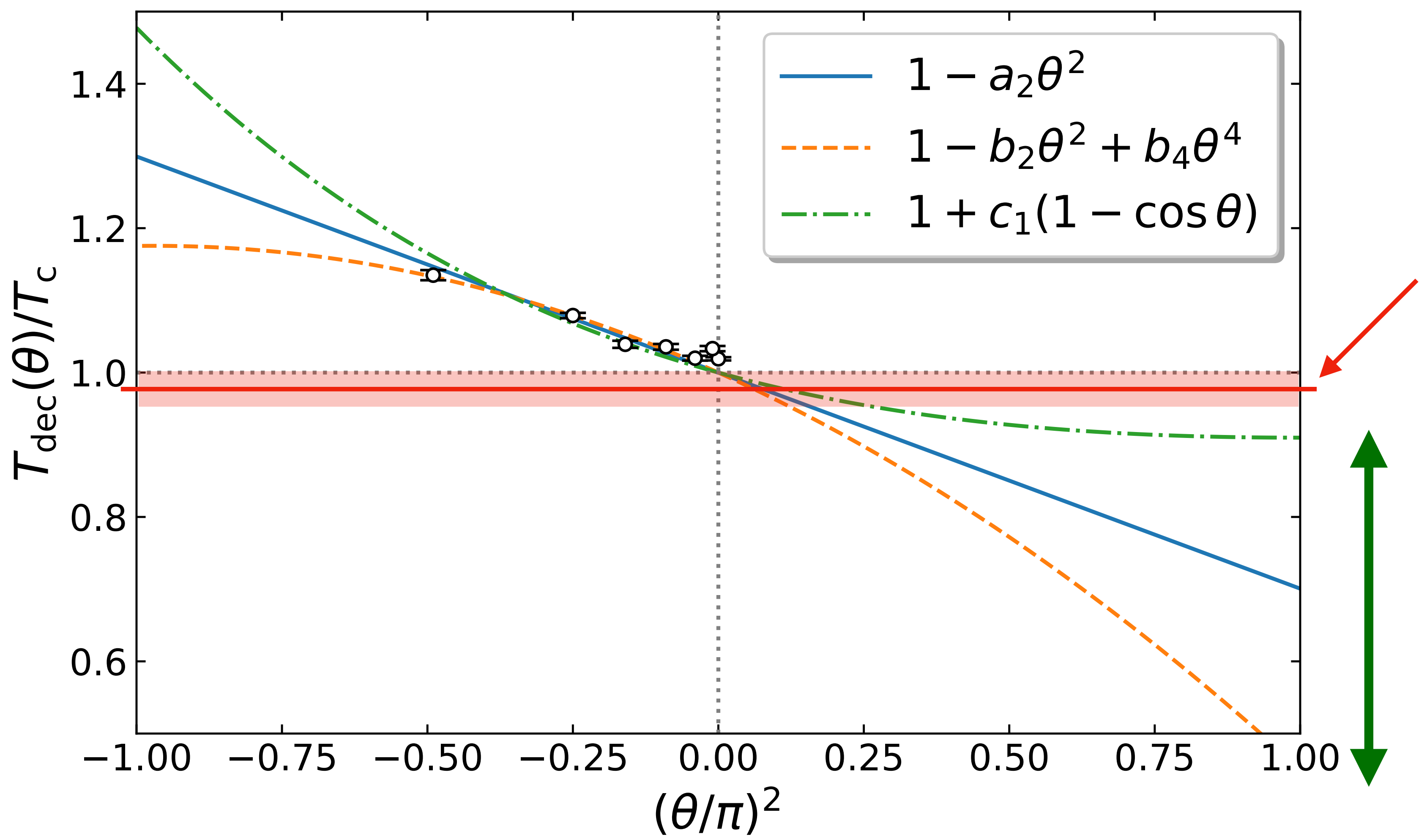
A peak appears near the critical temperature.

Polyakov loop susceptibility



($L_s = 24,\ L_t = 5,\ \tilde{\theta} = 0.3\pi$)

θ dependence of the deconfining temperature at $V = \infty$



$a_2 = 0.0303$
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T_{CP}

$T_{dec}(\theta = \pi) \lesssim 0.91\ T_c$

pure imaginary θ → real θ

analytic continuation

SU(3) case

$$\frac{T_{dec}(\theta)}{T_{dec}(\theta = 0)} \sim 1 - 0.0178\ \theta^2 + \dots$$

[M. D'Elia, F. Negro(2013)]
[N. Otake, N. Yamada(2022)]

The results suggest $T_{CP} > T_{dec}(\theta = \pi)$

Summary

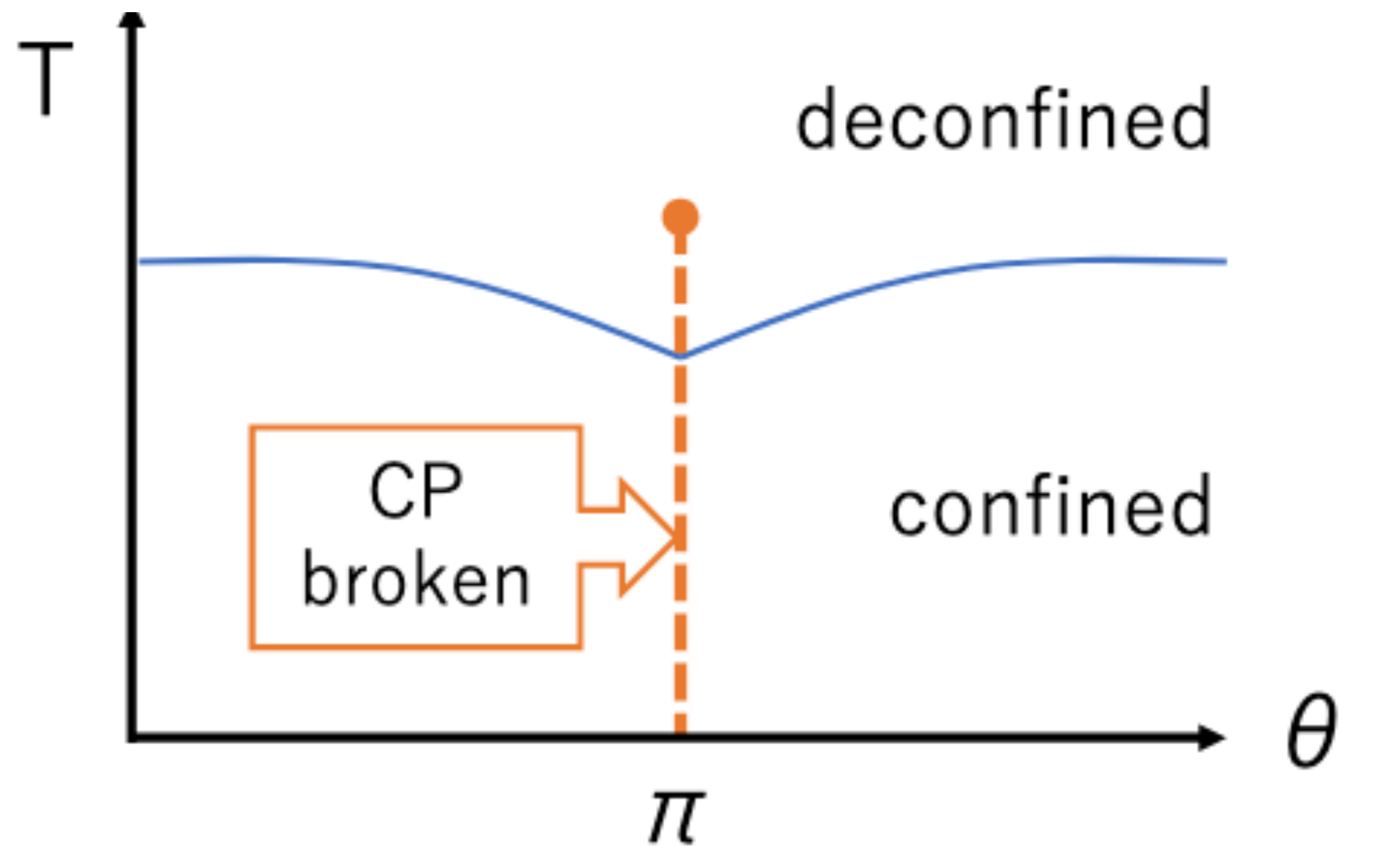
- We studied the phase structure at $\theta = \pi$ in 4D SU(2) Yang-Mills theory through imaginary θ simulations.
  - SSB of CP symmetry
    - ✳ θ dependence of $\langle Q \rangle$ was estimated by analytic continuation.
      - ✦ $\langle Q \rangle_{\theta=\pi} = 0$ at $0.99 < T/T_c \lesssim 1.00$

$$0.99 < T_{CP}/T_c \lesssim 1.00$$

  - θ dependence of the deconfining temperature
    - ✳ $T_{dec}(\theta = \pi)$ was also estimated by analytic continuation.

$$T_{dec}(\theta = \pi) \lesssim 0.91\ T_c$$

Our results suggest $T_{CP} > T_{dec}(\theta = \pi)$ in SU(2) Yang-Mills theory, unlike the large N case.



Future prospects

- 4D SU(2) Yang-Mills theory
  - taking the continuum limit
  - eliminating possible artifacts of the smearing

- 4D SU(3) Yang-Mills theory (on-going)
  - Do these phase transitions occur at the same temperature as predicted at large N ?

There is a possibility that the situation is different between SU(2) and SU(3) cases since the order of the deconfinement transition is different.

c.f.) deconfinement phase transition
- 2nd order for $N = 2$
- 1st order for $N \geq 3$

Thank you for listening!