The gluino condensate of large-N SUSY Yang–Mills

Speaker:
Claudio Bonanno
IFT UAM/CSIC Madrid
(claudio.bonanno@csic.es)

EXCELENCIA SEVERO OCHOA
MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES
Cofinanciado por la Unión Europea
AGENCIA ESTATAL DE INVESTIGACIÓN
CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS
LATTICE 2024 LIVERPOOL

Lattice 2024
41th International Symposium on Lattice Field Theory
28/07 — 03/08/2024, Liverpool (UK)

Based on: "*Non-perturbative determination of the $\mathcal{N}=1$ SUSY Yang–Mills gluino condensate at large N*", **CB**, P. Butti, M. García Pérez, A. González-Arroyo, K.-I. Ishikawa, M. Okawa [arXiv:2406.08995]

Introduction — The gluino condensate

$\mathcal{N} = 1$ Supersymmetric (SUSY) SU(N) Yang–Mills theory
[SU(N) gauge theory coupled to 1 massless adjoint Majorana (gluino)]
features a non-vanishing Gluino Chiral Condensate

$$\mathrm{U}(1)_{\mathrm{A}} \quad \xrightarrow[\text{anomaly}]{} \quad \mathbb{Z}_{2N} \quad \xrightarrow[\text{SSB}]{} \quad \mathbb{Z}_2$$

Value of gluino condensate has been **subject of debate** since first exact **instanton** calculations in the Strong-/Weak-Coupling (SC/WC) regimes by Novikov–Shifman–Vainshtein–Zakharov (NSVZ):

$$\frac{1}{(4\pi)^2 b_0 N} \left|\langle \mathrm{Tr}\lambda^2 \rangle\right| = \begin{cases} 2e\,\Lambda^3/N & \text{[NPB\textbf{229} 407 (1983) — SC]} \\ \Lambda^3 & \text{[NPB\textbf{260} 157 (1985) — WC]} \end{cases}$$

Recently, SU(2) result $\mathbf{2\Lambda^3}$ found in [Anber & Poppitz JHEP**01** (2023) 118] using **fractional instantons**: the authors argue $2 \to N$ for SU(N)

$\Longrightarrow$ controversy about **value and N-scaling still unsolved**

Lattice status

In the last 10+ years, there was a tremendous progress in lattice simulations of SUSY Yang–Mills. See, e.g., **recent reviews**: [Bergner–Catterall (2016) 1603.04478; Bergner–Münster–Piemonte (2022) 2212.10371; Schaich (2023) 2208.03580].

Despite this progress, determining the gluino condensate has proven to be a highly non-trivial numerical challenge. So far, only few SU(2) determinations [Giedt et al. (2008) 0810.5746; JLQCD (PoS Lattice2011) 1111.2180; Bergner et al. (2019) 1902.08469; Piemonte et al. (2020) 2005.02236]

- **Massless gluino limit (chiral limit)** — ✓
- **No continuum limit** — ×
- **No matching with analytic NSVZ scheme** — ×

In [arXiv:2406.08995] we performed the first large-N lattice calculation of the **value and leading N-dependence** of gluino condensate: chiral-continuum limit + matching lattice with NSVZ scheme

$\Longrightarrow$ comparison between numerical and analytic results for the first time

Observables

Analytic NSVZ calculations done in scheme with **exact** β-function

$$\beta_{\text{NSVZ}}(\lambda_{\text{NSVZ}}) = -\frac{b_0 \lambda_{\text{NSVZ}}^2}{1 - \frac{b_1}{b_0}\lambda_{\text{NSVZ}}} \qquad b_0 = 3/(4\pi)^2 \qquad b_1 = 6/(4\pi)^4$$

$$\Lambda_{\text{NSVZ}}^3 \equiv \frac{\mu^3}{b_0 \lambda_{\text{NSVZ}}(\mu)} \exp\left[\frac{-8\pi^2}{\lambda_{\text{NSVZ}}(\mu)}\right] \quad \longleftarrow \quad \text{dynamical scale in NSVZ scheme}$$

$\langle \mathrm{Tr}\lambda^2 \rangle \longleftarrow$ **RGI (renorm. group invariant) condensate**

$$\Sigma_{\text{RGI}} \equiv \frac{1}{(4\pi)^2 b_0 N} \left| \langle \mathrm{Tr}\lambda^2 \rangle \right| = \frac{\lambda_{\text{NSVZ}}(\mu)}{N\left[1 - \lambda_{\text{NSVZ}}(\mu)/(8\pi^2)\right]} \Sigma_{\text{R}}^{(\text{NSVZ})}(\mu)$$

with $\Sigma_{\text{R}}^{(\text{s})}(\mu) = \langle \overline{\psi}\psi \rangle_{\text{R}}^{(\text{s})}(\mu)$.

What's the RGI condensate? Key to match NSVZ and lattice results

Matching NSVZ and the lattice

$\Lambda_s \leftarrow$ scheme-dependent RGI integr. constant of Callan–Symanzik eq. for λ_t

Analogously, Callan–Symanzik eq. for renorm. gluino mass $m_R^{(s)}$ has **RGI scheme-independent integration constant** m_{RGI}

$$\tau_s(\lambda_s) = \frac{d\log\left[m_R^{(s)}(\mu)\right]}{d\log(\mu)} \qquad \tau_s(\lambda_s) = d_0\lambda_s + \dots \qquad d_0 = 2b_0$$

$$m_{RGI} = \tilde{\mathcal{A}}\, m_R^{(s)}(\mu)\ [2b_0\lambda_s(\mu)]^{-\frac{d_0}{2b_0}} \times \exp\left[-\int_0^{\lambda_s(\mu)} dx \left(\frac{\tau_s(x)}{2\beta_s(x)} - \frac{1}{x}\right)\right]$$

with $\tilde{\mathcal{A}}$ an arbitrary constant.

Since $m_R^{(s)}(\mu)\Sigma_R^{(s)}(\mu)$ **is RGI** $\implies$ the following quantity is also RGI:

$$\Sigma_{RGI} = \mathcal{A}\,\Sigma_R^{(s)}(\mu)\ [2b_0\lambda_s(\mu)]^{\frac{d_0}{2b_0}} \times \exp\left[\int_0^{\lambda_s(\mu)} dx \left(\frac{\tau_s(x)}{2\beta_s(x)} - \frac{1}{x}\right)\right]$$

In NSVZ scheme (where τ is exactly known too) Σ_{RGI} above coincides with the quantity obtained in analytic calculations for $\mathcal{A} = 8\pi^2/(9N^2)$

Large-N volume independence

Our lattice calculation is based on **large-N twisted volume reduction**

Large-N equivalence of space-time and color degrees of freedom of YM theories

[Eguchi & Kawai PRL 48 (1982) 1063]

Large-N lattice gauge theory can be reduced on a one-point lattice with twisted boundary conditions $\Longrightarrow$ **Twisted Eguchi–Kawai** (TEK) matrix model

[A. González-Arroyo & M. Okawa PRD**27** (1983) 2397; JHEP**07** (2010) 043]

Including adjoint matter within the TEK model has been successfully done

[A. González-Arroyo & M. Okawa Phys.Rev.D 88 (2013) 014514]

Large-N TEK SUSY YM can be simulated [Butti et al. JHEP**07** (2022) 074 2205.03166] using well-established techniques developed in the last 10+ years by DESY–Jena–Münster–Regensburg collaboration

[see, e.g., Bergner et al. PRD**100** 074501 (2019) 1902.08469]

Lattice setup

- Dynamical massive gluino (Wilson fermion)
- Sign-quenched Pfaffian (no occurrence of negative signs)
- SUSY limit = continuum + massless gluino (chiral) limit (Kaplan/Curci–Veneziano)
- Chiral limit = "*adjoint pion*" massless limit (SUSY + valence quenched gluino)

Numerical strategies: same methods already successfully applied in large-N YM [CB et al. JHEP**12** (2023) 034 2309.15540] and in $N = 3$ QCD$_{N_f=2+1}$ [CB et al. JHEP**11** (2023) 013 2308.01303]

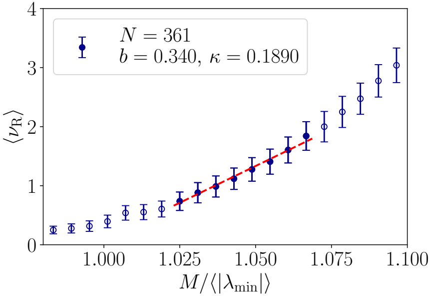
- Spectral method (Banks–Casher): $\frac{\Sigma_{\mathrm{R}}^{(\mathrm{s})}}{2\pi} = \lim_{\substack{V\to\infty \\ m\to 0 \\ \lambda\to 0}} \rho_{\mathrm{R}}^{(\mathrm{s})}(\lambda, m)$
- Part. Quenched Chiral Pert. Theory (Gell-Mann–Oakes–Renner):

$$m_\pi^2 = 2\frac{\Sigma_{\mathrm{R}}^{(\mathrm{s})}}{F_\pi^2} m_{\mathrm{R}}^{(\mathrm{s})}$$

Method I: Banks–Casher formula

Condensate from Banks–Casher via Giusti–Lüscher method [JHEP**03** (2009) 013]:

- Solve numerically $\left(\gamma_5 D_{\mathrm{W}}^{(\mathrm{adj})}[U_{\mathrm{adj}}]\right) u_\lambda = \lambda\, u_\lambda$ for first $\mathcal{O}(100)$ eigenvalues
- Count modes below threshold M to obtain **mode number** $\langle \nu(M) \rangle$
- $\Sigma = \frac{\pi}{4V}\sqrt{1-\frac{m^2}{M^2}}\left[\frac{\mathrm{d}\langle\nu(M)\rangle}{\mathrm{d}M}\right] \longleftarrow$ slope of $\langle \nu(M) \rangle$ vs M from linear fit



Renormalization:

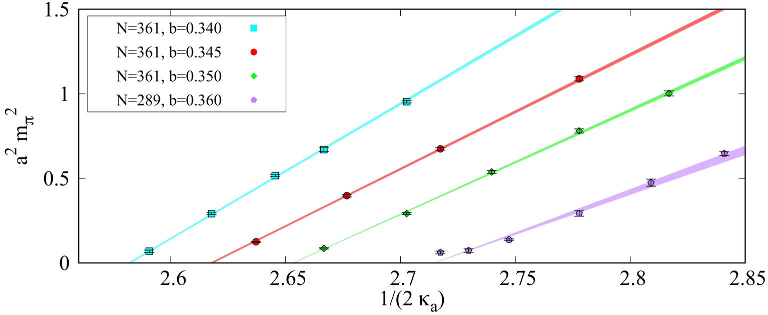
$\langle \nu \rangle = \langle \nu_{\mathrm{R}} \rangle \qquad M_{\mathrm{R}} = M/Z_{\mathrm{P}}$

$\Longrightarrow$ Slope fit yields $\Sigma = \Sigma_{\mathrm{R}}/Z_{\mathrm{P}}$

From the eigenvectors u_λ we obtained $Z_{\mathrm{P}}/Z_{\mathrm{S}}$ to renormalize M/m

$[\ am = 1/(2\kappa) - 1/(2\kappa_{\mathrm{crit}}) = aZ_{\mathrm{S}} m_{\mathrm{R}}\ ]$

Method II: Gell-Mann–Oakes–Renner relation



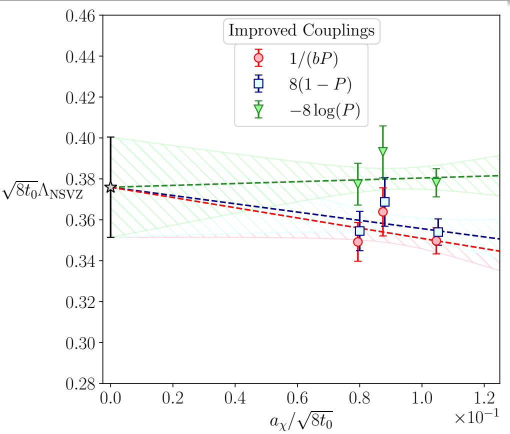
From [P. Butti, M. García Pérez, A. González-Arroyo, K.-I. Ishikawa, M. Okawa, JHEP 07 (2022) 074]

From the slopes of m_π^2 as a function of $aZ_{\mathrm{S}}m_{\mathrm{R}} = 1/(2\kappa) - 1/(2\kappa_{\mathrm{crit}})$

$+$ non-perturbative determination of $Z_{\mathrm{P}}/Z_{\mathrm{S}}$

$\Longrightarrow$ we obtain another determination of $\Sigma_{\mathrm{R}}/Z_{\mathrm{P}}$

The SUSY Yang–Mills Λ-parameter



Λ-parameter from 2-loop asymptotic scaling with 3 different **improved couplings**

$$\sqrt{8t_0}\Lambda_{\rm s} = \lim_{a_\chi \to 0} \frac{\sqrt{8t_0}}{a_\chi} \exp\{-f(\lambda_{\rm s})\}$$

$$f(x) = \frac{1}{2b_0}\left[\frac{1}{x} + \frac{b_1}{b_0}\log(b_0 x)\right]$$

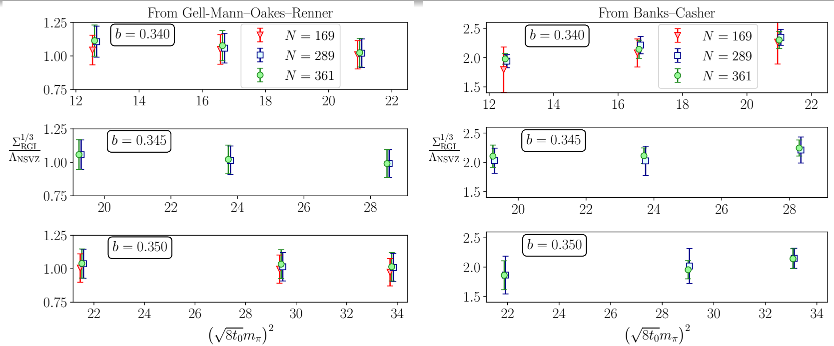
Improved couplings → lattice schemes where pert. theory converges faster

Ratio $\Lambda_{\rm impr}/\Lambda_{\rm L}$ could be obtained [Weisz PLB100 331 (1981); García Pérez et al. 1708:00841]

$$\Lambda_{\overline{\rm MS}}/\Lambda_{\rm L} \simeq 73.4667 \text{ and } \Lambda_{\rm NSVZ}/\Lambda_{\overline{\rm MS}} = e^{-1/18}$$

$$\sqrt{8t_0}\Lambda_{\rm NSVZ} = 0.376(25) \qquad \sqrt{8t_0}\Lambda_{\overline{\rm MS}} = 0.397(26)$$

The leading N-dependence of the gluino condensate

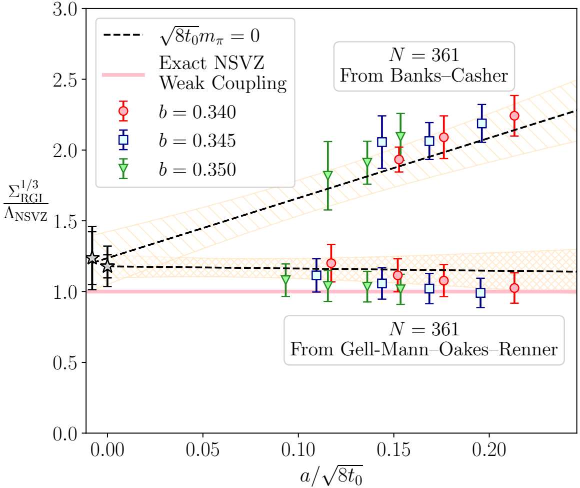


$\Sigma_{\mathrm{R}}/Z_{\mathrm{P}}$ renormalized via 2-loop perturbative $Z_{\mathrm{P}}^{(\overline{\mathrm{MS}})}(\mu = 1/a)$ in terms of improved couplings. Also RGI-conversion at 2-loop via improved couplings:

$$\Sigma_{\mathrm{RGI}} = \mathcal{A}\, 2b_0 \lambda_{\overline{\mathrm{MS}}}(\mu) \left[1 + \frac{d_1^{(\overline{\mathrm{MS}})} - 2b_1}{2b_0} \lambda_{\overline{\mathrm{MS}}}(\mu)\right] \Sigma_{\mathrm{R}}^{(\overline{\mathrm{MS}})}(\mu) \qquad \mathcal{A} = 8\pi^2/(9N^2)$$

Results for $N = 169, 289, 361$ fall on top of each other $\Longrightarrow$
our findings rule out all but the WC analytic NSVZ result

Gluino condensate in the SUSY (chiral-continuum) limit



Simultaneous chiral + continuum extrapolation of $N = 361$ determinations at finite values of lattice spacing and gluino mass $\longrightarrow$ SUSY limit

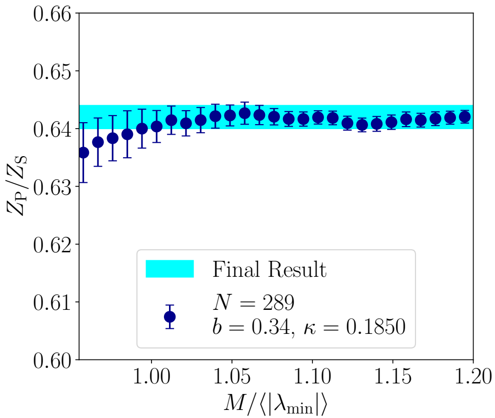
Final extrapolations have a conservative 30% systematic error due to the perturbative renormalization (dominant source of uncertainty)

$$\boldsymbol{\Sigma_{\mathrm{RGI}}/\Lambda^3_{\mathrm{NSVZ}} = [1.18(08)_{\mathrm{stat}}(12)_{\mathrm{syst}}]^3 = 1.64(60)}\ \textbf{(Lattice)}$$

$$\Sigma_{\mathrm{RGI}}/\Lambda^3_{\mathrm{NSVZ}} = 1\ \text{(Exact NSVZ analytic WC result)}$$

Back-up slides

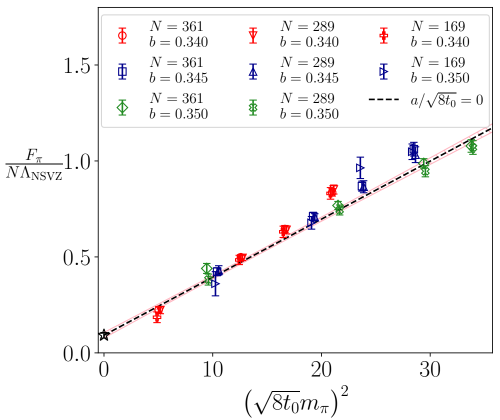
Calculation of $Z_{\mathrm{P}}/Z_{\mathrm{S}}$



From the same eigenproblem solved to obtain the mode number $\langle \nu(M) \rangle$ we also obtained $Z_{\mathrm{P}}/Z_{\mathrm{S}}$ non-perturbatively [Giusti & Lüscher JHEP**03** (2009) 013]

$$\left(\frac{Z_{\mathrm{P}}}{Z_{\mathrm{S}}}\right)^2 = \frac{\langle s_{\mathrm{P}}(M)\rangle}{\langle \nu(M)\rangle} \qquad s_{\mathrm{P}}(M) \equiv \sum_{|\lambda|,|\lambda'|\leq M} |u_\lambda^\dagger \gamma_5 u_{\lambda'}|^2,$$

Calculation of F_π in the SUSY limit



Too obtain the condensate from the GMOR relation, we need the pion decay constant, which can be obtained as usual from pion correlators.

Final result in the SUSY limit:

$$\frac{F_\pi}{N\Lambda_{\rm NSVZ}} = 0.092(14)$$