Test of a 2-level algorithm for the glueball spectrum in $SU(N_c)$ Yang-Mills theory

41st Lattice Conference
Liverpool July 29th 2024

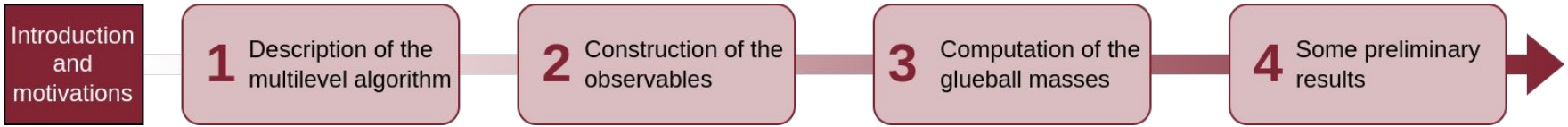
Andrea Falzetti

in collaboration with:
Mauro Papinutto
Francesco Scardino

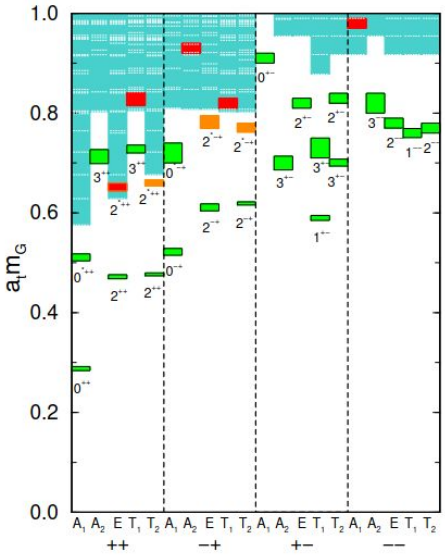
Abstract

We present preliminary results obtained using a new code for $SU(N_c)$ Yang-Mills theory which performs a 2-level sampling of glueball correlators obtained from a suitably chosen basis of (APE) smeared and unsmeared operators. The code builds loop operators of any shape and length and classifies them according to the irreducible representations of the cubic group. We report on the performances of the algorithm and on the computation of the first low-lying glueball state choosing $N_c = 3$ as a reference to compare our results with the literature.

Program of the talk:



Introduction and motivations



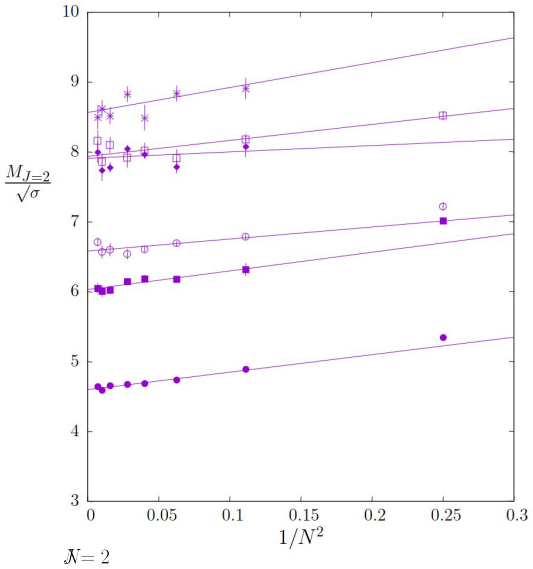
Schematic representation of the glueball spectrum.

"Glueball spectrum from an anisotropic lattice study", Morningstar, Peardon.

In Quantum Chromodynamics glueballs are bound states composed only of gluons, with no valence quarks. Their existence is a consequence of the non-abelianity of the gauge group $SU(3)$

These particles are also present in the spectra of pure gauge theories, such as Yang-Mills theories based on the Lie group $SU(N_c)$

Introduction and motivations



Large extrapolation of glueball masses in the channel.

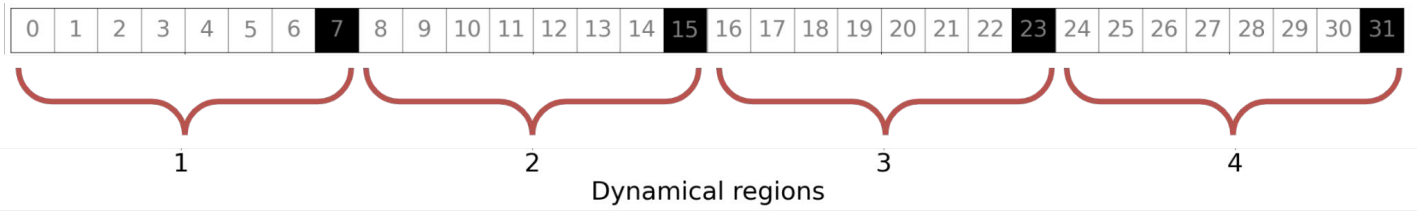
"SU(N) gauge theories in 3+1 dimensions: glueball spectrum, string tensions and topology"
Athenodorou, Teper (2021)

Detailed knowledge of the glueball spectrum in the $N_c \to \infty$ limit allows us to compare numerical results with models of analytic solutions of the theory.

Since the existence of these bound states is a non-perturbative phenomenon Montecarlo simulations are the preferred tool to compute the spectrum.

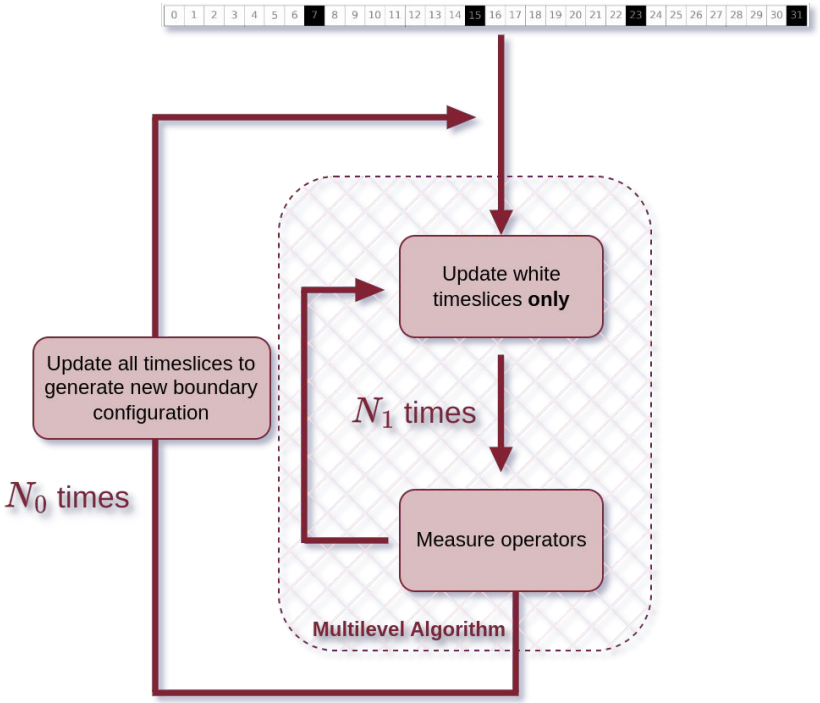
We are going to use a 2-level algorithm for 2 point correlators of gluonic operators.

The multilevel algorithm relies on the locality of the action that allows the path integral to be factorized in different spacetime regions over which one averages at fixed boundary conditions and subsequently one averages over the different boundary conditions.



In the temporal direction the lattice is subdivided in a number of identical dynamical regions.

When measuring, the gauge configuration on the boundaries of these regions is kept fixed, while multiple sub-measurements are performed in the interior of the regions.



Because of the locality of the action, when we are measuring correlators of operators at time slices that reside in different regions of the lattice the path integral will factorize into an average over boundary configurations.

Example:

$$\begin{cases} t_0 = 3a \rightarrow t_0 \in R_1 \\ t_1 = 10a \rightarrow t_1 \in R_2 \end{cases}$$

$$\begin{aligned} \langle \mathcal{O}_i(t_0)\mathcal{O}_j(t_1)\rangle &= \int \mathcal{D}U e^{-S[U]}\mathcal{O}_i(t_0)\mathcal{O}_j(t_1) = \\ &= \int \mathcal{D}U_{\partial R} e^{-S_{\partial R}[U_{\partial_R}]} \left(\int \mathcal{D}U_{R_1} e^{-S_{R_1}[U_{R_1}]}\mathcal{O}_i(t_0)\right)\left(\int \mathcal{D}U_{R_2} e^{-S_{R_2}[U_{R_2}]}\mathcal{O}_j(t_1)\right) = \\ &= \left\langle \langle \mathcal{O}_i(t_0)\rangle_{R_1} \langle \mathcal{O}_j(t_1)\rangle_{R_2} \right\rangle_{\partial R} \end{aligned}$$

This greatly reduces the error on the correlator, especially in the large time-separation region, which is also the most critical for reliable extraction of the glueball masses.

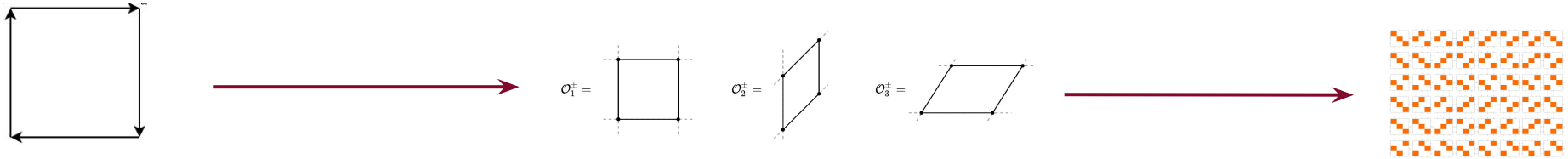
To compute the glueball spectrum we need operators transforming according to irreducible representations of the cubic group. This is done through the use of a Wolfram Mathematica code.

The user has to provide a list of “loop shapes” and the code will then proceed to generate all possible operators and classify them in the irreducible representations of O_h

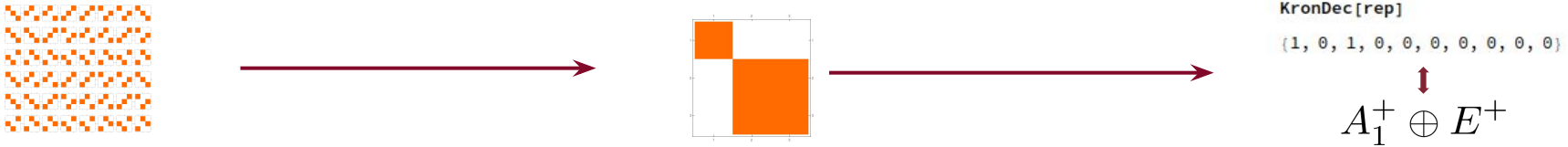
J	O Representations				
	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{E}$	$\mathbf{T_1}$	$\mathbf{T_2}$
0	1	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	1	0	1	1
4	1	0	1	1	1
5	0	0	1	2	1
6	1	1	1	1	2
7	0	1	1	2	2
8	1	0	2	2	2
9	1	1	1	3	2
10	1	1	2	2	3
11	0	1	2	3	3
12	2	1	2	3	3

Once we know the irreducible representation of the cubic group according to which an operator transforms we can use the subduction method to predict the possible spin quantum numbers of the operator in the continuum limit.

The code computes the action of all the group elements on the original “loop shape” to build a basis of wilson loop for the representation. Then proceeds to construct the full reducible representation.



The representation is then reduced, applying the usual methods of linear algebra. Once the representation is reduced in block-diagonal form, the code performs character table similarity analysis on each block to accurately assign each operator to the correct representation

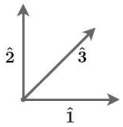


From the columns of the basis change matrix that brings the representation in block diagonal form we can read the linear combinations that corresponds to operator that transform according to irreducible representations of the cubic group.

Then we can confront with the Kronecker decomposition of the representation to assign each operator to the correct lattice quantum number.

$$\mathcal{O}_1^E =$$

$$\mathcal{O}_2^E =$$

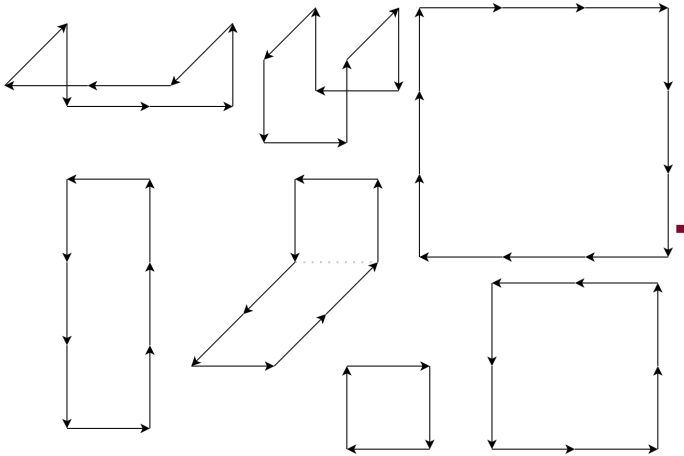


$$\mathcal{O}^{A_1} =$$

This process allows us to construct operators starting from wilson loops of any length and shape.

For the preliminary results presented in this talk we are using a basis made of wilson loops of length up to $\ell = 12$ computed at 4 different APE smearing levels. The basis has been chosen with the aim of having an acceptable number of operators in the most relevant representations for low-lying glueball states.

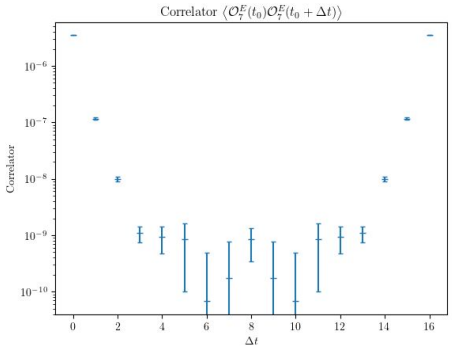
Our operator basis:



J^{PC}	N_{ops}
A_1^{++}	7
A_2^{++}	2
E^{++}	18
T_1^{++}	3
T_2^{++}	6
T_1^{-+}	9
T_2^{-+}	12

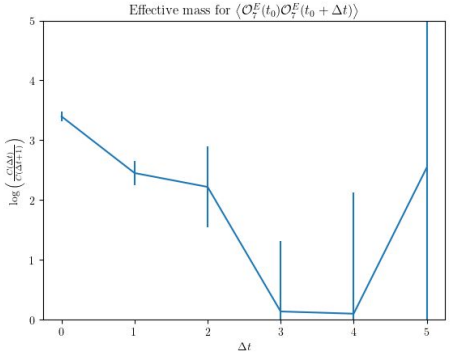
The glueball masses can be extracted from the exponential decay of the correlator in the large time-separation region.

$$
\begin{aligned}
C_{ij}(\Delta t) &= \langle \mathcal{O}_i(t_0)\mathcal{O}_j(t_0+\Delta t)\rangle \\
&= \sum_{n=1} \langle 0|\, \mathcal{O}_i(t_0)\, |n\rangle \, \langle n|\, \mathcal{O}_j(t_0+\Delta t)\, |0\rangle = \\
&= \sum_{n=1} 2e^{-am_n \cdot \frac{L_t}{2}} \, \langle 0|\, \mathcal{O}_i(0)\, |n\rangle \, \langle n|\, \mathcal{O}_j(0)\, |n\rangle \cosh\left(am_n\left(\Delta t - \frac{L_t}{2}\right)\right)
\end{aligned}
$$



Usually this is done by looking at the effective mass plateau.

$$
a \cdot m_{eff} = \log\left(\frac{C(t)}{C(t+1)}\right)
$$

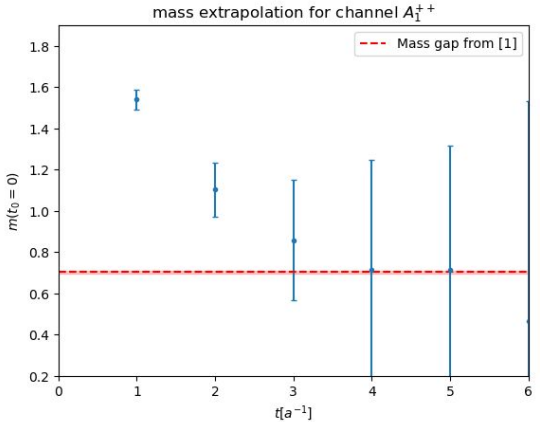


We will use the GEVP (**G**eneralized **E**igen**V**alue **P**roblem) method which is a variational technique used to extract the spectrum from the generalized eigenvalues of the correlator matrix:

$$C(t)\psi_b = \lambda_b(t,t_0)C(t_0)\psi_b \longrightarrow \lambda_b(t,t_0) = e^{-am_b(t-t_0)}$$

In the $t_0, t \gg a$ limit we will be able to get the spectrum from the relation:

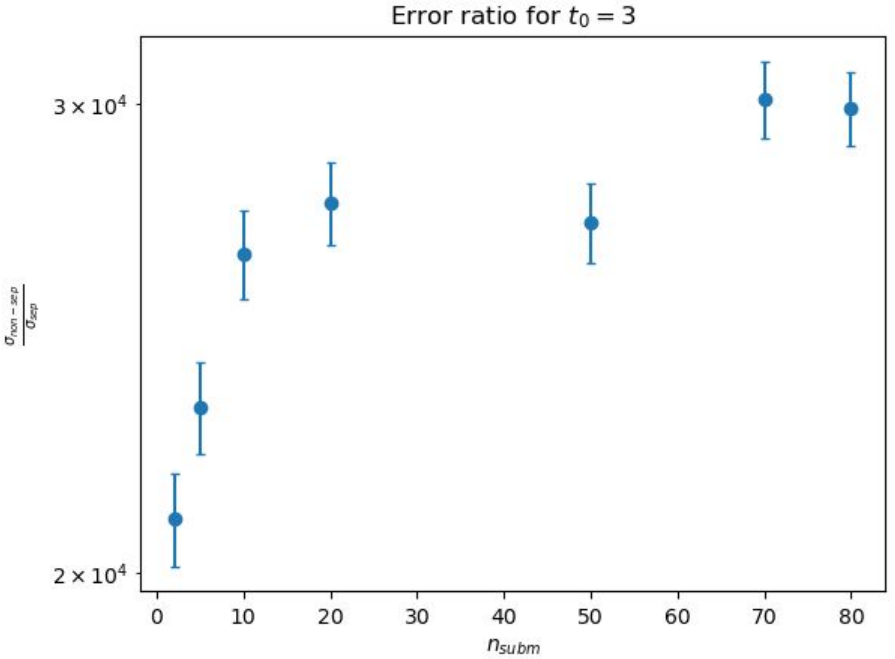
$$a \cdot m_b = \log\left(\frac{\lambda_b(t,t_0)}{\lambda_b(t+1,t_0)}\right)$$

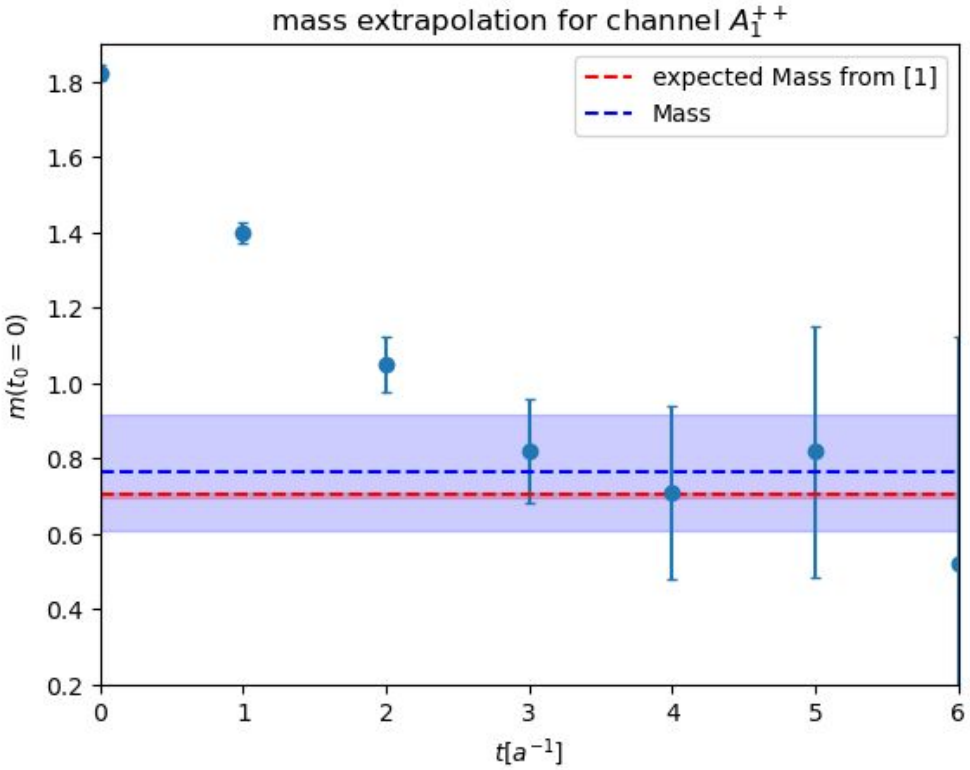


[1] SU(N) gauge theories in 3+1 dimensions: glueball spectrum, string tensions and topology, Andreas Athenodorou, Michael Teper

We compare the errors on the correlators obtained using the multilevel algorithm with the ones where t and $t_0$belong to the same region.

$$\begin{cases} \sigma_{\text{non-sep}} =< \sigma_C(t_0,t) > & t_0 \text{ and } t \text{ in the same region} \\ \sigma_{sep} =< \sigma_C(t_0,t) > & t_0 \text{ and } t \text{ in different regions} \end{cases}$$




mass extrapolation for channel A_1^{++}
expected Mass from [1]
Mass
$m(t_0 = 0)$
$t[a^{-1}]$
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6

Thank you for your kind attention!