Confining strings in three-dimensional gauge theories beyond the Nambu–Gotō approximation

Dario Panfalone

Based on the work: arXiv:2407.10678

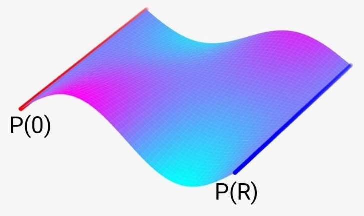
M. Caselle, N. Magnoli, A. Nada, M. Panero, D. Panfalone, L. Verzichelli.

University of Turin

Introduction and motivation

Polyakov loop correlator:

$$G(R) \sim \int DXe^{-S_{EFT}(X)} \equiv Z_{EST}$$



Nambu-Gotō (NG) string model:

$$S_{NG} = \sigma_0 \int_{\Sigma} d^2\xi \sqrt{g}$$

The Polyakov loop correlator at large distance is dominated by:

$$G(R) \sim K_0(E_0 R), \qquad E_0 = \sigma_0 L_t \sqrt{1 - \frac{\pi}{3\sigma_0 L_t^2}}$$

Introduction and motivation

In this picture it is natural to define:

$$\frac{T_{c,NG}}{\sqrt{\sigma_0}} = \sqrt{\frac{3}{\pi}}$$

- Critical index $\nu = 1/2$
- This prediction is close to the correct one, but wrong.

To obtain the correct EST describing the gauge theory, it is essential to go **beyond the NG approximation**...
Discovering the terms beyond the Nambu-Gotō approximation is one of the major open challenges in this context.

EST corrections beyond Nambu-Gotō

- **Low-energy universality of the EST** $\Longrightarrow$ First correction BNG can only appear at the order $1/N_t^7$
- **S-matrix approach** $\Longrightarrow$ First two correction ($1/N_t^7$ and $1/N_t^9$ terms) are controlled by the same parameter k_4, next parameter k_5 appears at $1/N_t^{11}$ order.

In the high-temperature regime, the corrections to the NG approximation up to $1/N_t^{11}$ order can be parameterized as:

$$aE_0(N_t) = N_t\sigma_0 a^2\sqrt{1-\frac{\pi}{3N_t^2\,\sigma_0 a^2}} + \frac{k_4}{(\sigma_0 a^2)^3 N_t^7} + \frac{2\pi k_4}{3(\sigma_0 a^2)^4 N_t^9} + \frac{5\pi^2 k_4}{16(\sigma_0 a^2)^5 N_t^{11}} + \frac{k_5}{(\sigma_0 a^2)^5 N_t^{11}} + \dots$$

EST corrections BNG on the lattice

- Following the work in SU(2) [arXiv:2109.06212] from $G(R)$ we extracted the ground state as $E_0 = 1/\xi$ and we studied the behavior of E_0 varying N_t.

In the SU(3) case we evalueted ξ:

- Starting from EST predictions $\Longrightarrow$ From long-distance fit with the EST functional form

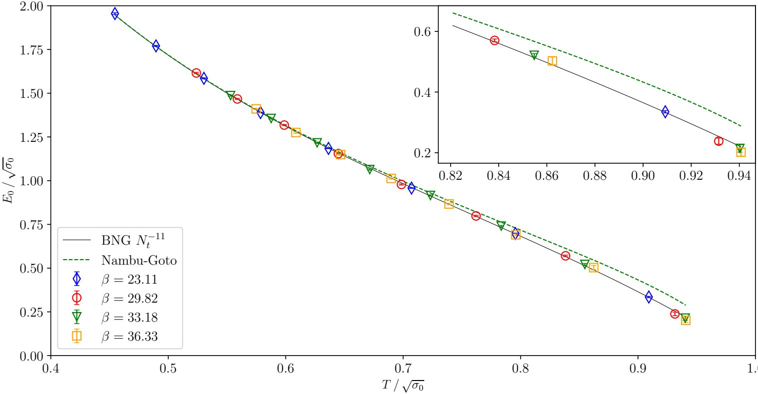
$$G(R) = k_l \left[K_0 \left(\frac{R}{\xi_l} \right) + K_0 \left(\frac{L_s - R}{\xi_l} \right) \right] \qquad \xi_l \equiv 1/E_0$$

- Svetitsky–Yaffe mapping $\Longrightarrow$ Short-distance fit with the spin-spin correlator of the three state Potts model in $(1+1)d$

[See talk by L.Verzichelli]

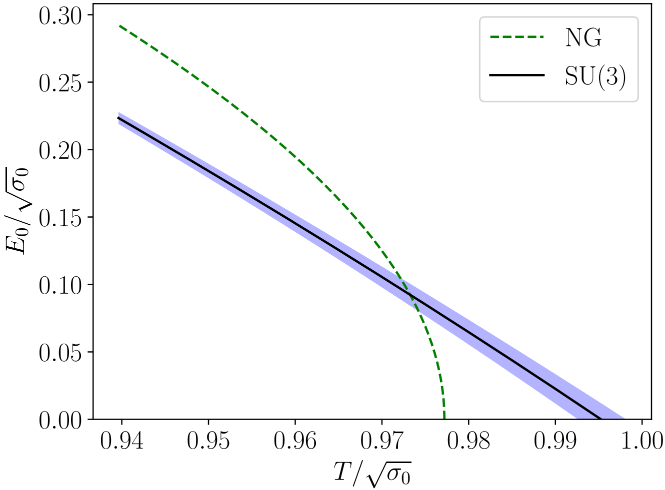
The agreement supports the validity of S-Y mapping and the robustness of our determination of E_0.

SU(3) numerical results



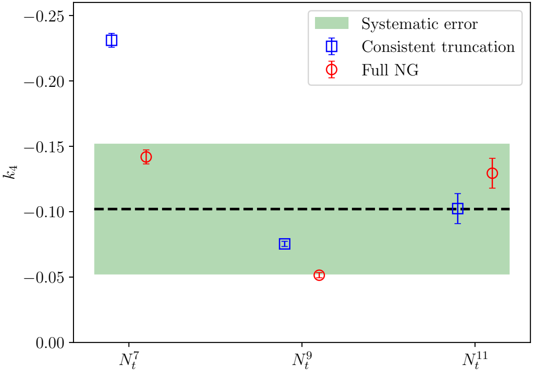
Combined best fits of the SU(3) data including all terms up to $1/N_t^{11}$

SU(3) numerical results



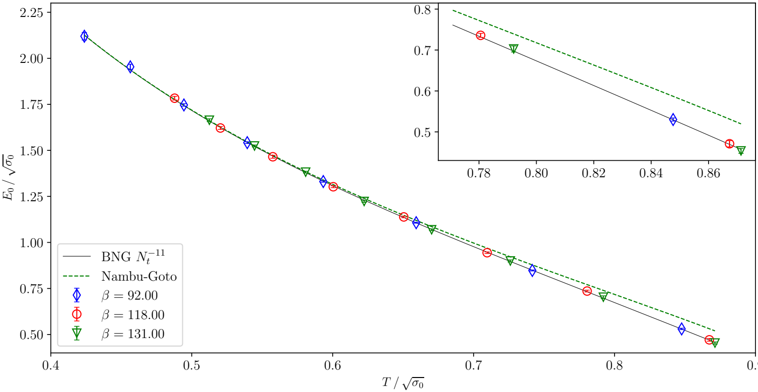
$T_{E_0=0} = 0.995(5)\sqrt{\sigma_0}$
compatible with $T_c = 0.9890(31)\sqrt{\sigma_0}$ found in [arXiv:0803.2128]

SU(3) numerical results



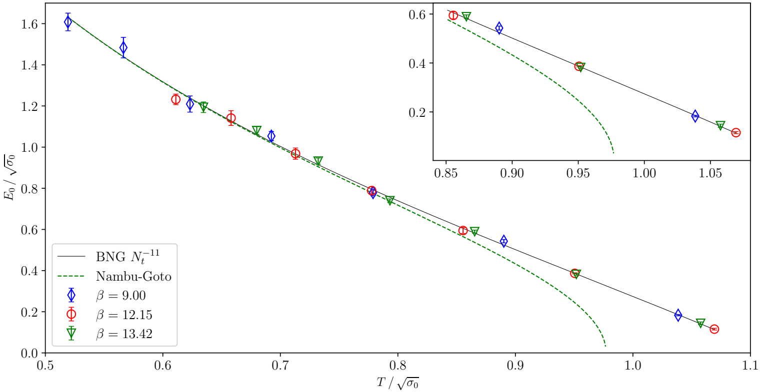
Final results: $k_4 = -0.102(11)[50], \quad k_5 = 0.45(8)[25]$

SU(6) numerical results



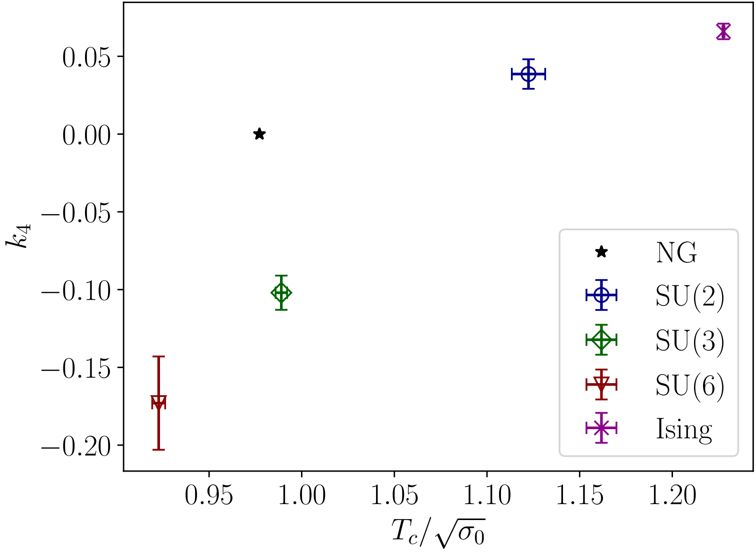
Final results: $k_4 = -0.173(30)[79], \quad k_5 = 0.98(23)[15].$

Reanalysis of SU(2)



Final results: $k_4 = 0.0386(95)[121], \quad k_5 = -0.123(52).$

Summary of SU(N) corrections



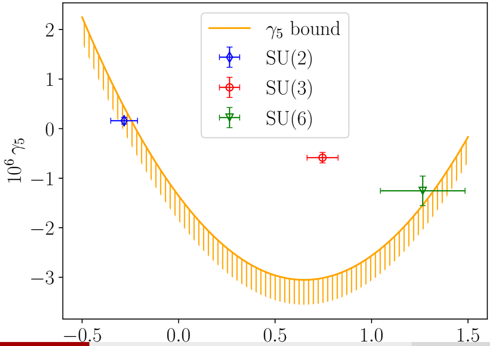
Comparison with bootstrap constraints

$$\gamma_3 = -\frac{225}{32\pi^6} k_4, \quad \gamma_5 = -\frac{3969}{32768\pi^{10}} k_5 :$$

	$\gamma_3 \times 10^3$	$\gamma_5 \times 10^6$
SU(2)	−0.282(70)[89]	0.159(66)
SU(3)	0.746(80)[365]	−0.58(11)[32]
SU(6)	1.26(22)[58]	−1.25(30)[19]

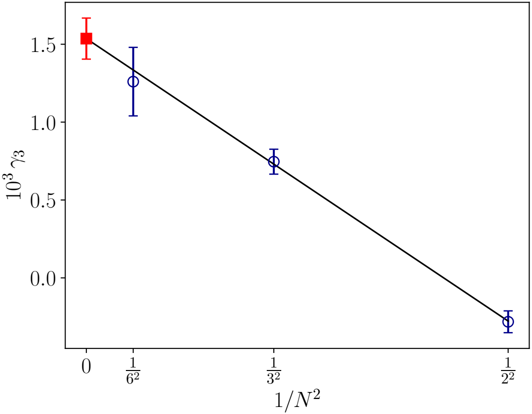
[ArXiv:1906.08098]

$$\gamma_3 > -\frac{1}{768} \simeq -0.0013$$



Comparison with bootstrap constraints

$$\gamma_3^{(N=\infty)} + \frac{c}{N^2}$$



$$\gamma_3^{(N=\infty)} = 1.54(13) \times 10^{-3}$$

Conclusion

- Following the approach used for the SU(2) gauge theory, we investigated the cases of the theories with $N = 3$ and 6 and improved the $N = 2$ case
- Final results are reported with their statistical and systematic uncertainties
- Our estimates are in agreement with the bounds obtained from the bootstrap analysis
- We also performed a new high-precision test of the Svetitsky–Yaffe conjecture

Conclusion

- Following the approach used for the SU(2) gauge theory, we investigated the cases of the theories with $N = 3$ and 6 and improved the $N = 2$ case
- Final results are reported with their statistical and systematic uncertainties
- Our estimates are in agreement with the bounds obtained from the bootstrap analysis
- We also performed a new high-precision test of the Svetitsky–Yaffe conjecture

Thank you for your attention!

SU(3) Numerical results

β	N_t	N_s	T/T_c	n_{conf}
36.33	11	240	0.95	6.2×10^4
	12	160	0.87	1.1×10^5
	13	96	0.81	3×10^5
	14	96	0.75	3×10^5
	15	96	0.70	3×10^5
	16	96	0.66	2.7×10^5
	17	96	0.62	2.6×10^5
	18	96	0.58	2.2×10^5

	β	$N_{t,min}$	$N_{t,max}$	k_4	k_5	$\sigma_0 a^2$	$\chi^2/N_{d.o.f.}$	$\sigma_0 a^2$ from lit.
up to N_t^{-11}	23.11	7	14	-0.126(17)	0.63(12)	0.024704(18)	1.9	0.024701(80)
	29.82	9	16	-0.10(3)	0.42(18)	0.014233(16)	2.9	0.014213(51)
	33.18	10	17	-0.04(3)	0.1(2)	0.011286(16)	1.4	0.011339(53)
	36.33	11	18	-0.10(3)	0.4(2)	0.009344(17)	1.6	0.009381(56)

SU(6) Numerical results

β	N_t	N_s	T/T_c	n_{conf}
131	10	160	0.95	1.2×10^5
	11	96	0.87	1.2×10^5
	12	96	0.79	1.2×10^5
	13	96	0.73	1.2×10^5
	14	96	0.68	1.2×10^5
	15	96	0.63	1.2×10^5
	16	96	0.60	1.2×10^5
	17	96	0.56	1.2×10^5

	β	$N_{t,min}$	$N_{t,max}$	k_4	k_5	$\sigma_0 a^2$	$\chi^2/N_{d.o.f.}$	$\sigma_0 a^2$ from lit.
up to N_t^{-11}	92	7	14	-0.20(5)	1.20(4)	0.02839(3)	0.5	0.02842(5)
	118	9	16	-0.16(5)	0.9(4)	0.01640(4)	0.7	0.016302(48)
	131	10	17	-0.16(6)	0.9(4)	0.01317(3)	0.8	0.013005(43)