Absence of CP Violation in the Strong Interaction

Gerrit Schierholz

Deutsches Elektronen-Synchrotron DESY

arXiv:2106.11369

arXiv:2212.05485

with Y. Nakamura

arXiv:2212.05485

arXiv:2307.08310

arXiv:2403.13508

The problem

QCD allows for a CP-violating term S_θ in the action

$$S = S_0 + S_\theta\,, \quad S_\theta = i\,\theta\,Q$$

called the θ term, where Q is the topological charge

$$Q = \frac{1}{32\pi^2}\,\epsilon_{\mu\nu\rho\sigma}\int d^4x\,\mathrm{Tr}\,[F_{\mu\nu}F_{\rho\sigma}] \;\in\; \mathbb{Z}$$

and $\theta \in [-\pi, \pi)$ is the vacuum angle

We consider Euclidean space-time. Lattice regularization in finite volume V is applied throughout (although continuum notion is used sometimes)

- ⋆ Path integral is properly regularized
- ⋆ Gauge invariant
- ⋆ Topological charge Q is well defined

A finite value of θ is expected to result in an electric dipole moment $\underline{d_n \propto \theta}$ of the neutron, which violates CP and P. To date the most sensitive measurements of d_n are compatible with zero. The current upper bound is $|d_n| < 1.8 \times 10^{-13} e\,\mathrm{fm}$, indicating that θ is anomalously small

Why should a parameter not forbidden by symmetry be essentially zero? In the narrower sense, this puzzle is referred to as the strong CP problem

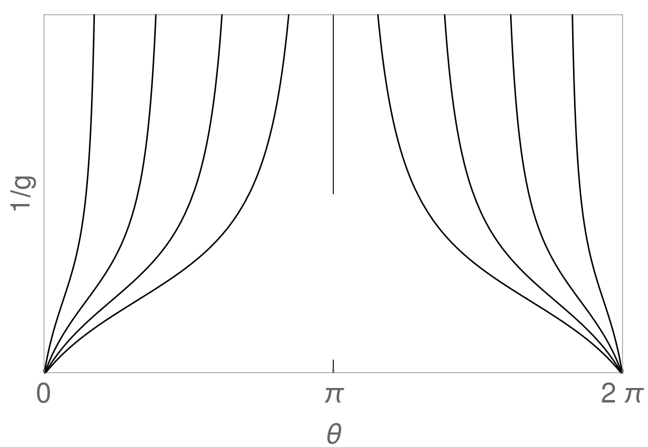
There are two separate issues

⋆ To solve the puzzle of the vanishing electric dipole moment of the neutron, it is sufficient to show that local operators, like the electromagnetic current, are not correlated with the topological charge – at least in the thermodynamic limit

⋆ If this is the case, it does not mean though that θ has no effect on the general properties of QCD, like confinement. The problem thus remains

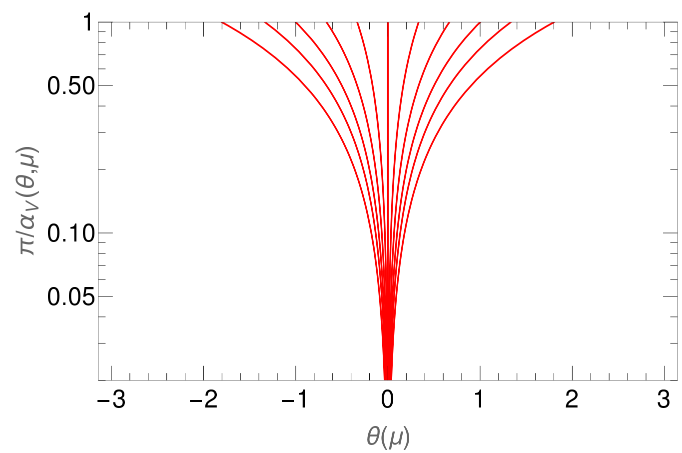
Actually, there is strong evidence for a highly nontrivial dependence of QCD on θ, based on secured knowledge

CP^N



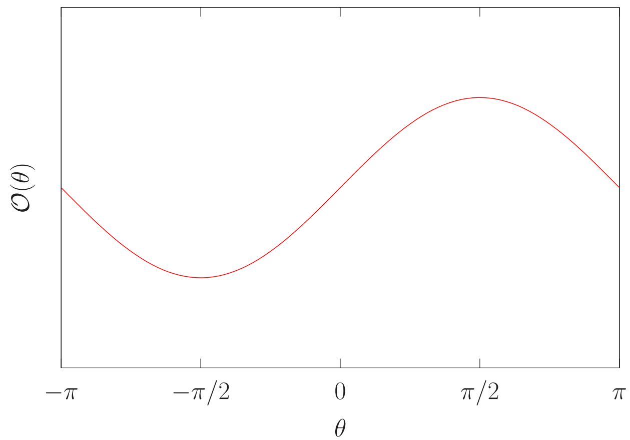
Pruisken, Levine, Libby; Knizhnik, Morozov

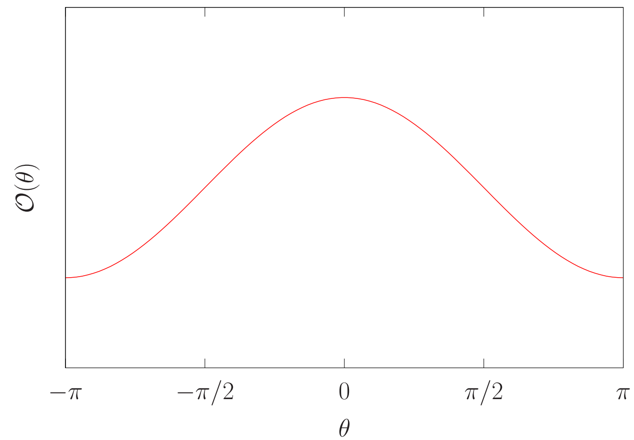
SU(3) YM



arXiv:2212.05485

Misconception of Strong CP problem





CP violated

$$\boxed{\propto \frac{\langle Q \rangle}{V} \simeq \frac{1}{\sqrt{V}}}$$

CP conserved

$$\boxed{\propto \frac{\langle Q^2 \rangle}{V} \simeq \text{const.}}$$

CP (Non)Violation

We are interested in n-point correlation functions of operators $\mathcal{O}_i$ at nonvanishing values of θ, which read

$$\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_\theta = \langle e^{i\,\theta\,Q}\,\mathcal{O}_1 \cdots \mathcal{O}_n \rangle = \sum_Q e^{i\,\theta\,Q}\,P(Q)\,\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_Q$$

where $P(Q) = Z_Q/Z$

disconnected sectors
of charge Q

Need to know Topological Charge

Continuum   on S_4, for example

$$Q = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\int d^4x\,\mathrm{Tr}\,[F_{\mu\nu}F_{\rho\sigma}]$$

Anomaly

$$Q = -\sum_i \int d^4x\,u_i^\dagger(x)\gamma_5 u_i(x)$$

$$\not{D} u_i(x) = 0$$

Lattice

Gradient flow

$$Q \underset{t>0}{=} \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\int d^4x\,\mathrm{Tr}\,[F_{\mu\nu}F_{\rho\sigma}]\;, \quad \partial_t Q = 0$$

$$Q = -\sum_{\{\lambda=0\}} (u_\lambda, \gamma_5 u_\lambda)\;, \quad \lambda = 1 - e^{i\phi}$$

$$D_N u_\lambda = \lambda u_\lambda$$ Hasenfratz, Laliena & Niedermayer

Index Theorem

Atiyah & Singer

Assuming there are n_+ zero modes of positive chirality, $\gamma_5 u_i(x) = +u_i(x)$, and n_- zero modes of negative chirality, $\gamma_5 u_i(x) = -u_i(x)$, it then follows that $Q = n_- - n_+$

Vanishing Theorem

It turns out that any gauge field configuration of positive (negative) charge Q has exactly n_- (n_+) zero modes, but never zero modes of both chiralities

In mathematical language: dim ker $i\not{D} = |n_+ - n_-|$

Proofs

2D — Nielsen & Schroer

4D (anti-)self dual — Brown, Carlitz & Lee

4D geneneric connections — Maier

Lattice — Chiu; Ilgenfritz et al; Di Giacomo & Hasegawa

Gradient flow

Effective theory — Leutwyler & Smilga

Assuming

$$P(Q) = \frac{1}{\sqrt{2\pi\langle Q^2\rangle}} e^{-Q^2/2\langle Q^2\rangle}\,, \;\; |Q| = n$$

It follows

$$\chi_t = \frac{\langle Q^2\rangle}{V}\,, \quad \frac{\langle n\rangle}{V} = \sqrt{\frac{2}{\pi}}\sqrt{\frac{\chi_t}{V}}$$

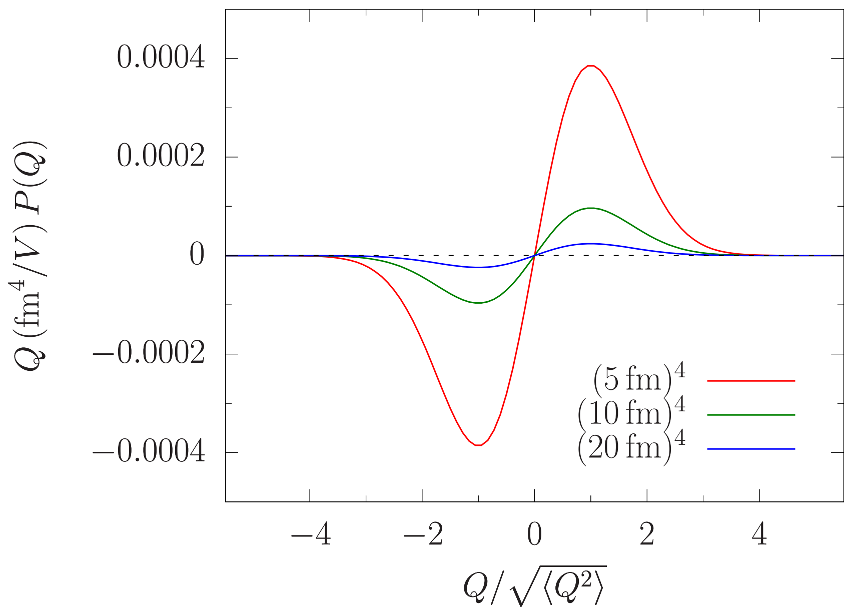
Dilute Instanton Gas

The topological charge distribution P_Q is obtained from a convolution of separate Poisson distributions for instantons and anti-instantons

$$P_Q = \sum_{n=0}^{\infty} \tilde{P}_n \, \tilde{P}_{Q-n} \,, \quad \tilde{P}_n = \tilde{P}_{-n} = \frac{\langle Q^2/2\rangle^n e^{-\langle Q^2/2\rangle}}{n!} \qquad \sum_Q Q^2 P_Q = \langle Q^2 \rangle$$

Thus, the toplogical charge (in form of $\langle Q^2 \rangle$) is still an important measure of the QCD vacuum

The fraction of zero modes to be expected in a subvolume of one fm^4 as a function of $Q = \pm n$ for total volumes V of $(5\,\mathrm{fm})^4$, $(10\,\mathrm{fm})^4$ and $(20\,\mathrm{fm})^4$:



Of particular interest is the electric dipole moment

$$\vec{d}_n = \frac{\int d^3\vec{x}\, d^3\vec{y}\, e^{i\vec{p}\vec{x}} \langle N(\vec{x}, x_0)\, \vec{y}\, J_0(\vec{y}, y_0)\, \bar{N}(0)\rangle_\theta}{\int d^3x\, e^{i\vec{p}\vec{x}} \langle N(\vec{x}, x_0)\, \bar{N}(0)\rangle_\theta}$$

Traces are suppressed

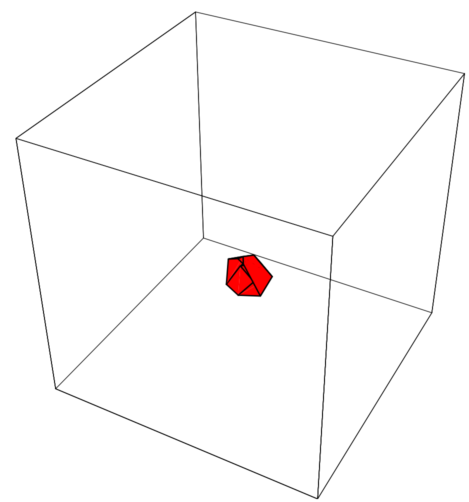
where J_0 is the time component of the electromagnetic current, and x_0, y_0 are chosen so that $x_0 \gg y_0 \gg 0$. Treating the θ term as a perturbation at first order, this reduces to

$$\vec{d}_n = i\,\theta\, \frac{\int d^3\vec{x}\, d^3\vec{y}\, e^{i\vec{p}\vec{x}} \langle (\sum_i \int d^4z\, u_i^\dagger(z)\gamma_5 u_i(z))\, N(\vec{x}, x_0)\, \vec{y}\, J_0(\vec{y}, y_0)\, \bar{N}(0)\rangle}{\int d^3x\, e^{i\vec{p}\vec{x}} \langle N(\vec{x}, x_0)\, \bar{N}(0)\rangle}$$

A nonvanishing dipole moment arises from the interaction of the nucleon with the zero modes, which have been shown to be highly localized

Ilgenfritz et al

Thus, the interaction can be visualized by a correlation function of two rather local operators

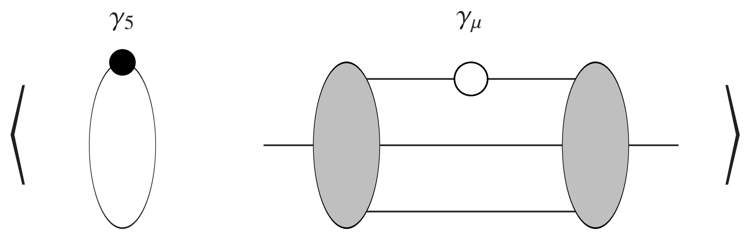


$|u_i(x)|^2$

Chiral Ward identity

Zero mode

$$\downarrow$$

$\vec{d}_n \;=$ 

Disconnected diagram

Quark propagator

$$S(x,y) = \sum_\lambda \frac{u_\lambda(x) u_\lambda^\dagger(y)}{i\lambda + m}$$

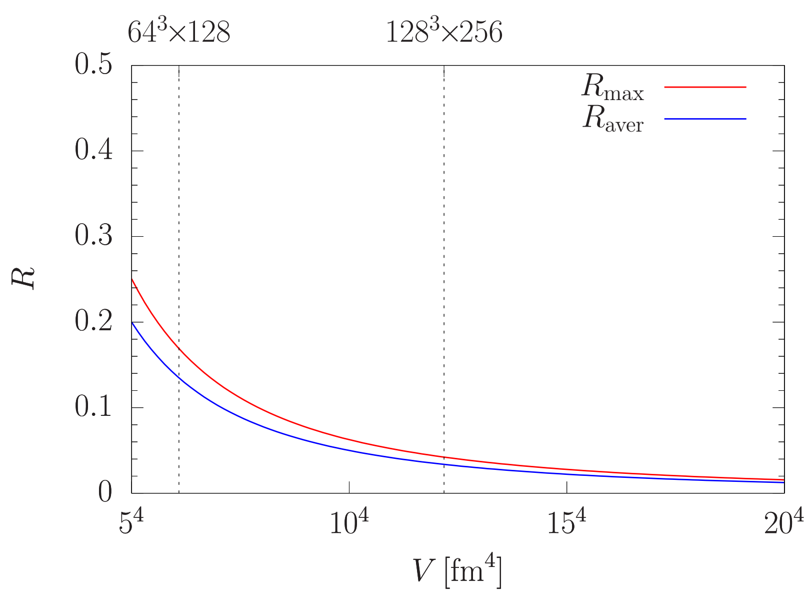
$$(u_\lambda, \gamma_5 u_\lambda) = 0 \quad \text{for} \quad \lambda \neq 0$$

Guadagnoli et al.

We can assume that the strong interactions are largely confined to a lattice volume of $V_0 = (2.5\,\mathrm{fm})^4$. At a lattice spacing of $a = 0.08\,\mathrm{fm}$ that corresponds to a 32^4 sublattice

The all-important question is: What is the probability to find a zero mode (or instanton) in the interacting range V_0 of the nucleon?

We can expect at most $R_{\mathrm{max}} = \sqrt{\langle Q^2 \rangle}(V_0/V)$ zero modes in V_0. The average number is a little smaller



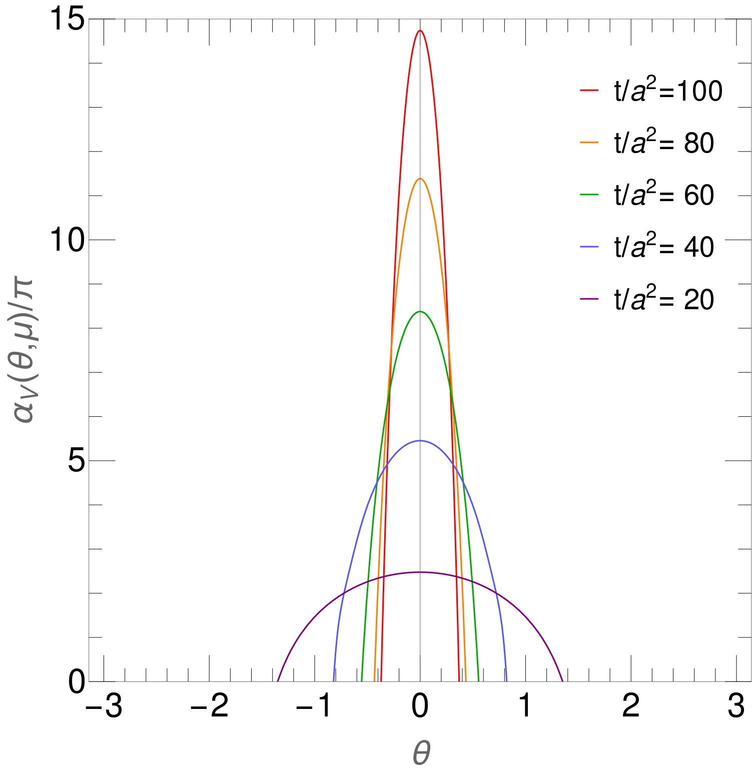
$$\chi_t = (79\,\mathrm{MeV})^4$$

As a result, the vacuum of V_0 is becoming more and more CP-neutral, making the dipole moment vanish proportional to $1/\sqrt{V}$, i.e.

$$|d_n| \propto \sqrt{\frac{\chi_t}{V}}\,\theta$$

Corresponding estimates apply to other hadronic observables, for example the CP-violating pion-nucleon coupling constant $\bar{g}_{\pi NN}$. This leads us to conclude that CP is conserved in the strong interactions

Back to RG flow



$\uparrow$

$$\frac{\alpha}{\pi}\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle \quad \text{RGinv}$$

- ⋆ The flow to confinement is constricted to the inner part of the envelope of the curves, that is to stay within QCD
- ⋆ The color charge gets totally screened for $|\theta| > 0$ in the infrared limit, while it becomes gradually independent of θ as we approach the perturbative regime
- ⋆ Leads to the screening length of the color charge $\lambda_c = \sqrt{E_F/\rho e^2} \approx 0.5/\theta\,[\mathrm{fm}]$

$$e = \theta/2\pi \qquad \text{Witten}$$

Hadrons will disintegrate only once λ_c has reached a value smaller than the hadron radius

Conclusions

- ⋆ In the presence of the θ term one hast to distinguish between CP violation and changes of the vacuum. While CP violations are associated with observables that are an odd function of θ, effects on the vacuum structure are generally CP even

- ⋆ CP violation in hadronic processes arises from the interaction with excessive, unpaired (anti-)instantons, whose density tends to zero in the infinite volume

- ⋆ The vacuum will change as soon as the characteristics of the individual topological sectors are significantly different. This is the case for the energy density, which becomes proportional to |Q| in the infrared

- ⋆ The screening length of the color charge is $\lambda_c \propto 1/|\theta|$, so that hadrons will disintegrate only when λ_c has reached a value smaller than the hadron radius, similar to the finite temperature phase transition, where $\lambda_t \propto 1/|T - T_c|$

- ⋆ In an external field with, or equivalent to, $|\theta| > 0$, the color charge will be screened, leading perhaps to the 'oblique' phases advocated by 't Hooft

Literature

Reuter arXiv:hep-th/9604124

derives

$$\theta(\mu = 0) = 0 \quad \text{for} \quad \alpha_s(\mu = 0) = \infty$$

based on an exact RG evolution equation à la Wetterich

Knizhnik and Morozov JETP Lett. 39 (1984) 240

derive

$$\frac{\partial(1/g^2)}{\partial \ln \mu} = C + \bar{D}\cos\theta$$
$$\frac{\partial\theta}{\partial \ln \mu} = 8\pi^2 \bar{D}\sin\theta$$

from the instanton density in an external field à la SVZ. This result is in remarkable agreement with our result for $\bar{D} \approx 1/32\pi^2$ (corresponds to $D = 1/8$)

QHE (see Introduction) arXiv:cond-mat/0101003