Fractional Instantons and Confinement: a $T_2 \times \mathbb{R}^2$ roadmap

A. González-Arroyo
(in coll. with Georg Bergner(JENA) and Ivan Soler(PISA))

Instituto de Física Teórica UAM/CSIC
Departamento de Física Teórica, UAM

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Motivation

MAIN GOALS

- Explore the validity of a semiclassical approach that could account for some of the non-perturbative properties of the Yang-Mills vacuum
- One such proposal is the Fractional instanton liquid model proposed by me and my students (Garcia-Perez, Martinez and Montero) in 1994: Such a model can account for and relate the topological susceptibility and the string tension (Microscopic mechanism for confinement)
- Semiclassical ideas have received a new boost with concepts like ressurgence and several authors (Unsal, Poppitz, etc) advocated their relevance for certain gauge theories.

Old history

- A way to connect semiclassics with non-perturbative dynamics was proposed by Luscher(1982) for 2D non-gauge asymptotically free theories: Use the size of space as an interpolating parameter
- Luscher, van Baal and others applied this to Yang-Mills theory in a finite box (in particular to the glueball spectrum)
- Triggered by the volume independence ideas of large N Korthals-Altes and me started to do the same on a box with 't Hooft twisted boundary conditions.
- Perturbation theory is not enough. Some solutions of the classical equations are needed: they are fractional instantons. Certain NP properties can be computed analytically in terms of a 1D gas of FI for $l_s\Lambda << 1$.
- For larger sizes the gas becomes non-dilute (a liquid) and 4 dimensional. This transition is smooth and led to the FI-liquid model of the Yang-Mills vacuum ($l_s\Lambda \gg 1$).

Main strategy: Various alternatives

- $T_3 \times \mathbb{R}$ The old idea. No phase transition. An effective 1D model.
- $S_1 \times \mathbb{R}^3$ Several authors (Unsal, Poppitz, ...) have used a similar strategy using l_T as a tuning parameter connecting the semiclassical regime with infinite volume. Problems: No 't Hooft flux. Phase transitions in some theories
- $T_2 \times \mathbb{R}^2$ This is our present work. Allows 't Hooft flux which might prevent a phase transition. A study proposed by several authors including ourselves. An effective 2D theory.

Semiclassical methods

Path integral expanded as a sum over action extrema and fluctuations around.

$$Z = \sum_C e^{-\Gamma(C)} \quad ; \quad \Gamma(C) = S(C) + Q(C)$$

- This works well in low-dimensional examples as well as for weak coupling.
- In fact it is a TRANSSERIES and C is not necessarily a classical solution
- Very often the configuration is made up of basic local units $C = \bigcup_i C_i(x_i)$
- For dilute situations the distribution is Poissonian and the total weight is additive $\Gamma(C) = \sum \Gamma(C_i(x_i))$.

The basic building blocks (SU(2) case)

$T_3 \times \mathbb{R}$ (Garcia Perez-AGA 1993)

(anti)-self-dual conf has $Q = 1/2$ ($S = 4\pi^2/g^2$).
It tunnels between two classical vacua
Induces mass gap in Polyakov loop correlators (string tension)

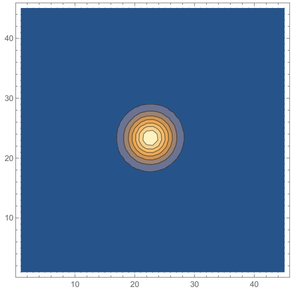
$T_2 \times \mathbb{R}^2$ (AGA-Montero 1998)

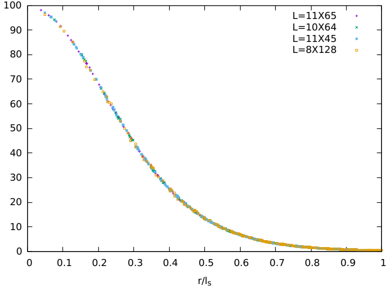
(anti)-Self-dual conf has $Q = 1/2$ ($S = 4\pi^2/g^2$).
Circular symmetric profile in $\mathbb{R}^2$ decaying exponentially with distance
At large distances from center behaves like a Z_2 vortex.
The configuration is unique modulo translations.

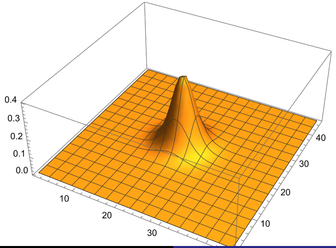
$S_1 \times \mathbb{R}$ (Kraan, vanBaal, Lee, Lu (1998)

Analytical formula for $Q = 1$. More complex since the fractional components are not necessarily $Q = 1/2$. At large distances the field decays as monopole (power-like)

Basic vortex-like fractional instanton (SU(2) case)

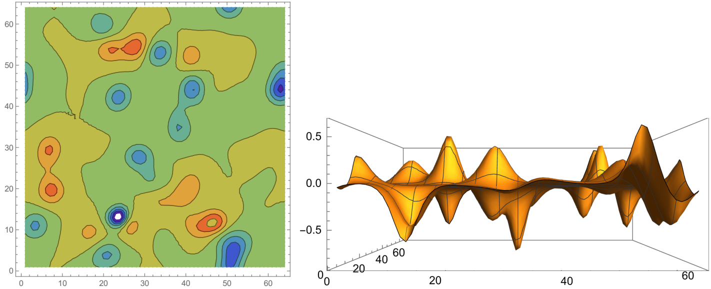




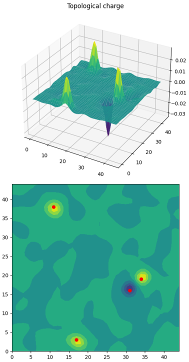


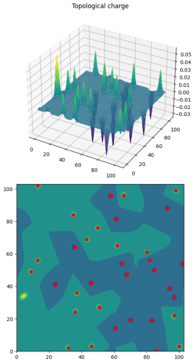
Our new results

We have Monte Carlo generated many SU(2) gauge Configurations with Wilson action and a large range of β values for a lattice $L_s^2 \times L_t^2$ with $L_s \ll L_t$ and twisted BC in the small torus



Snapshots

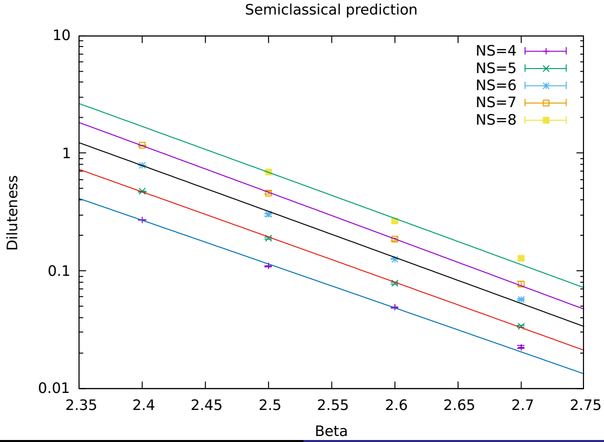




Topological charge density integrated over the small torus (2D distribution)

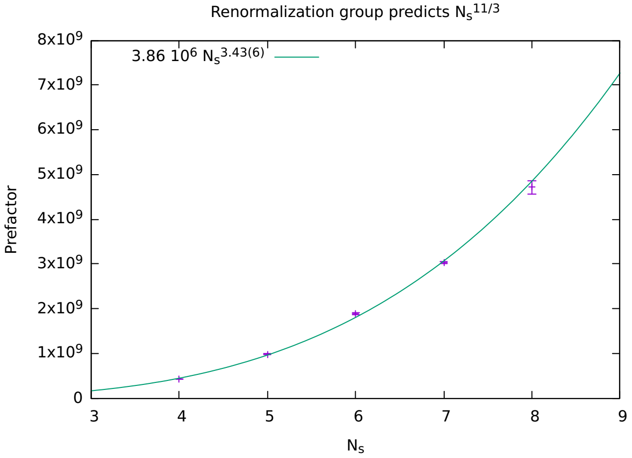
Distribution of the fractional instanton/vortex gas

Let us define Diluteness: $D(\beta, L_s) = (N_{FI} + N_{AFI}) L_s^2 / L_t^2$.
It is Dimensionless. We expect this quantity to decrease with the Boltzman factor: $D = \mathcal{A}(L_s)\beta^2 \exp\{-\beta\pi^2\}$.



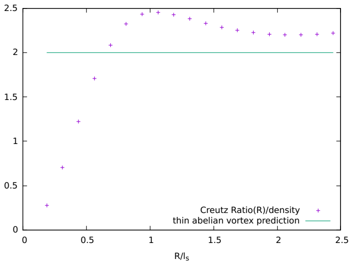
Continuum limit

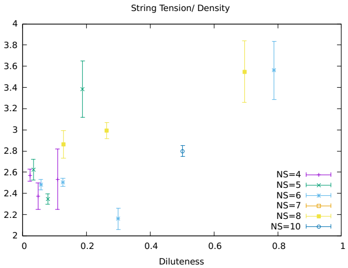
In the continuum this has to depend only on $l_s\Lambda = L_s\Lambda/a(\beta)$.
Thus, $\mathcal{A} \propto L_s^{4\pi^2 b_0}$. $(4\pi^2 b_0 = 11/3 = 3.66)$:



Confinement

In 2D a gas of thin abelian vortices confines $\sigma = -\log(1-2\rho) \sim 2\rho$ where ρ is the 2D density. Our thick non-abelian vortex-like object corrects this slightly





The Monte Carlo data (RIGHT) shows a strong correlation despite density changes by more than one order of magnitude

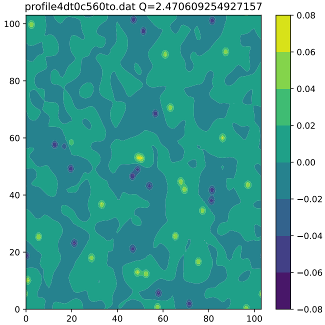
Conclusions

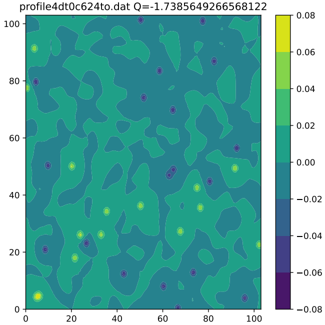
- We have shown how SU(2) Yang-Mills theory in $T_2 \times \mathbb{R}^2$ for small torus size behaves as a 2D gas of fractional instantons with diluteness growing with the small torus size as predicted by continuum semiclassical theory.
- When the system becomes non-dilute the string tension is close to its infinite volume value.
- The FILM predicts that the transition occurs as the fractional instantons have a size which decouples from the system size and the gas develops into a 4 dimensional liquid. Indeed the same as the one reached from $T_3 \times \mathbb{R}$.

See next talk

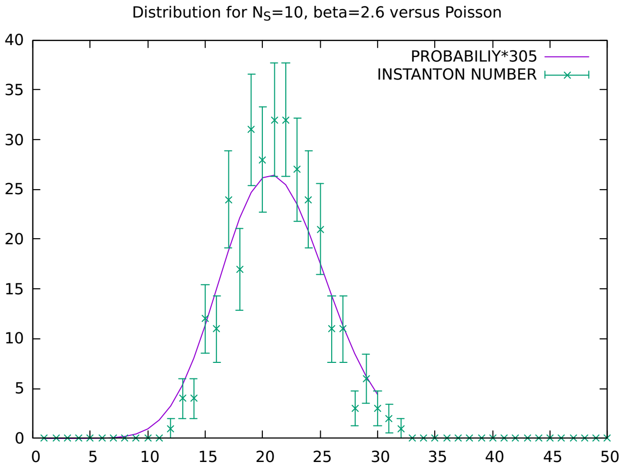
Backslides

Some more snapshots

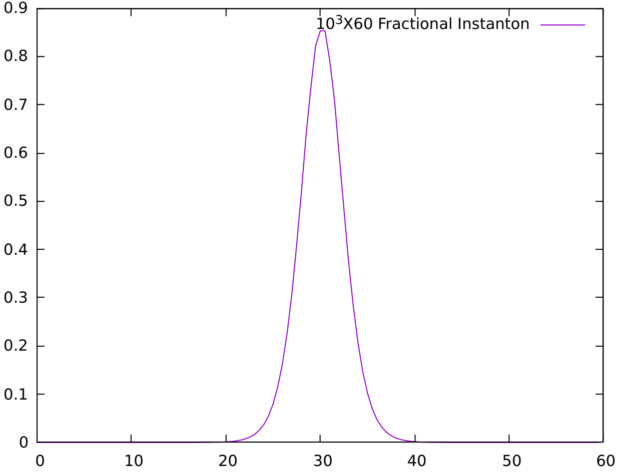




Poisson distribution



The $T_3 \times \mathbb{R}$ instanton



The 1998 plots of $T_2 \times \mathbb{R}^2$ fractional instanton