Scale setting of SU(N) Yang–Mills theories via Twisted Gradient Flow

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LATTICE 2024

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Motivations – Determining the Λ–parameter

Main goal: Λ–parameter of SU(N) Yang–Mills theories in the Twisted Gradient Flow (TGF) scheme

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for N=3,5

Determination of $\Lambda_{\mathrm{TGF}}/\mu_{\mathrm{had}}$ with low-energy scale μ_{had} through *step scaling method* (never done for large-N)

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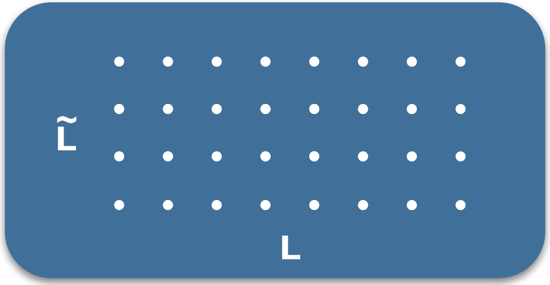
Second step

Preliminary results
in this work
for N=5

Determination of $\mu_{\rm had}\sqrt{8t_0}$ with *gradient-flow scale* $\sqrt{t_0}$

$$\Lambda_{\rm TGF}\sqrt{8t_0} = \Lambda_{\rm TGF}/\mu_{\rm had} \cdot \mu_{\rm had}\sqrt{8t_0}$$

Background – Twisted Gradient Flow scheme



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with $\tilde{L} = L/N$ on directions 1,2

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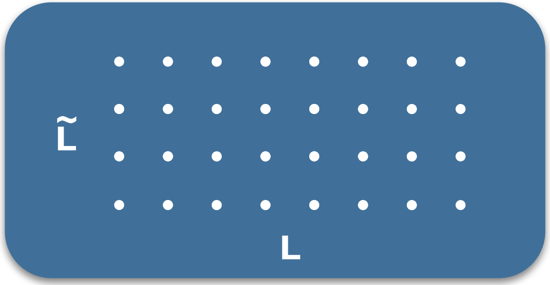


Twisted Boundary Conditions ('t Hooft, 1980; González-Arroyo and Okawa, 1983):
periodic up to a gauge transformation (the twist) for links $U_\mu(n)$
on plane (1,2). With appropriate choice of twist:

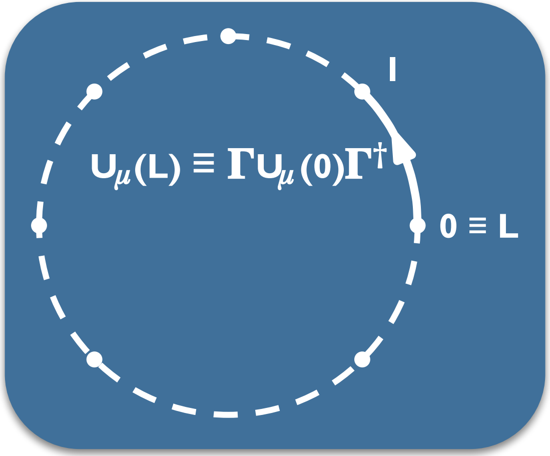
$$U_\mu(n + \tilde{L}\hat{\nu}) \equiv \Gamma_\nu U_\mu \Gamma_\nu^\dagger \quad (\mu, \nu = 1, 2)$$

Consistency relation:

$$\Gamma_1 \Gamma_2 = Z_{12} \Gamma_2 \Gamma_1 \, , \quad Z_{12} = e^{i \frac{2\pi}{N} k}$$

k, N coprime integers. To avoid instabilities in large-N limit,
best choice of k scales with N (Chamizo and González-Arroyo, 2017):

$$N = 3 \implies k = 1 \quad N = 5 \implies k = 2, \ldots$$



Objective – Setting the scale t_0

Gradient-flow scale t_0 defined for SU(3) as

$$\langle t^2 E(t)\rangle\big|_{t=t_0} = 0.3 \implies \sqrt{8t_0} \simeq 0.5 \text{ fm}$$

Definition can be extended to SU(N) as

$$\frac{N}{N^2-1}\langle t^2 E(t)\rangle\big|_{t=t_0} = 0.1125$$

with flowed energy density: $E(t) = \frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}(n,t)G_{\mu\nu}(n,t)\right]$

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Extraction of t_0 in the TGF scheme

Thermodynamic limit: at fixed bare coupling (fixed lattice spacing) infinite volume extrapolation

$$\sqrt{8t_0(a)}/a = \lim_{L\to\infty} \sqrt{8t_0(a,L)}/a$$

Continuum limit: at fixed renormalization scale μ_{had} (fixed LCP) continuum extrapolation

$$\mu_{\text{had}} \sqrt{8t_0} = \lim_{a\to 0} a\mu_{\text{had}} \times \sqrt{8t_0(a)}/a$$

Objective – Effect of topology

Problem

$E(t)$ of configuration correlated with its topological charge Q:
the flow drives the field to the lowest-action configuration in the Q-sector

Possible systematic to t_0 due to *topological freezing* :
standard algorithms get stuck at fixed Q as $a \to 0$

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Solution

Topological freezing mitigated with the *Parallel Tempering on Boundary Conditions* (PTBC) algorithm:
as studied in previous work for $N = 3$, PTBC allows to measure topological observables when standard algorithms fail

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Evaluation

To reproduce and evaluate the effect of freezing, projection on the $Q = 0$ sector and comparison of results:

$$\langle E(t) \rangle \longrightarrow \langle E(t)\delta_{Q,0} \rangle / \langle \delta_{Q,0} \rangle$$

Same thermodynamic limit expected (Brower et al., 2003)

Algorithm – PTBC

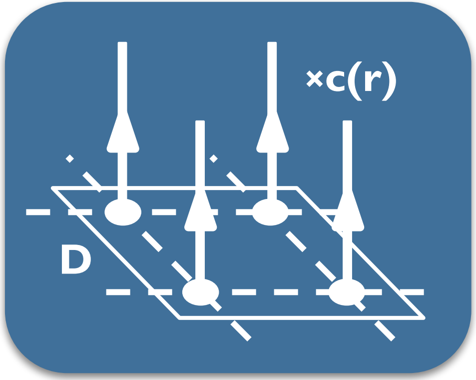
Proposed for 2d CP$^{(N-1)}$ (Hasenbusch, 2017), later for 4d SU(N) Yang–Mills (Bonanno et al., 2021) and full QCD (Bonanno et al., 2024)

N_r replicas of the lattice simulated in parallel

Replicas differ for boundary conditions on small 3d sub-lattice, the *defect* D:
links crossing D multiplied by $c(r) \quad (r = 0, 1, \ldots, N_r - 1)$
Periodic: $c(0) = 1$ Open: $c(N_r - 1) = 0$ Others: $0 < c(r) < 1$

Replicas are updated independently with standard methods for some steps, then swaps among configurations are proposed via Metropolis test: decorrelation of Q transferred from open to periodic replica

Observables computed only on periodic replica: easier to keep finite-size effects under control



To improve performance, the defect is translated randomly and updates are more frequent around it
Tuning of $c(r)$ and size of D to have 20% uniform acceptance of swaps

Results for SU(5) – Decorrelation of topological charge

Comparison of MC histories of Q with PTBC and standard algorithm (heathbath + overrelaxation)
With PTBC, lattice sweeps counted on all replicas to account for extra computational effort

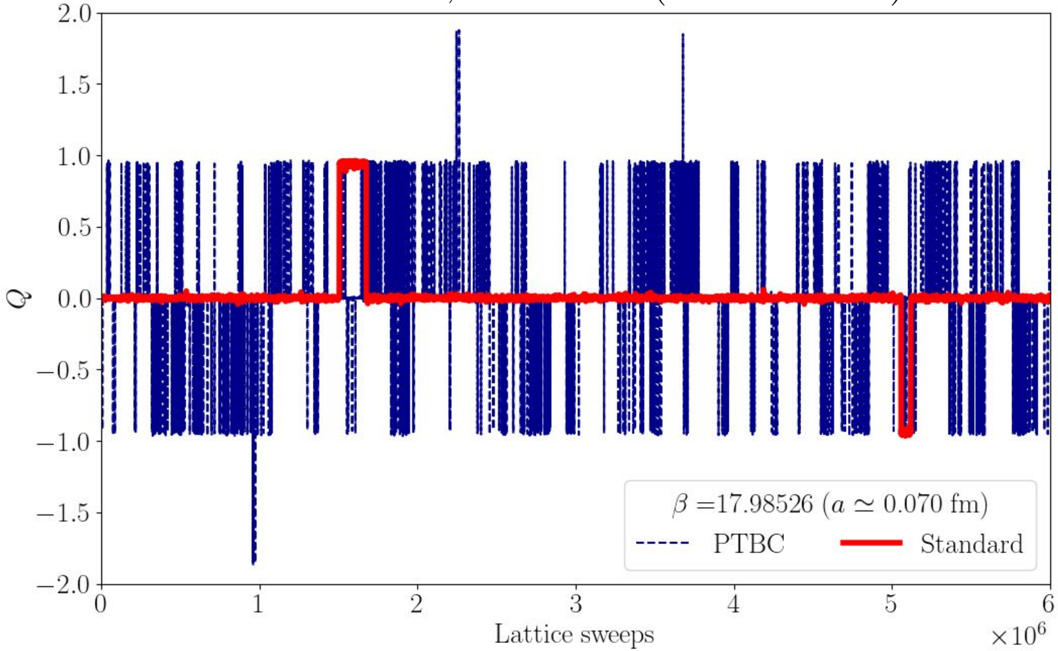
Standard algorithm frozen even with coarsest lattice spacing in this work

Integrated autocorrelation time τ_{Q^2} of Q^2 for quantitative comparison:

	PTBC	Standard
Q^2	$0.084(2)$	$0.08(2)$
τ_{Q^2}	$2.5(3)\cdot 10^2$	$> 10^5$

Gain of PTBC gets larger in the continuum limit

$l \simeq 1.4$ fm , $L = 20$ $(a \simeq 0.07$ fm$)$



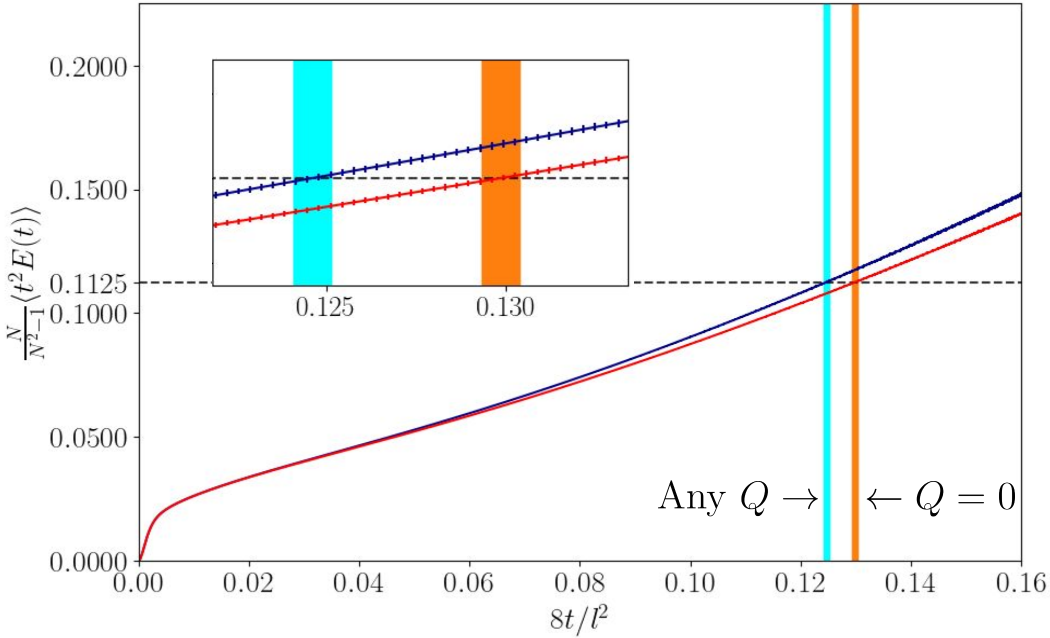
Results for SU(5) – Flow of the energy density

Flow of the energy density and determination of t_0, with and without projection to $Q = 0$

$\langle t^2 E(t) \rangle$ grows almost linearly with t around t_0 if the volume is large enough: a bending down signals large finite-size effects

With projection, larger t_0:
charged configurations have higher energy so threshold reached at later time when they are removed

$l \simeq 1.4$ fm, $L = 30$ $(a \simeq 0.047$ fm$)$

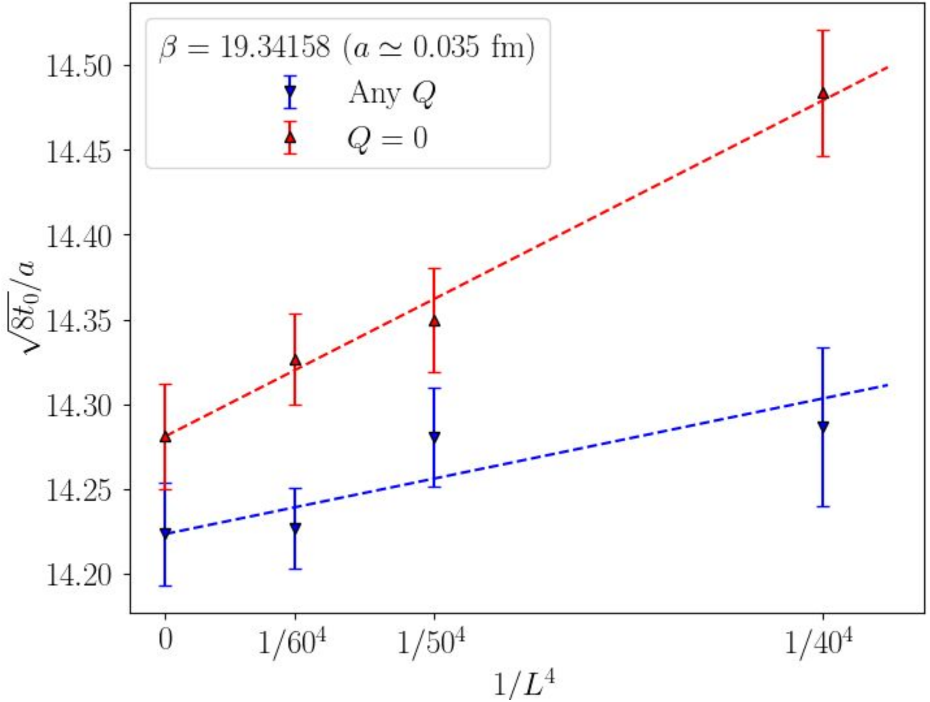


Results for SU(5) – Thermodynamic limit

Extrapolation to the thermodynamical limit of finite-volume determinations of $\sqrt{8t_0}$, with and without projection to $Q = 0$

Finite-size effects seem larger with projection

Without projection, no appreciable finite-size effects within our accuracy with volumes $1.7 \text{ fm} \lesssim l \lesssim 2.1 \text{ fm}$



Results for SU(5) – Continuum limit

Extrapolation to the continuum limit of $\mu_{\mathrm{had}}\sqrt{8t_0}$ without projection to $Q = 0$

Extrapolated value:

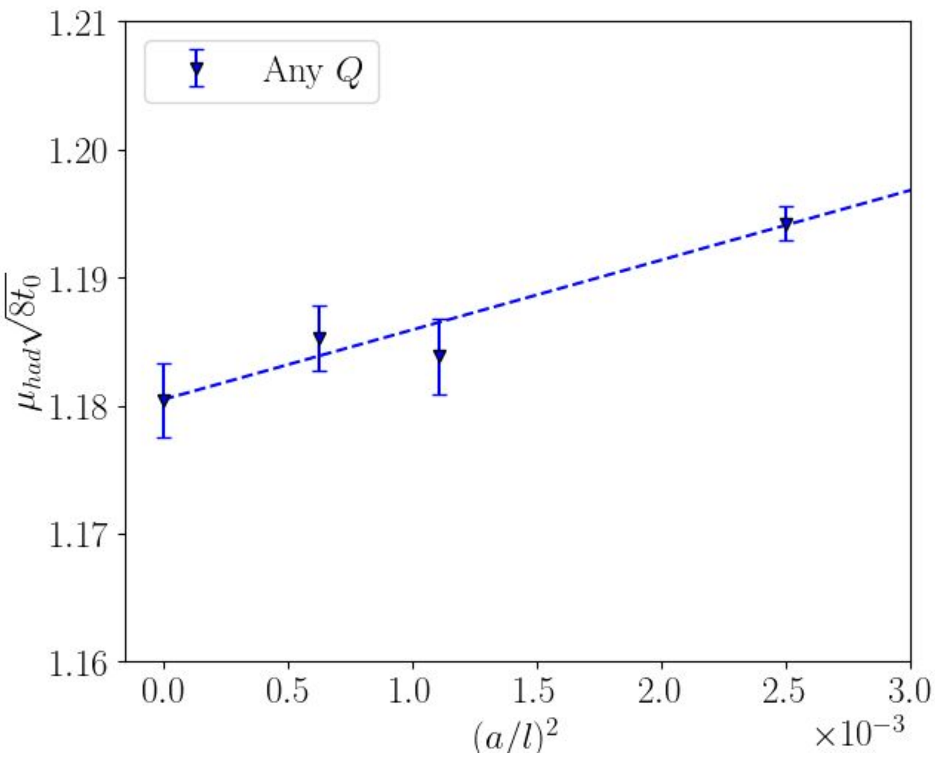
$$\mu_{\mathrm{had}}\sqrt{8t_0} = 1.180(3)$$

Previous work (Dasilva Golán et al., 2023):

$$\Lambda_{\overline{MS}}/\mu_{\mathrm{had}} = 0.56(2)$$

Final (preliminary) result:

$$\Lambda_{\overline{MS}}\sqrt{8t_0} = 0.66(3)$$



Conclusions

Take-home messages

Scale setting of the SU(5) Yang–Mills theory in the TGF scheme in order to determine the Λ–parameter in units of the gradient-flow scale

Parallel tempering on boundary conditions mitigates topological freezing, so better sampling of topological sectors and smaller finite-size effects

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Future outlooks

Refine results and compare with literature

Extend the study to SU(3) (ongoing) and SU(8) (not yet started)

Study of the volume dependence of the topological susceptibility using the PTBC algorithm

Backup – Gradient Flow

Regularization with *gradient flow* :
gauge links $U_\mu(n)$ evolved in a flow-time with

$$\partial_t V_\mu(n,t) = -g_0^2 \left[\partial_{n,\mu} S_W(V)\right] V_\mu(n,t)\,, \quad V_\mu(n,t=0) = U_\mu(n)$$

with g_0 bare coupling and S_W Wilson action
(Narayanan and Neuberger, 2006; Lüscher, 2010)

→ Fields smoothed in a radius $\sqrt{8t}$ (notice $[t] = [\mathrm{length}]^2$)

Renormalized coupling defined with the flowed energy density:

$$\lambda(\mu) = \mathcal{N}\langle t^2 E(t)\rangle\big|_{\sqrt{8t}=\mu^{-1}} = \lambda_{\overline{MS}}(\mu) + O(\lambda^2_{\overline{MS}})$$

$$E(t) = \frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}(n,t)G_{\mu\nu}(n,t)\right]$$

→ $\lambda(\mu) = \mathrm{const.}$ defines a line of constant physics (LCP)

With TBCs finite-volume effects controlled by effective size $l = aL$, so scale defined as

$$\mu = 1/(cl) \quad (c = 0.3)$$