Fractional instantons and Confinement: first results on $T^2 \times R^2$

Tests on Fractional Liquid Model

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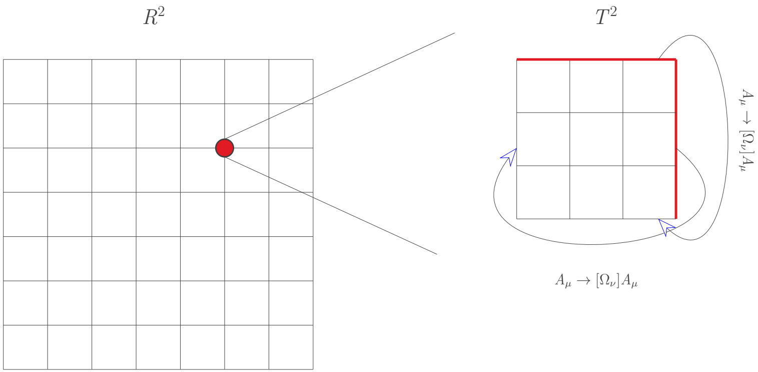
Fractional Instantons on $T^2 \times R^2$

Smooth fractional on $T^2 \times R^2$

No analytical solution available. Numerically, can be obtained on a twisted lattice:



Smooth fractional on $T^2 \times R^2$

Simulating at large beta $\rightarrow$ smooth configuration with $Q \sim 1/2$

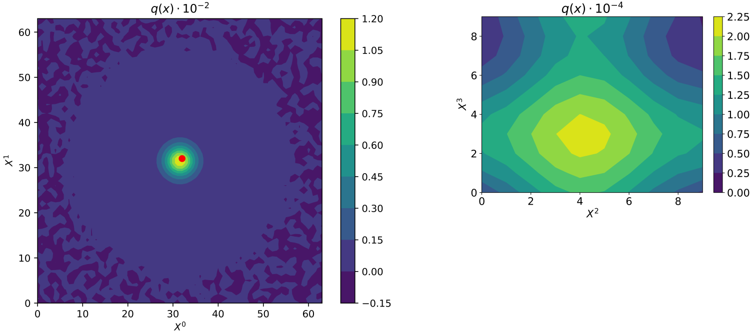


Fig. topological density $q(x)$ for a smooth configuration generated on a $T^2 \times R^2$ lattice.

Smooth fractional properties on $T^2 \times R^2$

We can study the properties of fractional instantons on smooth configurations.

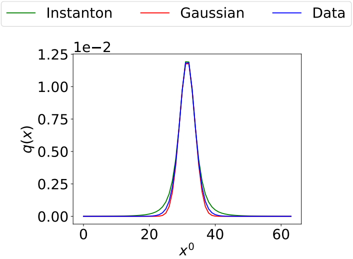


Fig. 1D section of topological density $q(x)$ for a smooth fractional

– Four degrees of freedom $\rightarrow$ position

– BPST Instanton shape

$$q(r) = Q\frac{\rho^2}{(r^2+\rho^2)^2}$$

but better fit given by:

$$q(r) = \frac{100}{N_s^2}\exp\left(\frac{A - Br^2 + Er^3}{1 + Cr + Fr^2}\right)$$

– ρ fixed by torus size N_s

For fits around 3 or 5 points they all give similar estimation of the size.

Smooth fractional

Torus size dependence

In the semiclassical regime one expects naive dimensional **scaling with torus size:**

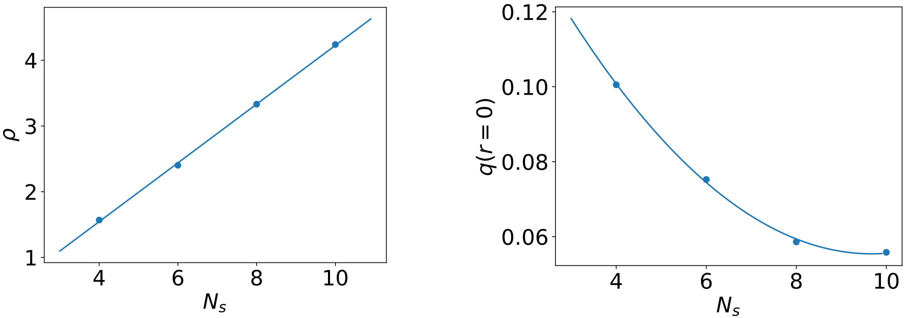


Fig. Scaling of the parameters ρ and $q(r=0)$ with the torus size N_s.

Monte-Carlo on $T^2 \times R^2$

Tab. Parameters used for the Monte-Carlo simulation.

β	a	t_{gf}
2.4	0.11530	3.52
2.5	0.08194	7.34
2.6	0.05938	15
2.7	0.04337	27.37

N_s	N_r	$\tau = \sqrt{8t_{gf}}\ a$	twist
4-13	64, 128	0.650 (fm)	(2,3)

Monte-Carlo

Identification

Identification

– Erase UV noise with a smoothing technique: Gradient Flow (GF).

– Find local maxima and minima .

– Perform a fit and accept parameters close to Smooth fractional.

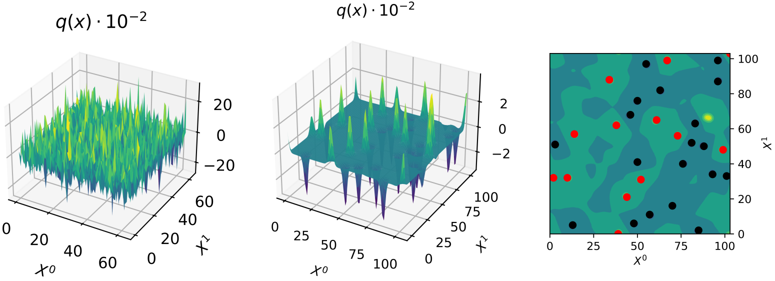


Fig. Topological density $q(x)$ for a lattice with: $\beta = 2.6$, $N_s = 6$.

Histograms: parameters of all maxima and minima

A peak on the population corrresponding to the **fractionals** is observed.

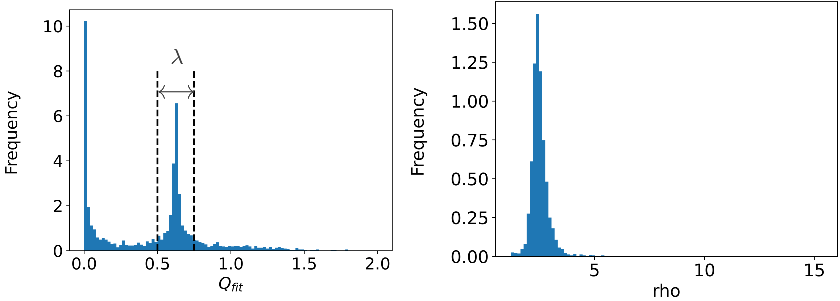


Fig. Histograms for Q_{fit} and ρ of the population of local maxima. Simulation at $\beta = 2.6, N_s = 6, \tau = 15$.

λ: Window of identification using Q_{fit}.

Monte-Carlo: Filtered Fractionals

Density

Result: We can test the semiclassical prediction $\sigma = 2n_{fi}$.

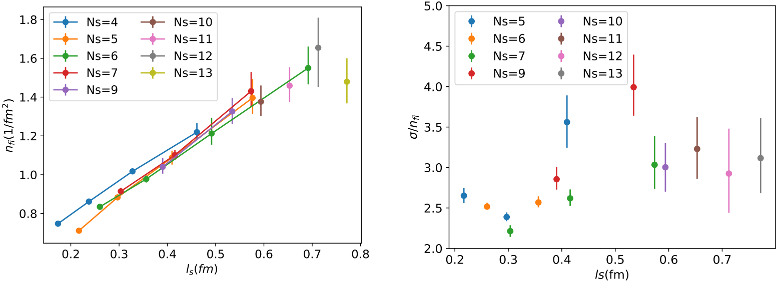
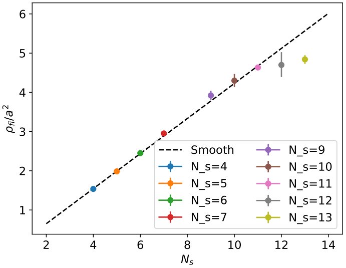


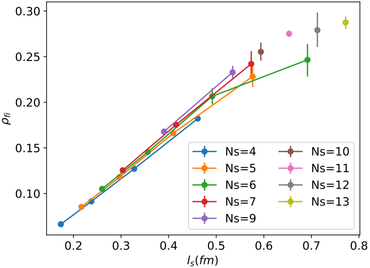
Fig. Left: Scaling of the density of fractionals with the T^2 size. Right: test of the semiclassical prediction $\sigma = 2n_{fi}$.

Monte-Carlo: Filtered Fractionals

Width

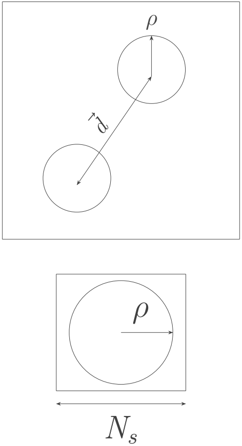
The points seems to scale with the box size.

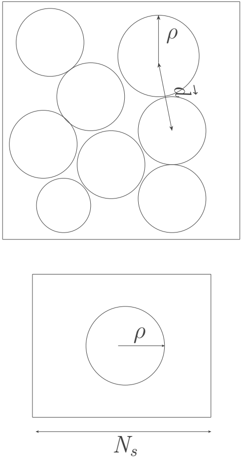




Why do the points depart from the naive scaling at $ls \sim 0.7$?

FLM: Large volume limit



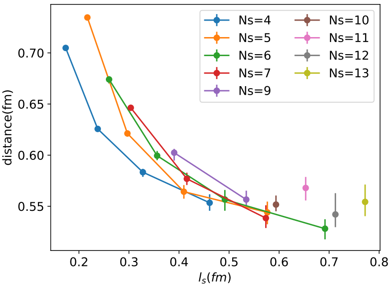


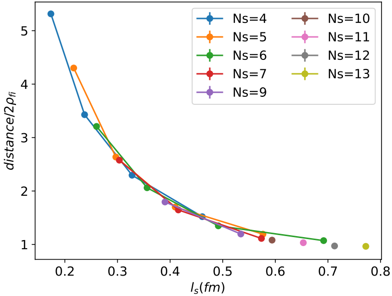
$\rho \sim N_s/2$

$2\rho \sim \vec{d}$

FLM: Large volume limit

At $l_s \sim 0.7$ the fractionals start to **decouple** from the box size.

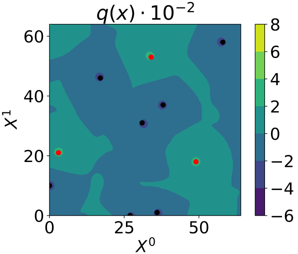


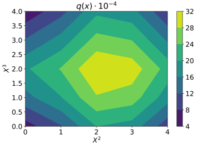


Result: $l_s \sim 0.7$ is the semiclassical edge. Consistent with $T^3 \times R$ $^{\text{GP}:hep-lat/9302007}$

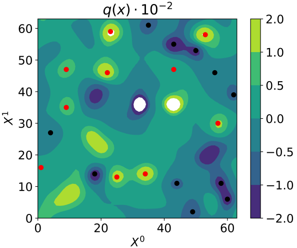
FLM: Large volume limit

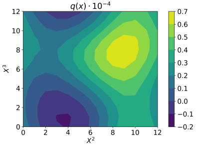
$l_s \sim 0.2$

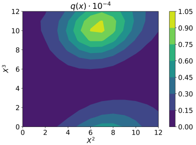




$l_s \sim 0.75$







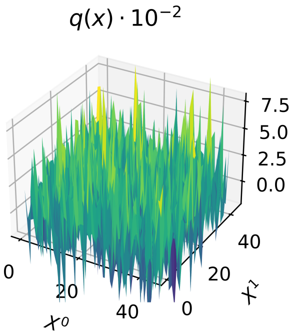
Gradient Flow dependence

Gradient flow

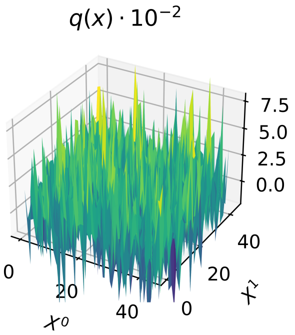
Instanton fall through

$$S(\{c_i\}) \equiv \sum_x \mathrm{Tr}\{c_0 \langle 1 - \square \rangle + 2c_1 \langle 1 - \square\square \rangle\}$$

Wilson Flow $c_0 = 1, c_1 = 0$



Overimprove $c_0 = 1, c_1 = -1/6$



Gradient flow

Instanton annihilation

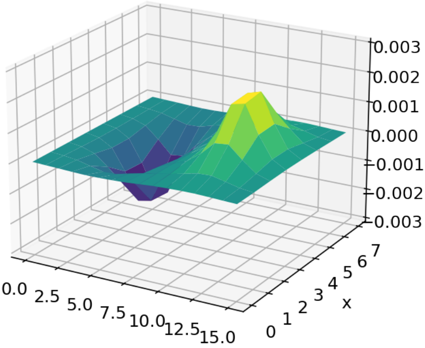
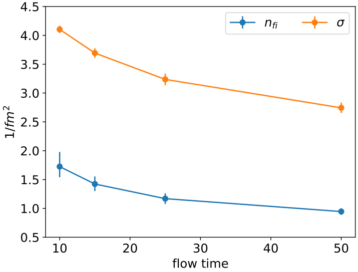


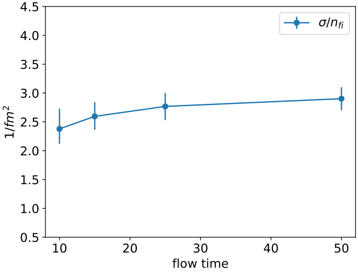
Fig. Annihilation of instantons due to minimization of the action along the gradient flow

Monte-Carlo

Gradient Flow dependence

The GF affects the density n_{fi} but also the string tension. The correlation between them remains constant $K = \frac{\sigma}{n_{fi}} \sim 2.6..$





Conclusions and outlook

Conclusions and outlook

Conclusions:

- We studied fractional instantons as underlying mechanism for confinement.
  - String tension and fractional instanton density correlated.
- Semiclassical regime on $T^2 \times R^2$ accessible by Monte-Carlo configurations

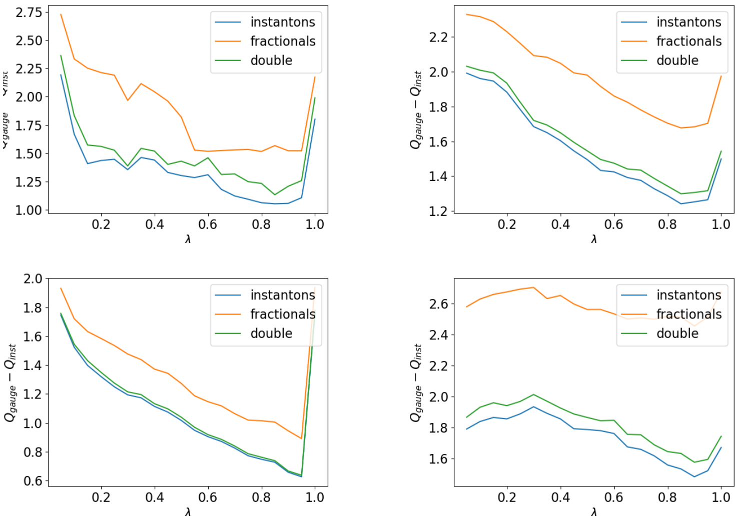
$$\sigma = 2.6 n_{fi}.$$

- Identification of the edge of semiclassical regime at $l_s \sim 0.7$

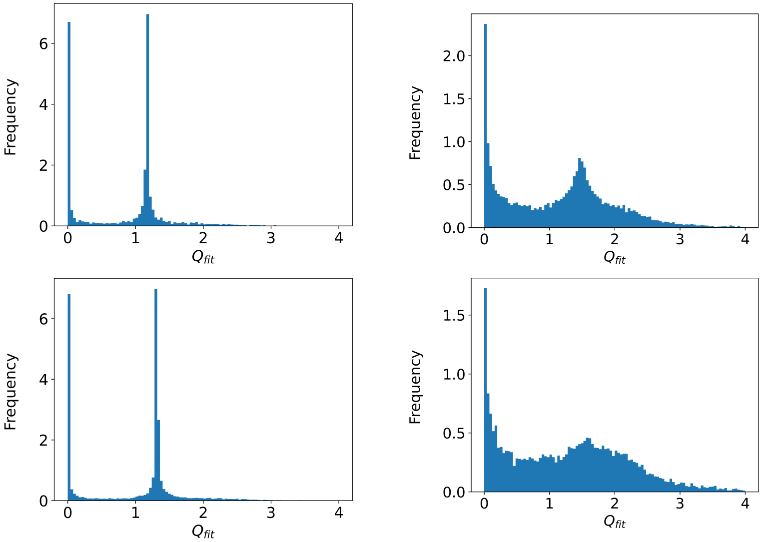
Outlook:

- Go beyond semiclassical regime $l_s > 0.7$.
  - Recover properties Yang-Mills vacuum: $\sigma = 5 fm^{-2}$.
  - Decoupling of the size of the fractionals with the box.
- Adjoint Filtering Method to reduce the amount of Gradient Flow

Back up slide:
Dependence with λ

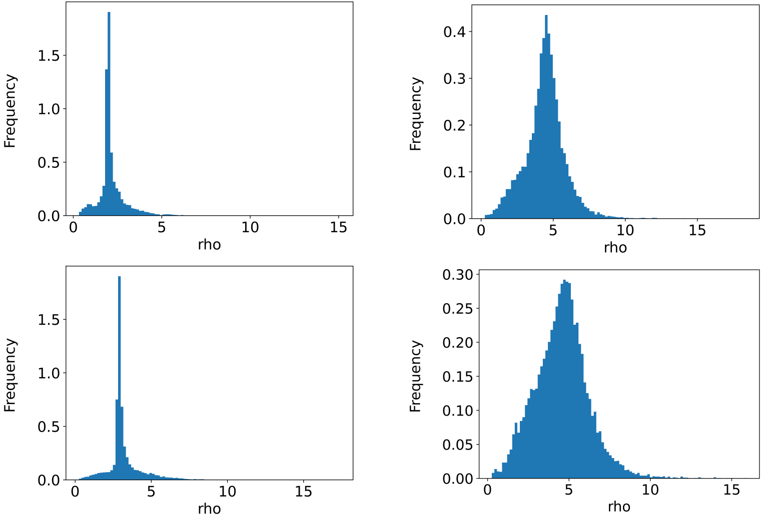


Back up slide:
Q_{fit} histograms filtering



Back up slide:

ρ histograms after filtering



Back up slide:
ρ histograms after filtering