Effective string description of the reconfined phase in the trace deformed SU(2) Yang-Mills theory in (2+1) dimensions[1] .

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work in collaboration with
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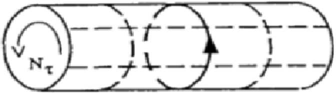
[1] C. Bonati, M. Caselle, A. Negro, D. Panfalone, L. Verzichelli in preparation.

Introduction and motivations

- An interesting approach to understand confinement is to compactify the theory on a $R^3 \times S$ manifold. The compactification radius sets the energy scale of the theory and the hope is, by squeezing the radius, to reach a weak coupling regime where perturbative methods can be used.



- The main problem in this direction is that at some critical value $N_{t,c}$ of the compactification radius N_t, the model undergoes a deconfinement transition.

Introduction and motivations

- A recent interesting proposal to avoid this problem is to add to the action a "trace deformation" so as to keep the model in the confined phase[1].

$$S^{\rm def} = S_{\rm YM} + h \int |{\rm Tr} P(\vec{x})|^2 {\rm d}^3 x \ ,$$

- The question is thus if this "reconfined" phase of the model shares the same properties with the original confining phase.
- Some properties seem to be conserved: the glueball spectrum, the localization/delocalizations transition of the Dirac eigenvalues the θ dependence of the free energy....
- Some other properties, like the monopole condensation seem to show a different behaviour.
- However the definitive test of similarity is on the behaviour of the confining flux tube.

[1] M. Unsal and L. G. Yaffe, Phys. Rev. D 78, 065035 (2008)
J. C. Myers and M. C. Ogilvie, Phys. Rev. D 77, 125030 (2008) .

The model: (2+1) $SU(2)$ in the reconfined phase.

$$S^{\mathrm{def}} = S_{\mathrm{W}} + h \sum_{\vec{x}} |P(\vec{x})|^2 \ ,$$

where S_{W} is the usual Wilson action

$$S_{\mathrm{W}} = -\frac{\beta}{2} \sum_{x} \sum_{0 \le \mu < \nu \le 2} \mathrm{Tr}\, U_{\mu\nu}(x)$$

and $P(\vec{x})$ is the Polyakov loop defined as

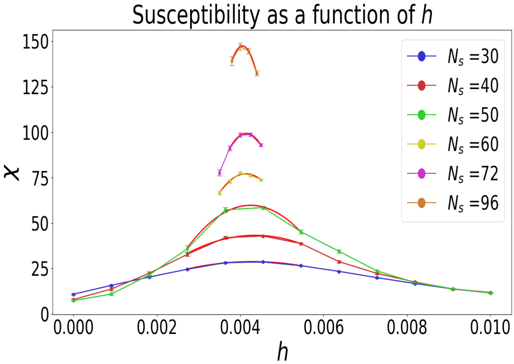
$$P\left(\vec{x}\right) = \mathrm{Tr} \prod_{t=0}^{N_t} U_0\left(t, \vec{x}\right).$$

We are interested in the two-point correlation function:

$$G(R) = \left\langle \sum_{\vec{x}} \ P\left(\vec{x}\right) P\left(\vec{x} + R\hat{k}\right) \right\rangle,$$

The phase diagram.

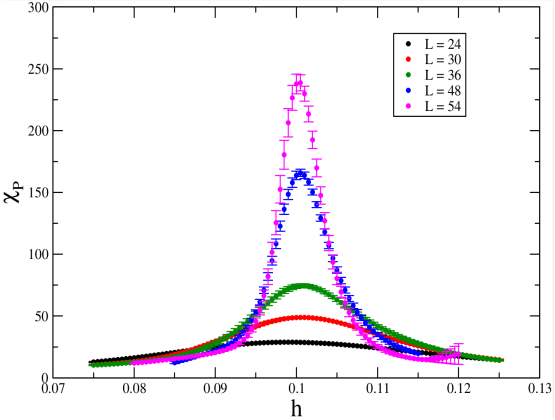
At a fixed value of β and N_t, chosen so as to have the model in the deconfined phase if we increase h at some point we find a "reconfinement transition" which in all the models studied up to now is always of the first order.



This is the Finite Size Scaling analysis of the susceptibility for the $SU(2)$ model in (2+1) dimensions with $N_t = 10$ and $\beta = 23.3805$ (for which the deconfinement transition is located at $N_{t,c} = 15$)

The phase diagram.

The same holds for $SU(3)$ in (3+1) dimensions at $\beta = 6.0$ and $N_t = 6$



Taken from *C. Bonati et al, Reconfinement, localization and thermal monopoles in SU (3) trace-deformed Yang-Mills theory, Phys. Rev.D 103 (2021) 034506*

(2+1) $SU(2)$ in the reconfined phase: data

So, to analyse the behaviour of the flux tube in the reconfined phase we select a set of values of β, N_t and h beyond the reconfinement transition.

β	h	N_t	N_s	$N_{t,c}$
23.3805	0.005	9,10,11,12,13,14	96	15
23.3805	0.006	8,9,10,11,12,13,14	96	15
23.3805	0.007	7,8,9,10,11,12,13,14	96	15
27.4745	0.004	11,12,13,14,15,16,17,18,19	96	20
27.4745	0.005	9,10,11,12,13,14,15,16,17,	96	20

Table: Some information on the simulations.

We evaluate for each combination of β, h and N_t the Polyakov loop correlator for several values of R and extract the ground state energy of the flux tube.

Effective String Theory

- For a generic EST in $D = 2 + 1$, we have[2] for $R >> N_t$

$$\langle P(0)P^{\dagger}(R)\rangle = \sum_{n=0}^{\infty} |v_n(N_t)|^2 \frac{E_n}{\pi} K_0(E_n R)$$

- In the large R limit the sum is dominated by the lowest state and from the exponential decay of the modified Bessel function thus we may fit the Polyakov loop correlators using

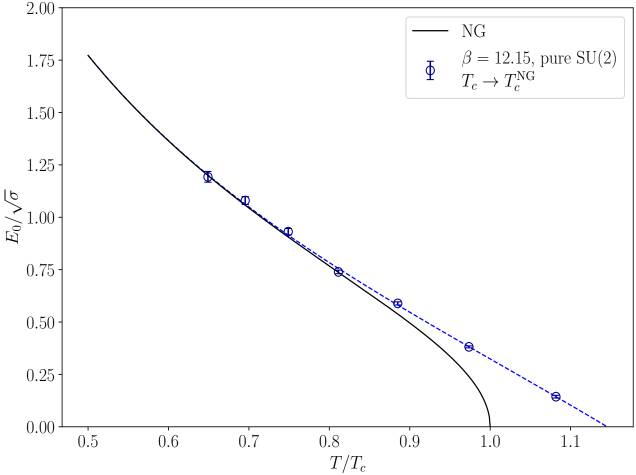
$$\langle P(0)P^{\dagger}(R)\rangle = AK_0(E_0(N_t)R)$$

 and extract from the fit the ground state energy $E_0(N_t)$ we may extract the correlation length $E_0(N_t)$. This is true for any EST. If we want to characterize the particular EST which describes our LGT we must look at the the N_t dependence of E_0 allows to distinguish between different effective strings
For instance, for the Nambu-Goto string we have:

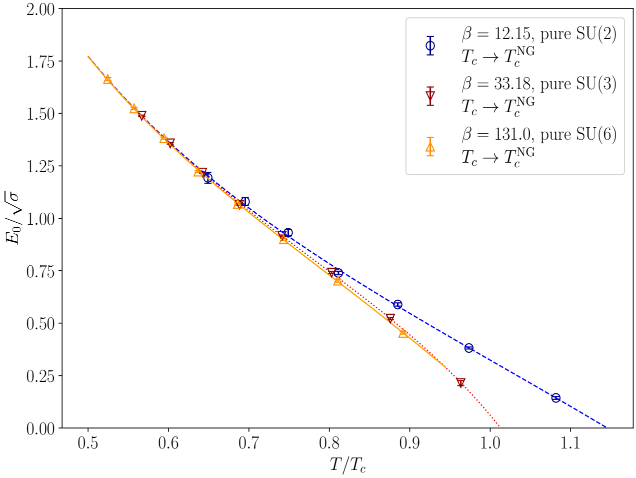
$$E_0 = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}}$$

[2] M. Lüscher and P. Weisz, *JHEP* 2004.07(2004), p. 014

Nambu-Goto expectation versus $SU(2)$ data.

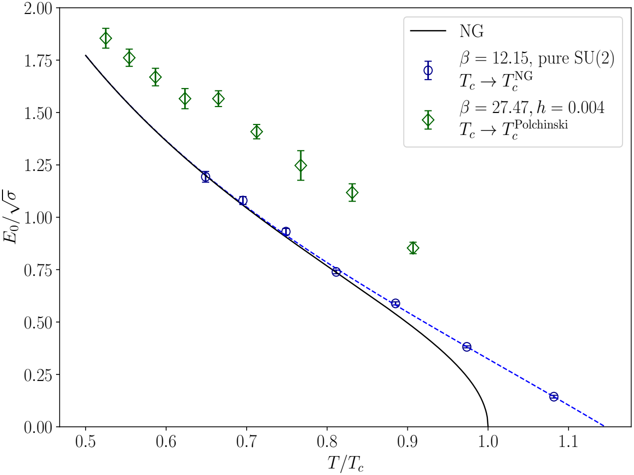


Nambu-Goto expectation versus $SU(N)$ data[3].



[3] See the talk by D. Panfalone

Confined versus reconfined data for $SU(2)$.



The "rigid string"[4]

The "rigid string" is obtained adding to the Nambu-Goto action a term proportional to (square of) the extrinsic curvature which has the effect of increasing the "stiffness" of the string.

$$S_R = \int_\Sigma d^2\xi \sqrt{g} \; , \left[\sigma + \alpha \mathcal{K}^2 + \ldots \right] \; ,$$

where $\mathcal{K}$ is the extrinsic curvature defined as $\mathcal{K} = \Delta(g) X$, with

$$\Delta(g) = \frac{1}{\sqrt{(g)}} \partial_a [\sqrt{(g)} g^{ab} \partial_b]$$

.

In the "physical gauge" in (2+1) dimensions, keeping only the first order terms this boils down to:

$$S_R = \int_\Sigma d^2\xi \;\; \sigma \; \partial X \partial X \; + \; \alpha \; \partial^2 X \partial^2 X \; + \; \ldots ,$$

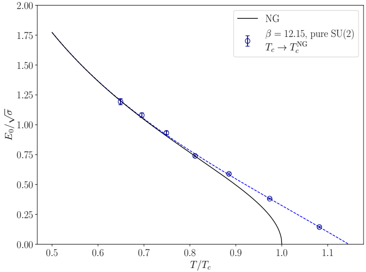
[4] A. M. Polyakov, Fine Structure of Strings, Nucl. Phys. B 268 (1986) 406
H. Kleinert, The Membrane Properties of Condensing Strings, Phys. Lett. B 174 (1986)
L. Peliti and S. Leibler, Effects of thermal fluctuations on systems with small surface tension Phys. Rev. Lett. **54**,1690 (1985). 335.

The "rigid string"

The standard approach to study the model was to treat the $\mathcal{K}^2$ term as a perturbation of the gaussian one. Comparing the corrections obtained in this way with simulations it turned out that α was compatible with zero for all ordinary non-abelian LGTs. In recent years it has been understood that this is due to the fact that the $\mathcal{K}^2$ term can be eliminated since it is proportional to the eq. of motion of the NG string and that a better proposal to describe higher order perturbations to the Nambu-Goto action is instead

$$S_{BNG} = \int_\Sigma d^2\xi \sqrt{g}\ , \left[\sigma + \gamma_3 \mathcal{K}^4 \dots\right] \ ,$$

This is the origin of the deviations with respect to NG observed in $SU(N)$ LGTs (see the talk by D. Panfalone)



The Polchinski-Yang proposal

- A completely different approach to study S_R was proposed in 1992 by Polchinski and Yang which described the rigid string assuming the quartic term as the dominant one and the quadratic NG term as a small perturbation.
- This corresponds to a completely different vacuum and requires $\alpha >> N_t^2\sigma$ and $N_t^2\sigma << 1$ and corresponds (apparently) to an unphysical regime since for **ordinary YM theories** $N_{t,c} \sim 1/\sqrt{\sigma}$, thus if $N_t^2\sigma << 1$ the model is deconfined and thus we do not expect the presence of a confining flux tube and thus of an effective string description.
- Despite this, this regime was studied in great detail for completely different reasons. The goal was to show that in this particular regime the (unphysical) high temperature behaviour of model was the same of that of QCD in the large N limit.

The Polchinski-Yang solution[5]

The Polchinski-Yang solution for generic values of the transverse dimensions is

$$E_0 = w\lambda$$

where

$$w = \sqrt{N_t^2 - \frac{(d-2)N_t}{2}\sqrt{\frac{1}{2\alpha\lambda}}}$$

and

$$\sqrt{\lambda} = \frac{3}{8}\frac{(d-2)}{N_t\sqrt{2\alpha}} + \sqrt{\frac{9}{128}\frac{(d-2)^2}{\alpha N_t^2} + \sigma - \frac{\pi(d-2)}{3N_t^2}}$$

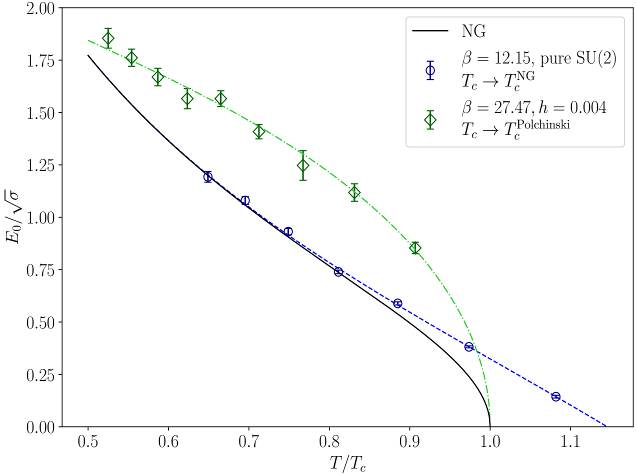
From this solution it is possible to obtain the new value of the deconfinement temperature, which is given by:

$$N_{t,c}^2\sigma = \frac{\pi(d-2)}{3} - \frac{(d-2)^2}{16\alpha}$$

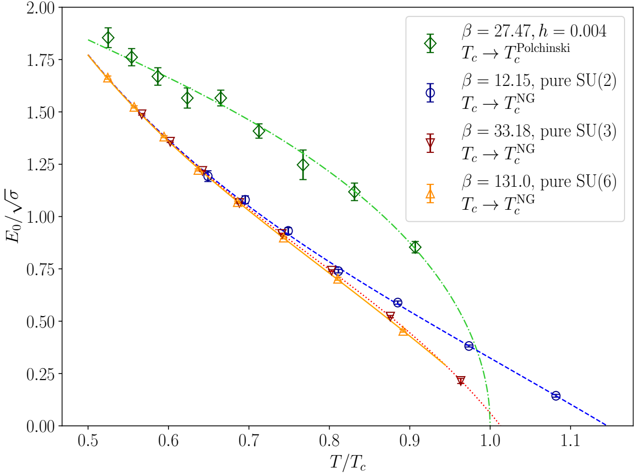
which, as expected, is higher than the standard one (which is $N_{t,c}^2\sigma = \pi(d-2)/3$)

[5] J. Polchinski and Z. Yang, High temperature partition function of the rigid string, Phys. Rev. D 46 (1992) 3667

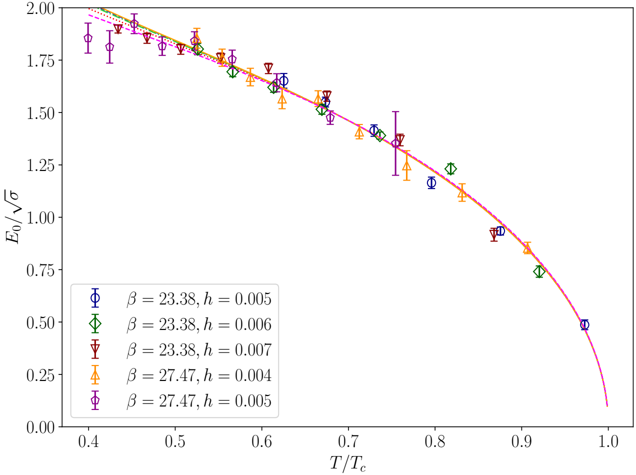
Rigid string versus Nambu-Goto: $SU(2)$



Rigid string versus Nambu-Goto: $SU(N)$.

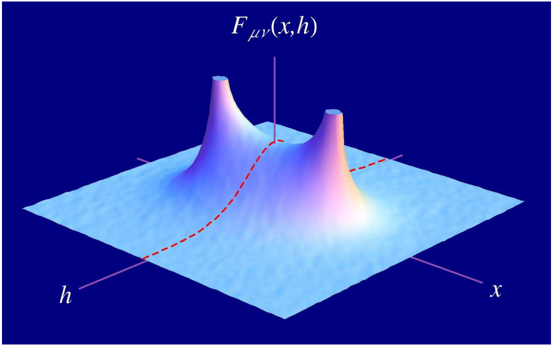


Rigid string: all data.



Flux tube profile.

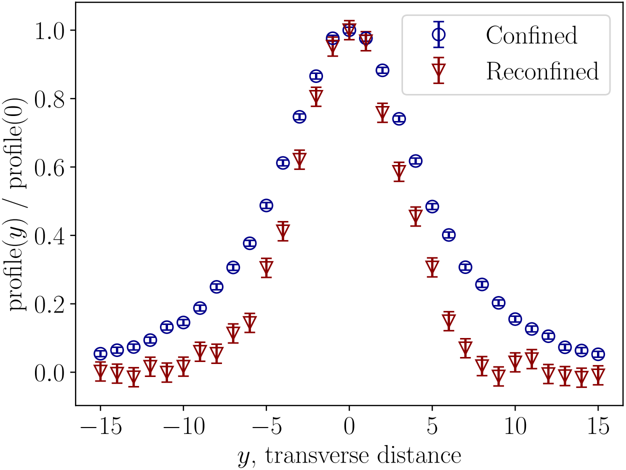
An independent test of the role of the rigidity term comes from the flux tube profile



(see the talk by L. Verzichelli)

Rigid string versus Nambu-Goto: Flux tube profile.

The effect of the rigidity term is to "squeeze" the flux tube fluctuations



Summary

- Most likely the reconfined flux tube is not of the Nambu-Goto type
  If this is the case, then understanding which are the physical degrees of freedom driving the formation of the confining flux tube in ordinary YM theories is still an open problem

- The presence of the rigidity term is directly related (by the Polyakov solution) to the condensation of monopoles, which is most likely the mechanism which drives confinement in the reconfined phase. This agrees with the original analysis of Unsal and Yaffe

- Deformed YM theories in the reconfined phase represent a lattice realization of the (apparently) unphysical solution of Polchinski and Yang

Acknowledgements

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