Entanglement entropy of a color flux tube in 2+1 D Yang-Mills theory

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Motivation

- QCD string is a pure spatially-extended state of gluons
- Dynamics of QCD string are important to hadronization in heavy-ion collisions & DIS, e.g. Lund model in Pythia
- Goal: study quantum correlations between parts of a static QCD string
  - Pure SU(2) 2+1 D Yang Mills at $T=\frac{1}{2} T_c$
  - Static heavy quarks as sources
  - Renyi entanglement entropy as measure of quantum correlations
  - Comparison to string model

Entanglement Entropy (Renyi Entropy)

$$\hat{\rho}_{\mathcal{A}} = \mathrm{tr}_{\bar{\mathcal{A}}}\left(\hat{\rho}\right)$$

Example:

$$\psi = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

$$|\uparrow\uparrow\rangle \; |\uparrow\downarrow\rangle \; |\downarrow\uparrow\rangle \; |\downarrow\downarrow\rangle$$

$$\rho = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\hat{\rho}_{\mathcal{A}} = \mathrm{Tr}_{\bar{\mathcal{A}}}\rho = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$S_{\mathrm{EE}} = -\,\mathrm{tr}_{\mathcal{A}}\left(\hat{\rho}_{\mathcal{A}} \log\left(\hat{\rho}_{\mathcal{A}}\right)\right)$$

$$S_{\mathrm{EE}} = -\frac{d}{dq}\log(\mathrm{tr}_{\mathcal{A}}(\hat{\rho}^{q}_{\mathcal{A}}))|_{q=1}$$

$$S^{(q)} = \frac{1}{1-q}\log\left(\mathrm{tr}_{\mathcal{A}}\left(\hat{\rho}^{q}_{\mathcal{A}}\right)\right) \forall\ q \in \mathbb{N},\ \ q \geq 2.$$

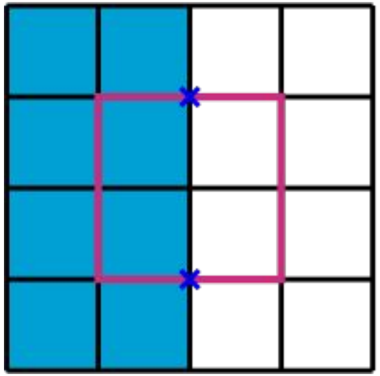
$$S_{\mathrm{EE}} = \lim_{q\to 1} S^{(q)}.$$

Difficulties with entanglement entropy in (gauge) field theories

- Entanglement entropy is UV-divergent in field theory
- Hilbert space of gauge invariant states is non-factorizable
- Need gauge invariant definition and lattice implementation
- Want a measure of entanglement for physical object (flux tube) that is **gauge-invariant and finite**

Entanglement Entropy in Gauge Theories

- Hilbert space of gauge invariant states is non-factorizable in general
- Factorizable within “superselection sectors” (determined by field state on the boundary)
  (P. Buividovich et al arxiv:0806.3376, H. Casini et al arxiv:1312.1183, D. Radecivic et al arxiv:1404.1391, S. Ghosh et al 1501.02593, S. Aoki et al arxiv:1502.04267, W. Donnelly arxiv:1109.0036)
- We work with “electric center”



$$\mathcal{H} \neq \mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\bar{\mathcal{A}}}$$

$$\rho_{\mathcal{A}\mathcal{A}'} = \begin{pmatrix} p_1\rho_{\mathcal{A}_1\mathcal{A}'_1} & 0 & \dots & 0 \\ 0 & p_2\rho_{\mathcal{A}_2\mathcal{A}'_2} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & p_m\rho_{\mathcal{A}_m\mathcal{A}'_m} \end{pmatrix} \qquad \rho_{\mathcal{A}} = \begin{pmatrix} p_1\rho_{\mathcal{A}_1} & 0 & \dots & 0 \\ 0 & p_2\rho_{\mathcal{A}_2} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & p_m\rho_{\mathcal{A}_m} \end{pmatrix}$$

(H. Casini et al arxiv:hep-th/1312.1183)

UV-finite Entanglement Entropy

$$\frac{1}{|\partial A|} S = \frac{1}{|\partial A|} S_{UV} + \frac{1}{|\partial A|} S_f$$

(P. V. Buividovich, M. I. Polikarpov arXiv:0802.4247)
(S. Ryu et al arXiv:hep-th/0603001)
(T. Nishioka et al arXiv:hep-th/0611035)
(I. Klebanov et al arXiv:0709.2140)

$$\tilde{S}^{(q)}_{|Q\bar{Q}} \equiv S^{(q)}_{|Q\bar{Q}} - S^{(q)}$$

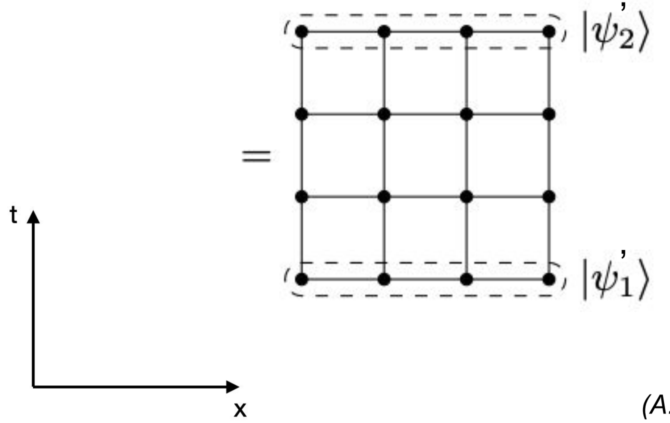
(Flux Tube Entanglement Entropy)

Can be realized on the lattice using Polyakov lines and the replica method

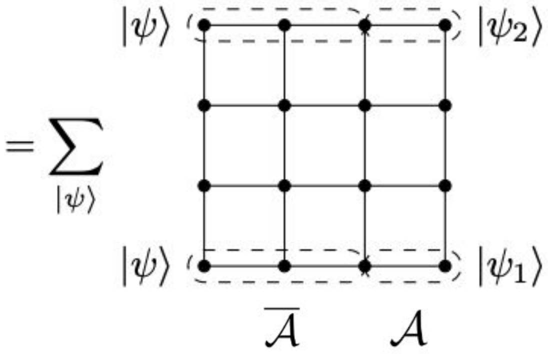
Reduced Density Matrix in Lattice Field Theory

$$\hat{\rho}_{\mathcal{A}} = \mathrm{tr}_{\bar{\mathcal{A}}}\left(\hat{\rho}\right)$$

$$\langle\psi_1^{'}|\,\hat{\rho}\,|\psi_2^{'}\rangle = \frac{1}{Z}\,\langle\psi_1^{'}|\,e^{-\beta\hat{H}}\,|\psi_2^{'}\rangle$$



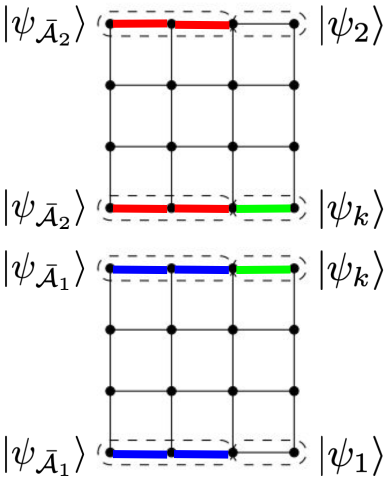
$$\langle\psi_1|\,\hat{\rho}_{\mathcal{A}}\,|\psi_2\rangle$$



(A. Rabenstein et al arXiv:1812.04279)

Reduced Density Matrix squared

$$\langle \psi_1 | \hat{\rho}_{\mathcal{A}}^2 | \psi_2 \rangle = \langle \psi_1 | \hat{\rho}_{\mathcal{A}} | \psi_k \rangle \langle \psi_k | \hat{\rho}_{\mathcal{A}} | \psi_2 \rangle = \sum_{|\psi_k\rangle, |\psi_{\bar{\mathcal{A}}_1}\rangle, |\psi_{\bar{\mathcal{A}}_2}\rangle}$$



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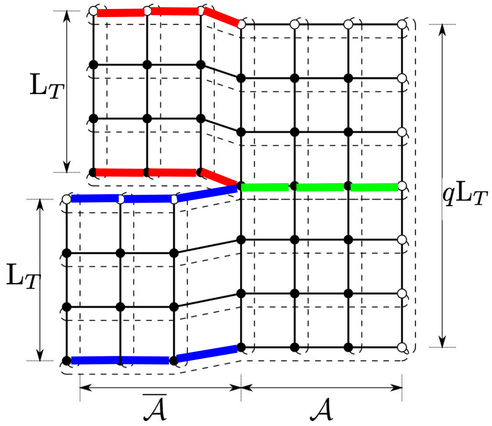
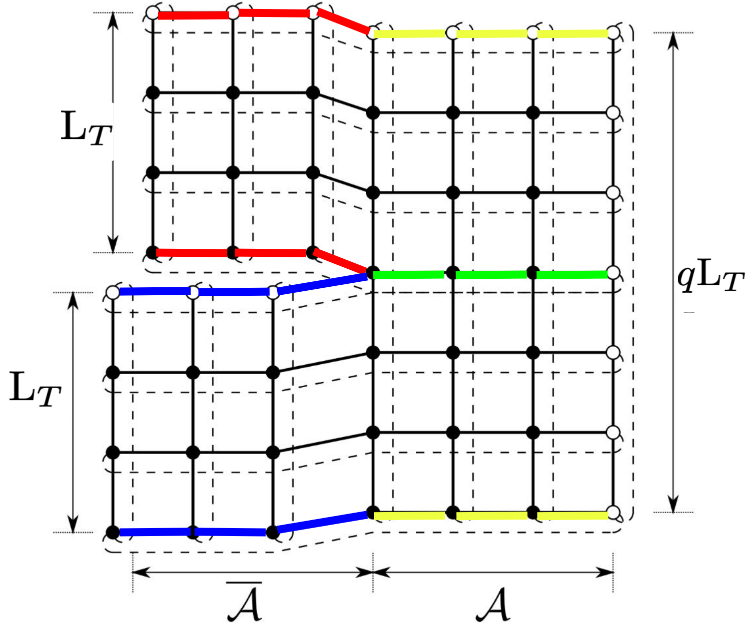


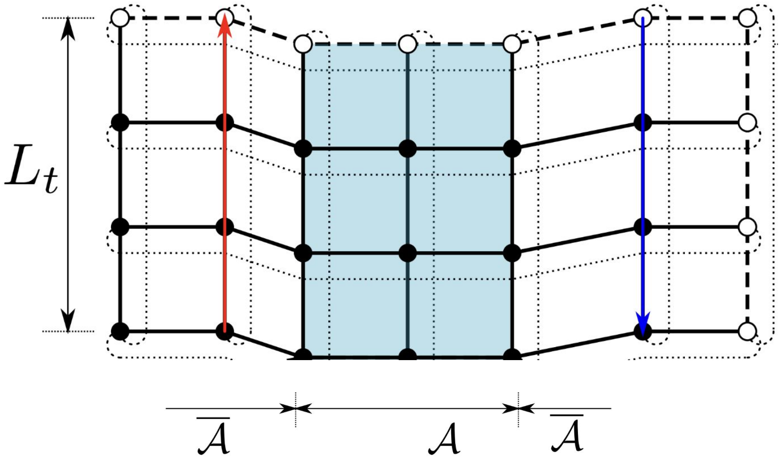
Reduced Density Matrix squared

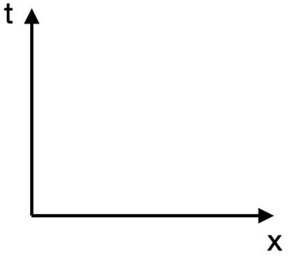
$$\mathrm{Tr}\hat{\rho}_{\mathcal{A}}^2 = \sum_{\psi_{\mathcal{A}}} \langle \psi_{\mathcal{A}} | \hat{\rho}_{\mathcal{A}}^2 | \psi_{\mathcal{A}} \rangle = $$

$$= \frac{Z_2}{(Z_1)^2}$$



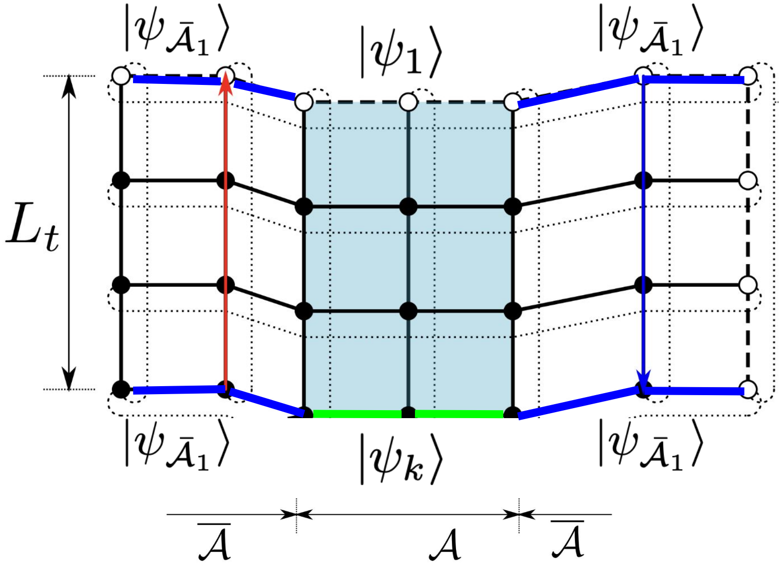
Polyakov Lines

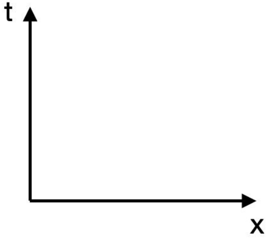




Polyakov Lines: Reduced Density Matrix

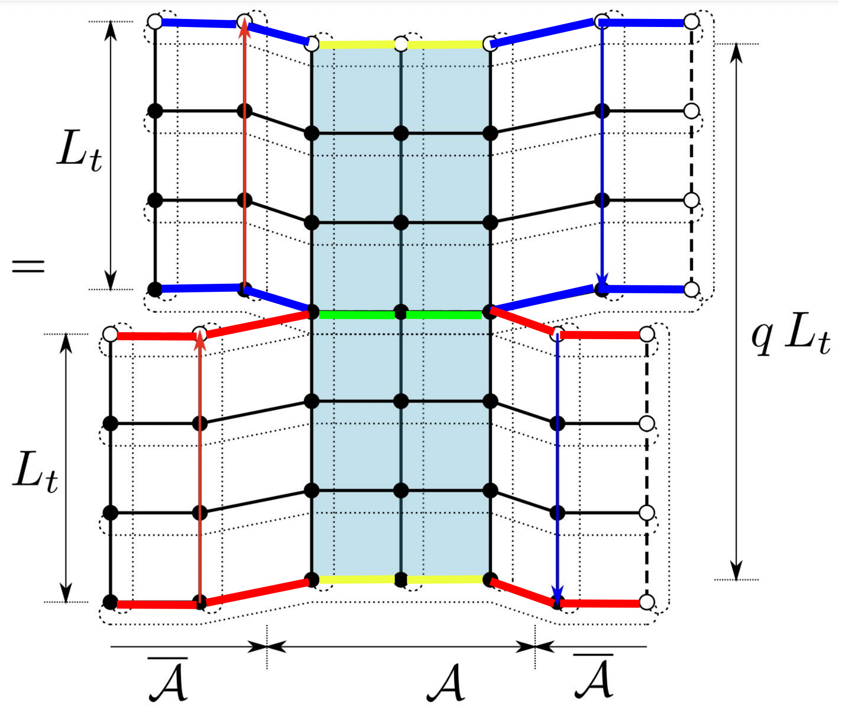
$$\langle\psi_1|\hat{\rho}_{\mathcal{A}}|\psi_k\rangle =$$

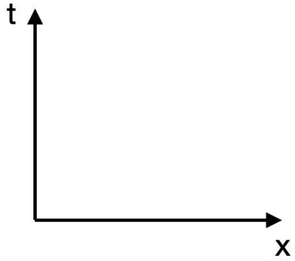




Polyakov Lines: Reduced Density Matrix squared

$$\mathrm{Tr}\hat{\rho}_{\mathcal{A}}^2 = \sum_{\psi_{\mathcal{A}}} \langle \psi_{\mathcal{A}} | \hat{\rho}_{\mathcal{A}}^2 | \psi_{\mathcal{A}} \rangle =$$





UV-finite Entanglement Entropy

$$\frac{1}{|\partial A|} S = \frac{1}{|\partial A|} S_{UV} + \frac{1}{|\partial A|} S_f$$

(P. V. Buividovich, M. I. Polikarpov arXiv:0802.4247)
(T. Nishioka, T Takayanagi arXiv:hep-th/0611035)

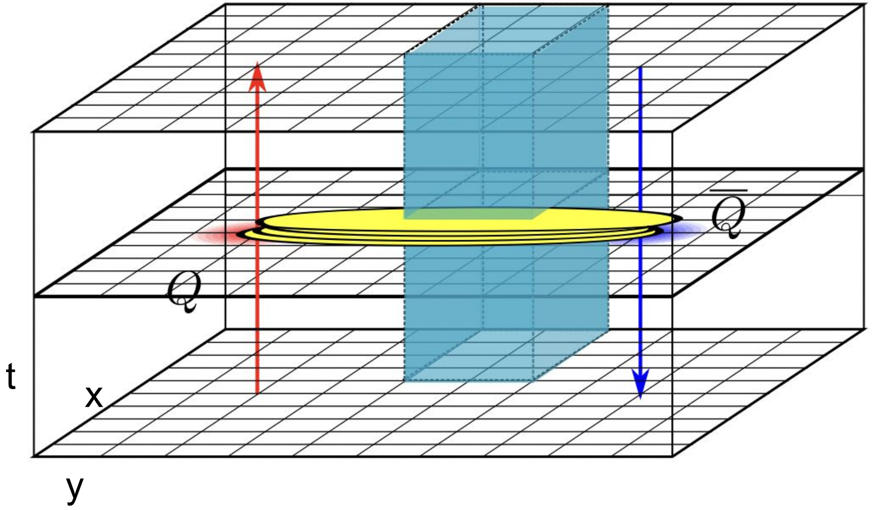
$$\tilde{S}^{E(q)}_{|Q\bar{Q}} \equiv S^{E(q)}_{|Q\bar{Q}} - S^{E(q)}$$

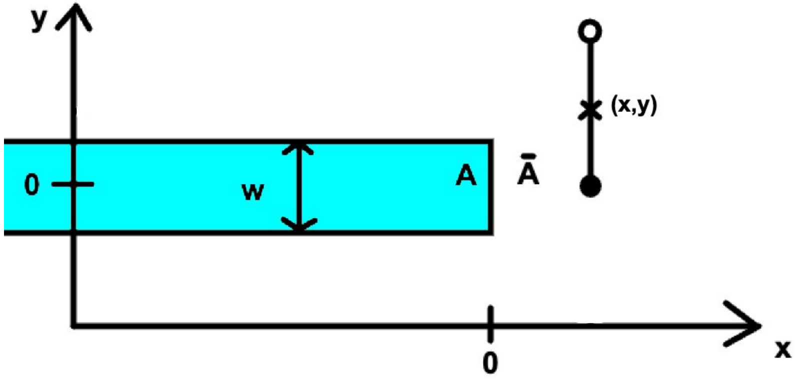
(Flux Tube Entanglement Entropy)

$$S^{E(q)}_X = -\frac{1}{q-1} \log \mathrm{Tr}_A \left[\left(\mathrm{Tr}_{\bar{A}} \rho_X \right)^q \right] = -\frac{1}{q-1} \log \frac{Z^{(q)}_X}{(Z_X)^q}$$

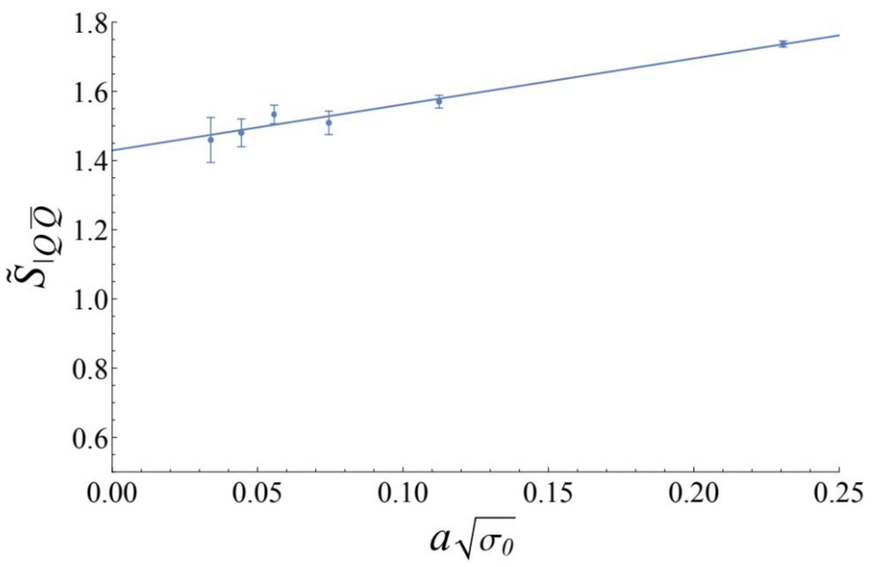
$$-\frac{1}{q-1} \log \frac{\langle \prod_{r=1}^{q} \mathrm{Tr} P_0^{(r)} \mathrm{Tr} P_{\vec{x}}^{(r)\dagger} \rangle}{\langle \mathrm{Tr} P_0 \mathrm{Tr} P_{\vec{x}}^{\dagger} \rangle^q} = -\frac{1}{q-1} \log \frac{Z^{(q)}_{|Q\bar{Q}}/Z^{(q)}}{(Z_{|Q\bar{Q}}/Z)^q} = -\frac{1}{q-1} \log \left(\frac{Z^{(q)}_{|Q\bar{Q}}}{(Z_{|Q\bar{Q}})^q} \cdot \frac{Z^q}{Z^{(q)}} \right) = S^{E(q)}_{|Q\bar{Q}} - S^{E(q)}$$

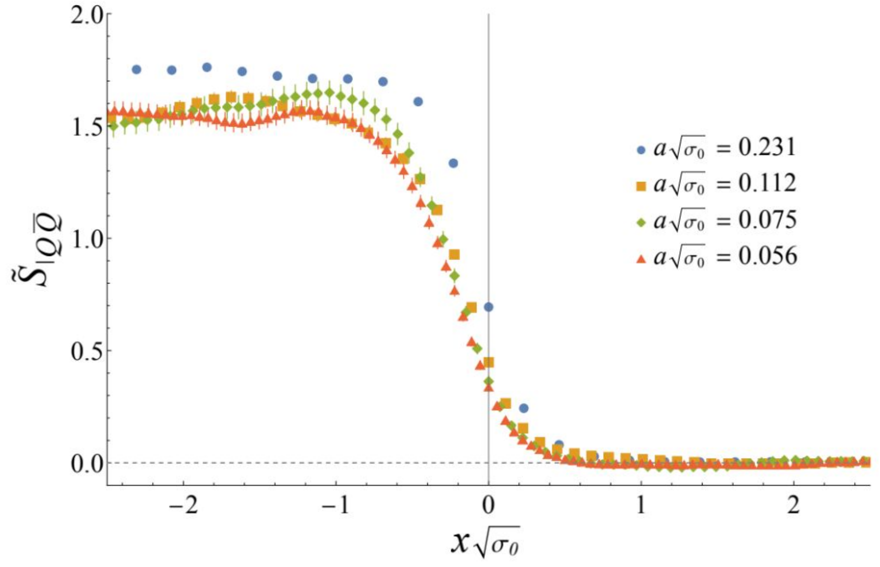
Lattice Setup: A = Half-slab





Finite Entanglement Entropy





$$\tilde{S}^{(q)}_{|Q\bar{Q}} \equiv S^{(q)}_{|Q\bar{Q}} - S^{(q)}$$

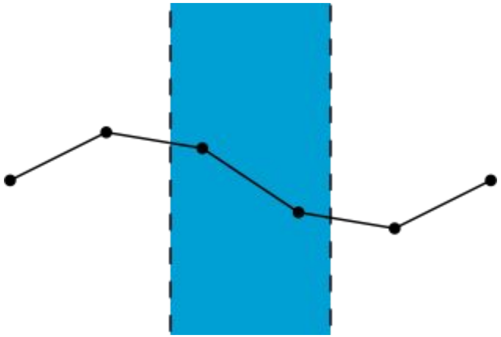
String Model Predictions

- Thin relativistic string in long string limit (1 transverse dimension)
- Solely entanglement due to vibration of flux tube when fully cross cut
- UV cutoff epsilon
- Follow procedure of (L. Bombelli et al PhysRevD.34.373)

$$H = M^2 L + \frac{\pi}{2M^2 L} \int_0^\pi ds (p^2 + M^4 x'(s)^2)$$

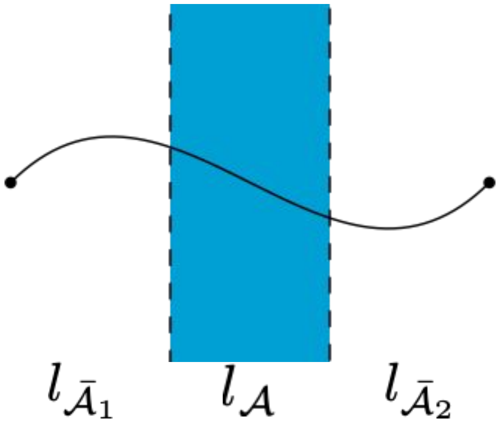
M. Luscher et al , Nucl. Phys. B 180, 1 (1981)

String tension

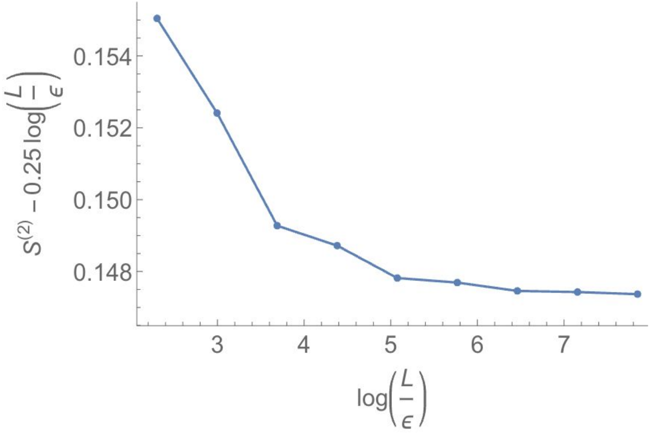


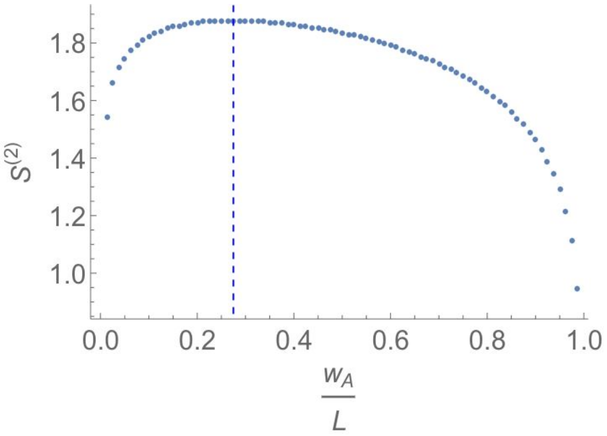
String Model Predictions

- Numerically calculate entropy taking L/ε to infinity
- Splits into cutoff scaling and string proportion terms

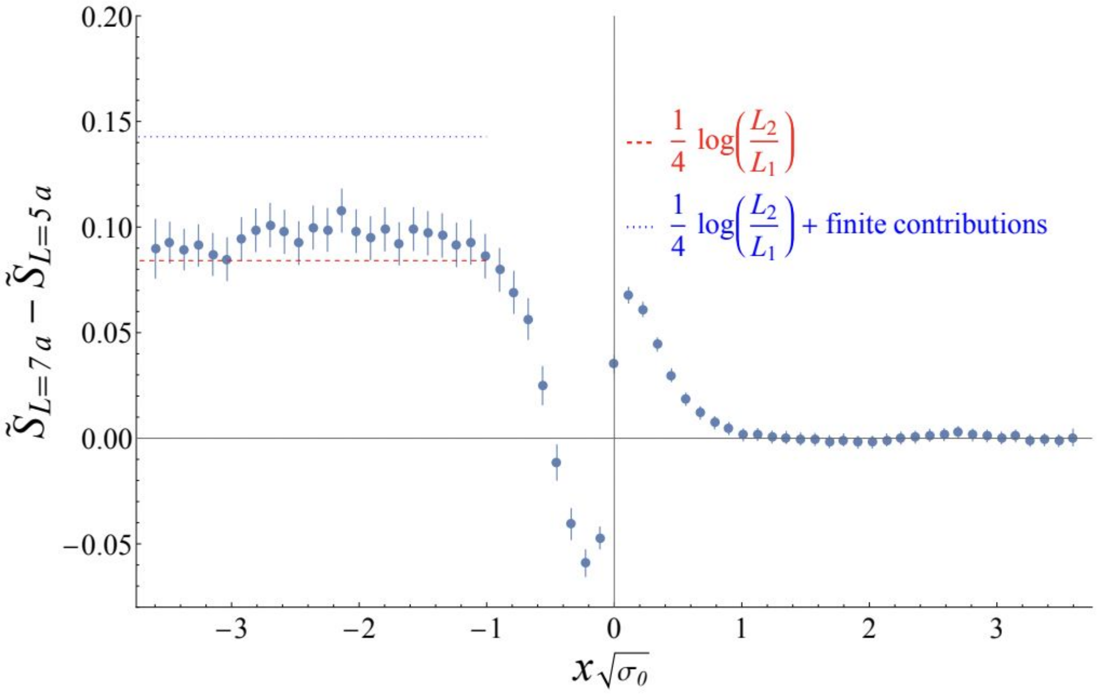


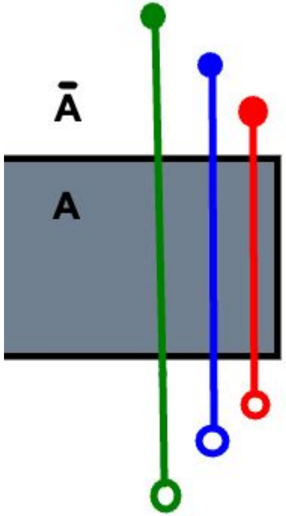
$$S^{(2)} \approx \frac{1}{4}\ln(L/\epsilon) + \ln(l_{\mathcal{A}}^{0.114}(l_{\bar{\mathcal{A}}_1} l_{\bar{\mathcal{A}}_2})^{0.136}) + 0.566$$



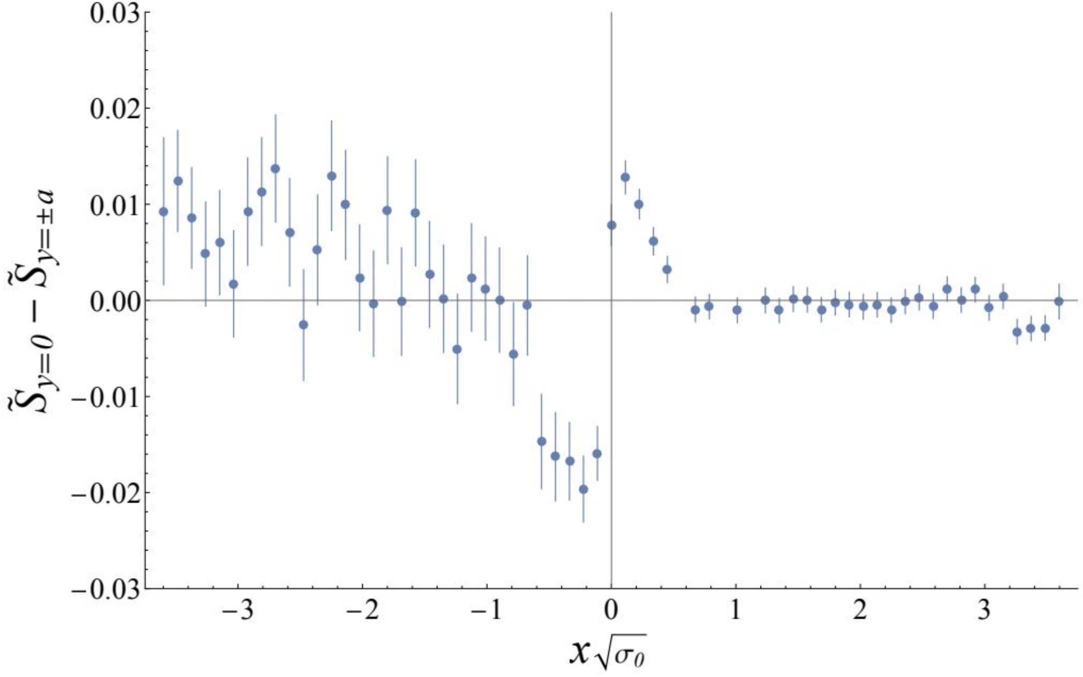


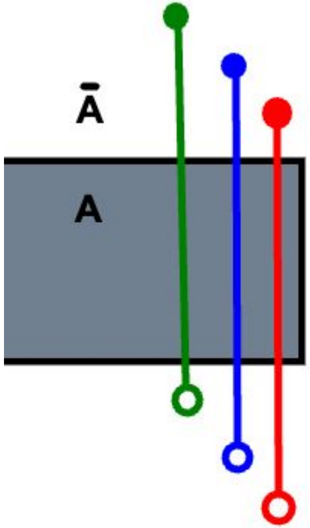
Entropy String Length Dependence





Entropy Location along the String Dependence





Conclusions

Introduced Flux Tube Entanglement Entropy, a UV-finite gauge invariant measure of entanglement entropy due to color flux tube using Polyakov lines and replicas

Used half-slab geometry to explore entanglement entropy of different regions of the QCD string

Found entanglement entropy is finite, follows thin-string model predictions*

Further Directions:

- Longer quark separation
- SU(N) N>2
- D=3+1
- Deconfinement
- Static-quark Baryons: dynamics of the string junction

Backup

Entropy Region Width Dependence