

# Entanglement entropy of a color flux tube in 2+1 D Yang-Mills theory

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# Motivation

- QCD string is a pure spatially-extended state of gluons
- Dynamics of QCD string are important to hadronization in heavy-ion collisions & DIS, e.g. Lund model in Pythia
- Goal: study quantum correlations between parts of a static QCD string
  - Pure SU(2) 2+1 D Yang Mills at  $T = \frac{1}{2} T_c$
  - Static heavy quarks as sources
  - Renyi entanglement entropy as measure of quantum correlations
  - Comparison to string model

# Entanglement Entropy (Renyi Entropy)

$$\hat{\rho}_{\mathcal{A}} = \text{tr}_{\bar{\mathcal{A}}}(\hat{\rho})$$

Example:

$$\psi = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

$$\rho = \begin{pmatrix} \langle \uparrow\uparrow | & \langle \uparrow\downarrow | & \langle \downarrow\uparrow | & \langle \downarrow\downarrow | \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\hat{\rho}_{\mathcal{A}} = \text{Tr}_{\bar{\mathcal{A}}}\rho = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$S_{\text{EE}} = -\text{tr}_{\mathcal{A}}(\hat{\rho}_{\mathcal{A}} \log(\hat{\rho}_{\mathcal{A}}))$$

$$S_{\text{EE}} = -\frac{d}{dq} \log(\text{tr}_{\mathcal{A}}(\hat{\rho}_{\mathcal{A}}^q))|_{q=1}$$

$$S^{(q)} = \frac{1}{1-q} \log(\text{tr}_{\mathcal{A}}(\hat{\rho}_{\mathcal{A}}^q)) \forall q \in \mathbb{N}, q \geq 2.$$

$$S_{\text{EE}} = \lim_{q \rightarrow 1} S^{(q)}.$$

# Difficulties with entanglement entropy in (gauge) field theories

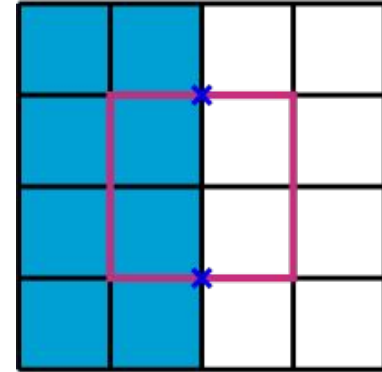
- Entanglement entropy is UV-divergent in field theory
- Hilbert space of gauge invariant states is non-factorizable
- Need gauge invariant definition and lattice implementation
- Want a measure of entanglement for physical object (flux tube) that is **gauge-invariant and finite**

# Entanglement Entropy in Gauge Theories

- Hilbert space of gauge invariant states is non-factorizable in general
- Factorizable within “superselection sectors” (determined by field state on the boundary)

(P. Buividovich et al arxiv:0806.3376, H. Casini et al arxiv:1312.1183, D. Radecivic et al arxiv:1404.1391, S. Ghosh et al 1501.02593, S. Aoki et al arxiv:1502.04267, W. Donnelly arxiv:1109.0036)

- We work with “electric center”



$$\mathcal{H} \neq \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$$

$$\rho_{AA'} = \begin{pmatrix} p_1 \rho_{\mathcal{A}_1 \mathcal{A}'_1} & 0 & \dots & 0 \\ 0 & p_2 \rho_{\mathcal{A}_2 \mathcal{A}'_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & p_m \rho_{\mathcal{A}_m \mathcal{A}'_m} \end{pmatrix} \quad \rho_{\mathcal{A}} = \begin{pmatrix} p_1 \rho_{\mathcal{A}_1} & 0 & \dots & 0 \\ 0 & p_2 \rho_{\mathcal{A}_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & p_m \rho_{\mathcal{A}_m} \end{pmatrix}$$

(H. Casini et al arxiv:hep-th/1312.1183)

# UV-finite Entanglement Entropy

$$\frac{1}{|\partial A|} S = \frac{1}{|\partial A|} S_{UV} + \frac{1}{|\partial A|} S_f$$

(P. V. Buividovich, M. I. Polikarpov arXiv:0802.4247)

(S. Ryu et al arXiv:hep-th/0603001)

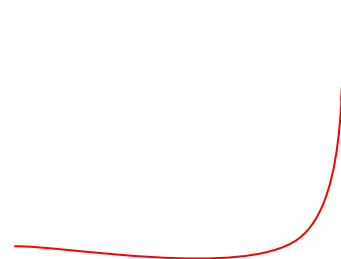
(T. Nishioka et al arXiv:hep-th/0611035)

(I. Klebanov et al arXiv:0709.2140)

$$\tilde{S}_{|Q\bar{Q}}^{(q)} \equiv S_{|Q\bar{Q}}^{(q)} - S^{(q)}$$

(Flux Tube Entanglement Entropy)

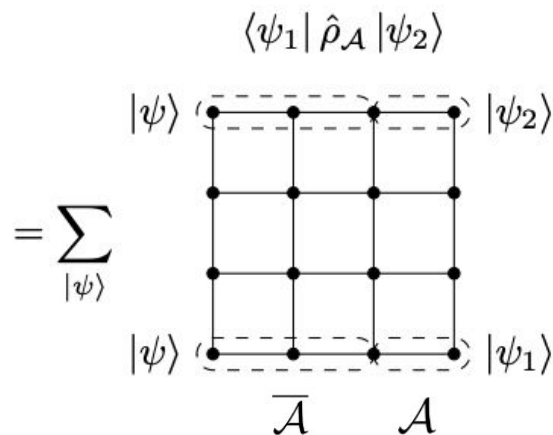
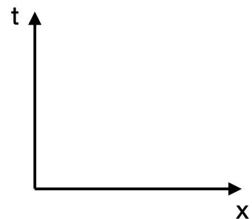
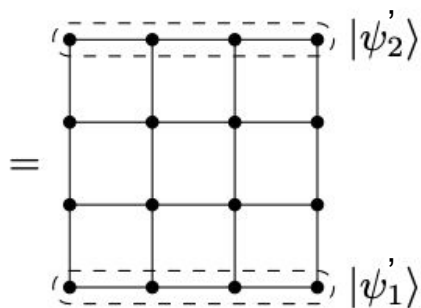
Can be realized on the lattice using  
Polyakov lines and the replica method



# Reduced Density Matrix in Lattice Field Theory

$$\hat{\rho}_{\mathcal{A}} = \text{tr}_{\bar{\mathcal{A}}}(\hat{\rho})$$

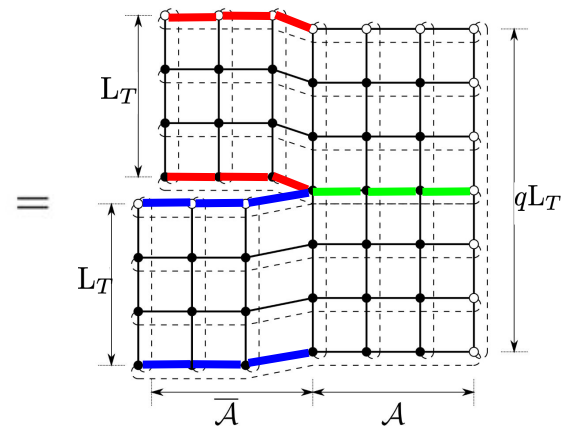
$$\langle \psi'_1 | \hat{\rho} | \psi'_2 \rangle = \frac{1}{Z} \langle \psi'_1 | e^{-\beta \hat{H}} | \psi'_2 \rangle$$



(A. Rabenstein et al arXiv:1812.04279)

# Reduced Density Matrix squared

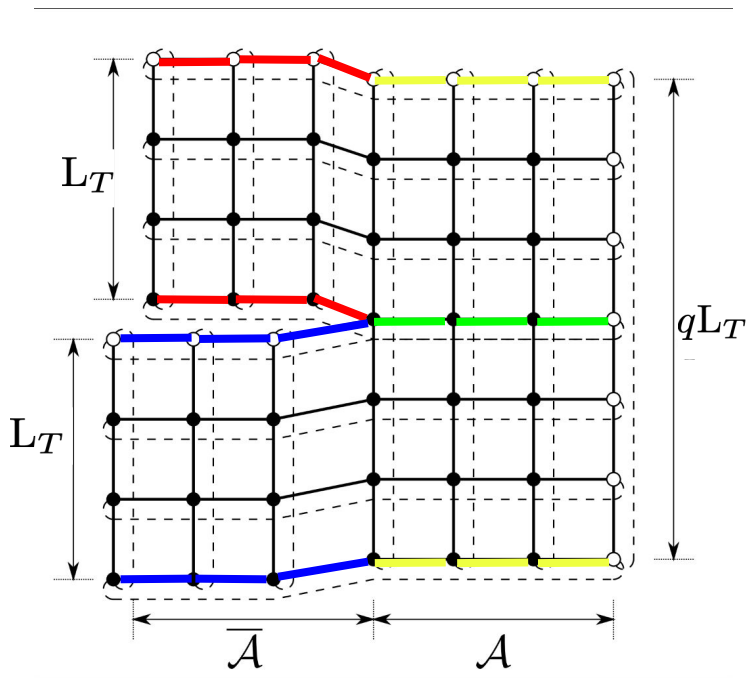
$$\langle \psi_1 | \hat{\rho}_{\mathcal{A}}^2 | \psi_2 \rangle = \langle \psi_1 | \hat{\rho}_{\mathcal{A}} | \psi_k \rangle \langle \psi_k | \hat{\rho}_{\mathcal{A}} | \psi_2 \rangle = \sum_{|\psi_k\rangle, |\psi_{\bar{\mathcal{A}}_1}\rangle, |\psi_{\bar{\mathcal{A}}_2}\rangle} \langle \psi_{\bar{\mathcal{A}}_2} | \langle \psi_{\bar{\mathcal{A}}_1} | \langle \psi_k | \langle \psi_k | \langle \psi_{\bar{\mathcal{A}}_1} | \langle \psi_{\bar{\mathcal{A}}_2} | \langle \psi_1 | \langle \psi_2 |$$



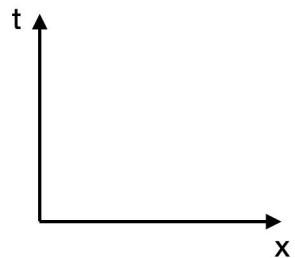
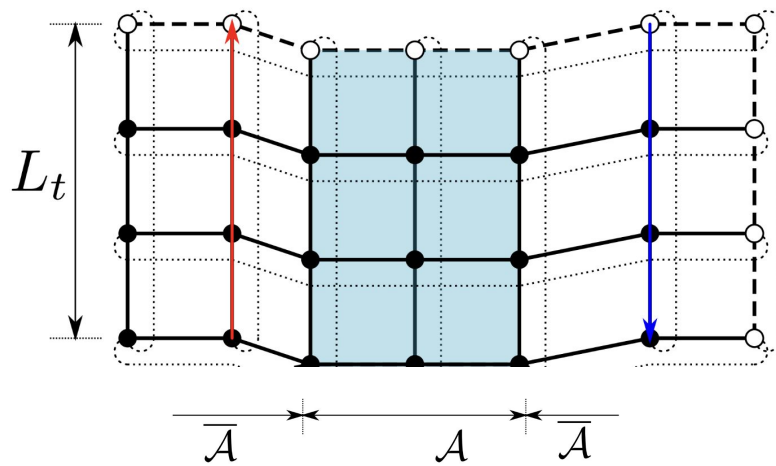


# Reduced Density Matrix squared

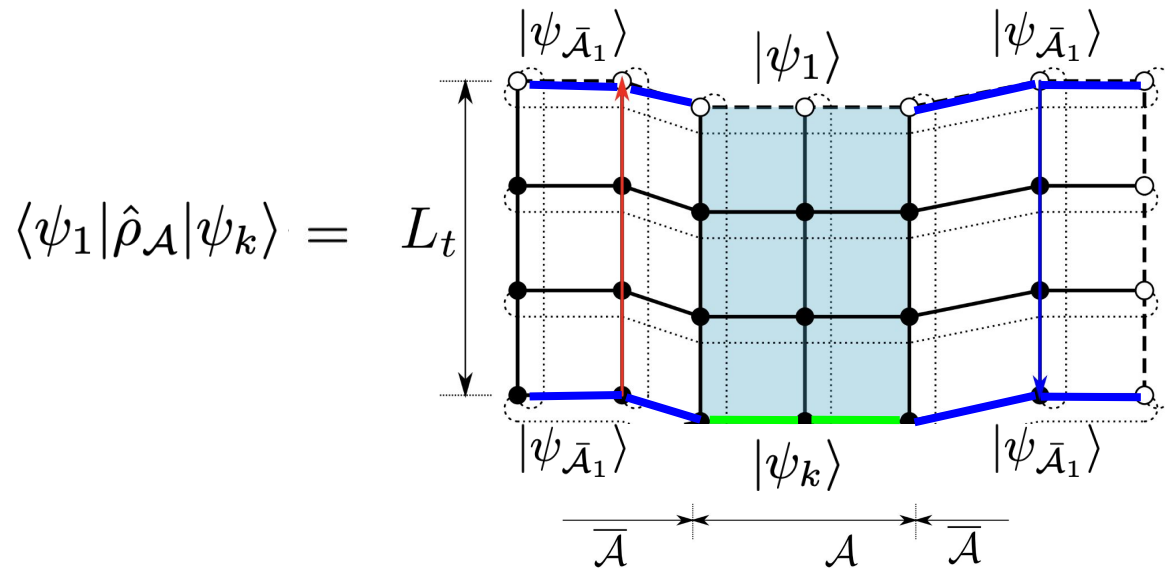
$$\begin{aligned}\text{Tr} \hat{\rho}_A^2 &= \sum_{\psi_A} \langle \psi_A | \hat{\rho}_A^2 | \psi_A \rangle = \\ &= \frac{Z_2}{(Z_1)^2}\end{aligned}$$



# Polyakov Lines

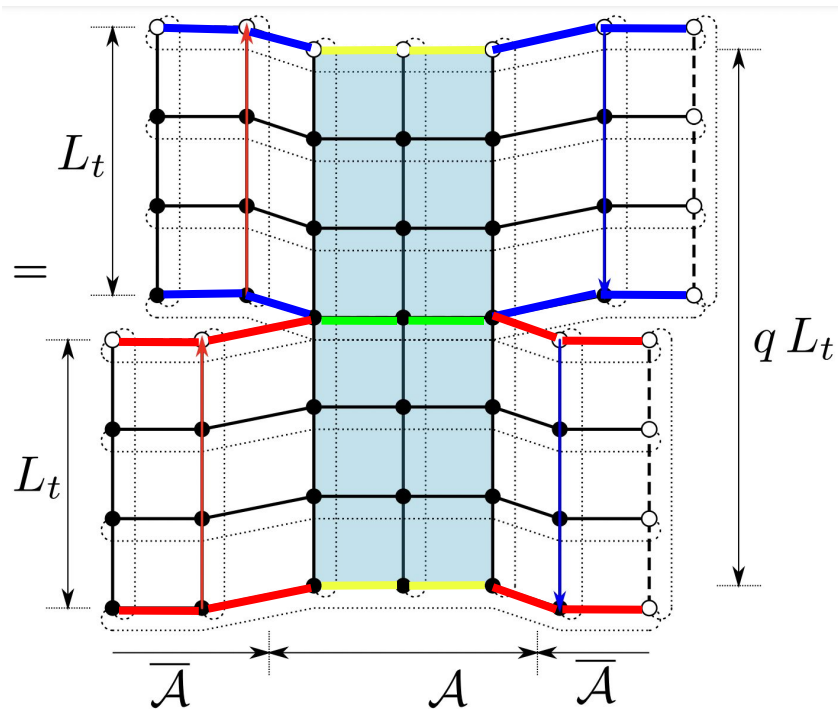
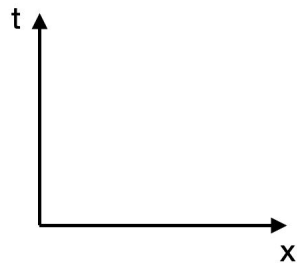


# Polyakov Lines: Reduced Density Matrix



# Polyakov Lines: Reduced Density Matrix squared

$$\text{Tr} \hat{\rho}_{\mathcal{A}}^2 = \sum_{\psi_{\mathcal{A}}} \langle \psi_{\mathcal{A}} | \hat{\rho}_{\mathcal{A}}^2 | \psi_{\mathcal{A}} \rangle =$$



# UV-finite Entanglement Entropy

$$\frac{1}{|\partial A|} S = \frac{1}{|\partial A|} S_{UV} + \frac{1}{|\partial A|} S_f$$

(P. V. Buividovich, M. I. Polikarpov arXiv:0802.4247)  
 (T. Nishioka, T Takayanagi arXiv:hep-th/0611035)

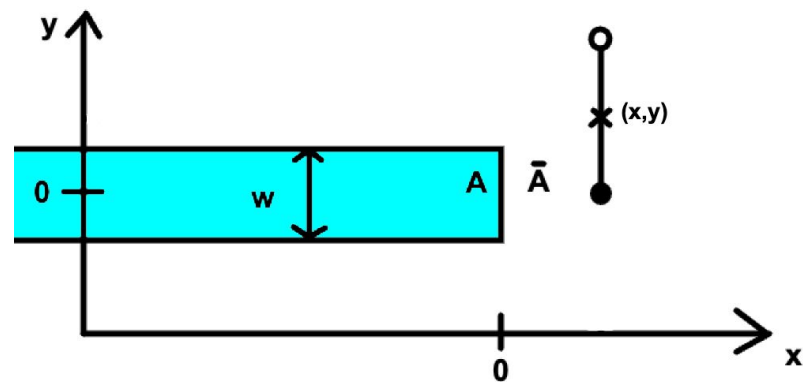
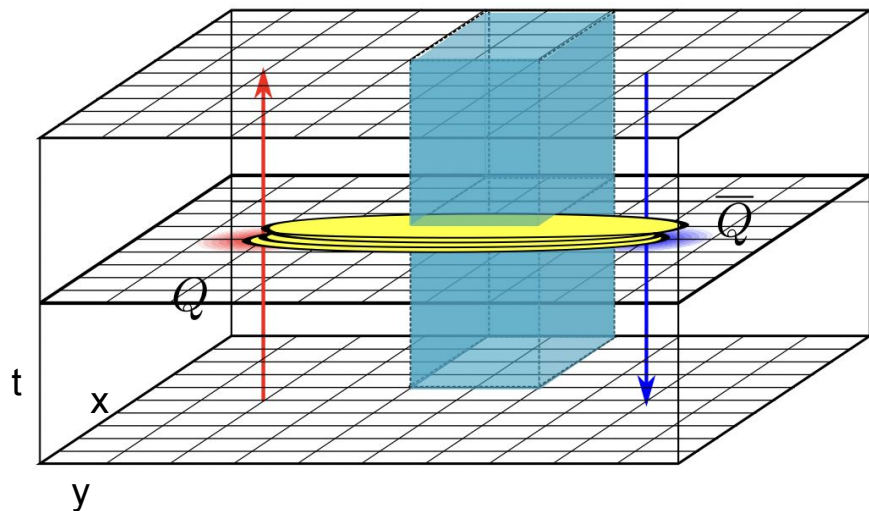
$$\tilde{S}_{|Q\bar{Q}}^{E(q)} \equiv S_{|Q\bar{Q}}^{E(q)} - S^{E(q)}$$

(Flux Tube Entanglement Entropy)

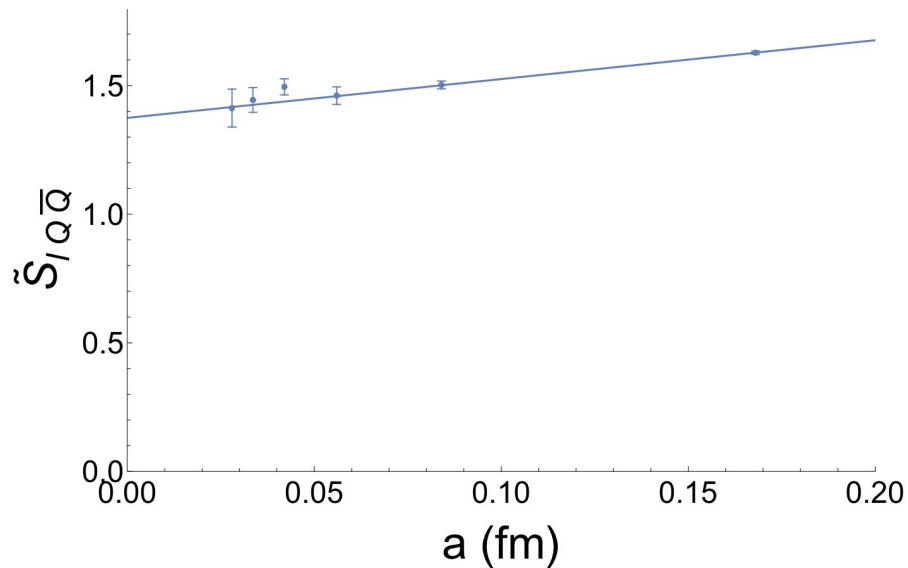
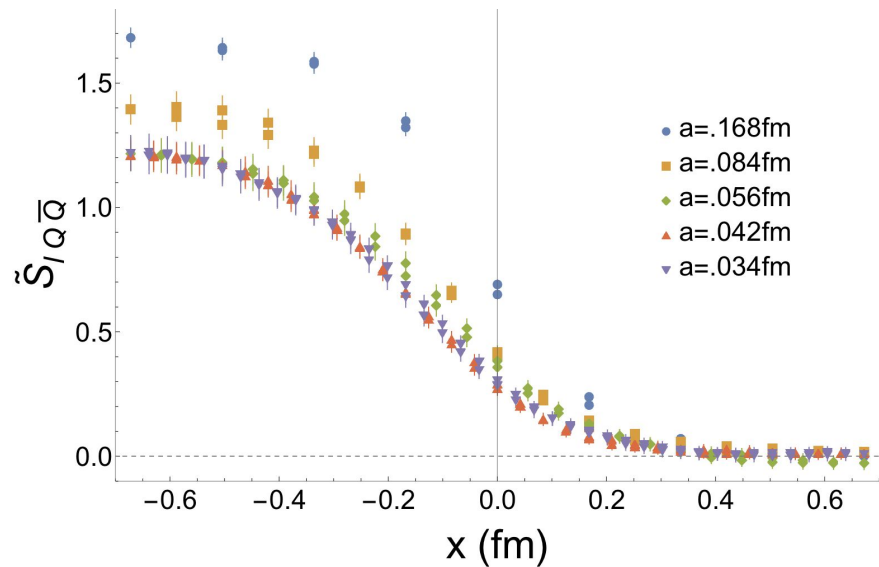
$$S_X^{E(q)} = -\frac{1}{q-1} \log \text{Tr}_A \left[ (\text{Tr}_{\bar{A}} \rho_X)^q \right] = -\frac{1}{q-1} \log \frac{Z_X^{(q)}}{(Z_X)^q}$$

$$\boxed{-\frac{1}{q-1} \log \frac{\langle \prod_{r=1}^q \text{Tr} P_0^{(r)} \text{Tr} P_{\bar{x}}^{(r)\dagger} \rangle}{\langle \text{Tr} P_0 \text{Tr} P_{\bar{x}}^\dagger \rangle^q}} = -\frac{1}{q-1} \log \frac{Z_{|Q\bar{Q}}^{(q)} / Z^{(q)}}{(Z_{|Q\bar{Q}} / Z)^q} = -\frac{1}{q-1} \log \left( \frac{Z_{|Q\bar{Q}}^{(q)}}{(Z_{|Q\bar{Q}})^q} \cdot \frac{Z^q}{(Z)^q} \right) = S_{|Q\bar{Q}}^{E(q)} - S^{E(q)}$$

# Lattice Setup: A = Half-slab



# Finite Entanglement Entropy



$$\tilde{S}_{|Q\bar{Q}}^{(q)} \equiv S_{|Q\bar{Q}}^{(q)} - S^{(q)}$$

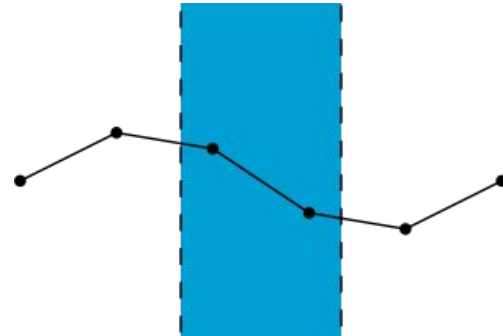
# String Model Predictions

- Thin relativistic string in long string limit (1 transverse dimension)
- Solely entanglement due to vibration of flux tube when fully cross cut
- UV cutoff epsilon
- Follow procedure of (L. Bombelli et al PhysRevD.34.373)

$$H = M^2 L + \frac{\pi}{2M^2 L} \int_0^\pi ds (p^2 + M^4 x'(s)^2)$$

M. Luscher et al, Nucl. Phys. B 180, 1 (1981)

String tension

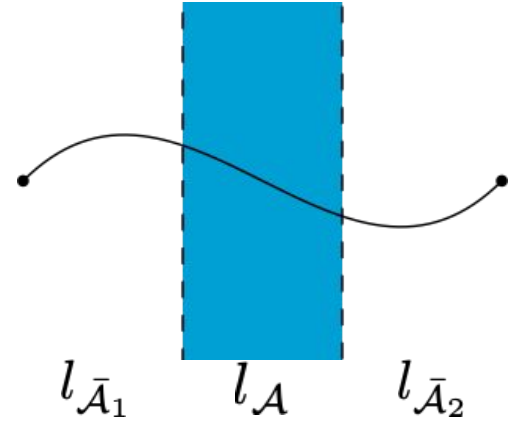
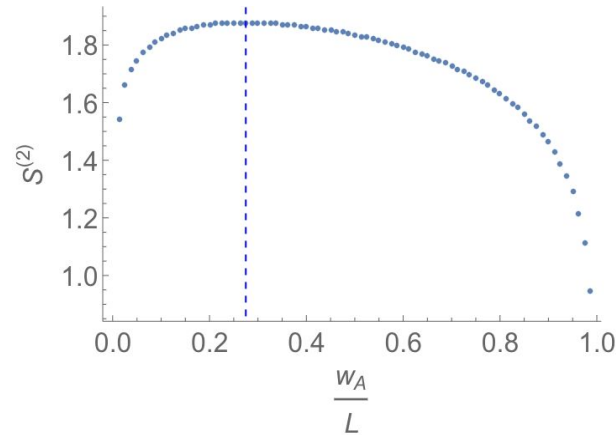
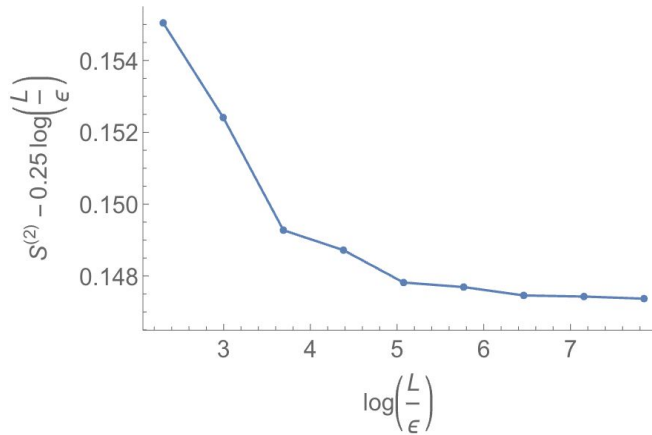




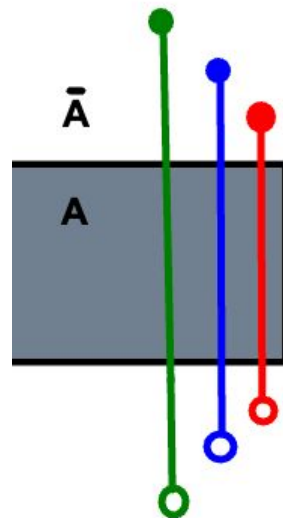
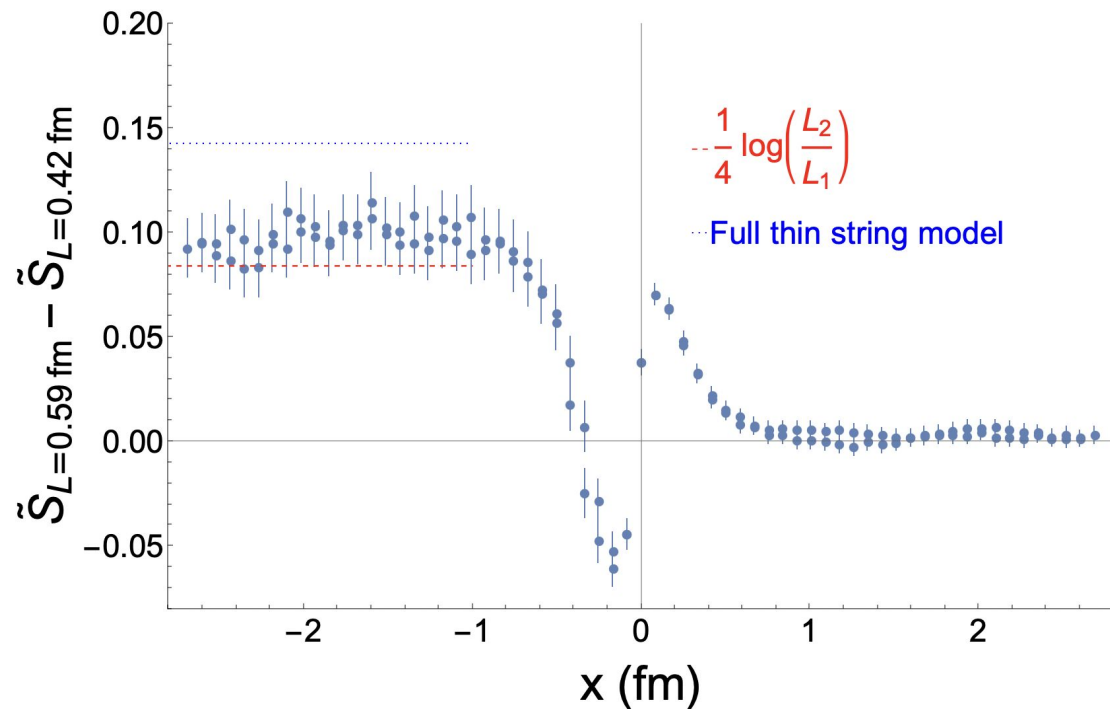
# String Model Predictions

- Numerically calculate entropy taking  $L/\epsilon$  to infinity
- Splits into cutoff scaling and string proportion terms

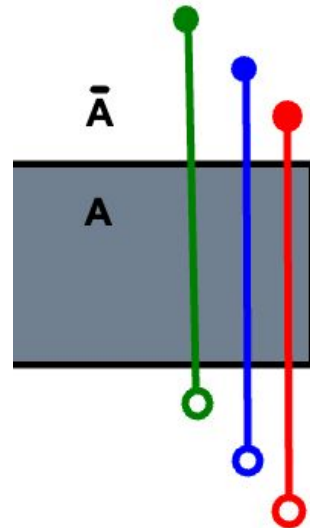
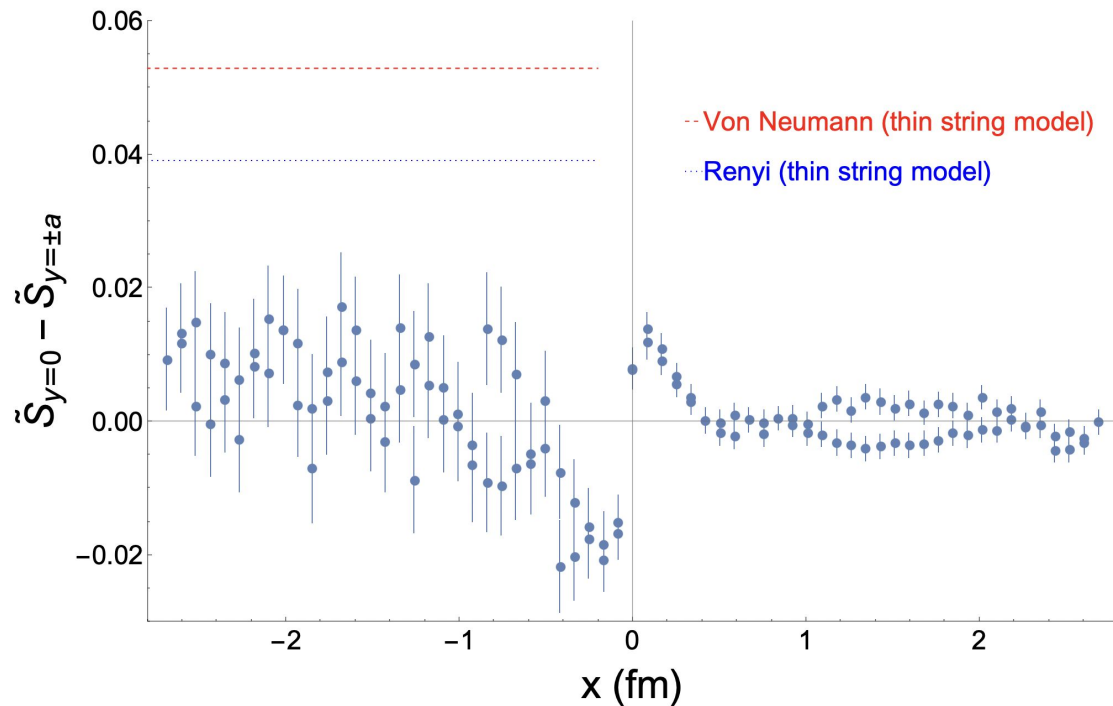
$$S^{(2)} \approx \frac{1}{4} \ln(L/\epsilon) + \ln(l_{\mathcal{A}}^{0.114} (l_{\bar{\mathcal{A}}_1} l_{\bar{\mathcal{A}}_2})^{0.136}) + 0.566$$



# Entropy String Length Dependence



# Entropy Location along the String Dependence



# Conclusions

Introduced Flux Tube Entanglement Entropy, a UV-finite gauge invariant measure of entanglement entropy due to color flux tube using Polyakov lines and replicas

Used half-slab geometry to explore entanglement entropy of different regions of the QCD string

Found entanglement entropy is finite, follows thin-string model predictions\*

Further Directions:

- Longer quark separation
- $SU(N)$   $N > 2$
- $D=3+1$
- Deconfinement
- Static-quark Baryons: dynamics of the string junction

Backup

# Entropy Region Width Dependence

