Investigating the flux tube structure within full QCD

Leonardo Cosmai

INFN

ICSC
Centro Nazionale di Ricerca in HPC,
Big Data and Quantum Computing

in collaboration with:

Marshall Baker *(Univ. of Washington, Seattle)*, **Paolo Cea** *(INFN, Bari)*, **Volodymyr Chelnokov** *(Goethe Universität, Frankfurt)*, **Alessandro Papa** *(Univ. Calabria and INFN, Cosenza)*

LATTICE 2024
LIVERPOOL

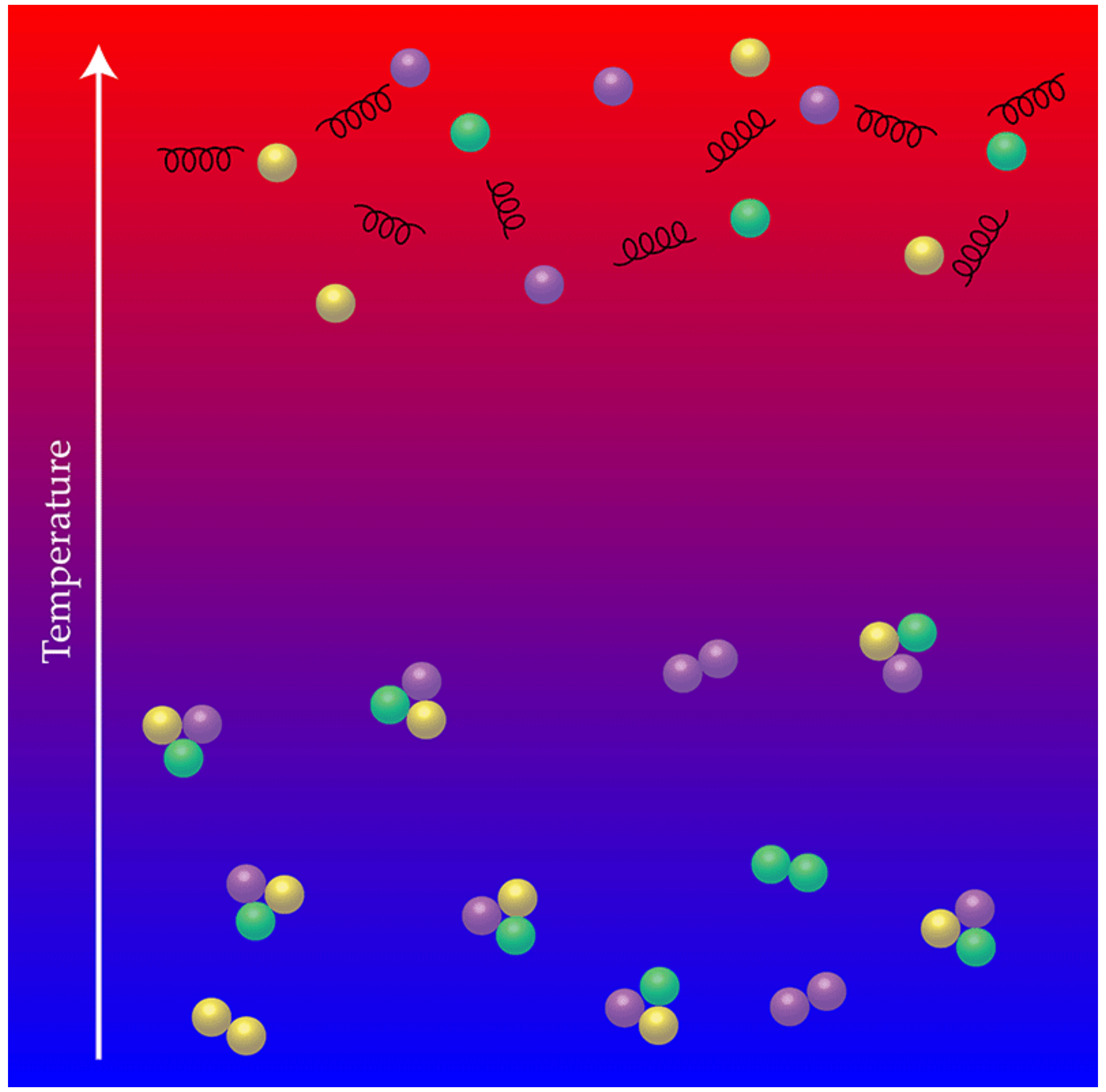
Liverpool, 31 July 2024

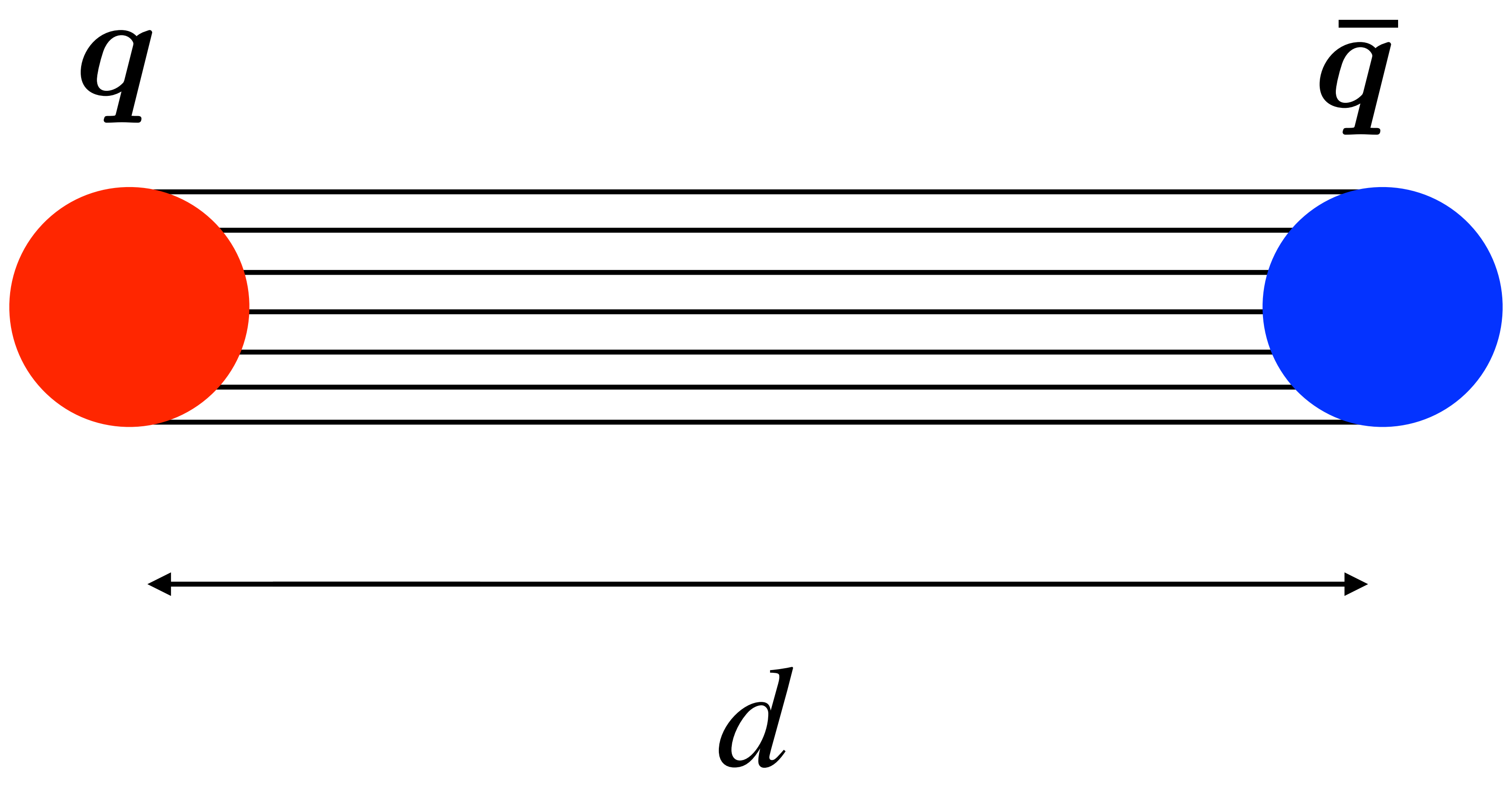
INTRODUCTION

- Achieving a detailed understanding of **color confinement** remains a central goal for nonperturbative studies of QCD

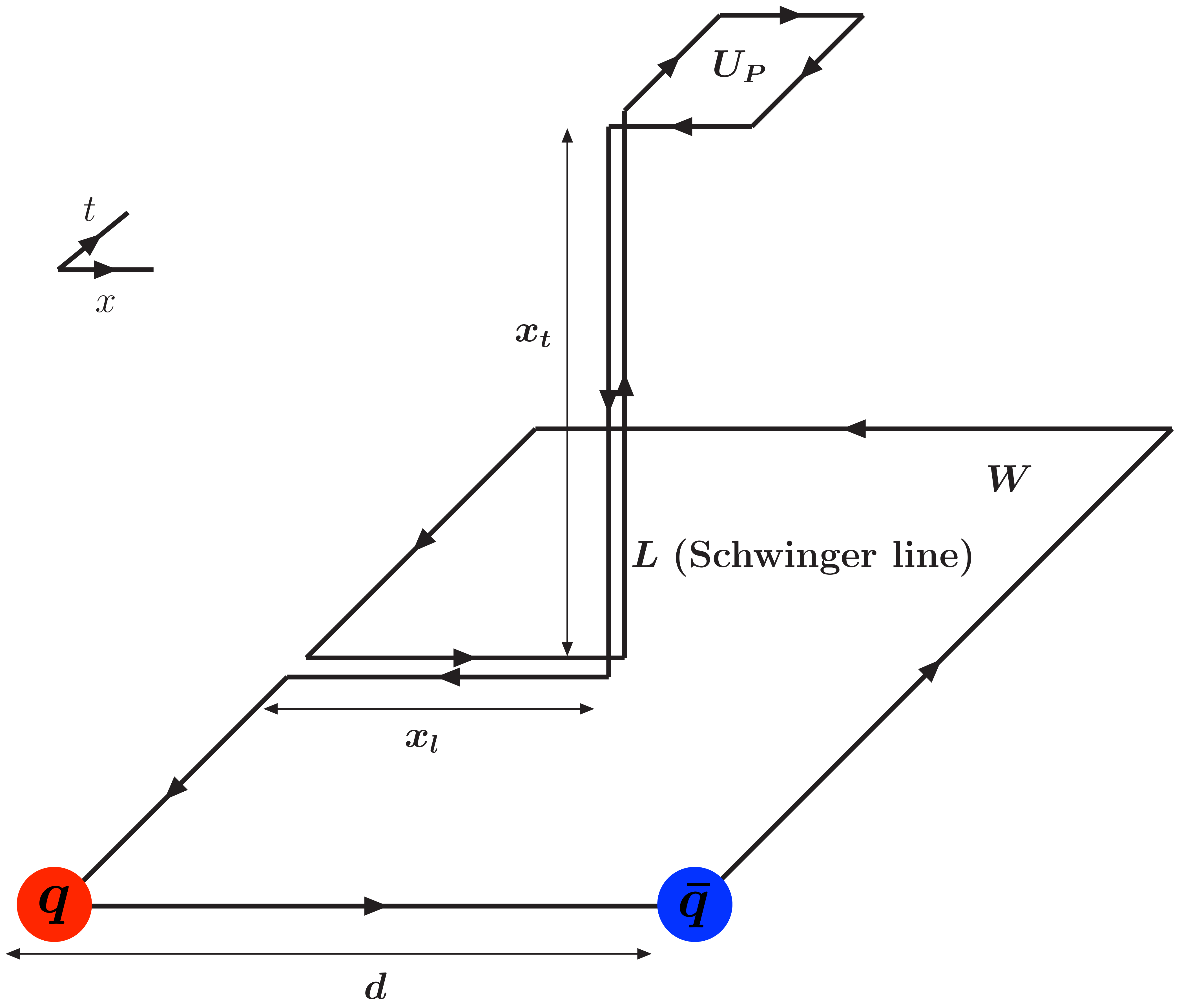
- Lattice numerical simulations have long revealed the emergence of **tube-like structures** when analyzing the chromoelectric fields between static quarks.

- The observation of these **tube-like structures** in lattice simulations is related to the linear potential between static color charges and provides **direct numerical evidence for color confinement**.





THE SPATIAL DISTRIBUTION OF THE COLOR FIELDS



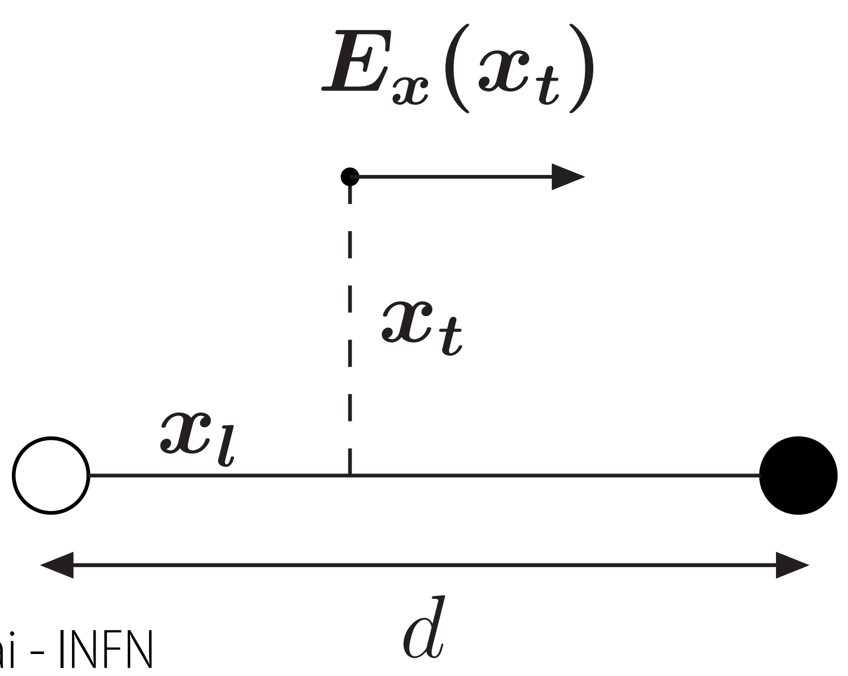
- **lattice measurements of the connected correlation function**

$$\rho^{\text{conn}}_{\mathbf{W},\mu\nu} = \frac{\langle \text{tr}(\mathbf{W L U_P L^\dagger}) \rangle}{\langle \text{tr}(\mathbf{W}) \rangle} - \frac{\mathbf{1}}{\mathbf{N}} \frac{\langle \text{tr}(\mathbf{U_P}) \text{tr}(\mathbf{W}) \rangle}{\langle \text{tr}(\mathbf{W}) \rangle}$$

- lattice definition of the **gauge-invariant field strength tensor**

$$\rho^{\text{conn}}_{\mathbf{W},\mu\nu} \equiv \mathbf{a^2 g} \langle \mathbf{F}_{\mu\nu} \rangle_{\mathbf{q\bar{q}}} \equiv \mathbf{a^2 g\, F}_{\mu\nu}$$

- rotating the plaquette relative to the plane of the Wilson loop allows us to extract the **components of the field tensor:**
  - plaquette U_P in the plane $(\hat{\mu} = 4, \hat{\nu} = 1) \longrightarrow E_x$
  - plaquette U_P in the plane $(\hat{\mu} = 4, \hat{\nu} = 2) \longrightarrow E_y$
  - plaquette U_P in the plane $(\hat{\mu} = 4, \hat{\nu} = 3) \longrightarrow E_z$
  - plaquette U_P in the plane $(\hat{\mu} = 2, \hat{\nu} = 3) \longrightarrow B_x$
  - plaquette U_P in the plane $(\hat{\mu} = 3, \hat{\nu} = 1) \longrightarrow B_y$
  - plaquette U_P in the plane $(\hat{\mu} = 4, \hat{\nu} = 2) \longrightarrow B_z$



SYMMETRY: The fields takes on the same values at spatial points connected by rotations around the axis on which the sources are located

OUR RESULTS FOR SU(3) PURE GAUGE

Structure of the color fields around a static quark-antiquark pair for $\mathbf{T=0}$ and $\mathbf{T \neq 0}$.

- The chromomagnetic field around the sources is compatible with zero within statistical errors.
- The dominant component of the chromoelectric field is longitudinal.
- The components of the chromoelectric field transverse to the line connecting the sources are also smaller than the longitudinal component and can be matched to an effective Coulomb-like field $\vec{\mathbf{E}}^{\mathrm{C}}(\vec{\mathbf{r}})$ satisfying the following conditions:
  - The transverse component $\mathbf{E_y}$ of the chromoelectric field is identified with the transverse component $\mathbf{E_y^C}$ of the perturbative field

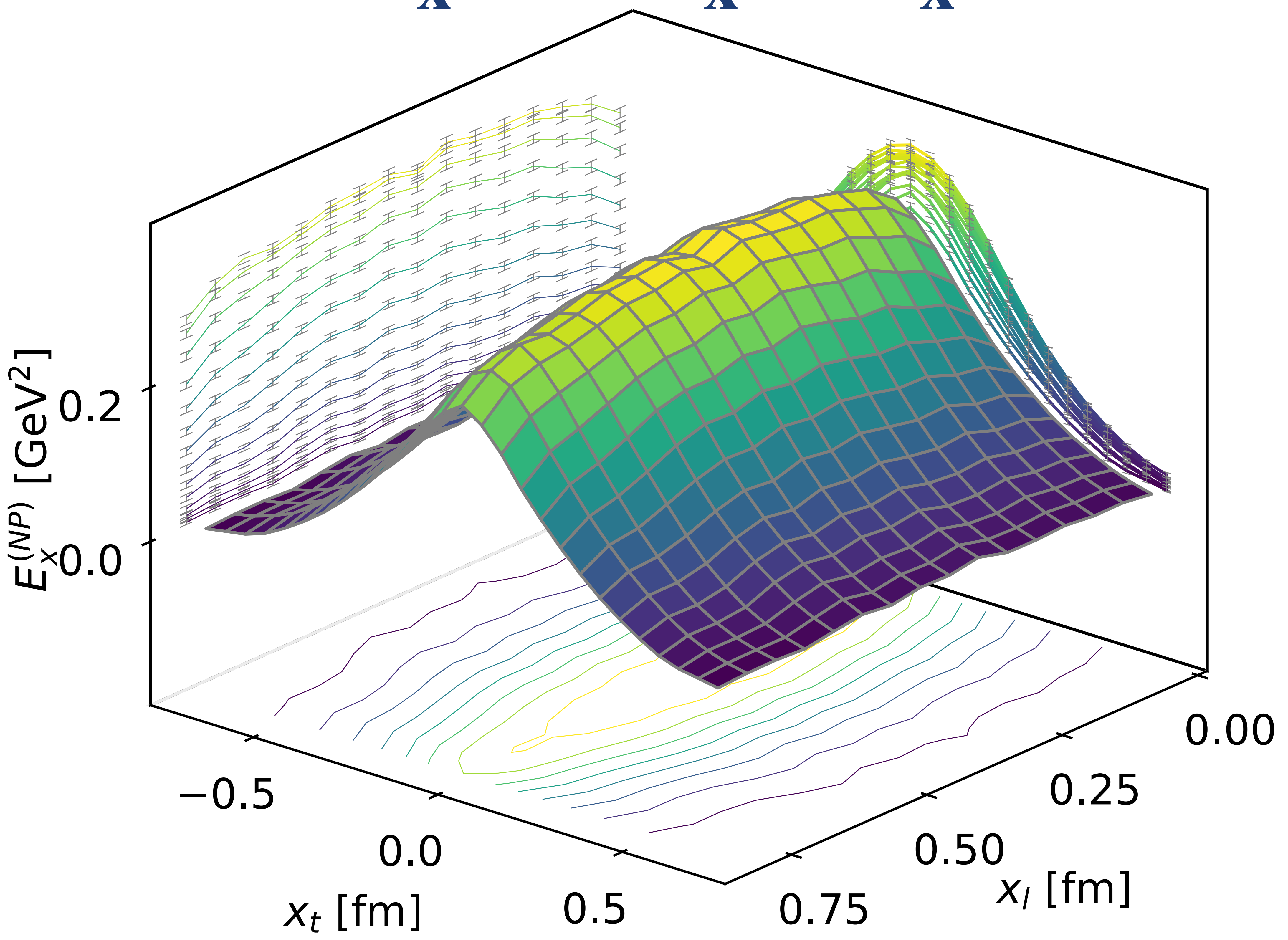
$$\mathbf{E_y^C \equiv E_y}$$

  - The perturbative field $\mathbf{E}^{\mathrm{C}}$ is irrotational

$$\vec{\nabla} \times \vec{\mathbf{E}}^{\mathrm{C}} = \mathbf{0}$$

The longitudinal $\mathbf{E_x}$ can be separated into the **perturbative**, short-distance part $\mathbf{E_x^C}$ and a **nonperturbative** term $\mathbf{E_x^{NP}}$, encoding the confining information, which is shaped as a **smooth flux tube**.

$$\mathbf{E_x^{NP} = E_x - E_x^C}$$



$$\mathrm{SU(3)} \quad \beta = 6.370 \quad \mathrm{d} = 0.85\ \mathrm{fm}$$

QCD (2+1): LATTICE SETUP

- Simulation of lattice **QCD with 2+1 flavors of HISQ (Highly Improved Staggered Quarks) quarks**, with the tree level improved Symanzik gauge action (HISQ/tree).

- Couplings are adjusted so as to move on a **line of constant physics** (LCP), as determined in Bazavov et al (arXiv:111.1710) with the strange quark mass $\mathbf{m_s}$ fixed at its physical value and a light-to-strange mass ratio $\mathbf{m_l/m_s = 1/20}$, corresponding to a **pion mass of 160 MeV** in the continuum limit.

- We fix the **lattice spacing** through the observable $\mathbf{r_1}$ as defined in Bazavov et al (arXiv:111.1710)

$$\frac{a}{r_1}(\beta)_{m_l=0.05m_s} = \frac{c_0 f(\beta) + c_2(10/\beta) f^3(\beta)}{1 + d_2(10/\beta) f^2(\beta)}$$

$$c_0 = 44.06,\ c_2 = 272102,\ d_2 = 4281,\ r_1 = 0.3106(20)\ \mathrm{fm}$$

- **MILC code** for producing **gauge configurations** (1 saved after 25 RHMC trajectories) and for the measurements of the chromoelectromagnetic field tensor. **Simulations on LEONARDO@Cineca.**

- **Smoothing of gauge configuration:** 1HYP on temporal links + n HYP3d on space links.

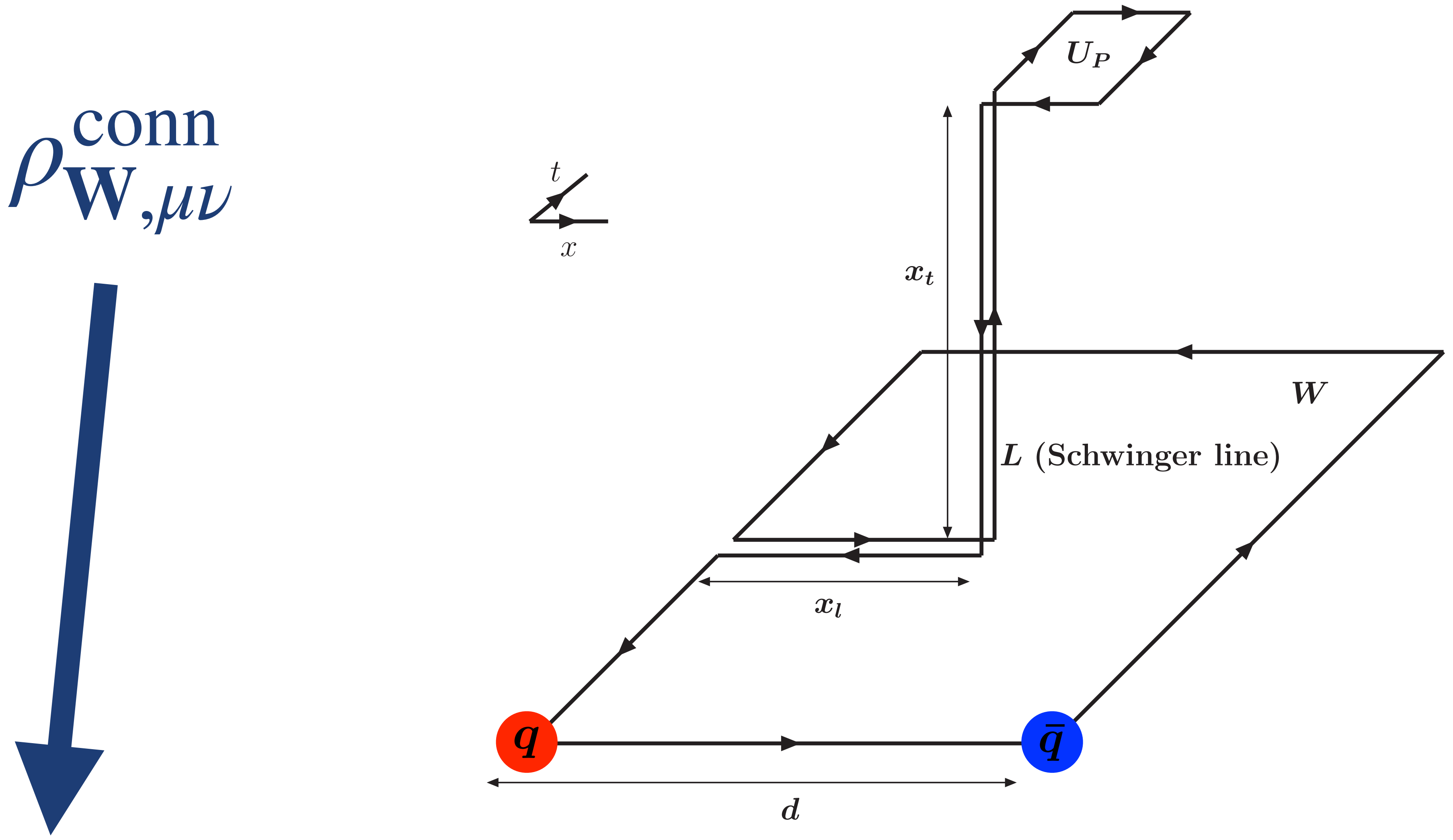
SUMMARY OF THE NUMERICAL SIMULATIONS

lattice size	beta	a(beta) [fm]	d [lattice spacings]	d [fm]	#of measurements
48^4	6.885	0.0949777	6	0.569866	500
32^4	7.158	0.0738309	8	0.590647	10064
24^4	6.445	0.144692	5	0.723462	3330
32^4	7.158	0.0738309	10	0.738309	10181
48^4	6.885	0.0949777	8	0.75982	779
32^4	6.885	0.0949777	8	0.759821	4409
32^4	6.5824	0.126658	6	0.759947	2667
32^4	6.3942	0.15203	5	0.760151	3000
32^4	6.885	0.0949777	9	0.854799	4347
32^4	6.25765	0.173715	5	0.868573	3545
32^4	6.5824	0.126658	7	0.886605	2667
32^4	6.3942	0.15203	6	0.912182	3000
48^4	6.885	0.0949777	10	0.949777	779
32^4	7.158	0.0738309	13	0.959801	10183
24^4	6.445	0.144692	7	1.01285	3330
32^4	6.5824	0.126658	8	1.01326	2666
32^4	7.158	0.0738309	14	1.03363	2107
32^4	6.25765	0.173715	6	1.04229	3549
32^4	6.885	0.0949777	11	1.04475	4408
32^4	6.3942	0.15203	7	1.06421	3000
32^4	6.33727	0.160714	7	1.125	3133
32^4	6.885	0.0949777	12	1.13973	4409
48^4	6.885	0.0949777	12	1.13973	769
32^4	6.5824	0.126658	9	1.13992	2667
32^4	6.314762	0.164286	7	1.15	3651
24^4	6.445	0.144692	8	1.157536	3330
32^4	6.28581	0.168999	7	1.18299	3148
32^4	6.25765	0.173715	7	1.216	3546
32^4	6.3942	0.15203	8	1.21624	3000
32^4	6.885	0.0949777	13	1.23471	4409
32^4	6.5824	0.126658	10	1.26658	2667
32^4	6.3942	0.15203	9	1.36827	3000

- **distance between the static sources:**

$$0.570 \le d \le 1.368\ \mathrm{fm}$$

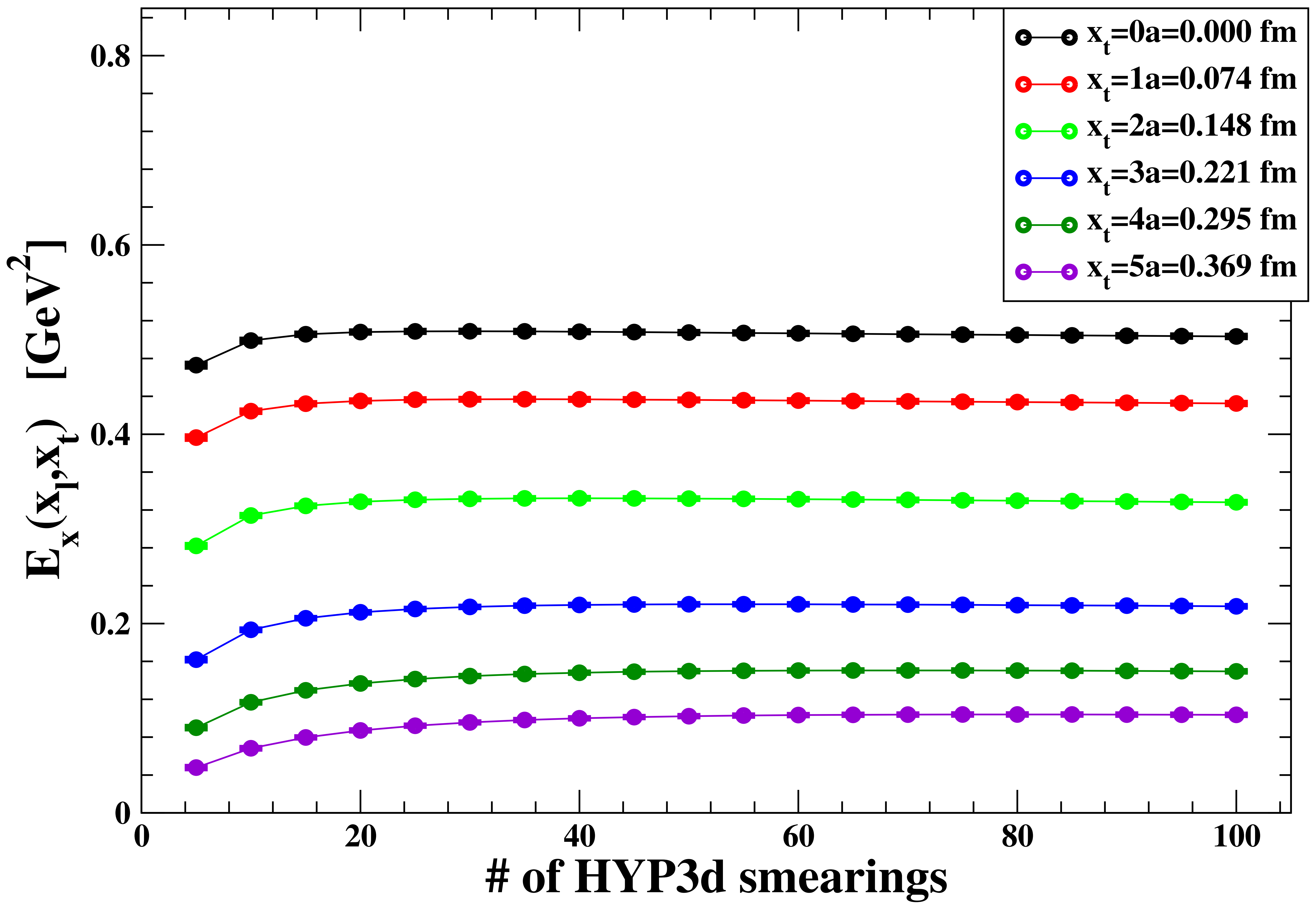
$$\rho^{\mathrm{conn}}_{\mathbf{W},\mu\nu}$$



- **Nontrivial renormalization** [N.Battelli, C.Bonati, arXiv:1903.10463] which depends on $\mathbf{x_t}$. By comparing our results we argued that **smearing** behaves as an **effective renormalization.**

- The smearing procedure can also be validated a posteriori by the **observation of continuum scaling.**

QCD (2+1): HYP3D SMEARING



Behavior under smearing of the "full" longitudinal electric field, $\mathbf{E_x}$, for different values of the transverse distance $\mathbf{x_t}$, at $\beta = 7.158$ and $\mathbf{d} = 10\mathbf{a} = 0.74\,\mathrm{fm}$ on a lattice 32^4.

- The connected correlator exhibits **large fluctuations** at the scale of the lattice spacing, which are responsible for a **small signal-to-noise ratio**.

- To extract the **physical information** carried by fluctuations at the physical scale (and, therefore, at large distances in lattice units), we smoothed out configurations by a **smearing procedure.**

- **Our setup** consists of
  - **one step** of 4-dimensional hypercubic smearing on the temporal links (**HYPt**), with smearing parameters $(\alpha_1, \alpha_2, \alpha_3) = (1.0, 1.0.0.5)$
  - **N steps** of hypercubic smearing (**HYP3d**) restricted to the three spatial directions with $(\alpha_1, , \alpha_3) = (0.75, 0.3)$.

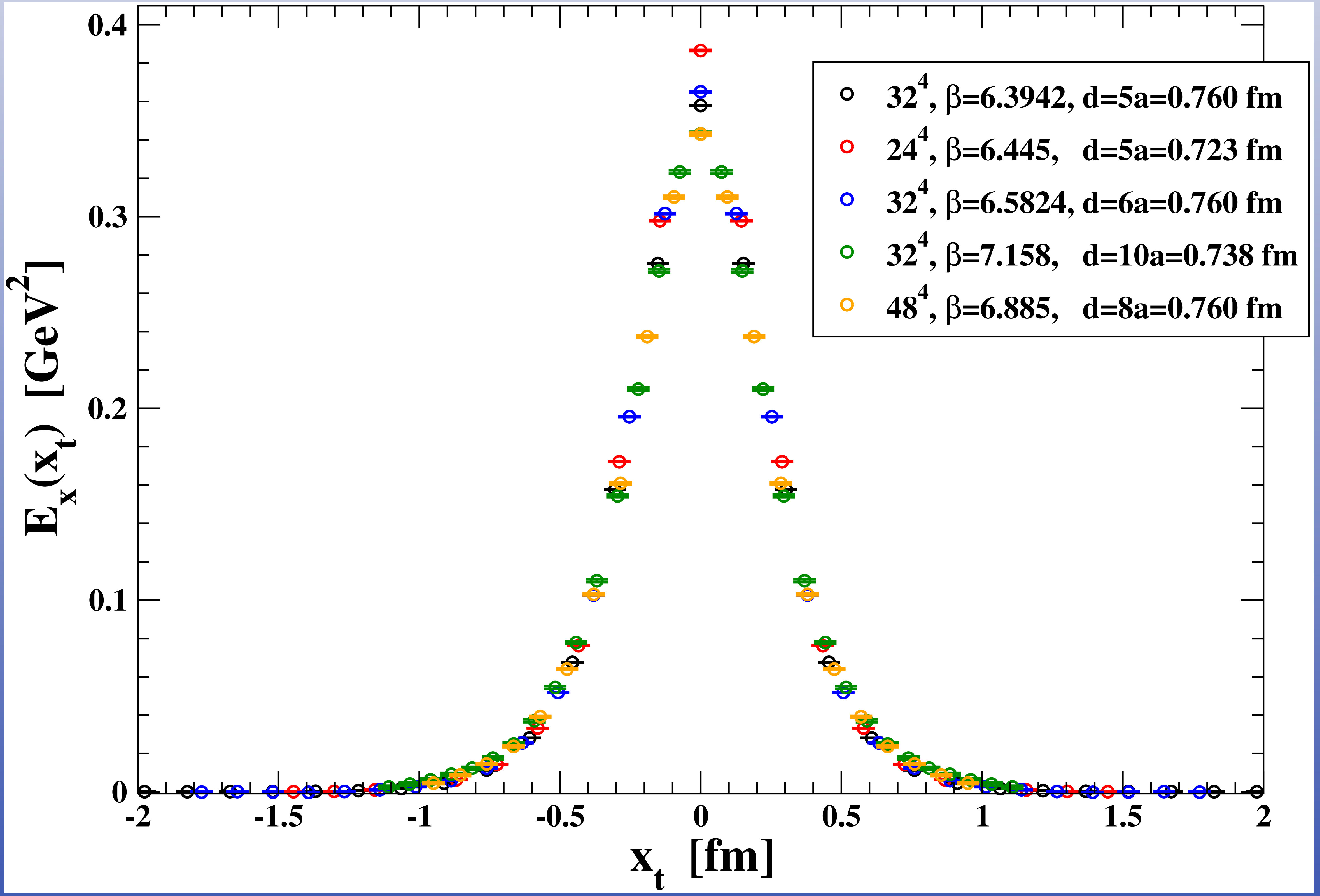
Optimal number of smearing steps: the field takes its maximum value.

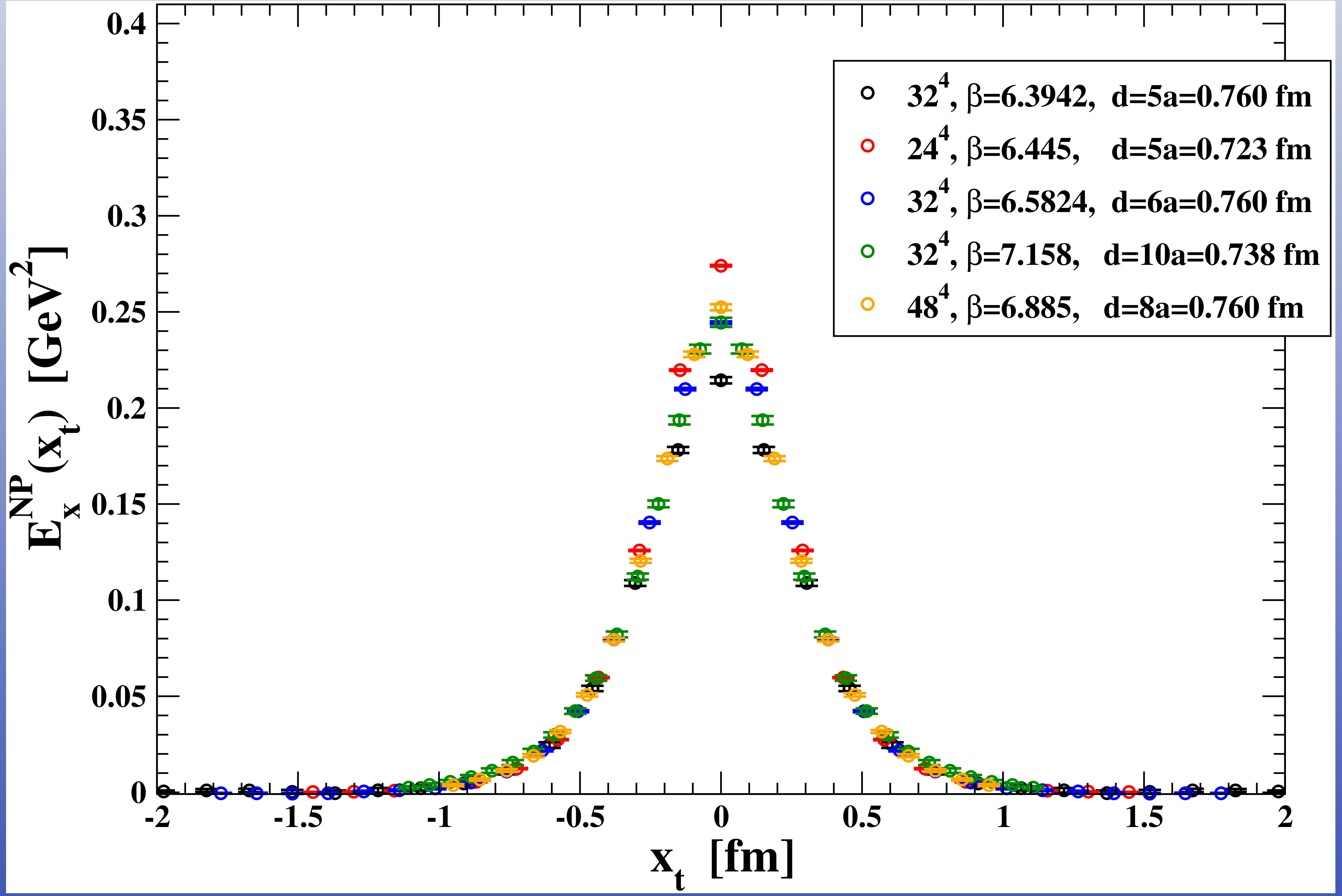
CONTINUUM SCALING

- We verified that our lattice setup is **close enough to the continuum limit**

➡ by checking that different choices of the lattice parameters, corresponding to the same physical distance **d** between the sources, lead to the same values of the relevant observables when measured in physical units.

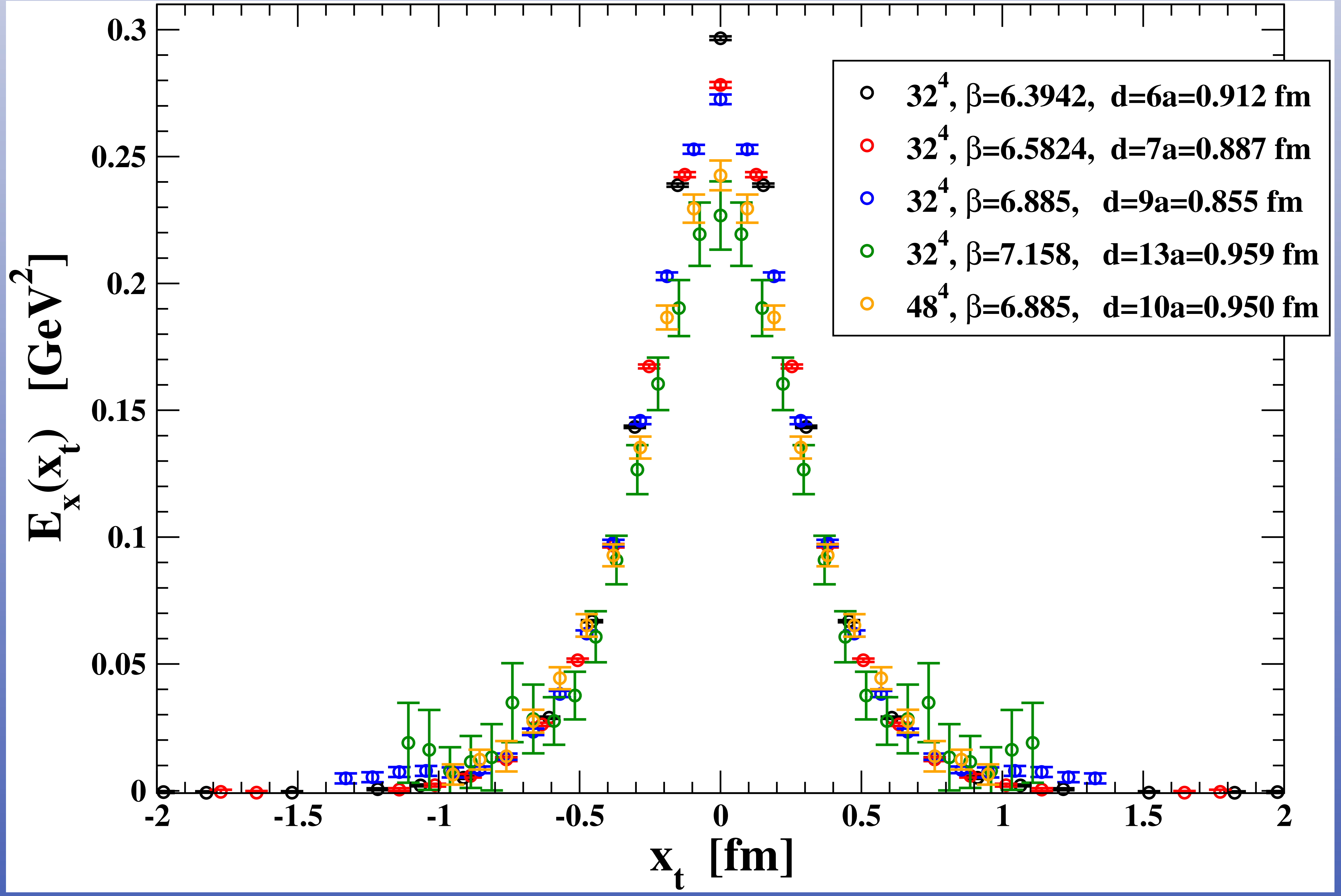
$0.723 \leq d \leq 0.760$ fm

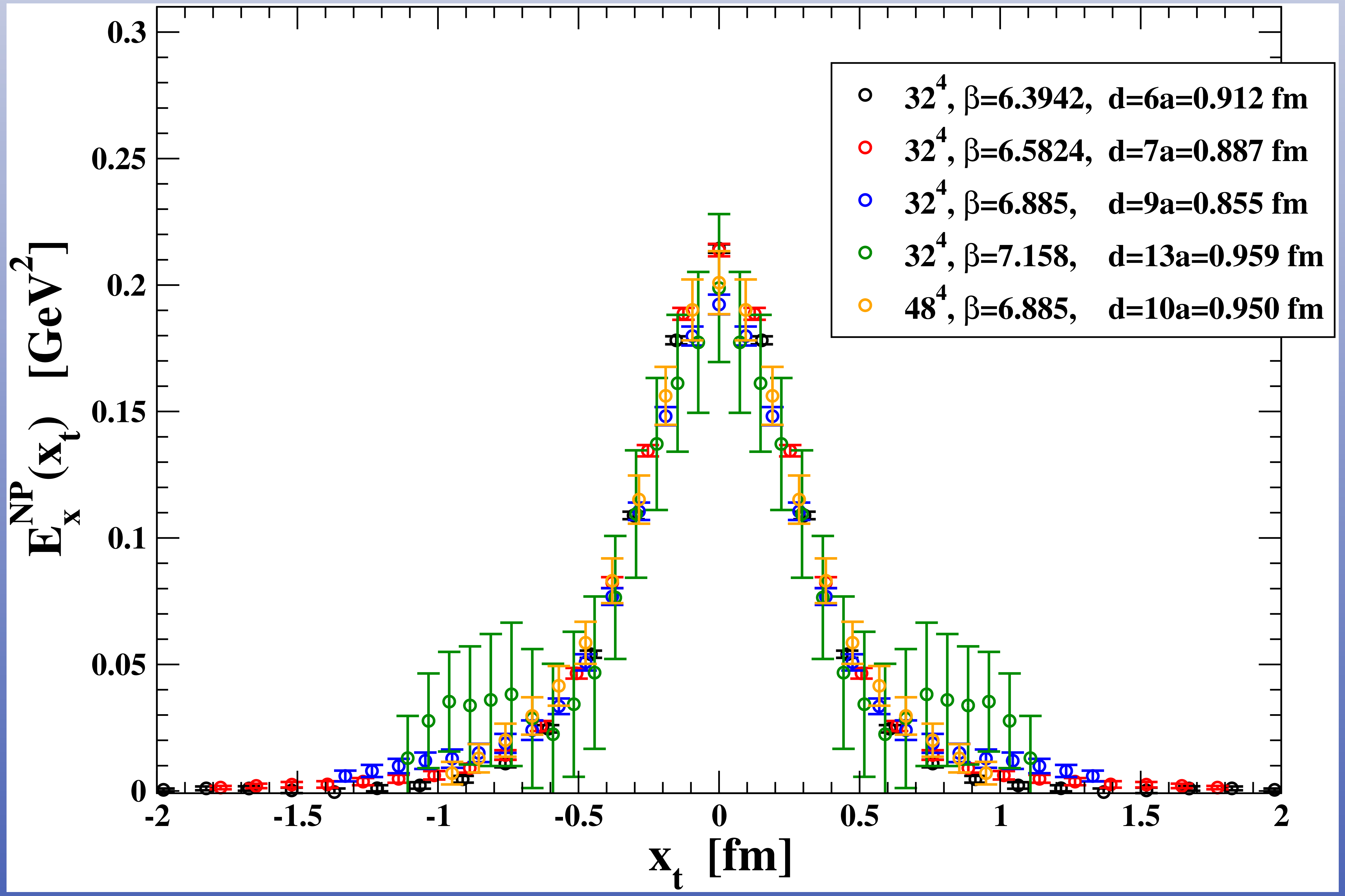




CONTINUUM SCALING (cont'd)

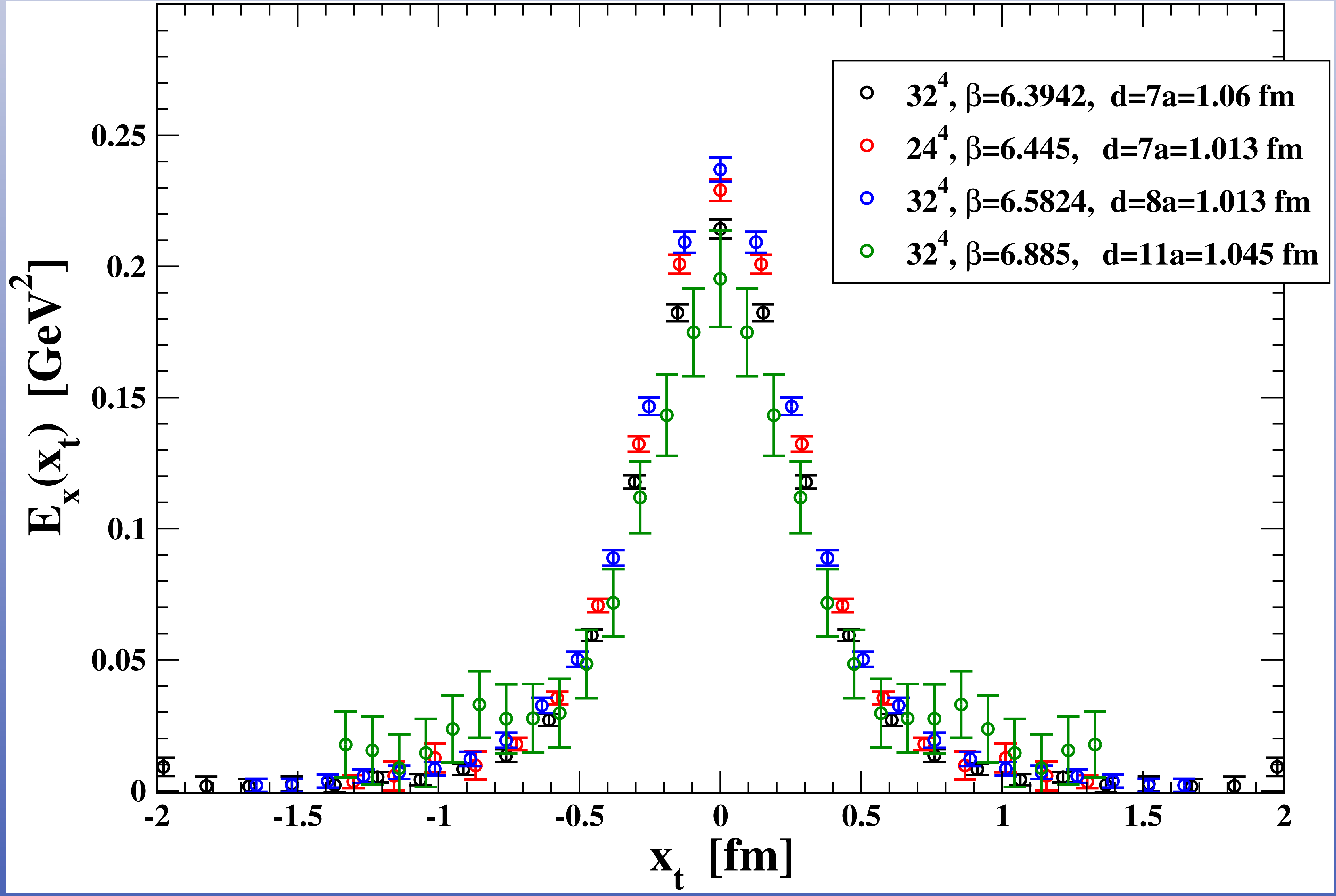
$$0.855 \leq d \leq 0.959\ \text{fm}$$

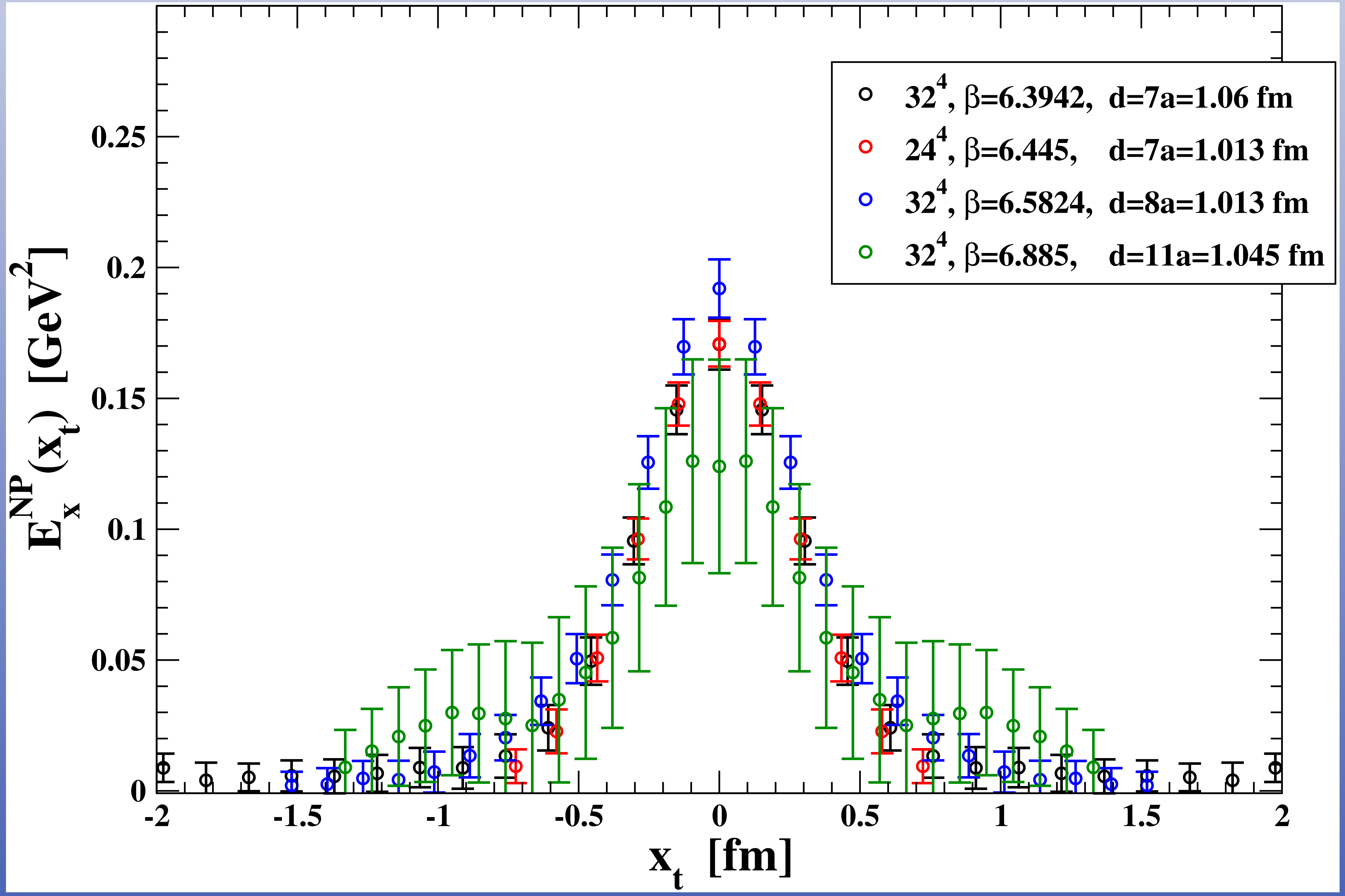




CONTINUUM SCALING (cont'd)

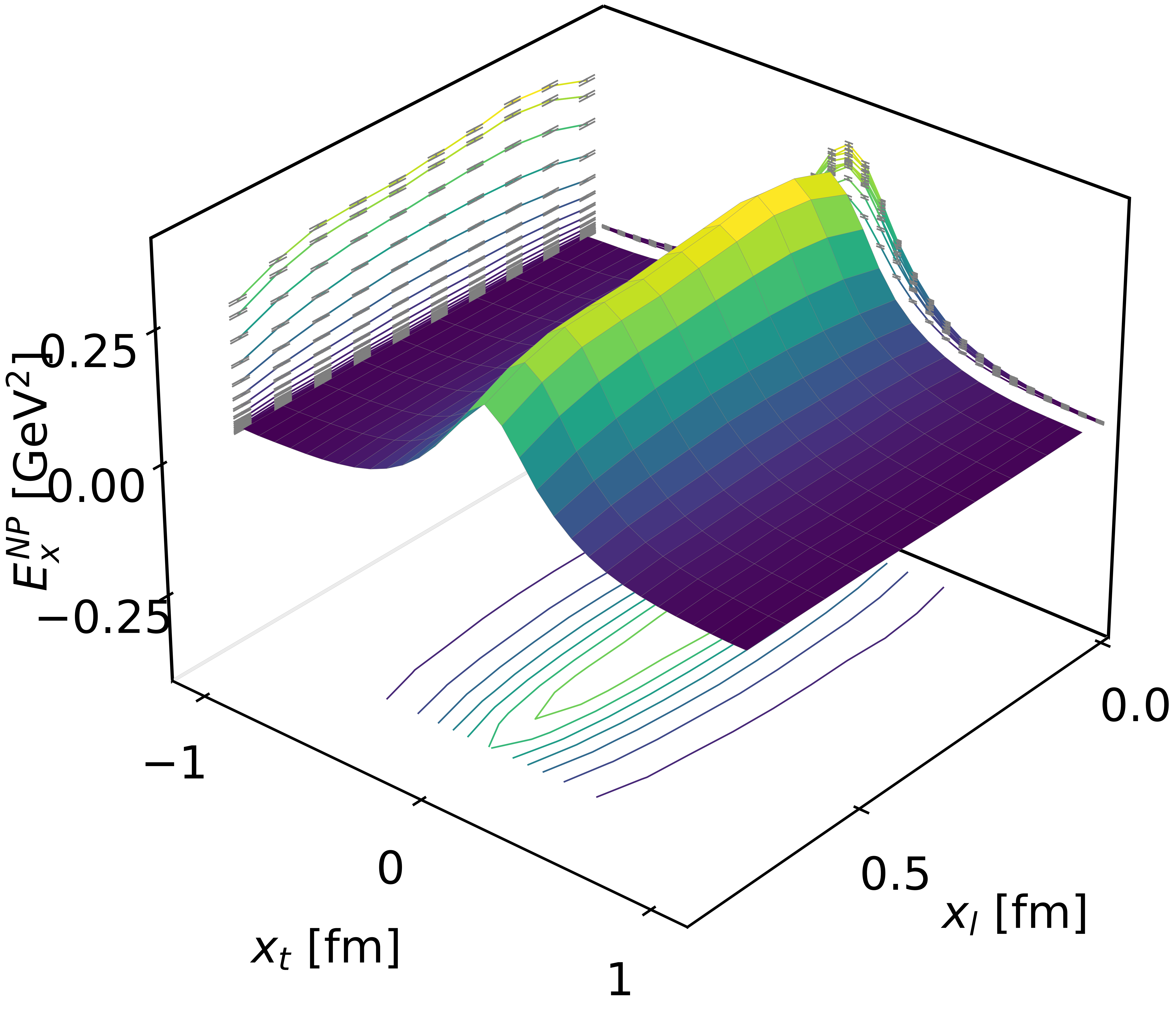
$1.013 \leq d \leq 1.060$ fm



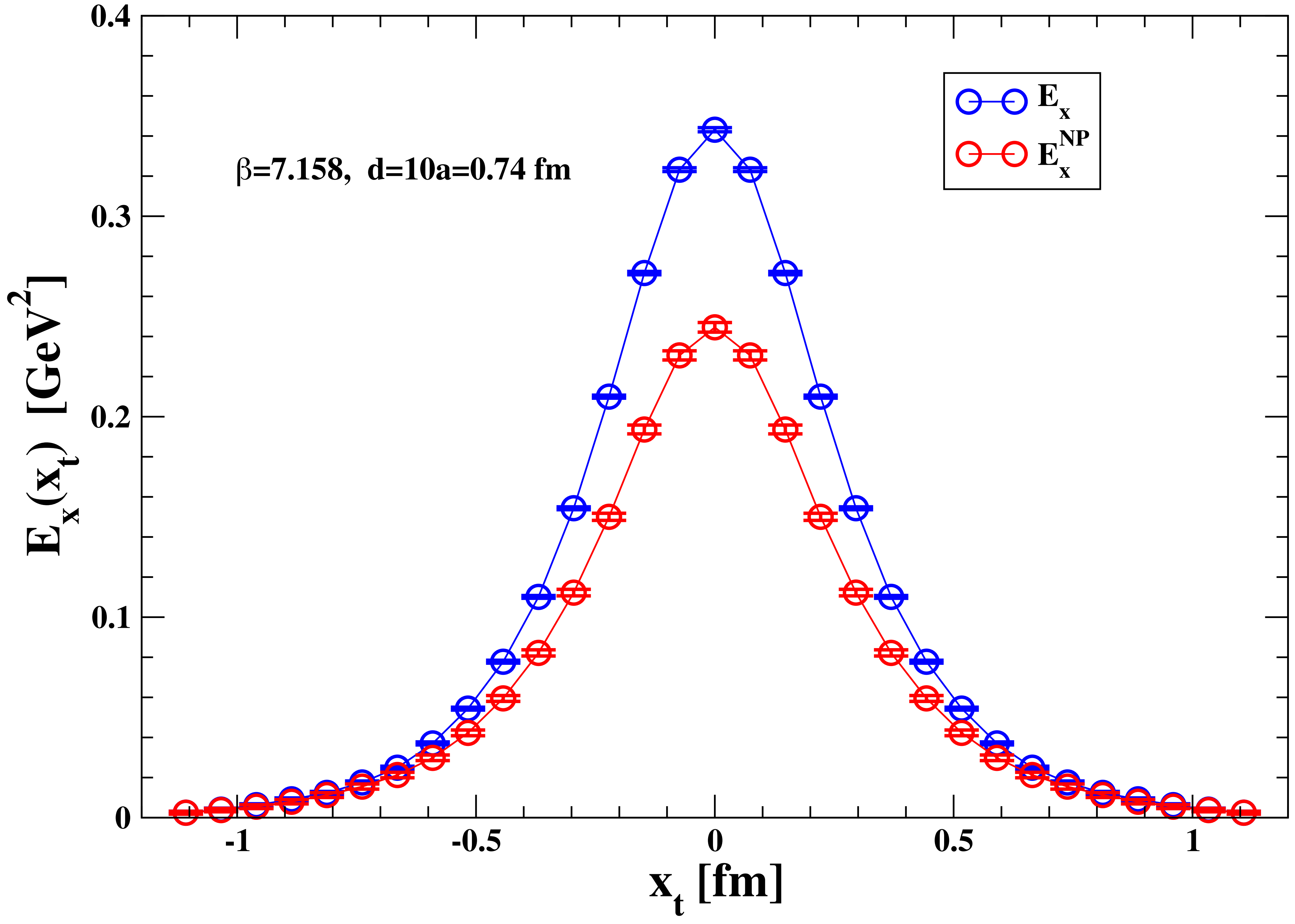


QCD (2+1) flavors: longitudinal chromoelectric field

$\beta = 7.158 \quad d = 10a = 0.74 \text{ fm}$



at midplane

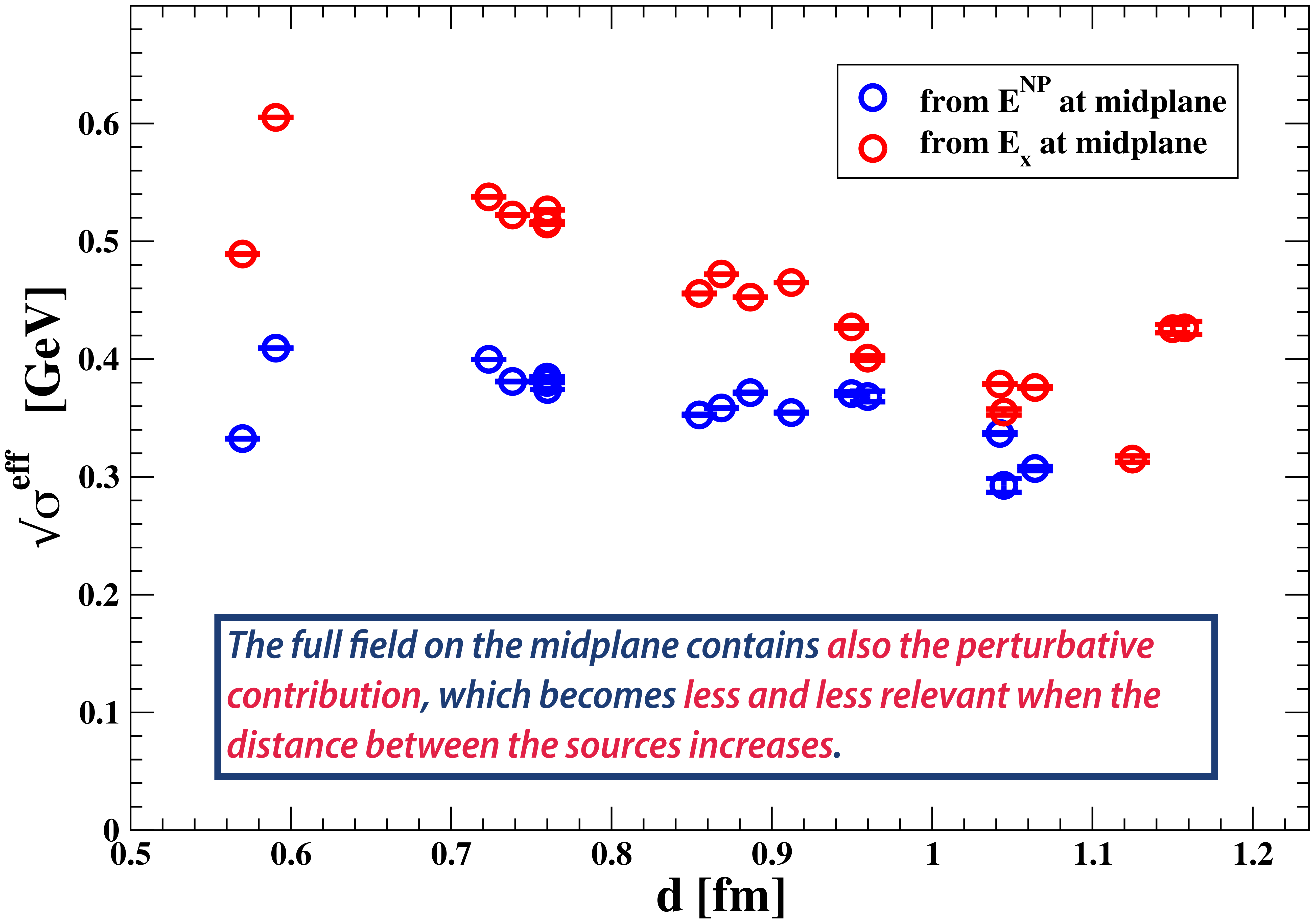


EFFECTIVE STRING TENSION

To characterize quantitatively the **shape** and some **properties of the flux tube** formed by the longitudinal electric field, we calculated numerically (at the midplane between the sources):

$$\sigma_{\text{eff}} = \int d^2x_t \, \frac{(E_x^{NP}(x_t))^2}{2}$$

"effective" string tension



The full field on the midplane contains also the perturbative contribution, which becomes less and less relevant when the distance between the sources increases.

E_x^{NP} at the midplane

d [fm]	$\sqrt{\sigma_{\text{eff}}}$
0.569866	0.332452 (212)
0.590647	0.409334 (58)
0.723462	0.399771 (77)
0.738309	0.380966 (255)
0.75982	0.384704 (187)
0.759821	0.380184 (205)
0.759947	0.382267 (117)
0.760151	0.374200 (68)
0.854799	0.352591 (637)
0.868575	0.358531 (117)
0.886605	0.371464 (449)
0.912182	0.354552 (319)
0.949777	0.370686 (1791)
0.959801	0.368236 (4516)
1.04229	0.336868 (810)
1.04475	0.292763 (5907)
1.06421	0.307063 (1845)

E_x at the midplane

d [fm]	$\sqrt{\sigma_{\text{eff}}}$
0.569866	0.489183 (177)
0.590647	0.605219 (33)
0.723462	0.537672 (46)
0.738309	0.522341 (128)
0.75982	0.514655 (113)
0.759821	0.515571 (100)
0.759947	0.526670 (53)
0.760151	0.516572 (32)
0.854799	0.455681 (282)
0.868575	0.472161 (51)
0.886605	0.452573 (170)
0.912182	0.464997 (124)
0.949777	0.427284 (957)
0.959801	0.400833 (1757)
1.04229	0.378825 (271)
1.04475	0.355045 (2577)
1.06421	0.375906 (586)
1.125	0.315084 (2681)
1.15	0.425761 (3482)
1.157536	0.426480 (5478)

$$\sqrt{\sigma_{\text{eff}}} \approx 0.4 \text{ GeV}$$

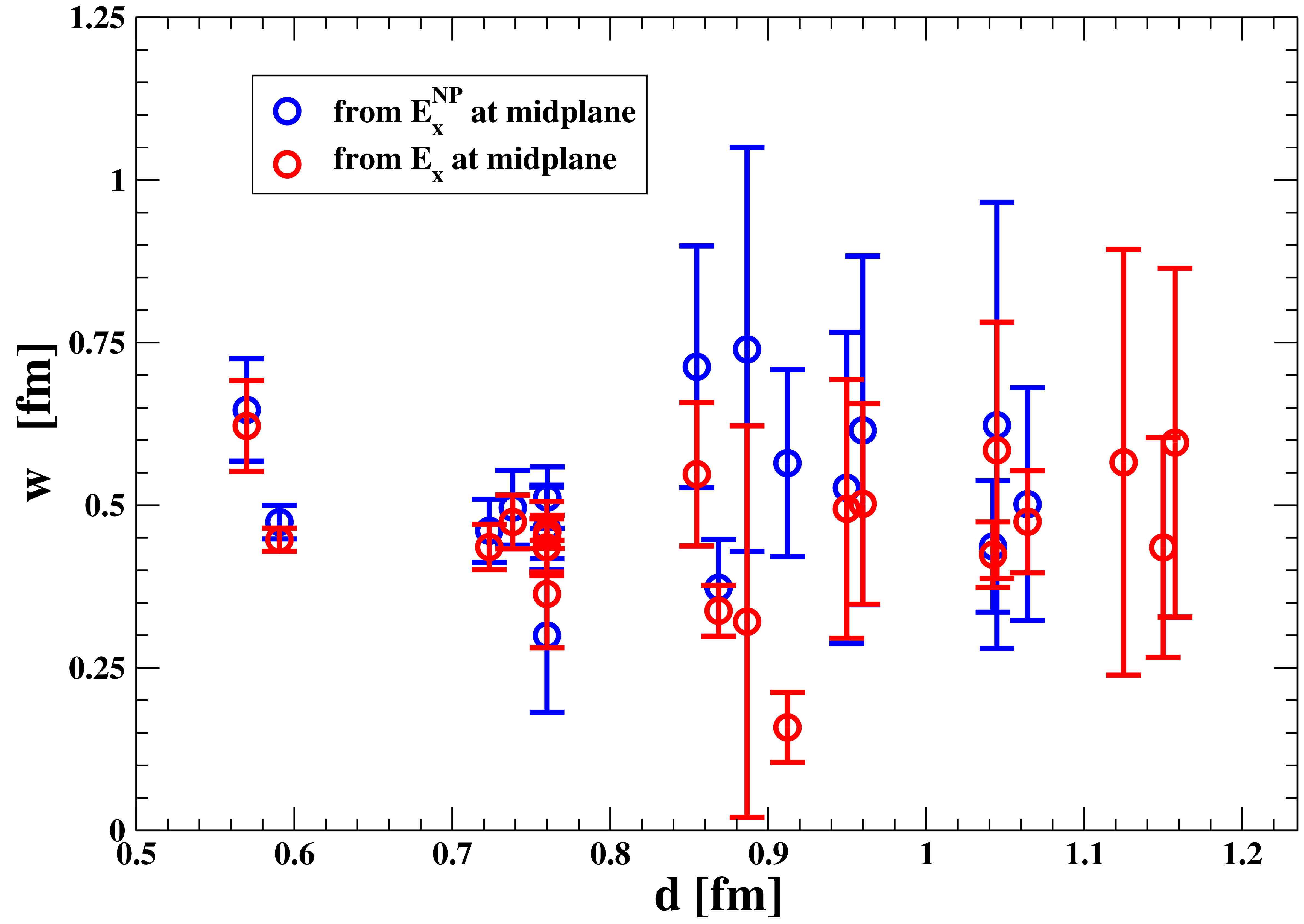
WIDTH OF THE FLUX TUBE

To characterize quantitatively the **shape** and some **properties of the flux tube** formed by the longitudinal electric field, we calculated numerically (at the midplane between the sources):

$$w = \sqrt{\frac{\int d^2x_t\, x_t^2 E_x^{NP}(x_t)}{\int d^2x_t\, E_x^{NP}(x_t)}}$$

width of the flux tube

$$w \approx 0.5 \text{ fm}$$



E_x^{NP} at the midplane

d [fm]	w
0.569866	0.646585 (78748)
0.590647	0.474086 (25839)
0.723462	0.460645 (48577)
0.738309	0.496320 (57512)
0.75982	0.463356 (67456)
0.759821	0.464393 (63411)
0.759947	0.299796 (117924)
0.760151	0.511873 (47241)
0.854799	0.712877 (185931)
0.868575	0.373149 (74361)
0.886605	0.739608 (310653)
0.912182	0.564672 (143845)
0.949777	0.526818 (239288)
0.959801	0.614954 (268025)
1.04229	0.436552 (100896)
1.04475	0.622952 (342927)
1.06421	0.501584 (178880)

E_x at the midplane

d [fm]	w
0.569866	0.621870(69886)
0.590647	0.447115(17753)
0.723462	0.435692(34762)
0.738309	0.474240(41283)
0.75982	0.451601(54312)
0.759821	0.435405(43580)
0.759947	0.363556(82688)
0.760151	0.459008(25524)
0.854799	0.547670(110240)
0.868575	0.337704(39088)
0.886605	0.321108(300997)
0.912182	0.158464(53710)
0.949777	0.494548(198891)
0.959801	0.502093(154215)
1.04229	0.423873(50377)
1.04475	0.584496(197020)
1.06421	0.474587(78494)
1.125	0.565908(327225)
1.15	0.435095(169046)
1.157536	0.596161(268178)

the width of the flux tube remains stable on a wide range of distances and is generally compatible for the full and the nonperturbative field.

POSSIBLE EVIDENCE FOR STRING BREAKING

- In the presence of **light quarks** it is expected that the **string between the static quark-antiquark** pair **breaks at large distance** due to ***creation of a pair of light quarks which recombine with the static quarks into two static-light mesons***.

- Usually, the **string breaking distance** is defined as the point where **the Wilson loop and the static-light meson operator have equal overlap onto the ground state**.

Evidences for string breaking:

$N_f = 2, m_\pi = 640$ MeV	$d^* = 1.248(13)$ fm	Bali et al., hep-lat/0505012
$N_f = 2 + 1$ (Wilson), $m_\pi = 280$ MeV	$d^* \approx 1.216$ fm	Kock et al., arXiv/1811.09289
$N_f = 2 + 1$ (Wilson), $m_\pi \in [200, 340]$ MeV	$d^* \approx 1.211(7)$ fm	Bulava et al., arXiv/2403.00754

- Our numerical setup is **not tailored for a clear-cut detection** of the expected **string breaking**.

⇒ However, **we can look directly at the nonperturbative gauge-invariant longitudinal electric field**, E_x^{NP}, in the region between two static sources that is responsible for the formation of a well-defined flux tube, characterized by nonzero effective string tension σ_{eff} and width w.

POSSIBLE EVIDENCE FOR STRING BREAKING (cont'd)

We tried to push our numerical simulations to distances as large as ~ 1.37 fm, searching for **hints of string breaking**.

- $0.570\ \mathrm{fm} \leq d \leq 1.064\ \mathrm{fm}$ ($d \approx 1.125$ fm under scrutiny)

  We are able to isolate the non perturbative part of the longitudinal electric field

- $1.140\ \mathrm{fm} \lesssim d < 1.368\ \mathrm{fm}$

  We find evidences for the full longitudinal electric field E_x on the midplane between two sources

  BUT

  there are not evidences for a sizeable nonperturbative longitudinal electric field E_x^{NP}.

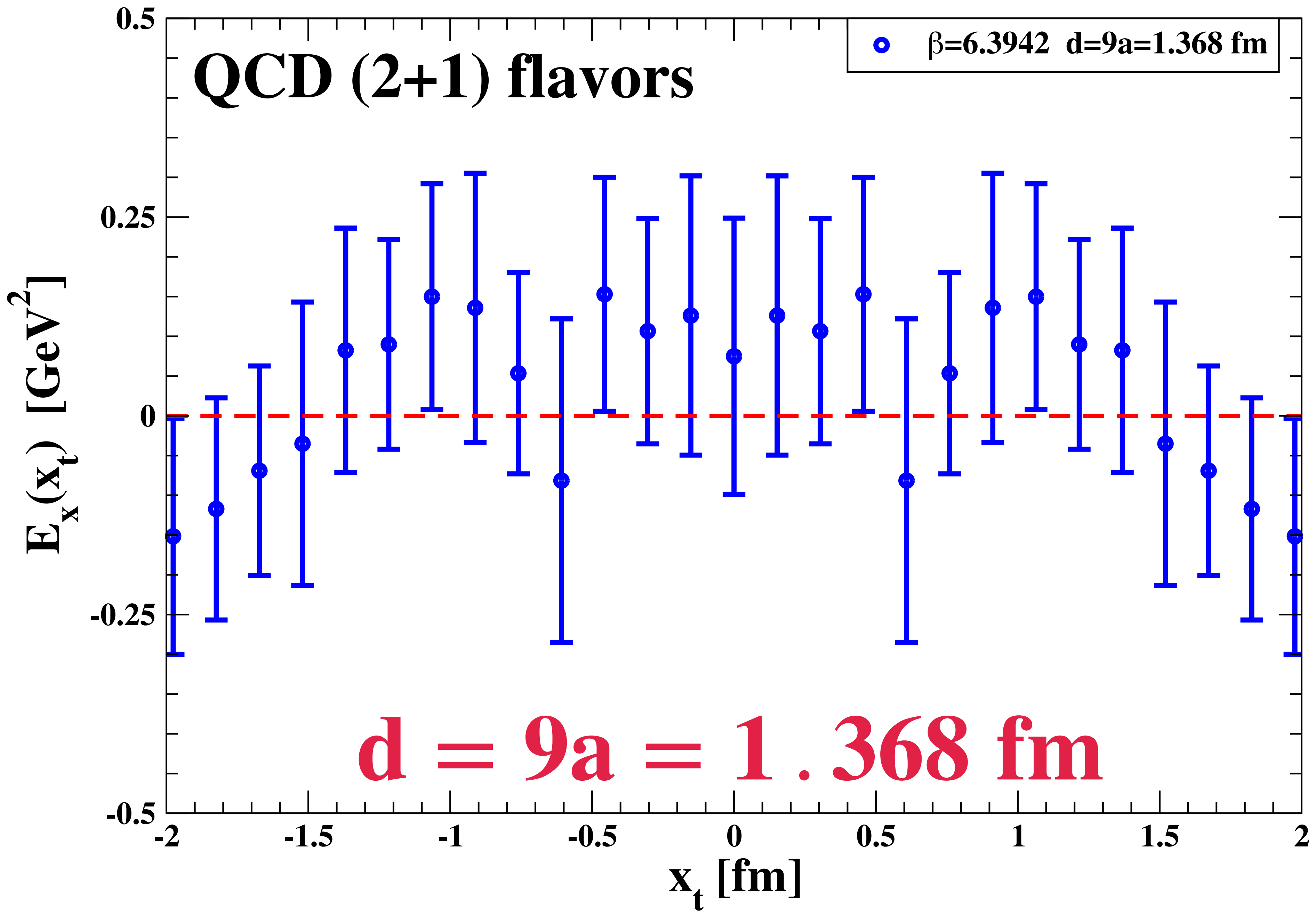
- For $d > 1.140$ fm

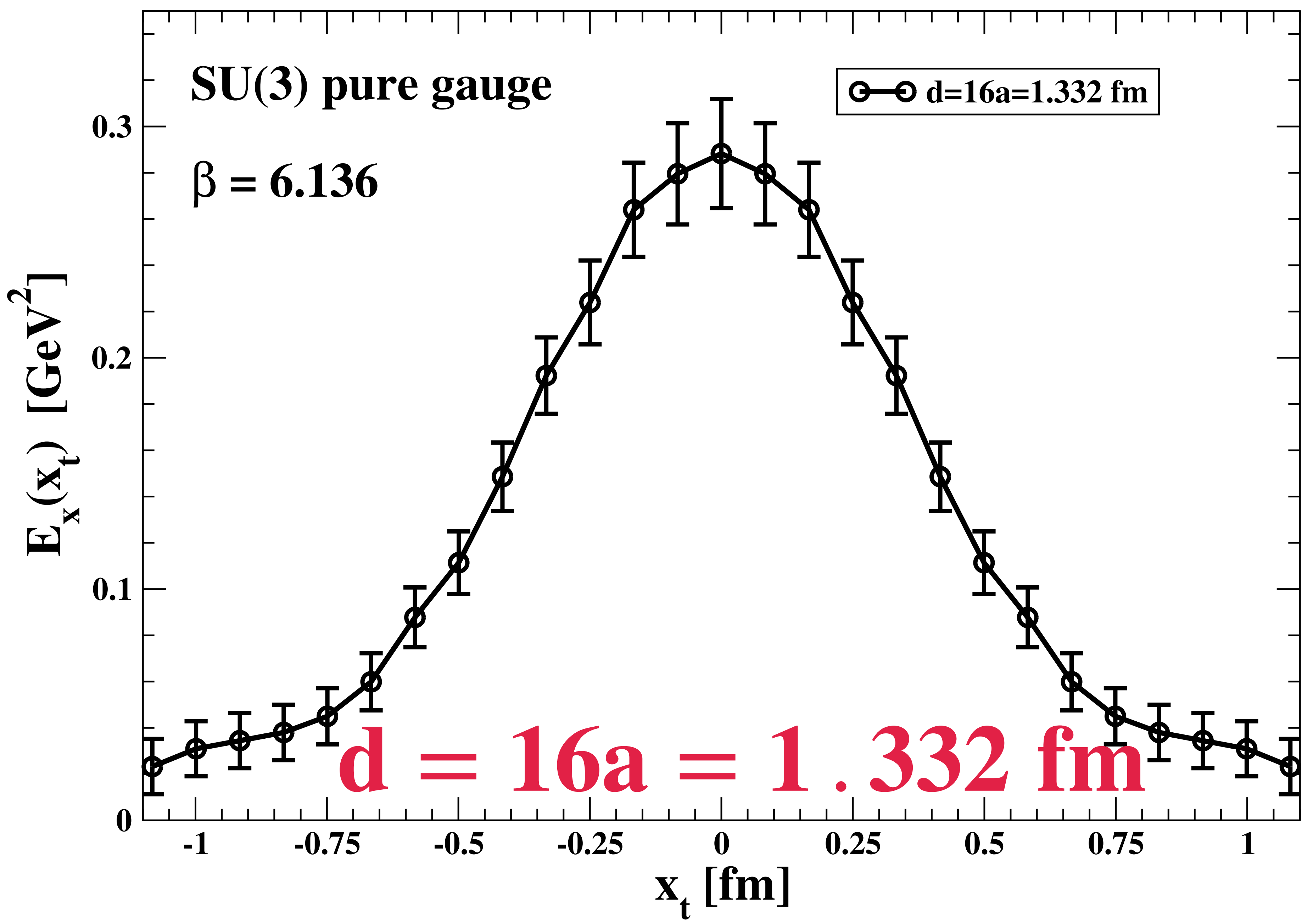
  No improvement in the signal can be observed **if the distance in lattice units between the two sources is reduced**, **keeping d fixed.**

- In **SU(3) pure gauge**, where the string remains unbroken by definition, **the signal for the longitudinal field is clear even at large distances both in physical and lattice units**.

Our preliminary estimate for the string breaking distance is:

$$1.064\ \mathrm{fm} \lesssim d^* \lesssim 1.140\ \mathrm{fm}$$





SUMMARY AND CONCLUSIONS

- We have investigated, by Monte Carlo numerical simulations of **QCD with (2+1) dynamical staggered fermions at physical masses**, the behavior of the nonperturbative gauge-invariant **longitudinal electric field** $\vec{E}^{NP}$ in the region between a quark and an antiquark.

- Numerical simulations for a range of values of the coupling **where continuum scaling is satisfied** and have considered several values of the **physical distance between the sources** $0.57\,\mathrm{fm} \leq d \leq 1.37\,\mathrm{fm}$.

- (After subtraction of the perturbative component), **the longitudinal electric field takes the shape of a flux tube**, whenever the distance between the sources **does not exceed a value** $d \simeq 1.064\,\mathrm{fm}$.

- This flux tube can be characterized by two quantities: σ_{eff} (related to the **string tension**), and the **width** w.

- Above $d \simeq 1.14\,\mathrm{fm}$, the longitudinal nonperturbative field E_x^{NP} is always compatible with zero, within large numerical uncertainties:

→ We have provided some numerical arguments in favour of the **string breaking** → $1.064\,\mathrm{fm} \lesssim d^* \lesssim 1.140\,\mathrm{fm}$

We plan to corroborate them with **further investigations**.