Intrinsic width of the flux tube in 2+1 dimensional Yang–Mills theories

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Soon on arXiv with
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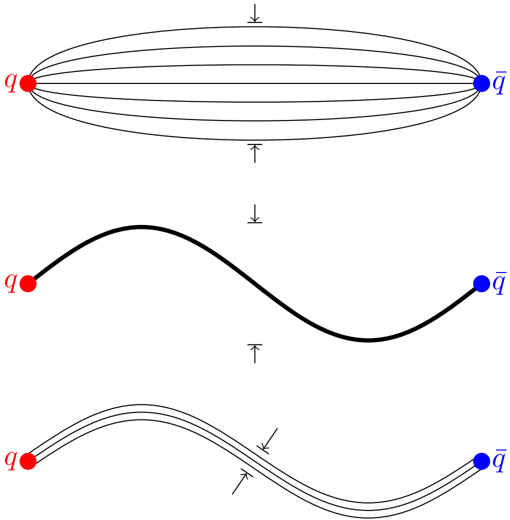
Flux tube as a string



Chromo-Electric and
Chromo-Magnetic fields
concentrated in a "tube"

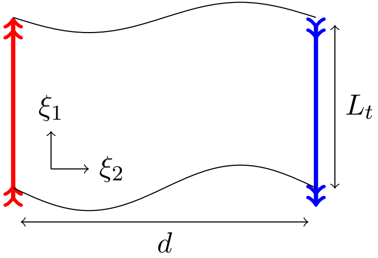
In effective string theory:
flux tube as vibrating string
$\implies$ finite width

Intrinsic width:
residual width when string
fluctuation are removed

EST predictions

$$w^2_{\mathsf{EST}} \sim \int \mathcal{D}X(\xi_1,\xi_2)\, X^2\, e^{-S_{\mathsf{EST}}[X]}$$

No intrinsic width involved! All due to fluctuations



At low temperature Lüscher-Munster-Weisz Nucl.Phys.B 180 (1981):

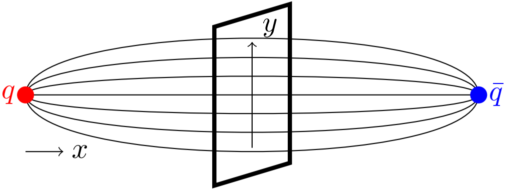
$$w^2_{\mathsf{EST}} = \frac{D-2}{2\pi\,\sigma} \log\left(\frac{d}{R_c}\right) + \dots$$

At high temperature Caselle-Allais [arXiv:0812.0284]:

$$w^2_{\mathsf{EST}} = \frac{d}{4\,L_t} + \frac{1}{2\pi} \log\left(\frac{L_t}{L_c}\right) + \dots$$

The flux tube profile: definitions

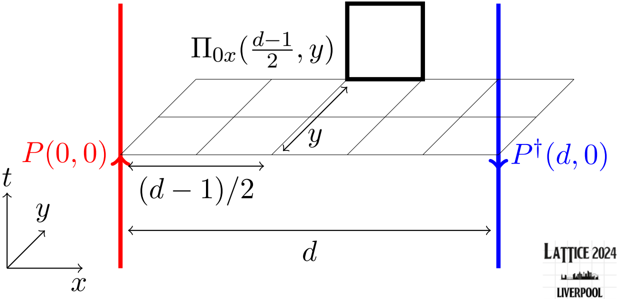
In order to see its width: we study the profile of the flux tube



Longitudinal Chromo-Electric field: $F_{0x}(y)$

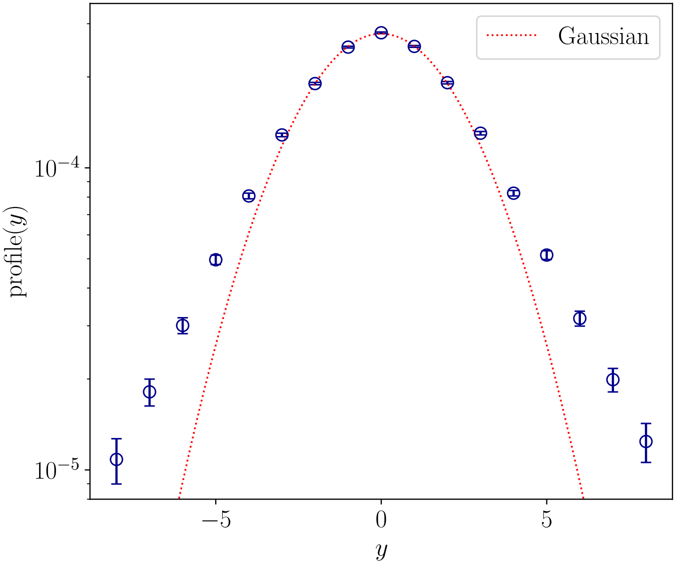
On the lattice: Polyakov-Polyakov-Plaquette correlator

$$\mathrm{profile(y,d)} = \frac{\langle P\,P^{\dagger}\,\Pi_{0x}\rangle}{\langle P\,P^{\dagger}\rangle} - \langle \Pi_{0x}\rangle$$



The flux tube: non-Gaussianity

Long vibrating string: we expect a Gaussian profile



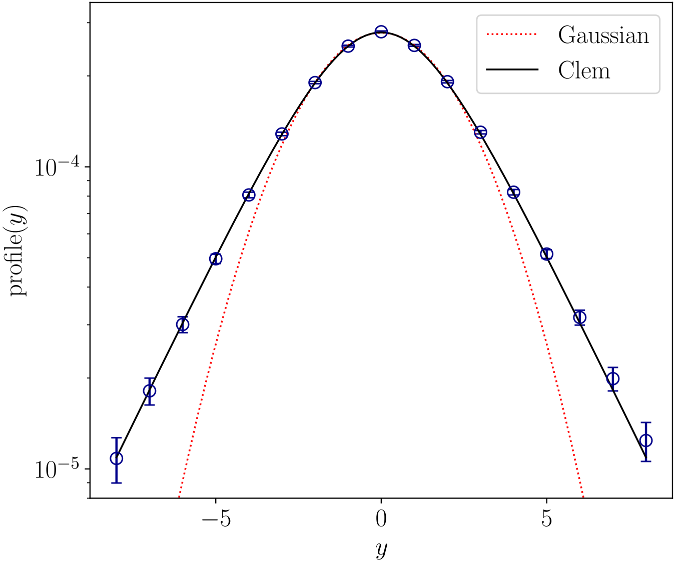
$$\mathrm{SU}(2) \text{ in } D = 2+1$$
$$\beta = 10.87$$
$$N_t = 30$$
$$T/T_c = 0.23$$

$$a\sqrt{\sigma} = 0.01728(23)$$
$$R_c/a = 3.9$$
$$d/a = 9$$

Data obtained with multilevel-algorithm as in Gliozzi-Pepe-Wiese [arXiv:1010.1373]

The flux tube: Clem fit

Long vibrating string: we expect a Gaussian profile



SU(2) in $D = 2+1$

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Clem:

$$p(y) = A\, K_0\left(2\mu\,\sqrt{y^2+\xi_v^2}\right)$$

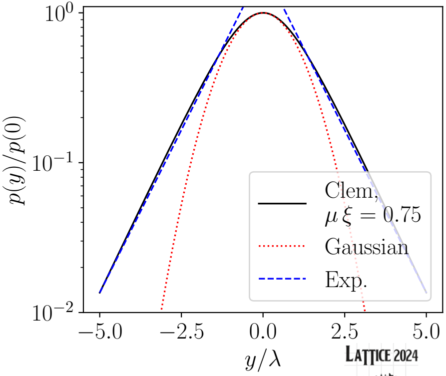
The Clem formula

$$p(y) = A\,K_0\left(2\mu\,\sqrt{y^2+\xi_v^2}\right) \approx \frac{A\sqrt{\pi}}{2}\,\frac{\exp\left(-2\mu\,\sqrt{y^2+\xi_v^2}\right)}{\sqrt{\mu}\,\left(y^2+\xi_v^2\right)^{1/4}}$$

Already used in $D = 3+1$, based on the dual superconductor description (e.g. Cea-Cosmai-Papa [arXiv:1208.1362])

ξ_v and $\lambda = 1/(2\mu)$:
the two length scales of the superconductor
In particular λ is the London length

For small $y \ll \xi_v$
Gaussian peak, with width $\sqrt{\xi_v/\mu}$
For large $y \gg \xi_v$
Exponential decay, with characteristic length $\lambda = 1/(2\mu)$



In this regime λ is the intrinsic width of the flux tube

Problems with the Clem formula in $D = 2 + 1$

- We could not find consistent values for ξ_v and μ to fit all our data
- Even the "type" of the superconductor (determined by ξ_v/λ) seems to vary
- This limits the applicability of the Clem model

According to Clem:

$$p(y) \approx \frac{A\sqrt{\pi}}{2} \frac{\exp\left(-2\mu\sqrt{y^2+\xi_v^2}\right)}{\sqrt{\mu}\,\left(y^2+\xi_v^2\right)^{1/4}}$$

- A high temperature: the exponent in the denominator seems to be bigger

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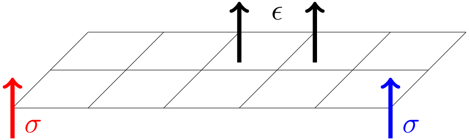
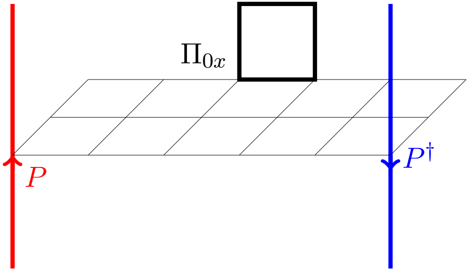
The profile and the Svetitsky–Yaffe mapping

We consider a D-dimensional $\mathrm{SU}(N)$ gauge theory with a second order phase transition, where the $\mathbb{Z}_N$ center symmetry is spontaneously broken.

We can extract the correlators of the gauge theory near the deconfinament temperature from those of the $\mathbb{Z}_N$-symmetric, $(D-1)$-dimensional spin model at the critical point.

- The Polyakov loop is mapped into the spin
- The plaquette is mapped into the energy





Further test of SY in the talk by Dario Panfalone

The profile near the phase transition

The spin-spin-energy correlator has been computed by Caselle and Grinza in arXiv:1207.6523

From those results:

$$\text{profile}(y,d) = A\,\frac{2\pi d}{K_0(md)}\,\frac{\exp\left(-m\sqrt{4y^2+d^2}\right)}{4y^2+d^2}$$

$\xi_v \to d$, not any more free parameter

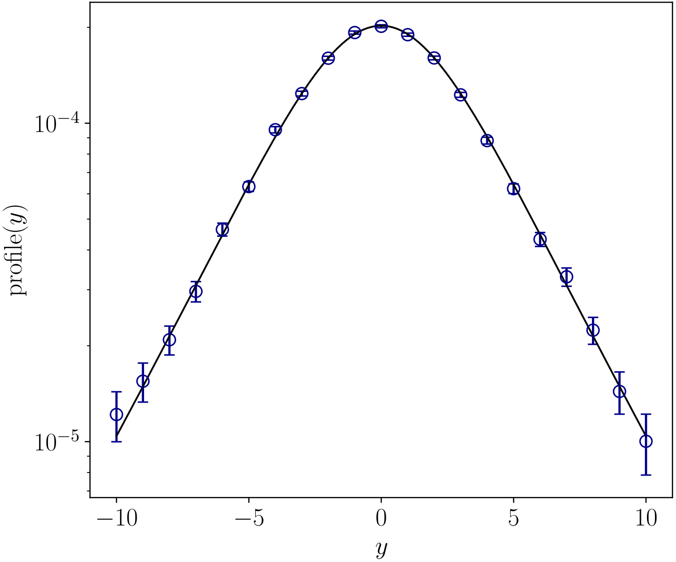
$$p_{\text{Clem}} \approx \frac{A\sqrt{\pi}}{2}\,\frac{\exp\left(-2\mu\,\sqrt{y^2+\xi_v^2}\right)}{\sqrt{\mu}\,\left(y^2+\xi_v^2\right)^{1/4}}$$

m: same mass scale as in $\langle P\,P^\dagger\rangle$:

$$\langle P(0,0)\,P^\dagger(0,d)\rangle \sim \exp(-m\,d)$$

We can test this assumption fitting high temperature numerical results
Only A and m are free parameters of the model (one less than Clem)
We can compare m to the value extracted from $\langle P\,P^\dagger\rangle$

Test for $T = 0.70T_c$



$$\beta = 10.87$$
$$N_t = 10$$
$$T/T_c = 0.70$$

$$a\sqrt{\sigma} = 0.13130(86)$$
$$R_c/a = 3.9$$
$$d/a = 11$$

$$m_{\text{fit}} = 0.1099(38)$$
$$m_{PP^\dagger} = 0.11064(24)$$

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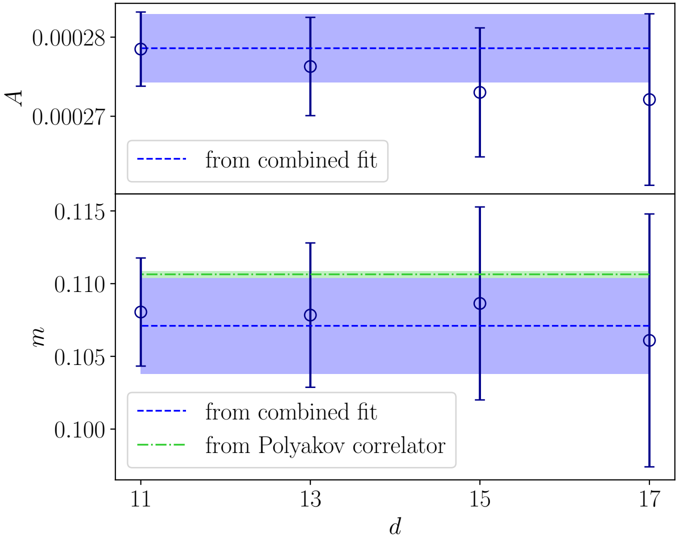
Test at multiple distances



m_{fit} agrees with $m_{PP^\dagger}$ for all the considered distances.

It is possible to perform a combined fit of all the data with a single value of A and m

Prediction:

$$\text{profile}(y, d) = \frac{A\, 2\pi\, d}{K_0(m\, d)} \ldots$$

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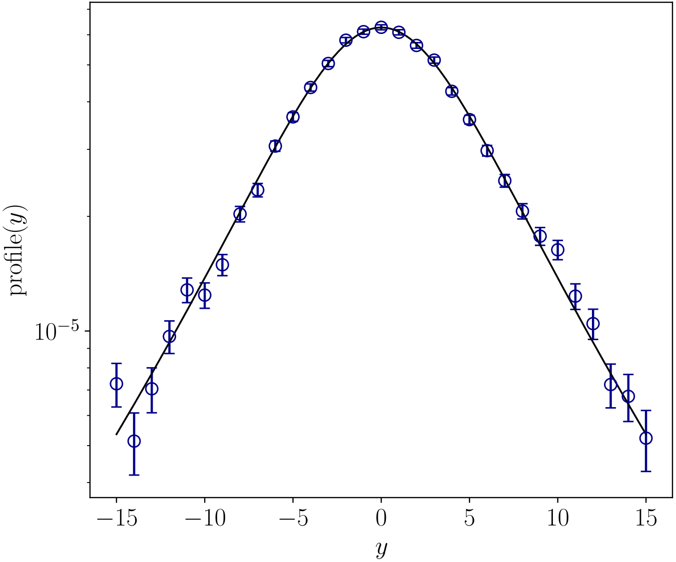
Test for $T = 0.87T_c$



$$\beta = 13.42$$
$$N_t = 10$$
$$T/T_c = 0.87$$

$$a\sqrt{\sigma} = 0.10490(51)$$
$$R_c/a = 4.9$$
$$d/a = 15$$

$$m_{\text{fit}} = 0.0387(27)$$
$$m_{PP^\dagger} = 0.04070(46)$$

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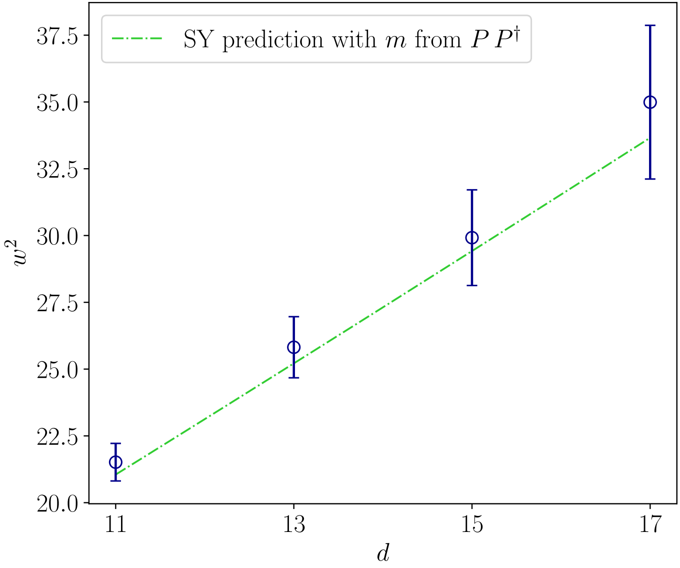
Linear broadening at high temperature



Gliozzi-Pepe-Wiese [arXiv:1010.1373]: linear broadening of the flux tube at large d:

$$w^2(d) = \frac{\int dy\, y^2\, \mathrm{profile(y,d)}}{\int dy\, \mathrm{profile(y,d)}} \sim d$$

In the model from SY:

$$\mathrm{profile}(y,d) \propto \frac{\exp\left(-m\sqrt{4y^2+d^2}\right)}{4y^2+d^2}$$

the intrinsic width is constant
but the Gaussian peak broadens

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Conclusions and Remarks

About the flux tube:

- The Clem formula seems to describe its profile at low temperature
- A high temperature deviations seem to appear
- The Svetitsky–Yaffe mapping provides a good model for the profile at high temperature
- In such model there is only one mass scale, determining the intrinsic width
- Its value is in good agreement with the mass scale in the Polyakov-Polyakov correlators

Outlooks

- Understand the crossover between Clem-like and Svetitsky–Yaffe-like regime
- Investigate how the Clem parameters depend on the temperature and the distance
- We are obtaining similar results for $\mathrm{SU}(3)$
- We will soon be able to compare these results with EST simulations (See talk by Elia Cellini)
- Svetitsky–Yaffe could be applied to the 4-dimensional $\mathrm{SU}(2)$ theory

Thank you for your attention!