Topological Data Analysis, Monopoles and Colour Confinement in SU(3) Yang-Mills

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Work in collaboration with X. Crean and J. Giansiracusa

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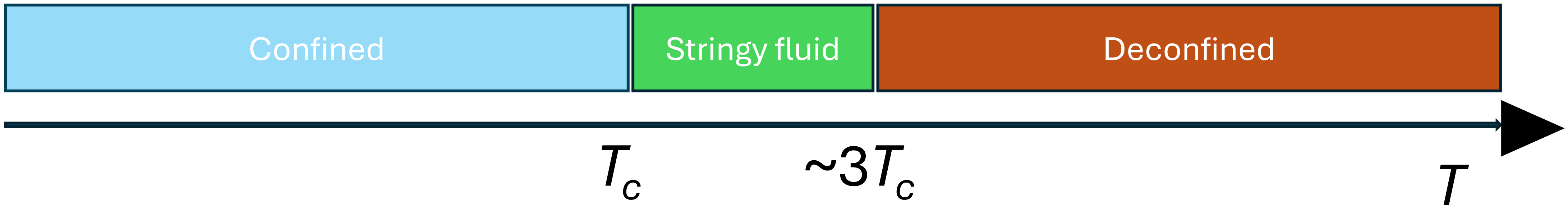
Homologies of currents as observables

- Homology is a powerful tool for describing the topological properties of a space (talk by Jeffrey Giansiracusa)
- The behaviour of the average homologies of monopole loop currents in U(1) Lattice Gauge Theory enables us to extract accurately the quantitative properties of the deconfinement phase transition in that system (talk by Xavier Crean)

This talk: test of the behaviour of homologies of Abelian monopole currents across the SU(3) deconfinement phase transition

Intermediate regime of QCD above T_c?

L. Ya. Glozman, Prog. Part. Nucl. Phys. 131 (2023) 104049, arXiv:2209.10235



- A stringy fluid regime has been conjectured in which chiral symmetry is restored but the system still confines
- Both soft boundaries of this regime can be phase transition points at large N
- Is there a good order parameter to identify the boundaries of this regime?

From QCD to SU(3) Yang-Mills

- Minimum requirement for the observable: being able to capture the deconfinement phase transition in SU(3) Yang-Mills
- The Polyakov loop satisfies that requirement, but is not a good order parameter for QCD in the chiral limit
- The observable could be based on the dynamics of vortices, but the corresponding Z_3 symmetry is explicitly broken in QCD
- We then consider Abelian monopoles

Outlook

- Introduction and motivations
- Abelian monopoles
- Numerical results
- Conclusions and future directions

Abelian monopoles in SU(3) Yang-Mills

Prifysgol Abertawe
Swansea University

- Classically, the existence of monopoles in gauge theories requires the presence of a self-interacting bosonic field transforming in the adjoint representation (e.g., the ‘t Hooft-Polyakov monopole in the Georgi-Glashow model)
- However, monopoles can arise if an effective dynamics develops in which an adjoint operator plays the role of a Higgs field
- Abelian monopoles are located at points in which two eigenvalues of this adjoint operator are degenerate

Maximal Abelian Gauge

Prifysgol Abertawe Swansea University

- Gauge fixing corresponding to the diagonalization of the adjoint operator

$$\tilde{X}(n) = \sum_{\mu} \left[U_\mu(n)\tilde{\lambda}U_\mu^\dagger(n) + U_\mu^\dagger(n-\mu)\tilde{\lambda}U_\mu(n-\mu) \right] \; , \qquad \tilde{\lambda} = \mathrm{diag}(1,0,-1)$$

- Equivalent gauge fixing condition

$$\{\tilde{g}\} = \arg\min_{\{g\}} \tilde{F}_{\mathrm{MAG}}(U,g)$$

$$\tilde{F}_{\mathrm{MAG}}(U,g) = \sum_{\mu,n} \mathrm{tr}\left(g(n)U_\mu(n)g^\dagger(n+\mu)\tilde{\lambda}g^\dagger(n+\mu)U_\mu^\dagger(n)g(n)\,\tilde{\lambda} \right)$$

[C. Bonati, M. D'Elia, Nucl. Phys. B877, 233 (2013) [arXiv:1308.0302]]

Abelian fields

- Gauge fixed configuration $\tilde{U}_\mu(n)$
- Diagonal elements $\tilde{U}_{ii} = r_i e^{i\varphi_i}$, $\sum_i \varphi_i = 2\pi n + \delta\varphi$
- Define $\phi_i = \varphi_i - \delta\varphi \frac{\left|\tilde{U}_{ii}\right|^{-1}}{\sum_j \left|\tilde{U}_{jj}\right|^{-1}}$
- Set $\theta_1 = \phi_1$ and $\theta_2 = -\phi_3$, and use the DeGrand and Toussaint prescription for identifying the monopoles associated to each Abelian field

[C. Bonati, M. D'Elia, Nucl. Phys. B877, 233 (2013) [arXiv:1308.0302]]
[T. DeGrand, D. Toussaint, Phys. Rev. D22, 2478 (1980)]

Numerical setup

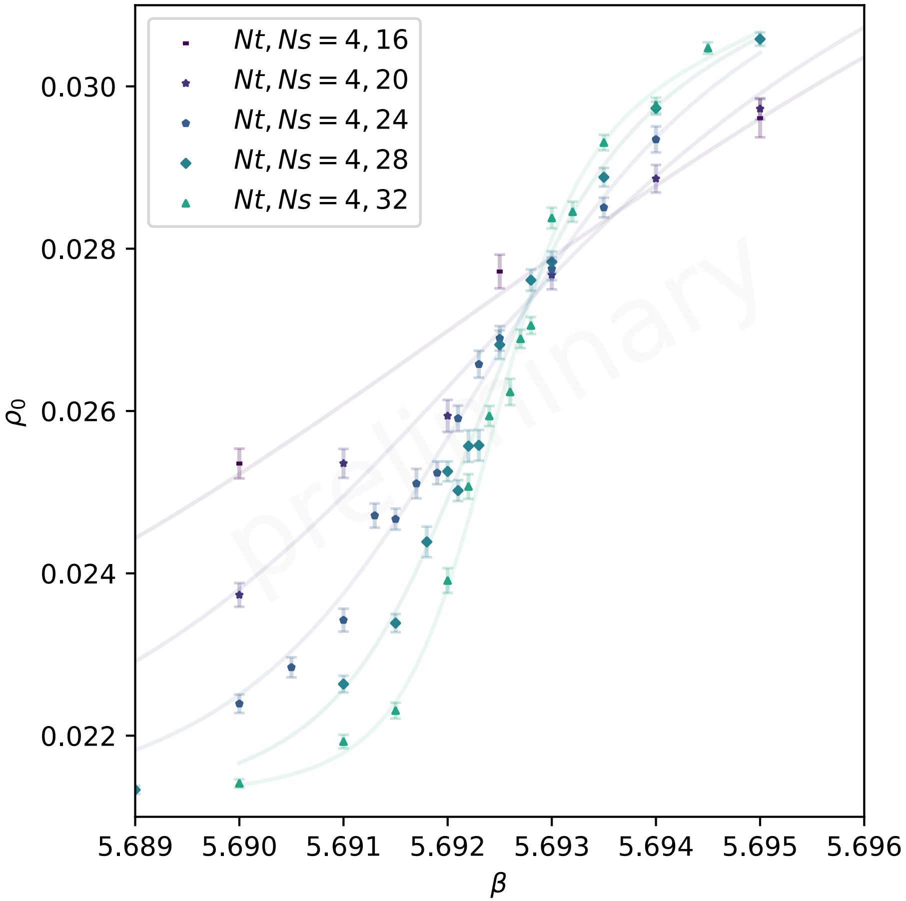
- Action $S = \beta \sum \left(1 - \frac{1}{3} Re\, Tr\, U_{\square} \right)$
- Asymmetric lattices of size $4 \times N_s^3 = 4 \times V,\, N_s = 16, 20, 24, 28, 32$
- Choose a set of β near the expected critical point(*) $\beta_c =$ 5.69236(15)
- Generate 600 thermalized configurations separated by 2000 composite sweeps (1 composite sweep = 1 heat bath + 4 over relaxation sweeps)

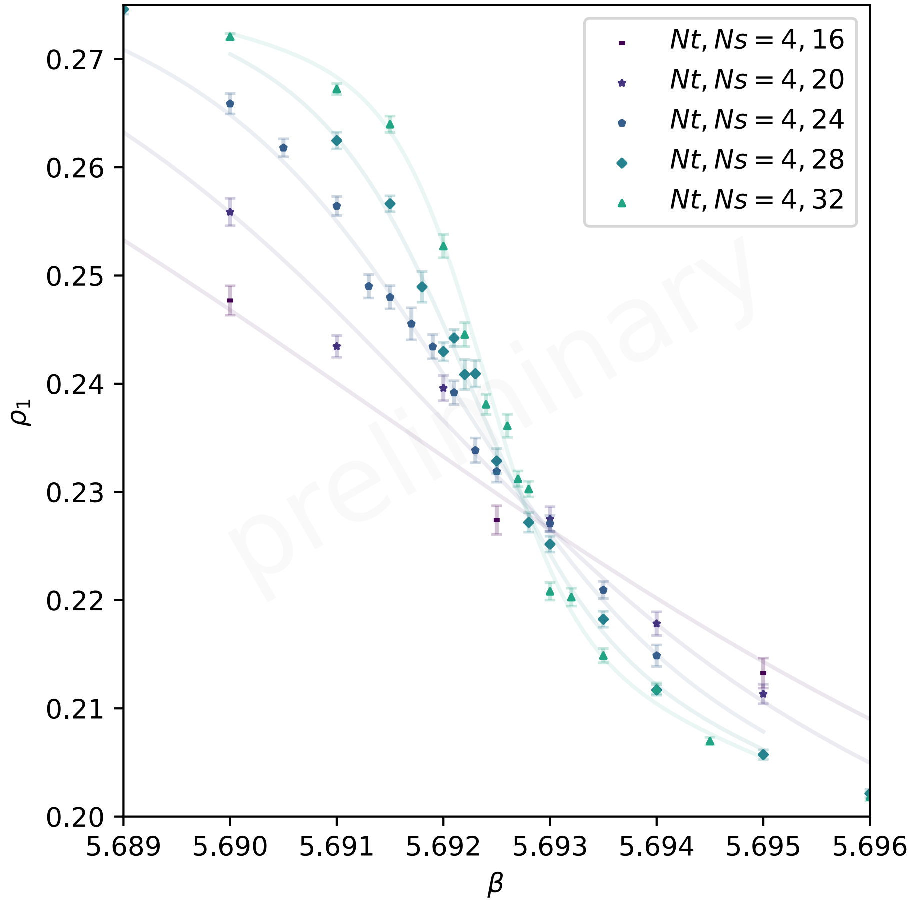
(*) E.g., B. Lucini, M. Teper and U. Wenger, JHEP 01 (2004) 061, arXiv: hep-lat/0307017

Measurements

- For each configuration, gauge-fix to the Maximal Abelian Gauge, extract the current loop graph on the dual lattice and compute its Betti numbers b_0 and b_1
- Compute the averages $\rho_0 = \frac{\langle b_0 \rangle}{V}$ and $\rho_1 = \frac{\langle b_1 \rangle}{V}$
- Compute the associated susceptibilities χ_0 and χ_1
- Reweight those observables and locate their peaks
- Define $\beta_c(N_s)$ as the position of those peaks
- Extrapolate using the ansatz $\beta_c(N_s) = \beta_c + \frac{a}{N_s^3}$

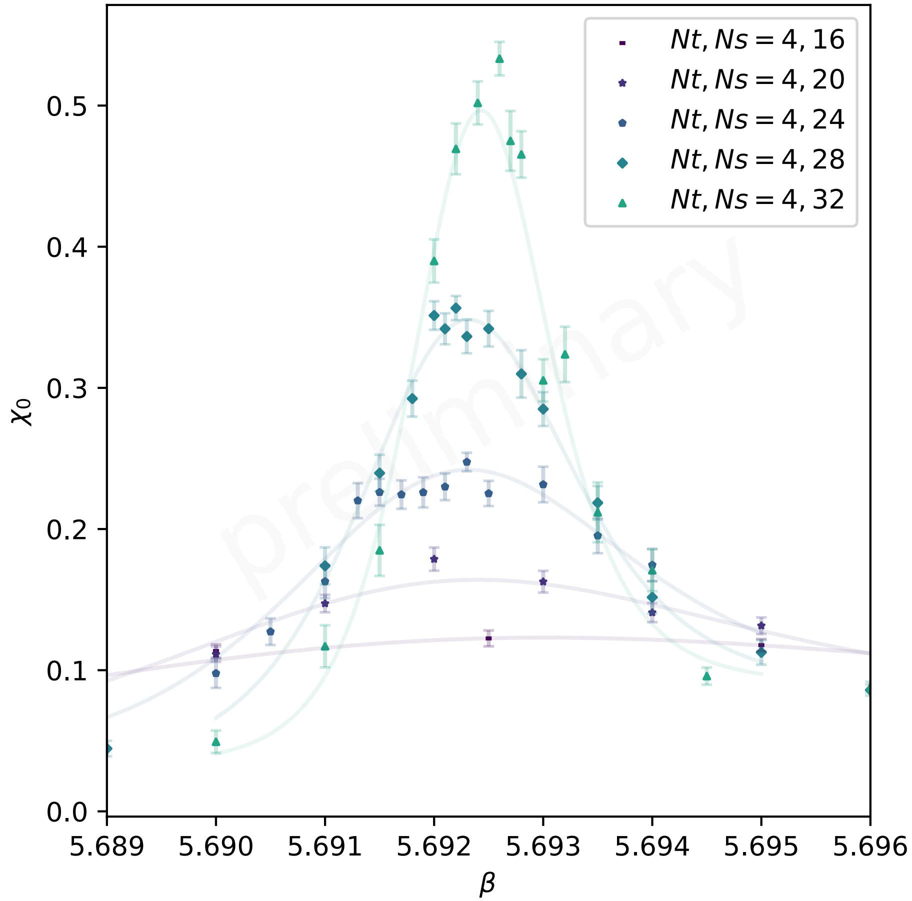
Betti numbers – SU(3) Yang-Mills

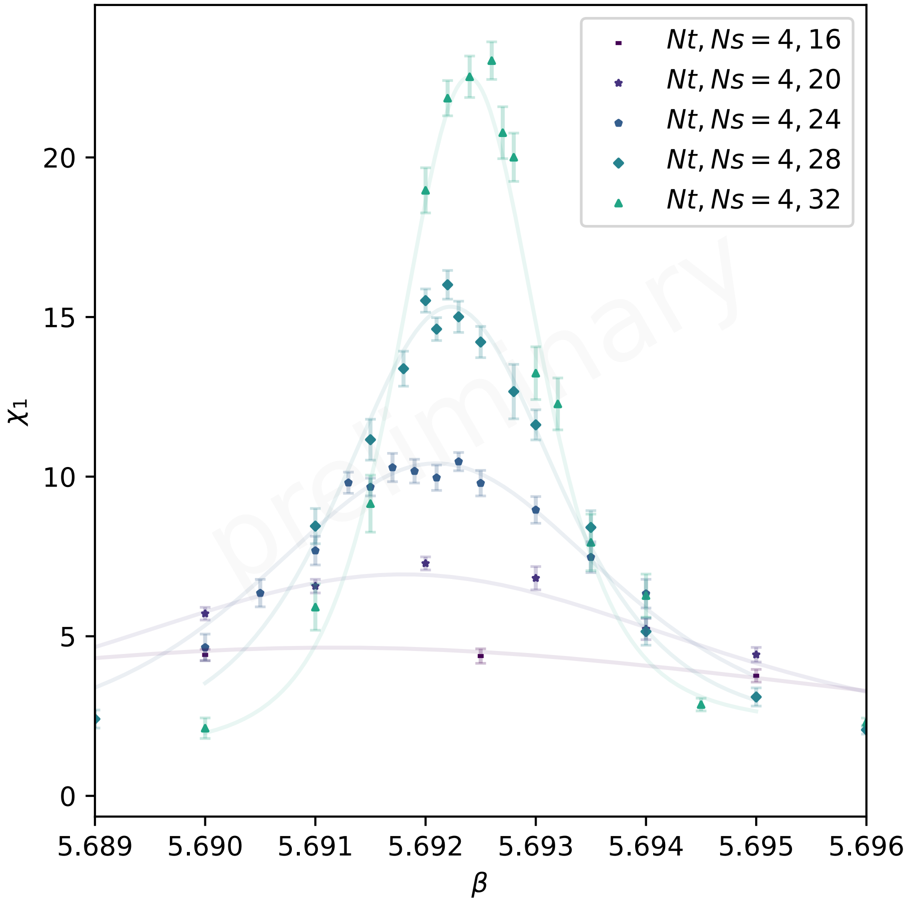




Singularity developing at the the phase transition as the volume increases

Susceptibilities of Betti numbers

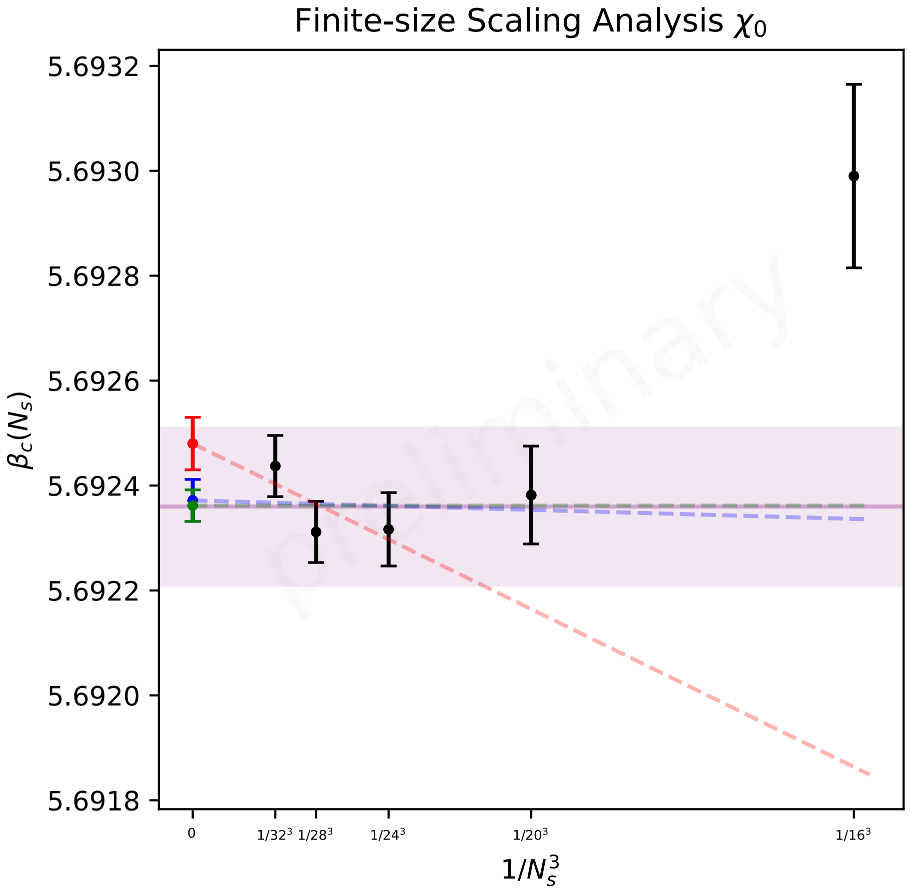


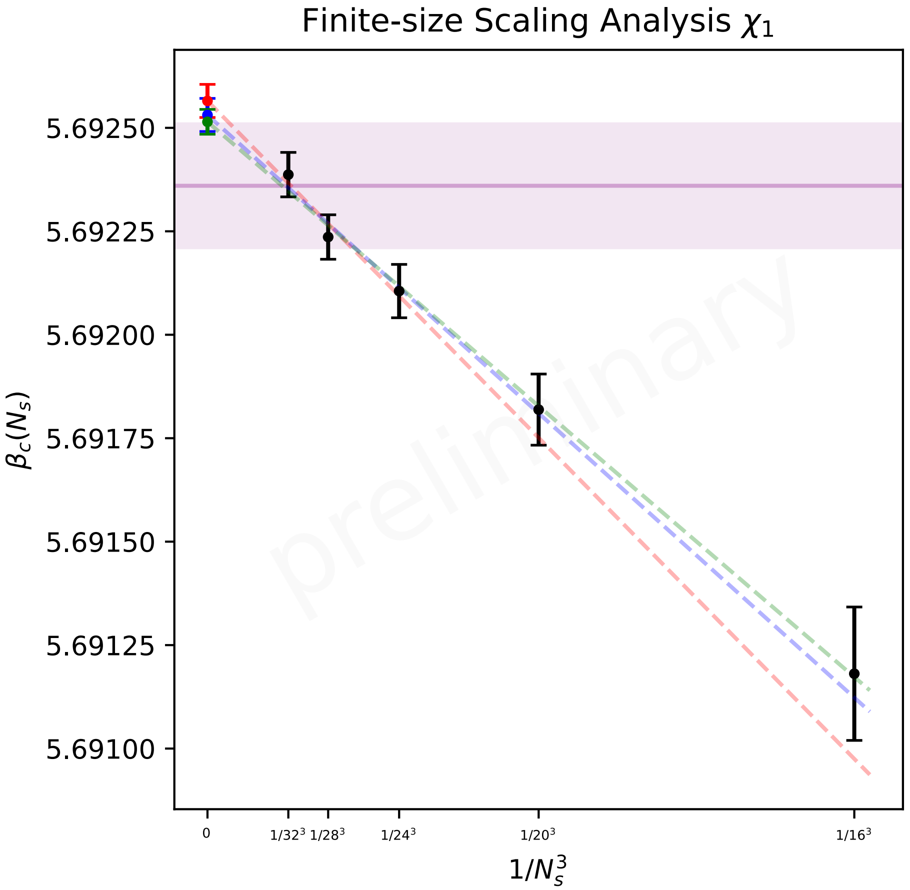


The peak is becoming sharper as the lattice size increases

[Horizontal band from B. Lucini, M. Teper and U. Wenger, JHEP 01 (2004) 061, arXiv: hep-lat/0307017]

Scaling of position of peaks





Results compatible with standard calculations, hints of better precision

Summary

Homologies of Abelian monopole current networks provide a quantitative precise description of the deconfinement phase transition in SU(3) theory (shown at a single lattice spacing)

Future work

- Perform an extrapolation to the continuum limit
- Apply the method to QCD at finite temperature