Proton and neutron electromagnetic form factors using N_f=2+1+1 twisted-mass fermions with physical values of the quark masses

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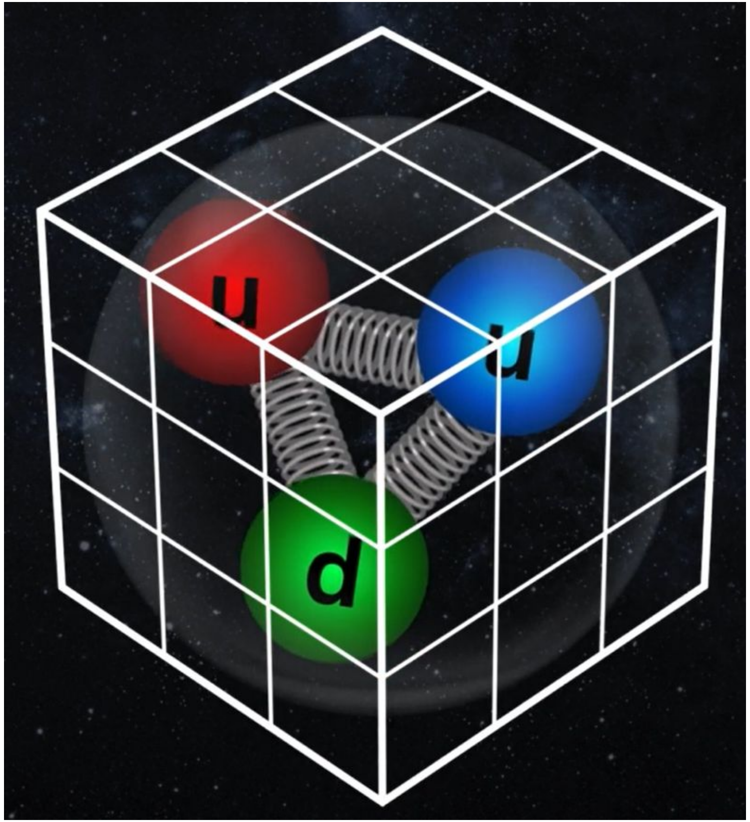
Electromagnetic form factors

- Interested in theoretically probing the structure of nucleons using lattice QCD.



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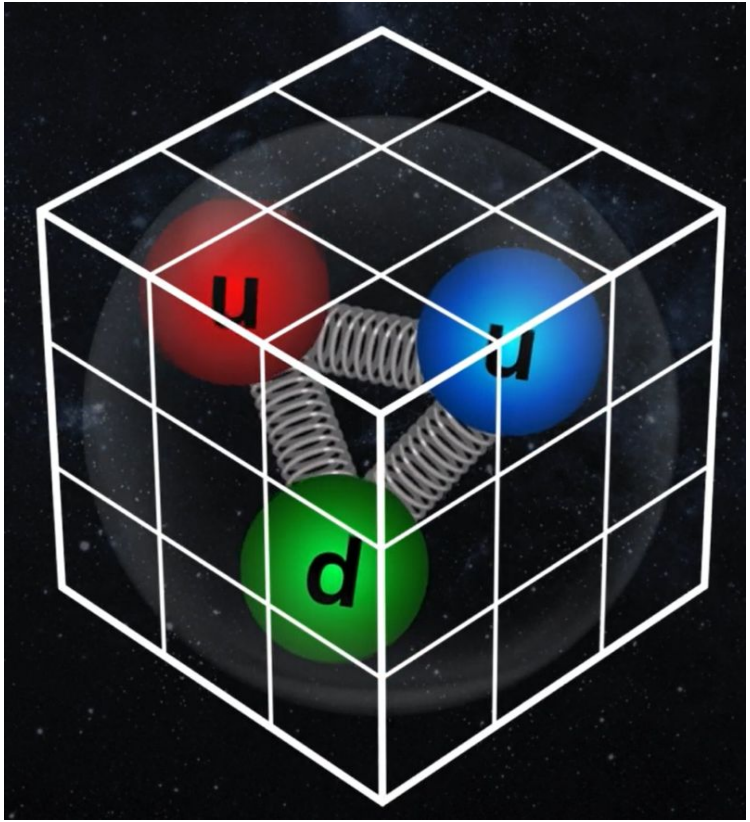

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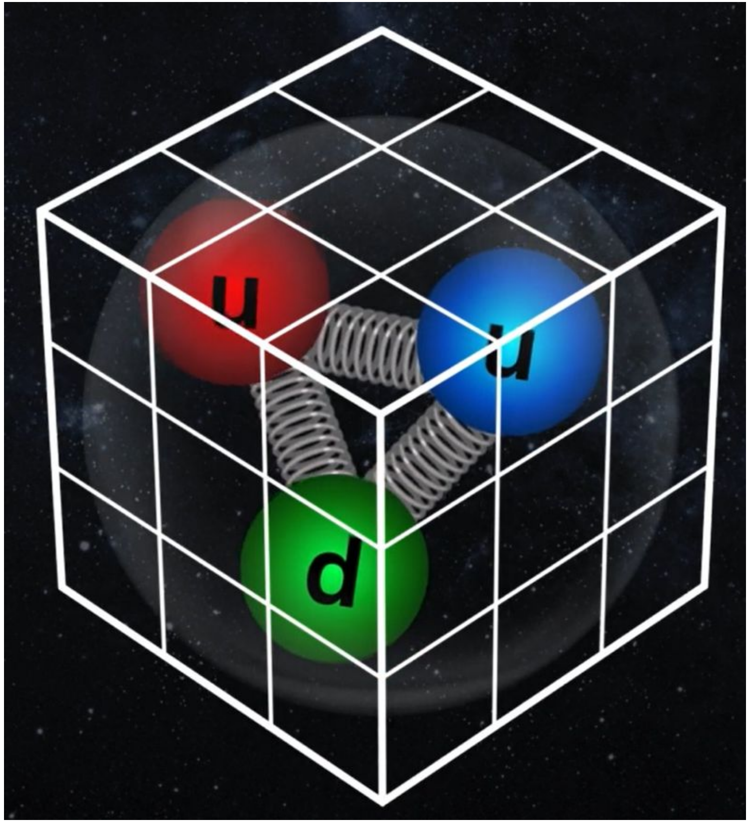

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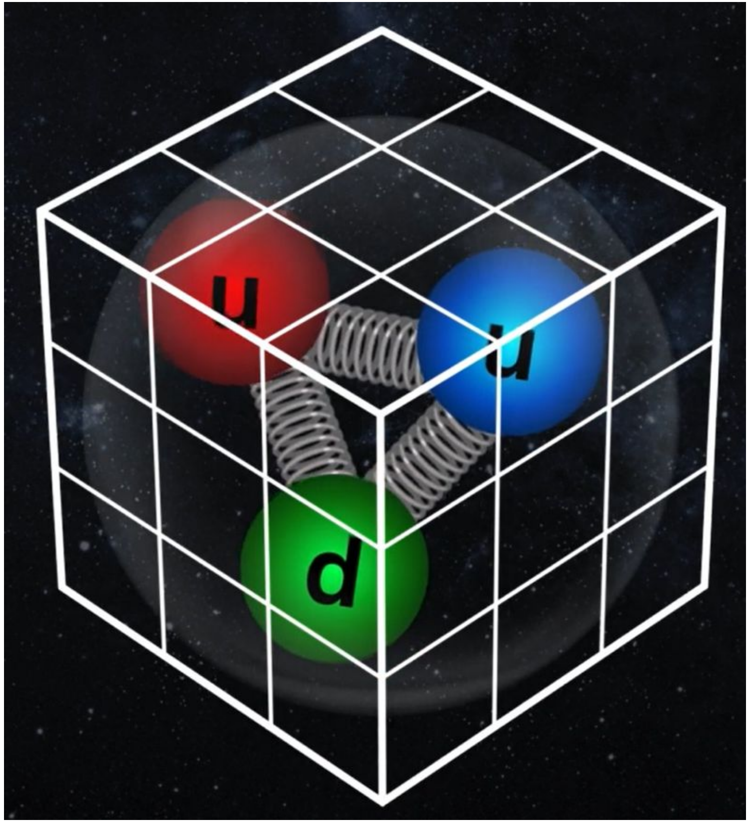
$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_N^2}F_2(q^2)$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$



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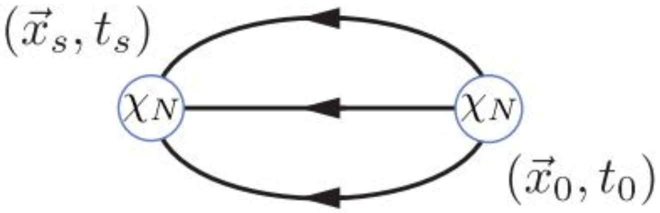

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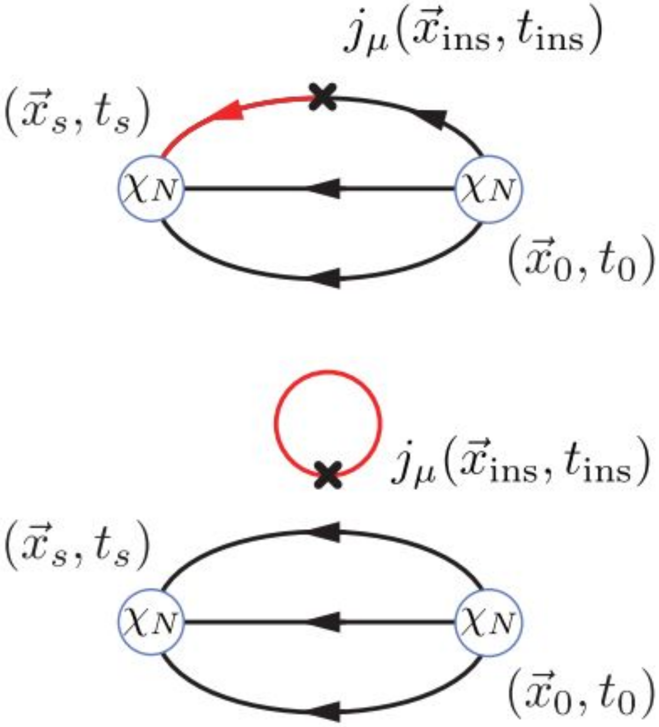
- The three point function is given by:

$$\mathcal{C}_\mu(\Gamma_\nu, \vec{q}, \vec{p}\,'; t_s, t_{\rm ins}, t_0) = \sum_{\vec{x}_{\rm ins}, \vec{x}_s} e^{i(\vec{x}_{\rm ins} - \vec{x}_0)\cdot\vec{q}} e^{-i(\vec{x}_s - \vec{x}_0)\cdot\vec{p}\,'} \times \mathrm{Tr}\left[\Gamma_\nu \langle \chi_N(t_s, \vec{x}_s) j_\mu(t_{\rm ins}, \vec{x}_{\rm ins}) \bar{\chi}_N(t_0, \vec{x}_0) \rangle\right].$$



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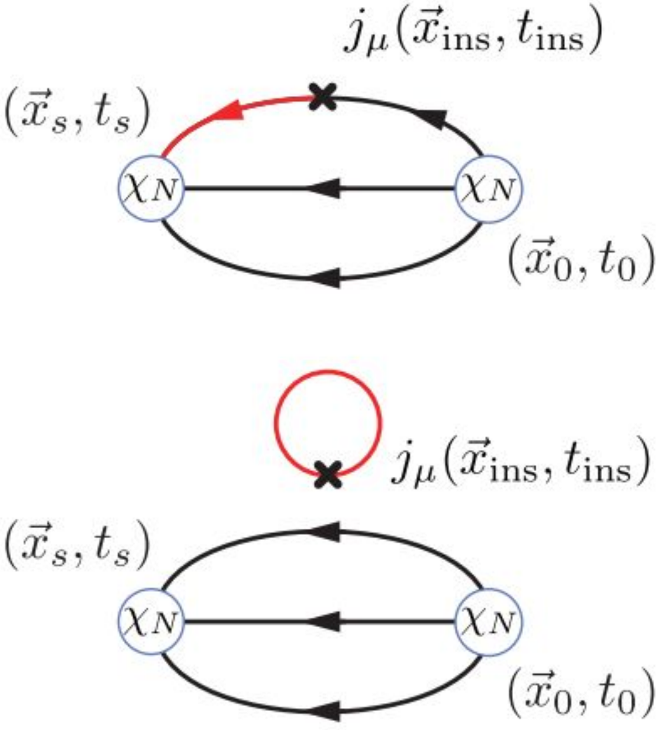
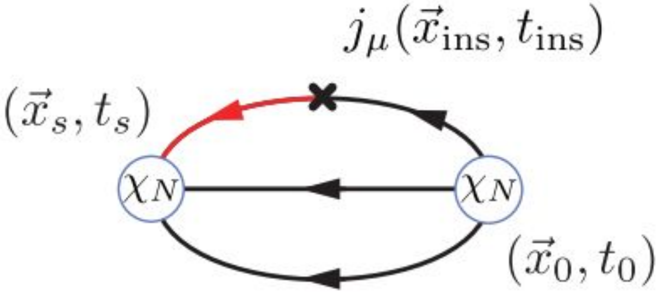
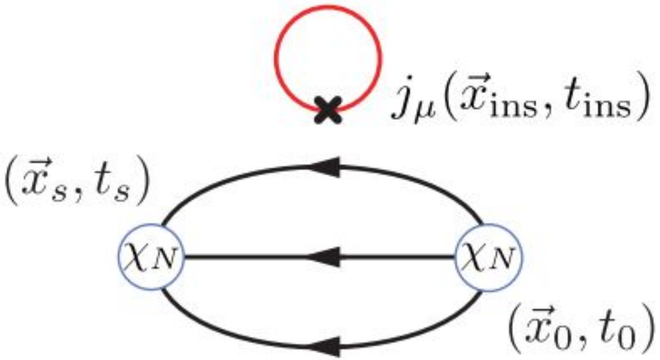


Nucleon matrix element on lattice

- We take the two-point and three-point functions to momentum space.
- We construct the following ratio to get rid of exponentials and overlaps.

$$\Pi_\mu(\Gamma_\nu,\vec{p}\,',\vec{p};t_s,t_{ins}) = \frac{C_\mu(\Gamma_\nu,\vec{p}\,',\vec{p};t_s,t_{ins})}{C(\Gamma_0,\vec{p}\,';t_s)} \times \sqrt{\frac{C(\Gamma_0,\vec{p};t_s-t_{ins})C(\Gamma_0,\vec{p}\,';t_{ins})C(\Gamma_0,\vec{p}\,';t_s)}{C(\Gamma_0,\vec{p}\,';t_s-t_{ins})C(\Gamma_0,\vec{p};t_{ins})C(\Gamma_0,\vec{p};t_s)}}$$





Lattice setup

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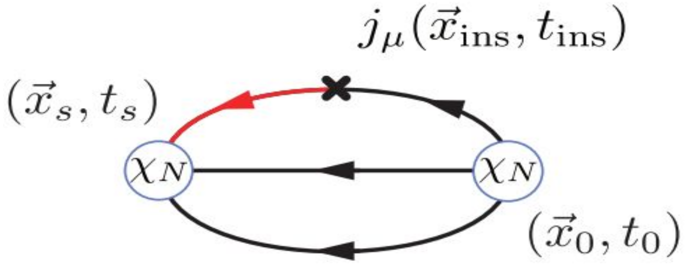
- We use three ensembles with N_f=2+1+1 from ETMC.

Ensemble	$(\frac{L}{a})^3 \times (\frac{T}{a})$	a [fm]	m_π [MeV]	$m_\pi L$
cB211.072.64	$64^3 \times 128$	0.07957(13)	140.2(2)	3.62
cC211.060.80	$80^3 \times 160$	0.06821(13)	136.7(2)	3.78
cD211.054.96	$96^3 \times 192$	0.05692(12)	140.8(2)	3.90

Connected and disconnected contributions



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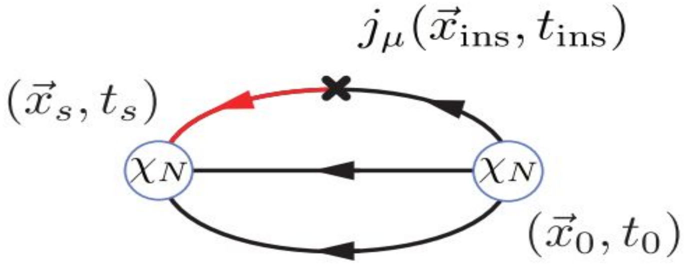

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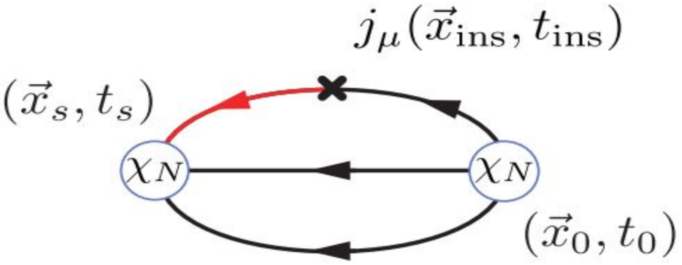

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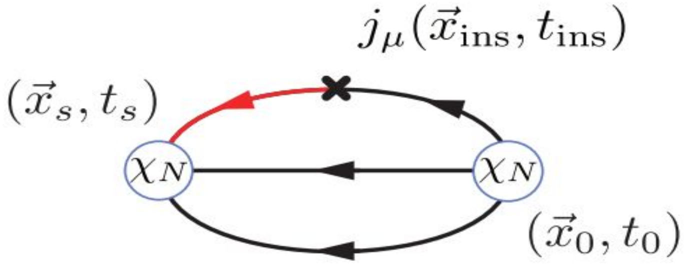

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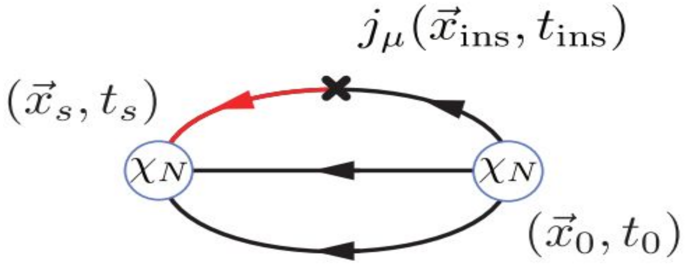

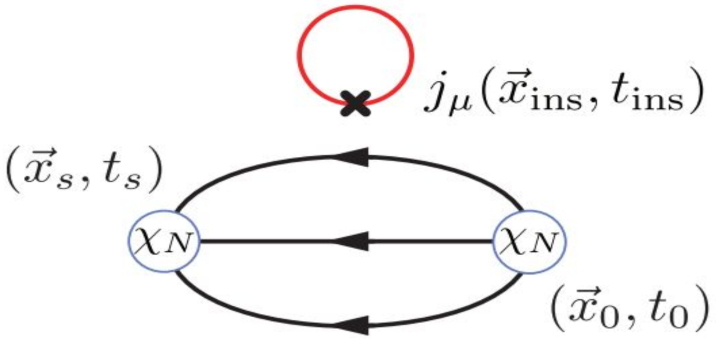

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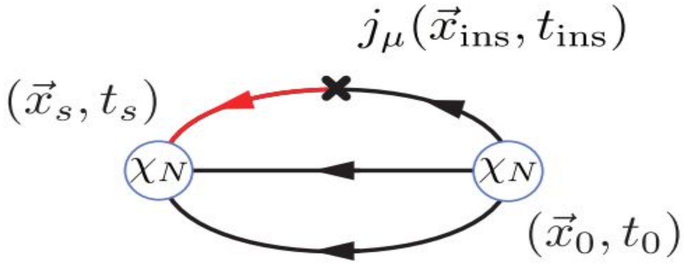

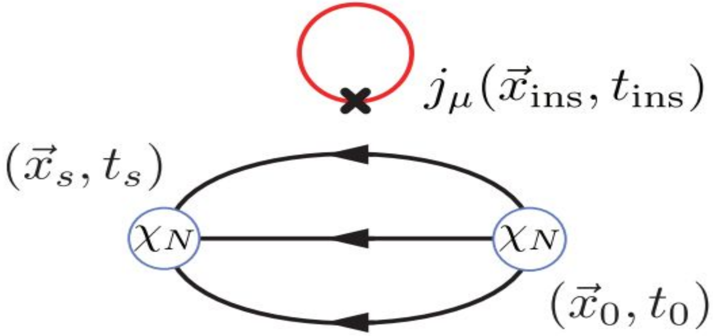

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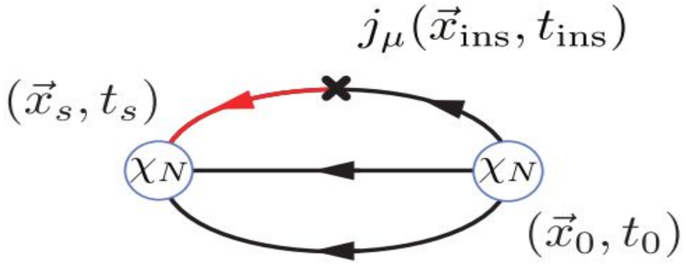

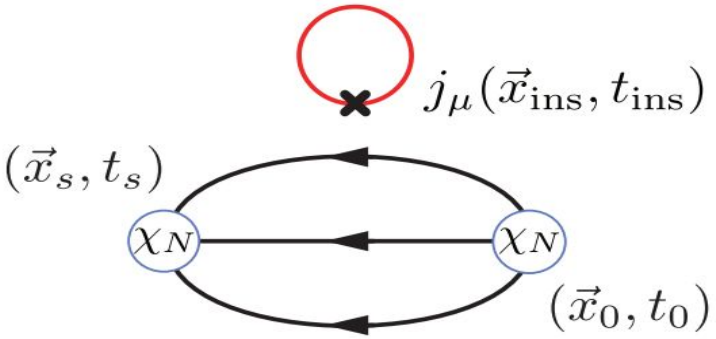

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Statistics

- Statistics for connected three point functions.

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n_{conf}=750		
t_s/a	t_s[fm]	n_{src}
8	0.64	1
10	0.80	2
12	0.96	5
14	1.12	10
16	1.28	32
18	1.44	112
20	1.60	128

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t_s/a	t_s[fm]	n_{src}
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8	0.55	2
10	0.69	4
12	0.82	10
14	0.96	22
16	1.10	48
18	1.24	45
20	1.37	116
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Ensemble	$n_{\rm conf}$	n_{ev}	$n_{\rm src}$
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cD211.054.96	500	-	480

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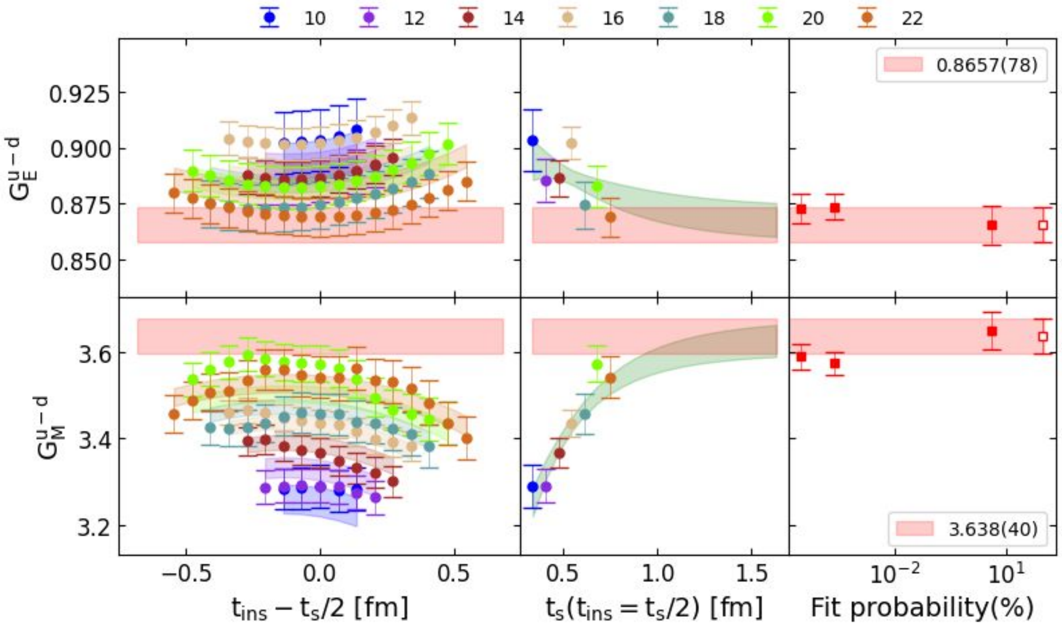
- The second excited state energy only appears in the two point function.

Extraction of Form Factors and Model Averaging

- The is done for each Q^2 value.



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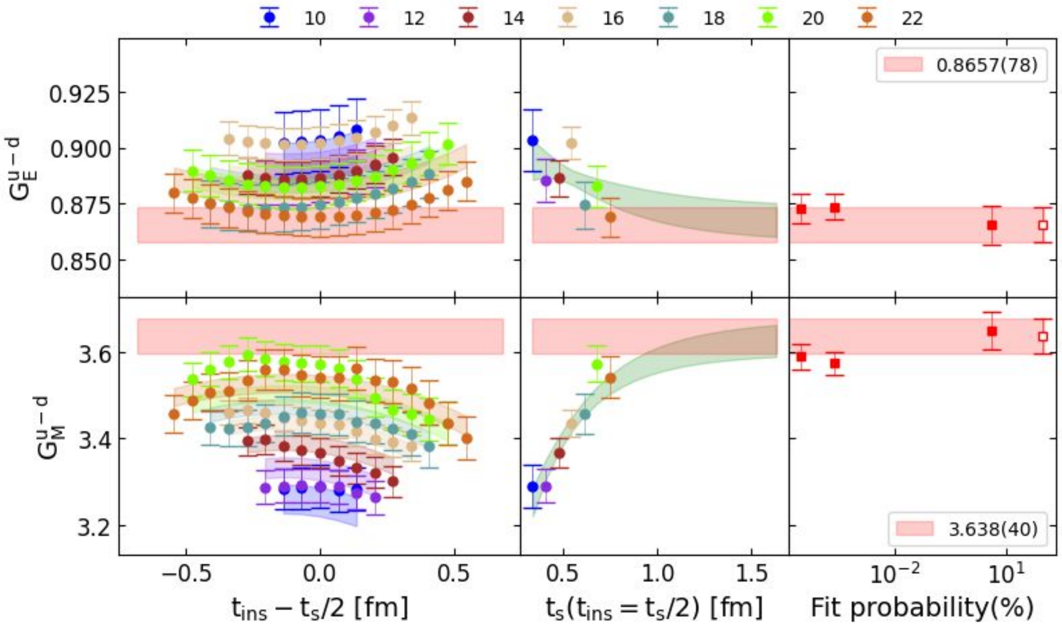

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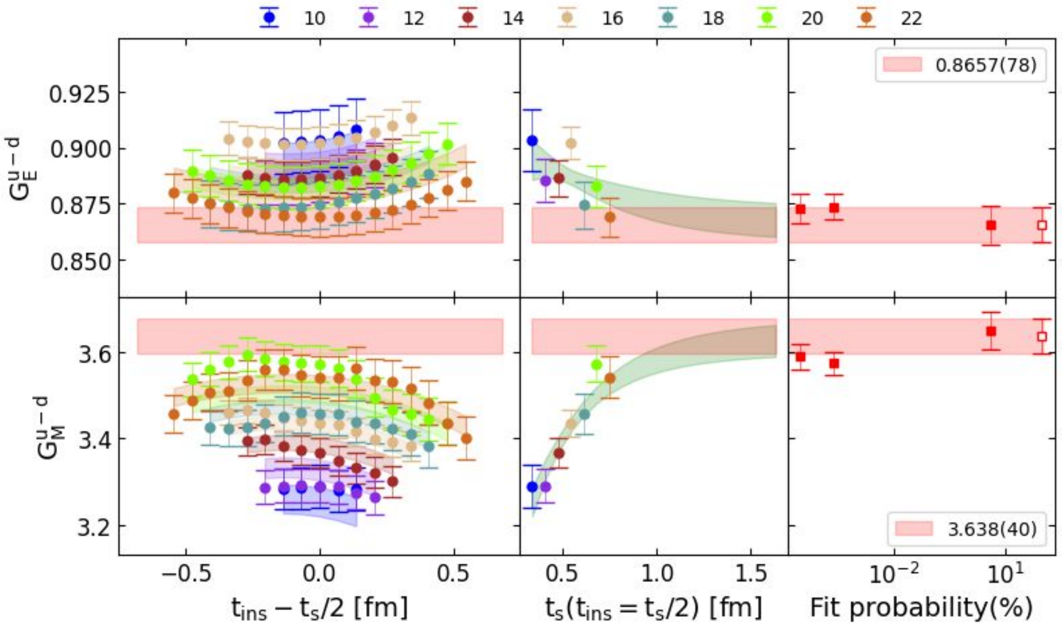

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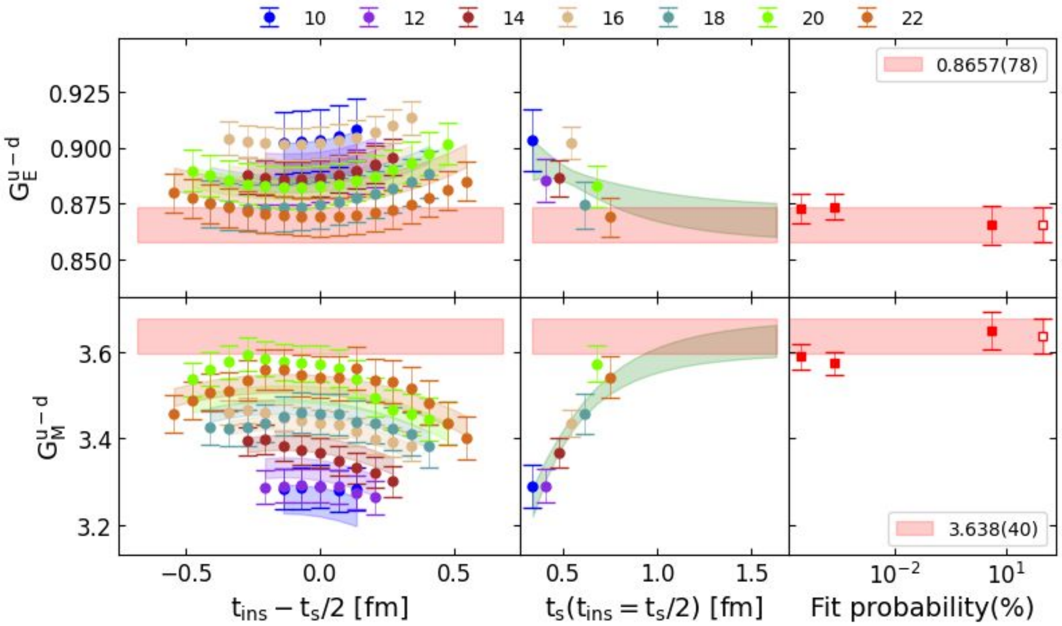

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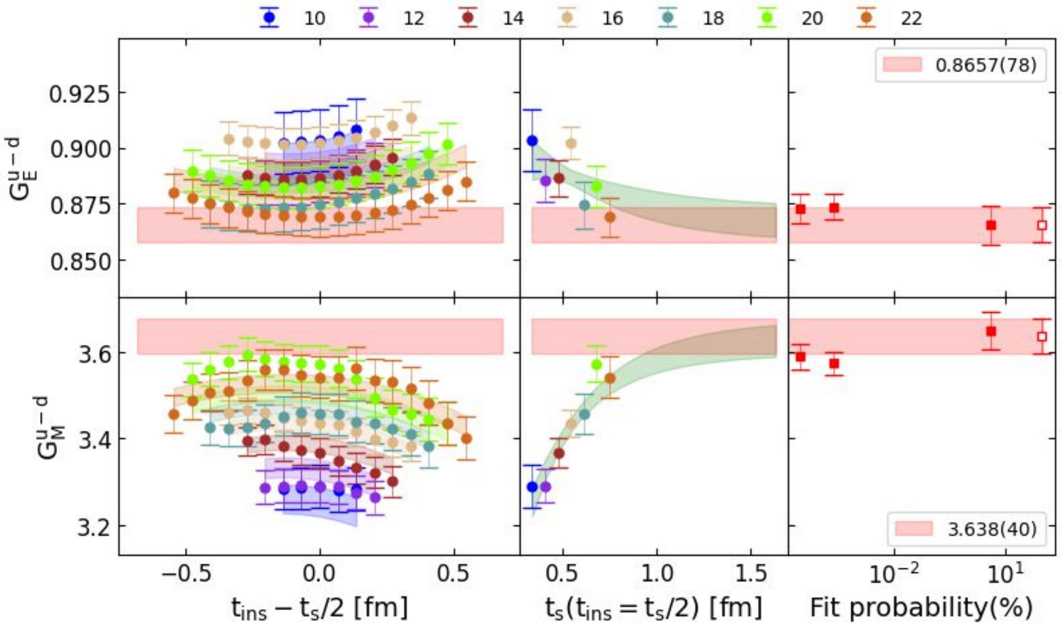
- On right is an example of isovector G_E and G_M for the cC211.060.80 ensemble for first non-zero Q^2.



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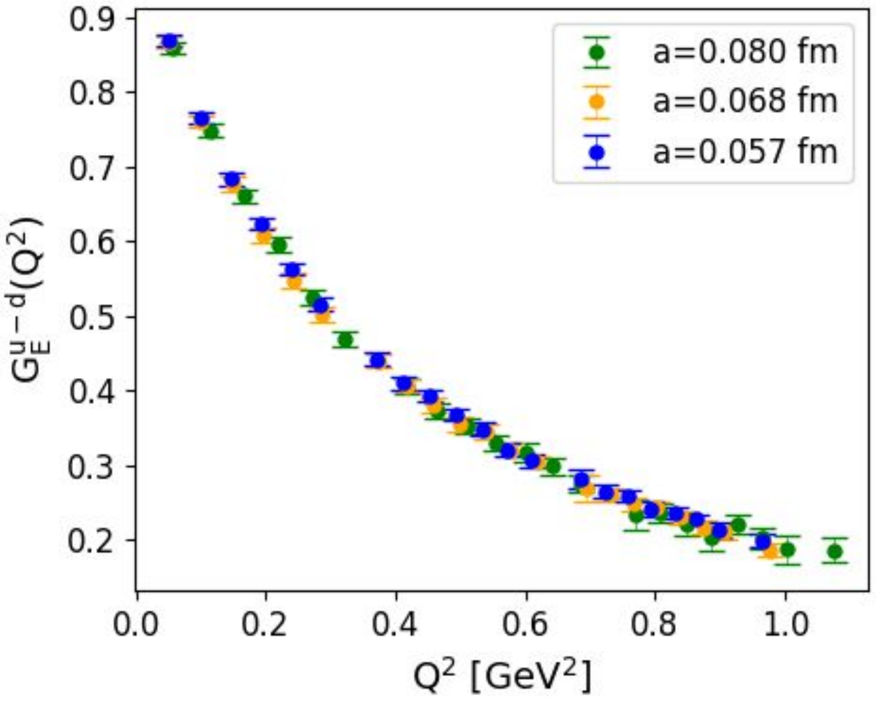

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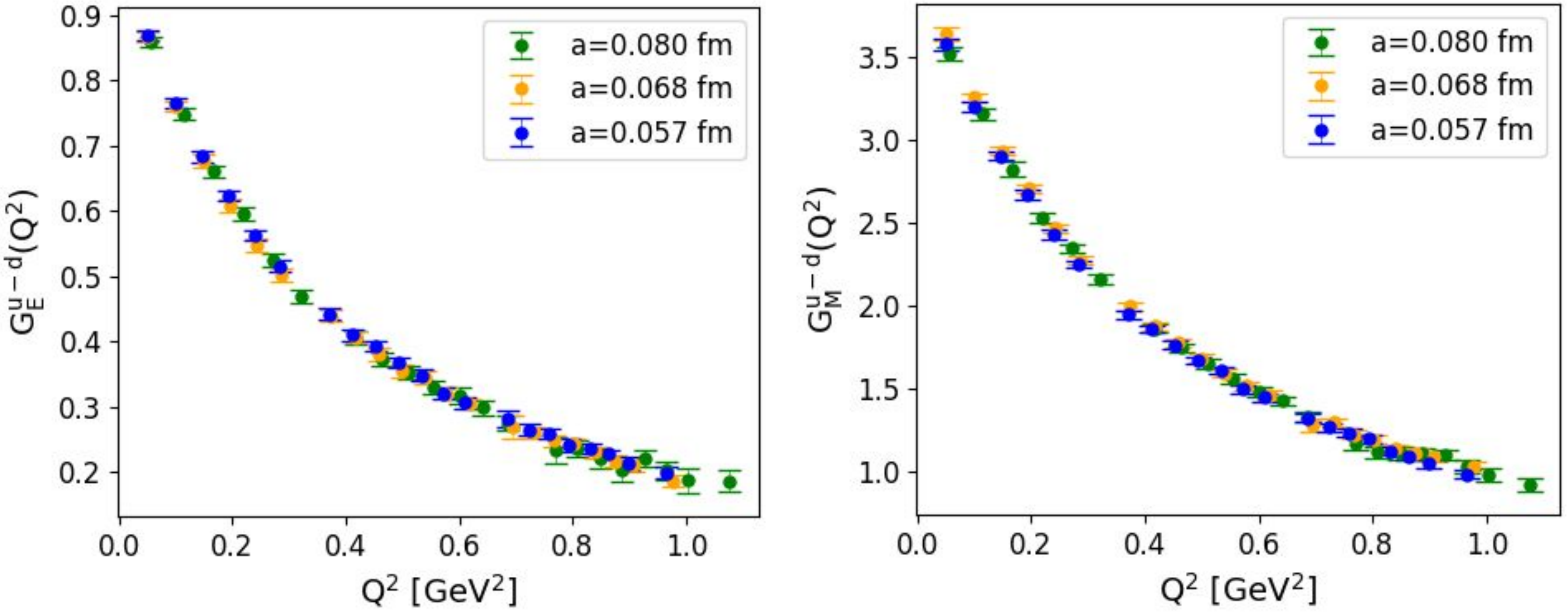

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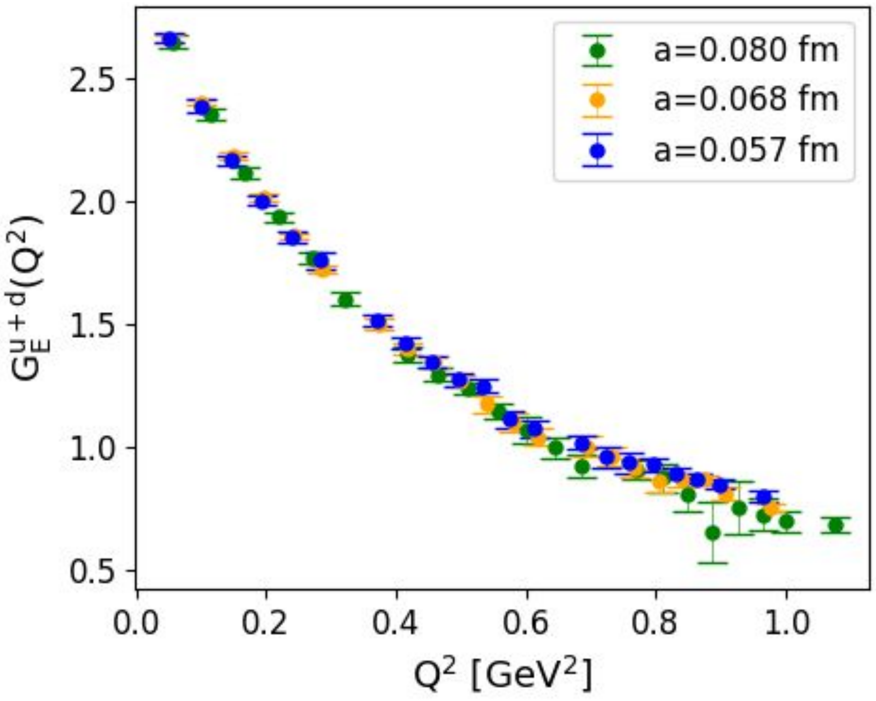

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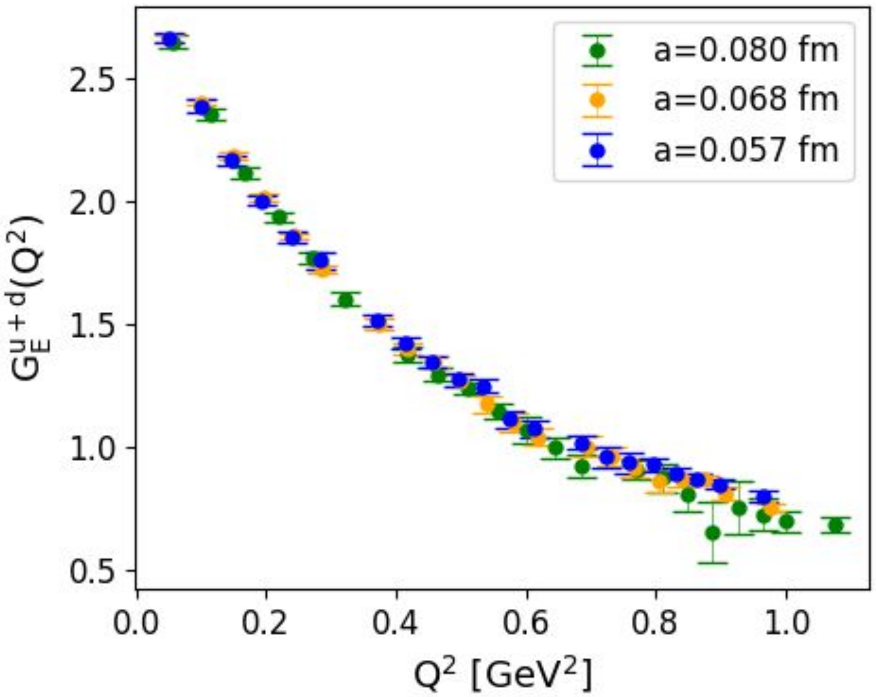

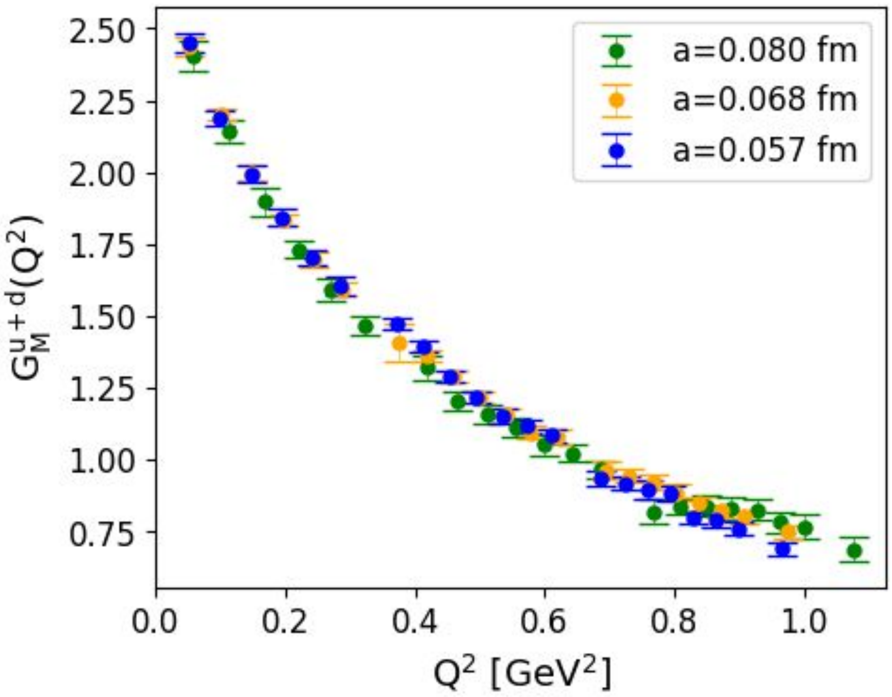

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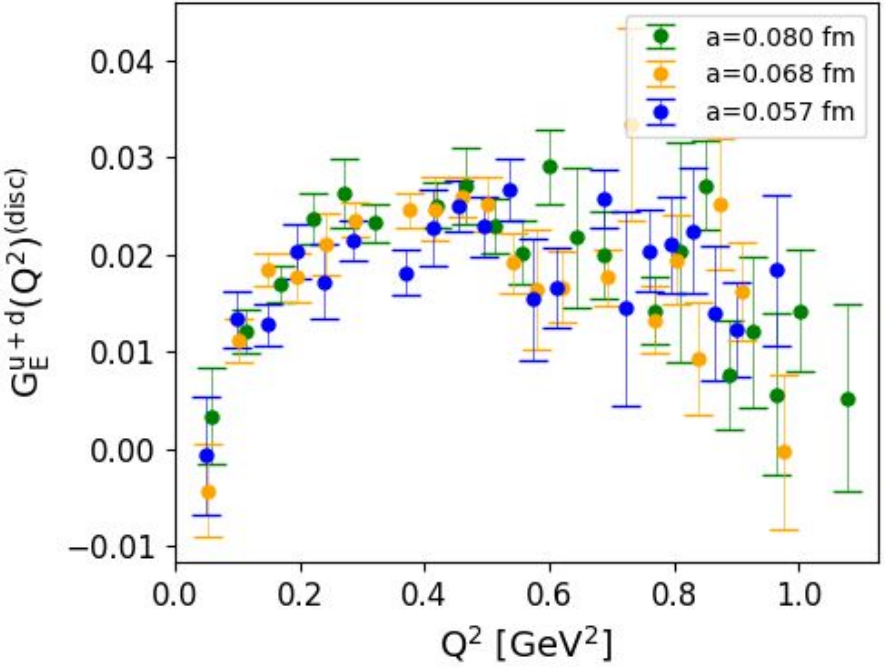

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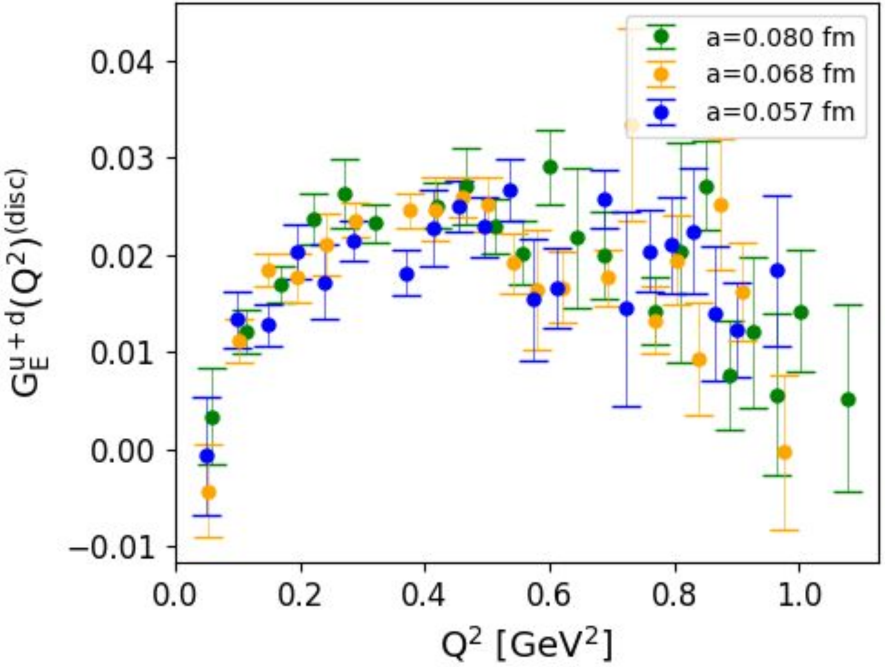

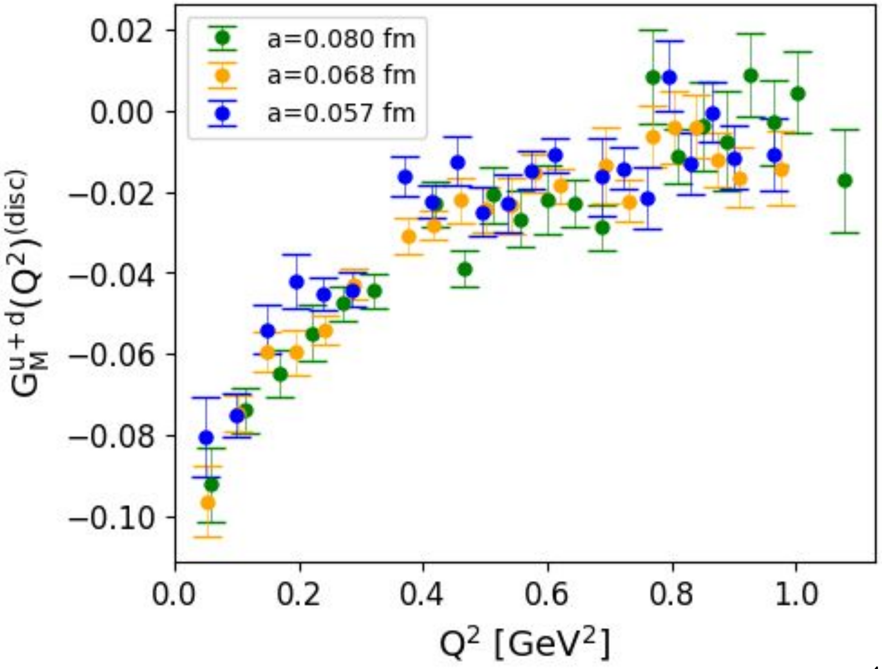

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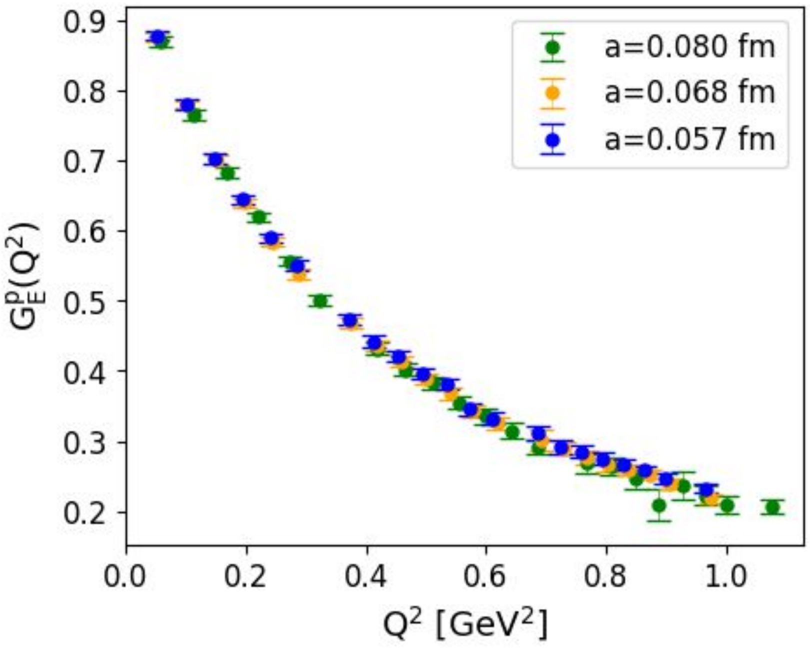

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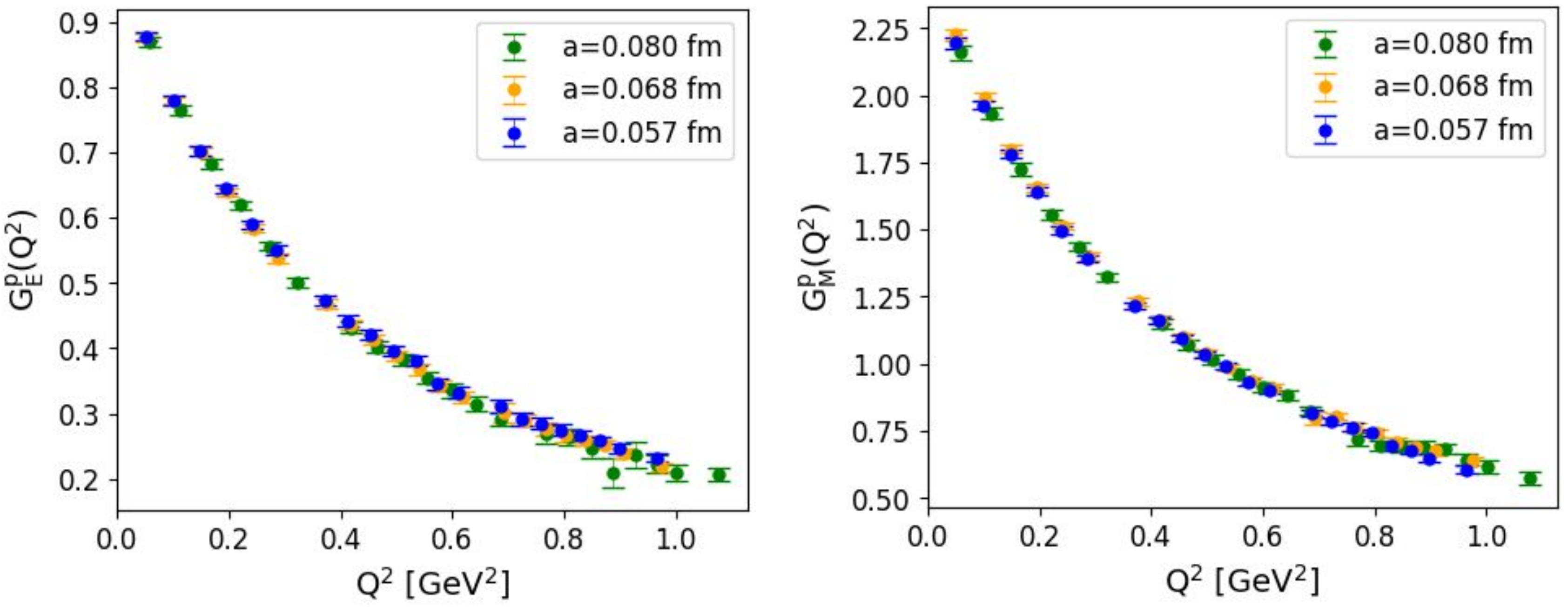

Neutron form factors

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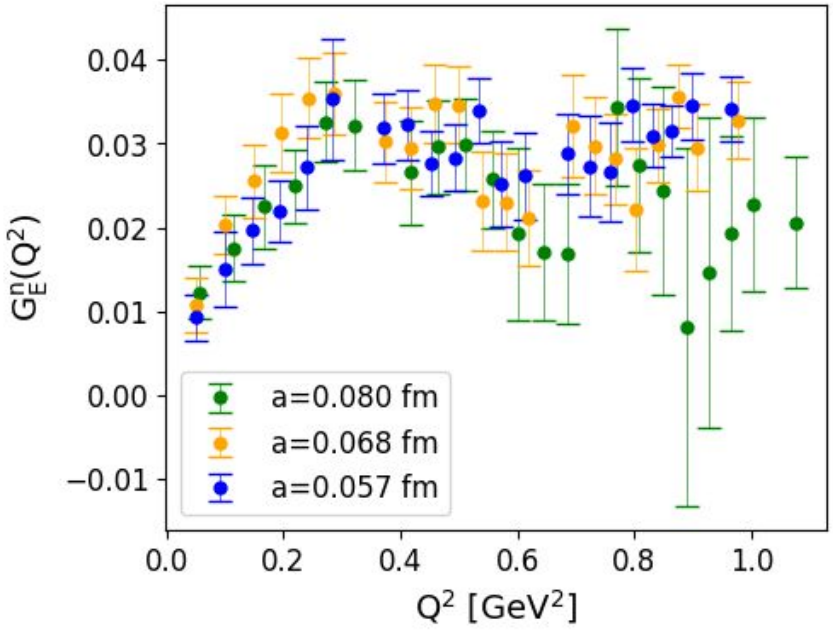

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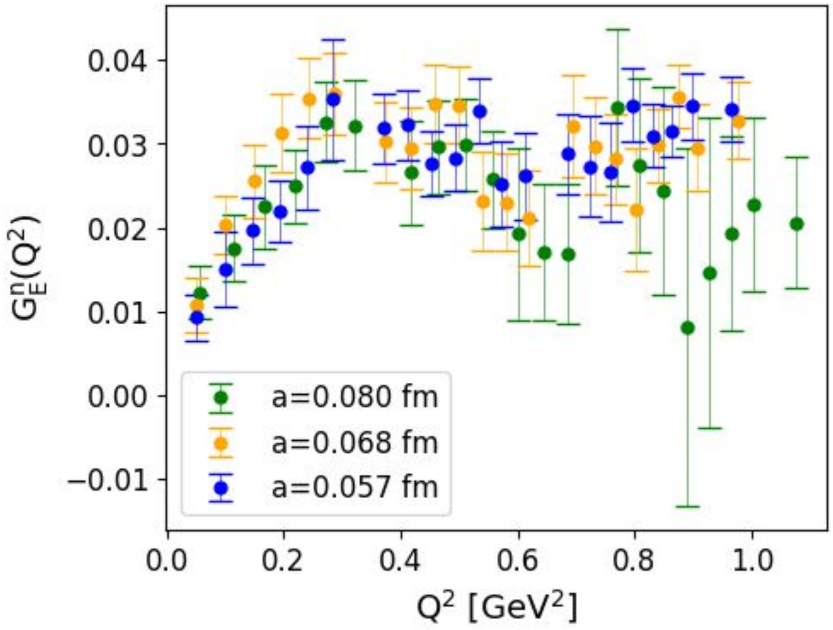

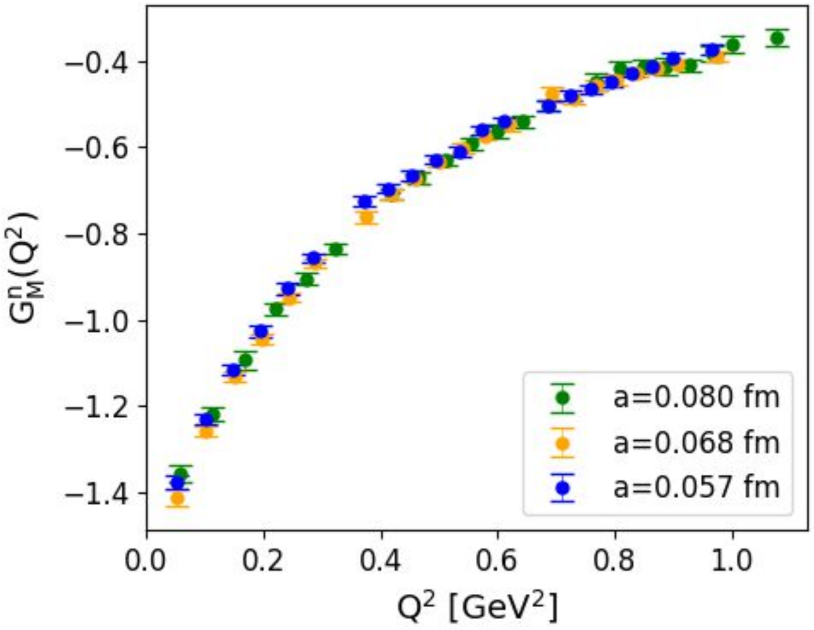

Parameterization of Q^2 Dependance and continuum limit

Dipole

$$G(Q^2) = \frac{g}{\left(1 + \frac{Q^2}{12} r^2\right)^2}$$

$$G(Q^2, a^2) = \frac{g(a^2)}{\left(1 + \frac{Q^2}{12} r^2(a^2)\right)^2}$$

$$g(a^2) = g_0 + a^2 g_2 \ , \ r^2(a^2) = r_0^2 + a^2 r_2^2$$

	Dipole	Z-expansion	Galster-like
Proton G_E	1 step + 2 step	1 step	-
Proton G_M	1 step + 2 step	1 step	-
Neutron G_E	-	-	1 step + 2 step
Neutron G_M	1 step + 2 step	1 step	-

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z-expansion

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Galster-like

$$G(Q^2) = \frac{Q^2 A}{4m_N^2 + Q^2 B} \frac{1}{\left(1 + \frac{Q^2}{0.71\,\text{GeV}^2}\right)^2}$$

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	Dipole	Z-expansion	Galster-like
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Determination of radius and magnetic moment

➢ Once we have the parameterization of Q^2 and a^2, the radius can be obtained by:

$$\langle r_X^2 \rangle^q = \frac{-6}{G_X^q(0)} \left. \frac{\partial G_X^q(q^2)}{\partial q^2} \right|_{q^2=0}$$

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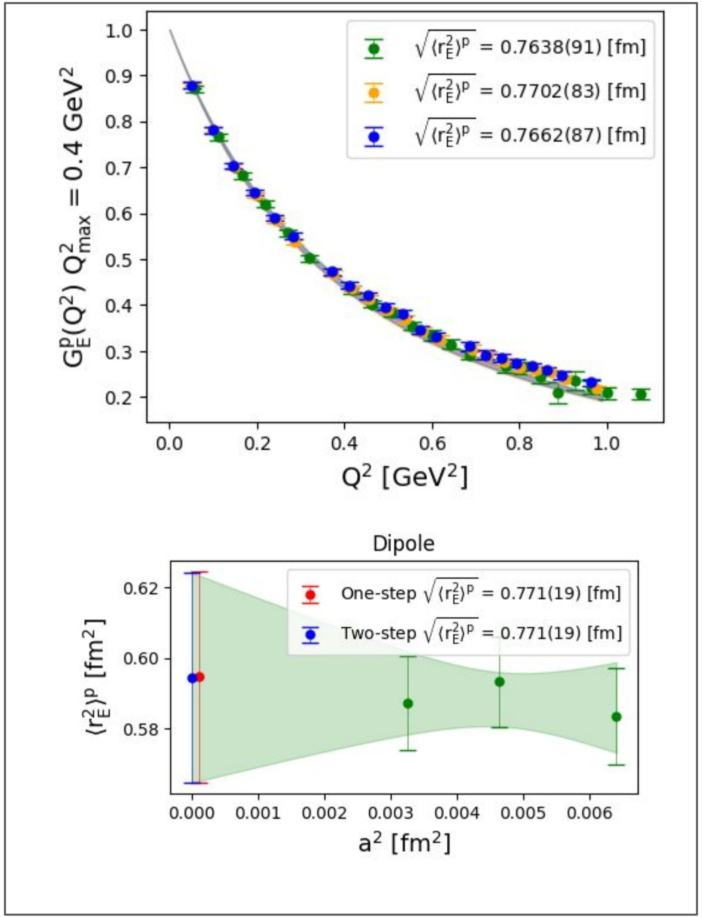
➢ The moments are obtained simply by taking the value at $Q^2 = 0$:

$$G_M^p(0) = \mu_p, \quad G_M^n(0) = \mu_n$$

Proton form factors with an example fit



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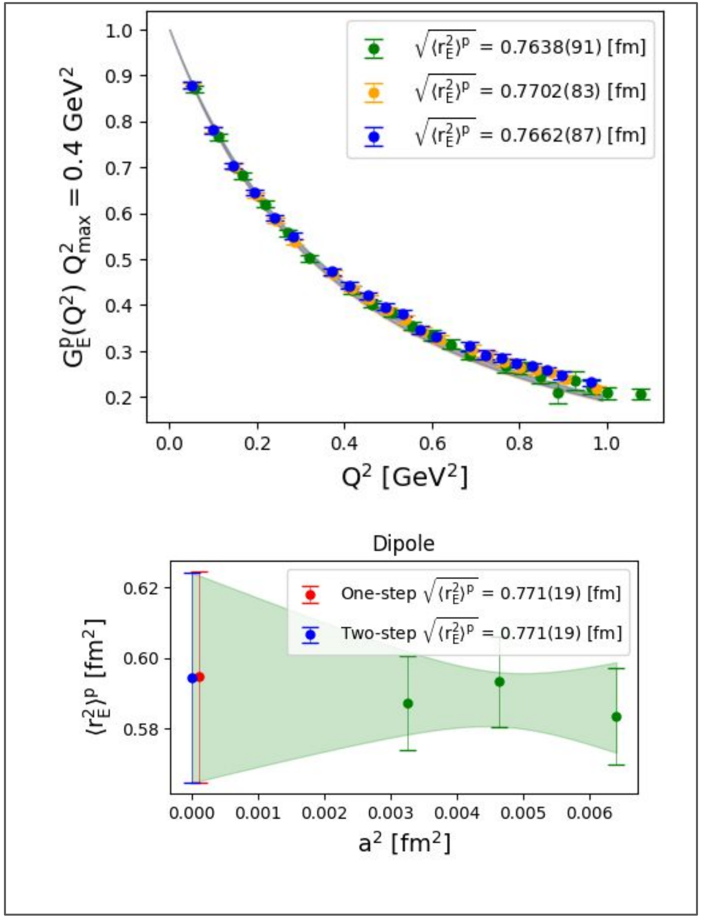
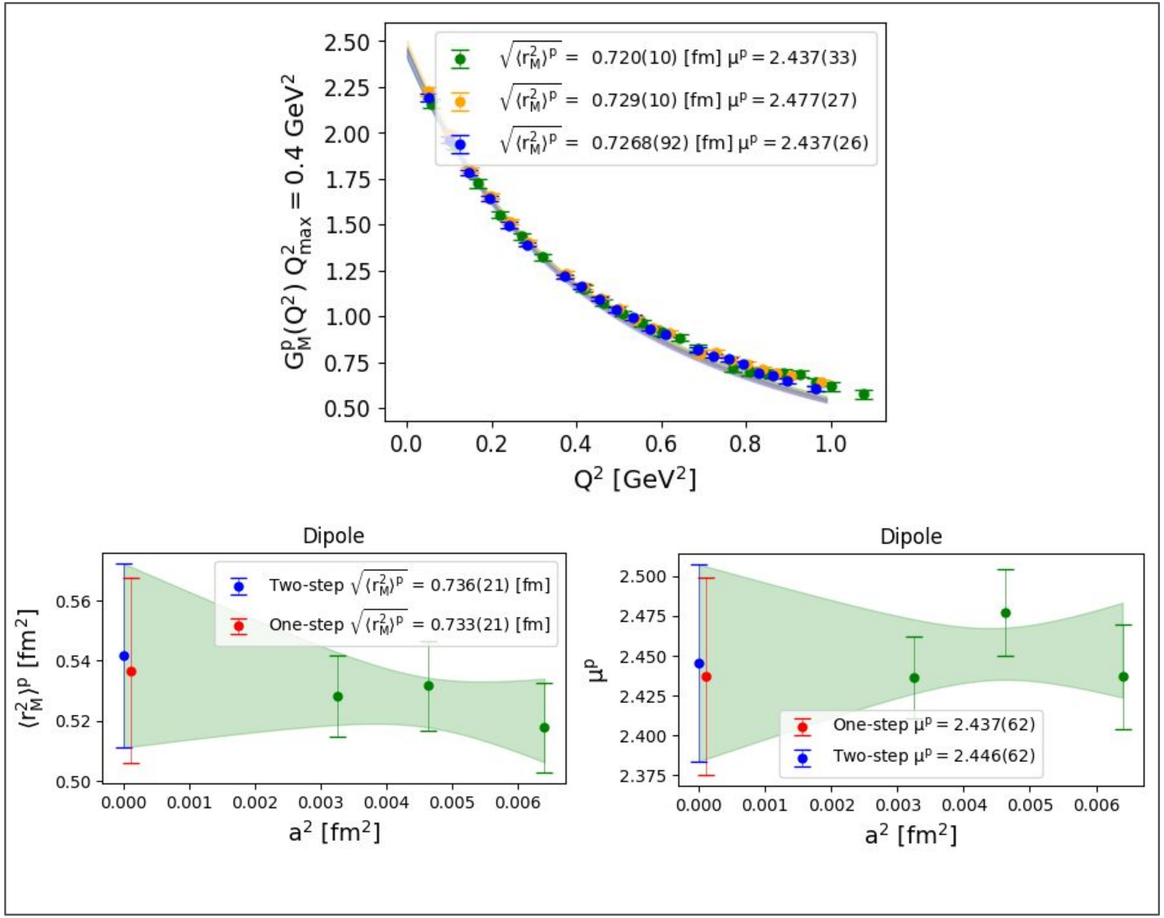
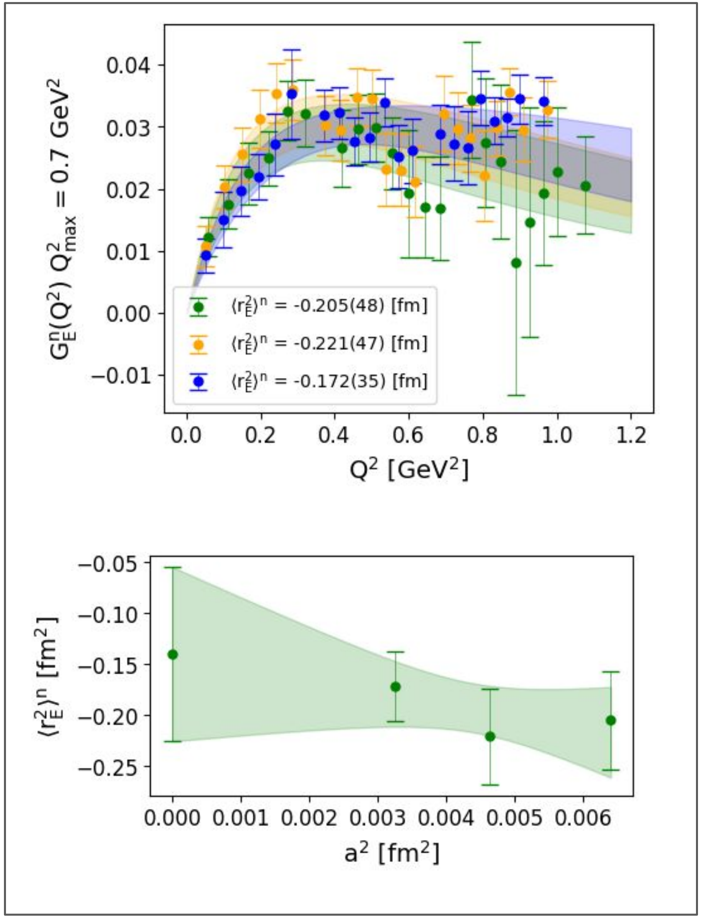
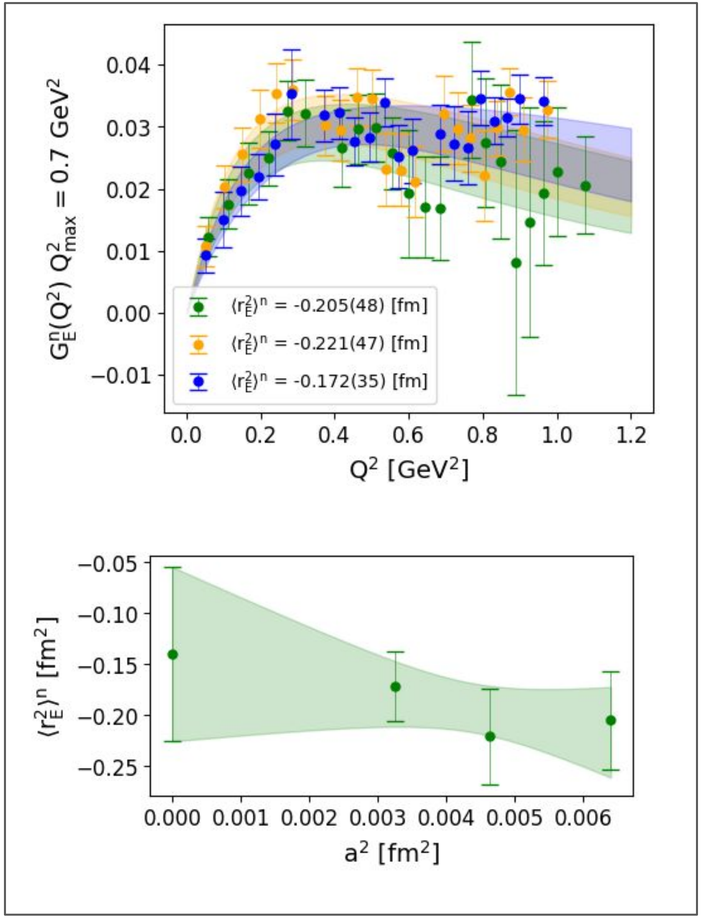


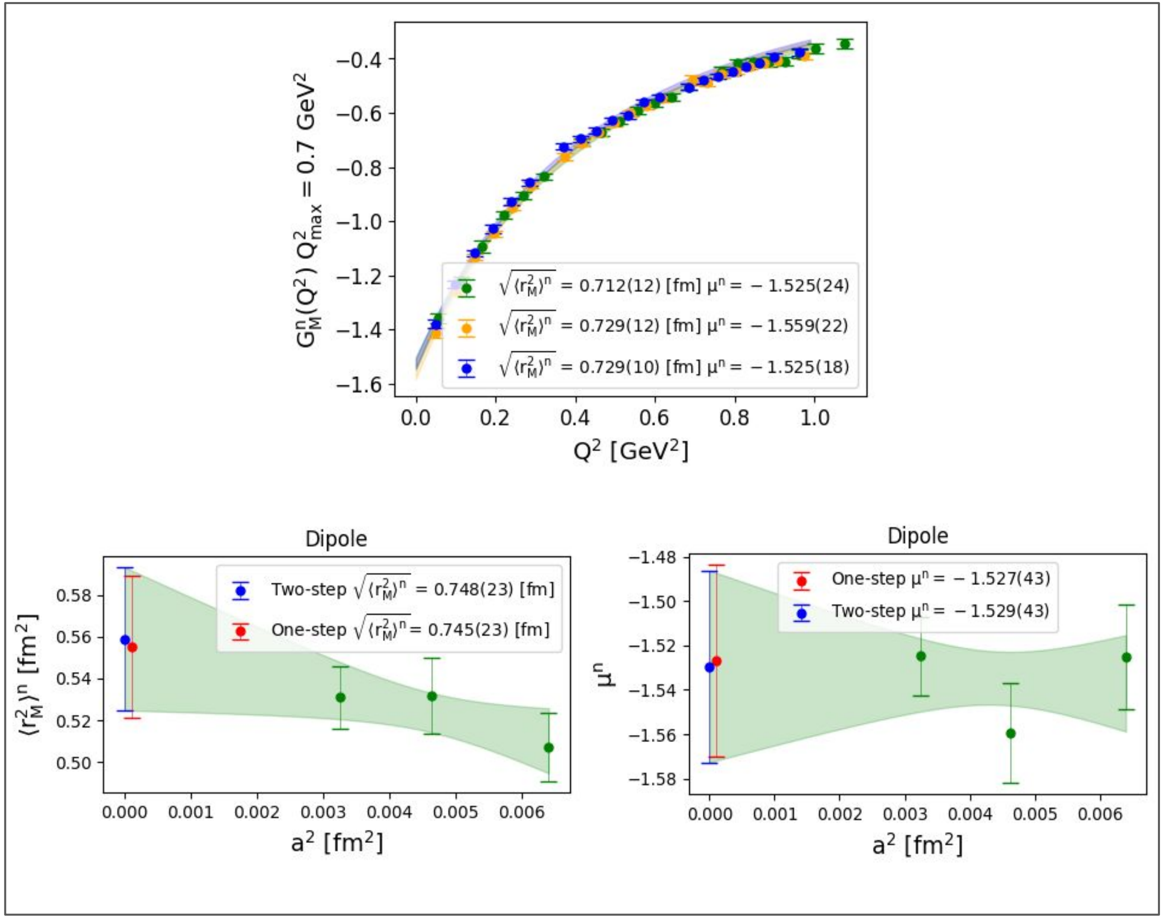


Neutron form factor with an example fit

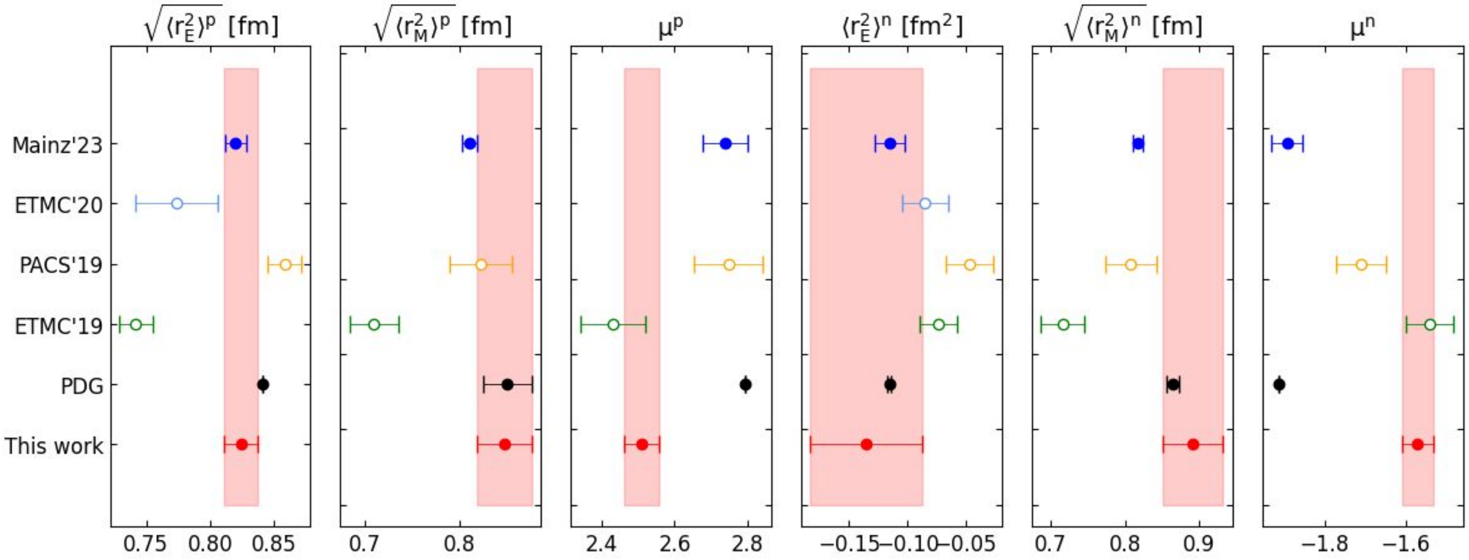


Neutron form factor with an example fit





Results



	$\sqrt{\langle r_E^2 \rangle}$	μ	$\sqrt{\langle r_M^2 \rangle}$
Proton	0.824(13) fm	2.509(48)	0.848(29) fm
Neutron	-0.135(47) fm	-1.570(39)	0.891(41) fm

Summary and Conclusion

- ➢ We have preliminary results at continuum limit, at physical point.
- ➢ Results include light disconnected contributions.
- ➢ Multistate fit ensuring ground state convergence.
- ➢ We will add more statistics to the disconnected contributions.
- ➢ Add analysis results from another lattice volume.

Thank you!

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