JOHANNES GUTENBERG UNIVERSITÄT MAINZ

The isoscalar non-singlet axial form factor of the nucleon from lattice QCD

Alessandro Barone

in collaboration with

D. Djukanovic, G. von Hippel, J. Koponen,
H. Meyer, K. Ottnad, H. Wittig

Lattice2024
29 July 2024, Liverpool

Introduction and motivation

Neutrino-nucleon low-energy scattering dominated by (quasi-)elastic scattering off single nucleon
$\Rightarrow$ need to determine nucleon matrix elements from theory to provide input for experiments

axial form factor

pseudoscalar form factor

$$\langle N(p',s')|\, A^a_\mu(0)\, |N(p,s)\rangle = \bar{U}_N(p',s') \left[\gamma_\mu\gamma_5 \boxed{G_A(Q^2)} - \frac{Q_\mu}{2M_N}\gamma_5 \boxed{G_P(Q^2)}\right] \frac{\lambda^a}{2} \boxed{U_N(p,s)}$$

$SU(3)_f$ spinor
$\lambda_a =$ Gell-Mann matrices

- $G_A(Q^2)$: relevant for neutrino scattering experiments - main source of errors in neutrino-nucleon interactions
- $G_P(Q^2)$: smaller impact on the cross-section, mainly relevant at small Q^2

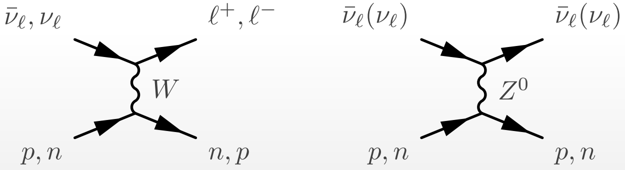
Nucleon interactions

Different $SU(3)$ flavour combinations play different roles in neutrino-nucleon scattering:

- $A^3_\mu \to G_A^{u-d}(Q^2)$:**isovector** $\to$ sensitive to $W^\pm$ (β decays)
- $\begin{cases} A^0_\mu \to G_A^{u+d+s}(Q^2) & \text{:isoscalar singlet} \\ A^8_\mu \to G_A^{u+d-2s}(Q^2) & \text{:isoscalar non-singlet} \end{cases}$ $\to$ sensitive to Z^0(elastic scattering)

  :isoscalar singlet, **:isoscalar non-singlet**

  ↳ focus of this talk



Theoretical input from LQCD

1. isovector well studied and in agreement between collaborations (see a review here [Meyer et al. (2022)[1]]), in slight tension with experiments [Meyer et al. (2016)[2]]
2. isoscalar and flavour decomposition still need more work [Alexandrou et al. (2021)[3]]

Nucleon structure

All these form factors provide information on the nucleon structure through various quantities

axial charge ← g_A ; axial radius ← r_A^2

$$G_A(Q^2) = g_A \left(1 - \frac{1}{6} r_A^2 Q^2 + \mathcal{O}(Q^4)\right) ,$$

In particular, the spin S_N can be decomposed [Ji (1997)[4]] as

$$\neq \sum_{q_{\rm val}} S_{q_{\rm val}} \longleftarrow S_N = \frac{1}{2}\sum_q \Delta\Sigma_q + \sum_{q_{\rm val}} L_{q_{\rm val}} + J_g$$

[Alexakhin et al. (2007)[5]]

$L_{q_{\rm val}}$ → valence quark angular momentum

J_g → gluon angular momentum

$\Delta\Sigma_q$ → intrinsic quark spin $\equiv g_A^q$ → include contribution from the sea

All G_A^f are relevant for nucleon structure! MicroBooNE aims to extract [Miceli et al. (2015)[6], Kim et al. (2019)[7]]

$$G_A^s(Q^2) , \quad Q^2[\text{GeV}^2] \in [0.08, 1]$$

Computation strategy on the lattice

We calculate two- and three-point correlation functions. In particular, we disinguish connected and disconnected contribution as

$$C_{3\mathrm{pt},i}(\boldsymbol{q},t,t_s) = -ia^6 \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{q}\cdot\boldsymbol{y}} \Gamma_{\beta\alpha} \left\langle \bar{\Psi}^{\alpha}(\boldsymbol{x},t_s)\, A_i^{u+d-2s}(\boldsymbol{y},t)\, \Psi^{\beta}(0) \right\rangle$$

$$= C^{\mathrm{conn}}_{3\mathrm{pt,i}}(\boldsymbol{q},t,t_s) + C^{\mathrm{disc}}_{3\mathrm{pt},i}(\boldsymbol{q},t,t_s)$$

(conn $\rightarrow$ $u+d$; disc $\rightarrow$ $u+d-2s$)

with

$$C^{\mathrm{disc}}_{3\mathrm{pt},i}(\boldsymbol{q},t,t_s) = \left\langle L_i(\boldsymbol{q},t) C_2(\boldsymbol{p}',t_s) \right\rangle, \qquad L_i(\boldsymbol{q},t) = -\sum_{\boldsymbol{z}} e^{i\boldsymbol{q}\cdot\boldsymbol{z}} \mathrm{Tr}\left[\mathrm{D}_{\mathrm{q}}^{-1}(\mathrm{z},\mathrm{z})\gamma_{\mathrm{i}}\gamma_5\right]$$

- $\boldsymbol{p}' = \boldsymbol{0} \Rightarrow \boldsymbol{q} = -\boldsymbol{p}$, i.e. rest frame of the final state nucleon
- smeared u, d fields
- we use APE-smeared gauge fields

The connected part contains only $u+d$; the strange s appears only in disconnected loops.

Ratio

The axial form factor G_A is addressed considering the transverse component $(\boldsymbol{q} \times \boldsymbol{A})$

$$C^{T}_{3\mathrm{pt},i}(\boldsymbol{q},t,t_s) = \epsilon^{ijk} q_j \, C_{3\mathrm{pt},k}(\boldsymbol{q},t,t_s) \propto (\boldsymbol{q} \times \boldsymbol{\gamma})_i \gamma_5 G_A(Q^2) + \cancel{(\boldsymbol{q} \times \boldsymbol{q})_i \gamma_5 G_P(Q^2)}$$

and then projecting into

$$C_{3\mathrm{pt}}(\boldsymbol{q},t,t_s) = \sum_i \frac{(\boldsymbol{q} \times \boldsymbol{s})_i}{|\boldsymbol{q} \times \boldsymbol{s}|^2} \, C^{T}_{3\mathrm{pt},i}(\boldsymbol{q},t,t_s)\,, \quad \boldsymbol{s} = \boldsymbol{e}_3\,, \quad \Gamma = \frac{1}{2}(1+\gamma_0)(1+i\gamma_5\gamma_3)$$

The signal is improved considering only momenta $|q_3| \leq \min\left(|q_1|, |q_2|\right)$ and the ratio is

$$R(\boldsymbol{q},t,t_s) = \frac{C_{3\mathrm{pt}}(\boldsymbol{q},t,t_s)}{C_{2\mathrm{pt}}(\boldsymbol{0},t_s)} \sqrt{\frac{C_{2\mathrm{pt}}(\boldsymbol{q},t_s-t) C_{2\mathrm{pt}}(\boldsymbol{0},t) C_{2\mathrm{pt}}(\boldsymbol{0},t_s)}{C_{2\mathrm{pt}}(\boldsymbol{0},t_s-t) C_{2\mathrm{pt}}(\boldsymbol{q},t) C_{2\mathrm{pt}}(\boldsymbol{q},t_s)}} \xrightarrow{t-t_s \gg 0} (\ldots) G_A^{\mathrm{eff}}(Q^2)$$

Summation method

[Maiani et al. (1987)[8], Capitani et al. (2012)[9]]

There are different approaches to address the extraction of the relevant form factor:

- plateau method
- two(three)-state fits
- **summation method**

The summation method assumes single-state dominance but the excited states are suppressed exponentially with t_s (instead of $t_s - t$)

$$S(\boldsymbol{q}, t_s) = a\sqrt{\frac{2E_q}{M_N + E_q}} \sum_{t=a}^{t_s - a} R(\boldsymbol{q}, t, t_s) \overset{t_s \gg 1}{=} b_0(\boldsymbol{q}) + t_s G_A(Q^2) + \mathcal{O}(t_s e^{-\Delta t_s})$$

Lattice setup: CLS $N_f = 2 + 1$ ensembles

[Bruno et al. (2015)[13]]

- non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions [Sheikholeslami and Wohlert (1985)[10], Bulava and Schaefer (2013)[11]]
- tree-level improved Lüscher-Weisz gauge action [Luscher and Weisz (1985)[12]]

ID	β	T/a	L/a	M_π [MeV]	$M_\pi L$	M_N [GeV]	N_{conf}	N_{meas}	t_s [fm]	N_{t_s}
H102	3.40	96	32	354	4.96	1.103	2005	32080	0.35..1.47	14
H105	3.40	96	32	280	3.93	1.045	1027	49296	0.35..1.47	14
C101	3.40	96	48	225	4.73	0.980	2000	64000	0.35..1.47	14
N101	3.40	128	48	281	5.91	1.030	1596	51072	0.35..1.47	14
S400	3.46	128	32	350	4.33	1.130	2873	45968	0.31..1.53	9
N451	3.46	128	48	286	5.31	1.045	1011	129408	0.31..1.53	9
D450	3.46	128	64	216	5.35	0.978	500	64000	0.31..1.53	17
N203	3.55	128	48	346	5.41	1.112	1543	24688	0.26..1.41	10
N200	3.55	128	48	281	4.39	1.063	1712	20544	0.26..1.41	10
D200	3.55	128	64	203	4.22	0.966	2000	64000	0.26..1.41	10
E250	3.55	192	96	129	4.04	0.928	400	102400	0.26..1.41	10
N302	3.70	128	48	348	4.22	1.146	2201	35216	0.20..1.40	13
J303	3.70	192	64	260	4.19	1.048	1073	17168	0.20..1.40	13
E300	3.70	192	96	174	4.21	0.962	570	18240	0.20..1.40	13

NB: full analysis for the conn. part, BUT addition of the disc. contribution is in progress!
*We use E300 and D200 to demonstrate our approach.

Analysis strategy (1): direct fit with z-expansion

[Djukanovic et al. (2022)[14]]

We use z-expansion to parametrize the form factor ($n = 2$)

$$G_A(Q^2) = \sum_{k=0}^{n} a_k z^k(Q^2)\,, \quad z(Q^2) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}}}}$$

Typical procedure (two-step fit) for every value of the $t_{s,\text{min}}$:

1. for each Q^2, linear fit to $S(\boldsymbol{q}, t_s)$ for all point $t_s \in \{t_{s,\text{min}}, \ldots\}$ to extract $G_A^{\text{eff}}(Q^2, t_{s,\text{min}})$
2. zfit in Q^2 range to extract a_0, a_1, a_2

HERE:

one-step **direct z-fit** on all data for all $t_s \in \{t_{s,\text{min}}, \ldots\}$, $Q^2 \in \{0, \ldots, Q^2_{\text{max}}\}$

$$t_{\text{cut}} = (4M_\pi)^2\,, \qquad Q^2_{\text{max}} = 0.7\,\text{GeV}^2$$

Analysis strategy (2): comparison of the approaches

We compare three fit approaches

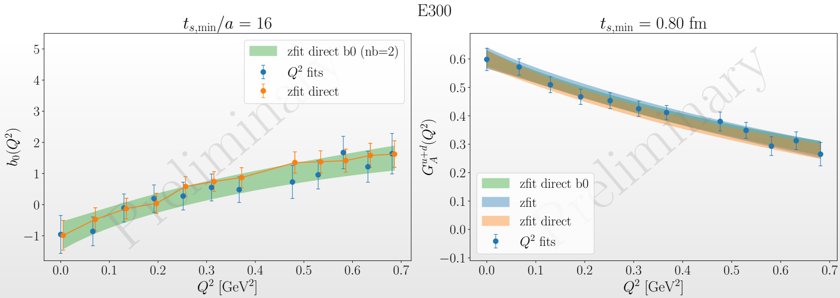
- two-step fit
- direct z-fit on G_A
- direct z-fit on both G_A and b_0

$$S(Q^2, t_s) = \boxed{b_0(Q^2)} + t_s \boxed{G_A(Q^2)} \rightarrow \sum_k^2 a_k z^k(Q^2)$$

$b_0(Q^2) \uparrow \sum_k^2 c_k z^k(Q^2)$

$b_0(Q^2) \downarrow \{b_0(Q_1^2), b_0(Q_2^2), ..., b_0(Q_n^2)\}$

CONNECTED CASE $(u + d)$



$\Rightarrow$ all approaches are consistent!

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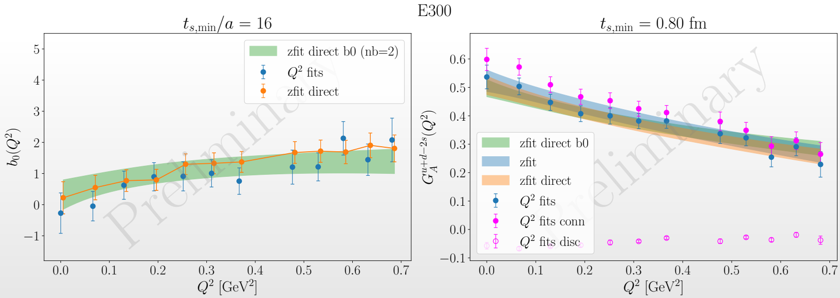
CONN+DISC CASE $(u + d - 2s)$



$\Rightarrow$ all approaches are consistent!

Analysis strategy (3): dealing with a large covariance matrix

Direct z-fit

- PRO: do only one fit and account for everything in one step
- CON: for small $t_{s,\mathrm{min}}$ the covariance matrix $C_{N\times N}$ is **large** with $N = (\#Q^2) \times (\#t_s)$ and possibly not optimally estimated ($\equiv$ error on the error is large)

To assess the stability of the fits we estimate the covariance of the data y with

- standard
- shrinkage (off-diagonal damping with a parameter $\alpha \in [0.985, 1]$, i.e. up to 1.5% damping)
- SVD cut (reduce the condition number of the matrix)

$$C_{ij} = \langle (y_i - \langle y_i \rangle)(y_j - \langle y_j \rangle) \rangle$$

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$$C_{ij} = \langle (y_i - \langle y_i \rangle)(y_j - \langle y_j \rangle) \rangle$$

$$C^s_{ij} = (1-\alpha)\delta_{ij} C_{ij} + \alpha C_{ij}$$

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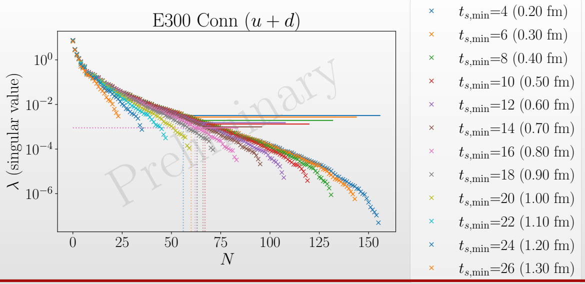
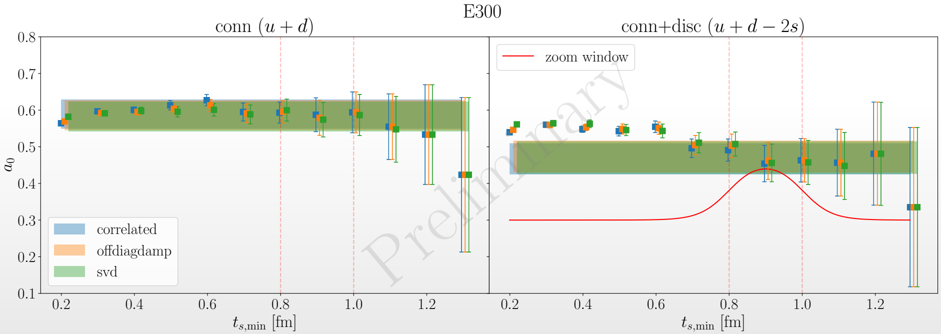


Strategy: scan progressively removing the lowest eigenvalues (setting them constant), stop when the χ^2 of the fit is acceptable.
Set set same condition number for all the $t_{s,\min}$

Analysis strategy (4): window average

We average over all the possible source-sink separation t_s with a window function

$$\text{window} = \frac{1}{N}\left[\tanh\left(\frac{t_s^{\min} - t_w^{\text{low}}}{\Delta t_w}\right) - \tanh\left(\frac{t_s^{\min} - t_w^{\text{up}}}{\Delta t_w}\right)\right], \qquad \begin{cases} t_s^{\text{low}} = 0.8\text{ fm} \\ t_s^{\text{up}} = 1\text{ fm} \\ \Delta t_w = 0.08\text{ fm} \end{cases}$$



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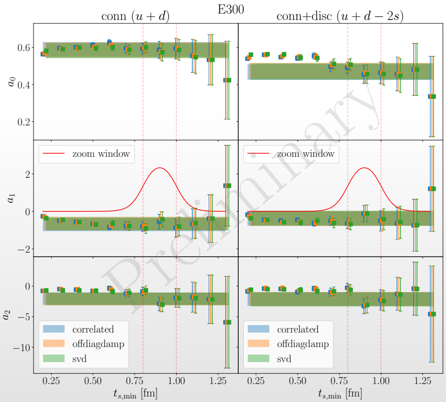

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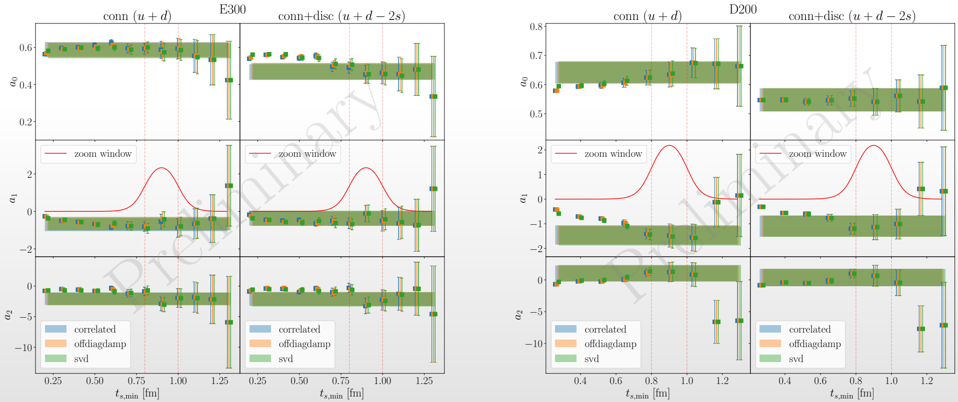


Chiral continuum extrapolation

General approach:

1. extract the z-expansion coefficients a_0, a_1, a_2 on all ensembles
2. extrapolate them to the chiral-continuum limit

We consider $(2\times)$3 fit ansatz (with/without FV term for the leading term $a_0 \equiv g_A$):

A1. linear in M_π^2 and a^2

$$a_i = p_0 + p_1 M_\pi^2 + s_0 a^2 + v_0 \left(\frac{M_\pi^2}{\sqrt{M_\pi L}} e^{-M_\pi L} \right) \delta_{i0}$$

A2. same as A1 for a_i, $i = 1, 2$

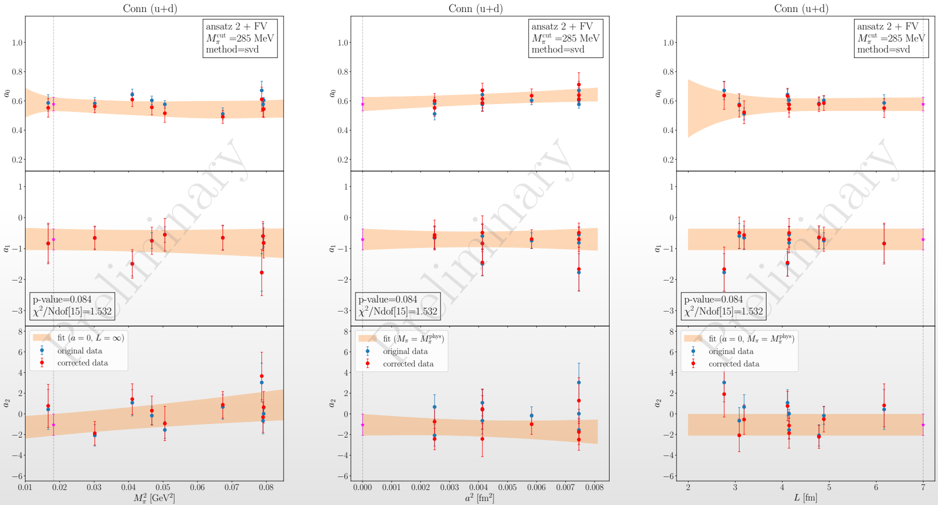
$$a_0 = p_0 + p_1 M_\pi^2 + p_2 M_\pi^3 - c_0 M_\pi^2 \ln \frac{M_\pi}{M_N} + v_0 \left(\frac{M_\pi^2}{\sqrt{M_\pi L}} e^{-M_\pi L} \right)$$

A3. same as A2 with M_π^3 terms for a_i, $i = 1, 2$

$$a_i = a_i + p_3 M_\pi^3$$

Chiral continuum extrapolation (isoscalar connected)

We show an example using ansatz **A2** +FV term ($\rightarrow$ no substantial FV effects)



Model average: Akaike Information Criterion (AIC)

We assign AIC weights [Borsanyi et al. (2021)[15]] to all the fits as

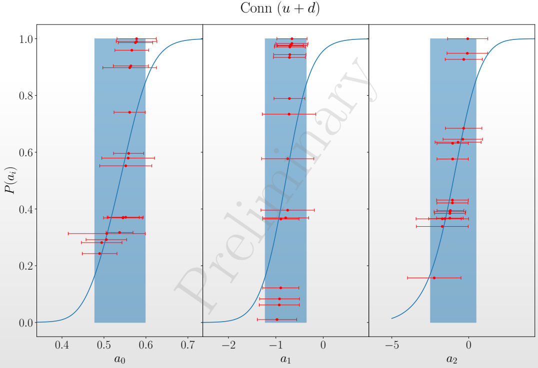
$$w_k^{\mathrm{AIC}} \propto e^{-\frac{1}{2}(\chi_k^2 + 2n_{\mathrm{par},k} - n_{\mathrm{data},k})} \longrightarrow P(a_i) = \int_{-\infty}^{a_i} \sum_k w_k^{\mathrm{AIC}} \mathcal{N}(a_i; \mu_k, \sqrt{\lambda}\sigma_k)$$

k → $\forall$ fits

$P(a_i)$ → cumulative distribution function (CDF)

weighted normal

λ is used to disantangle statistical and systematic error



Final result using the CDF percentiles as

$$a_i = a_i\big|_{50} \pm \sigma_i$$

$$P(a_i\big|_{50}) = 0.50$$

$$\begin{cases} \sigma_i^2 = \left[\frac{1}{2}\left(a_i\big|_{84} - a_i\big|_{16}\right)\right]^2 \\ \sigma_i^2 = \lambda \sigma_{\mathrm{stat},i}^2 + \sigma_{\mathrm{sys},i}^2 \end{cases}$$

λ → to disantangle the systematic error

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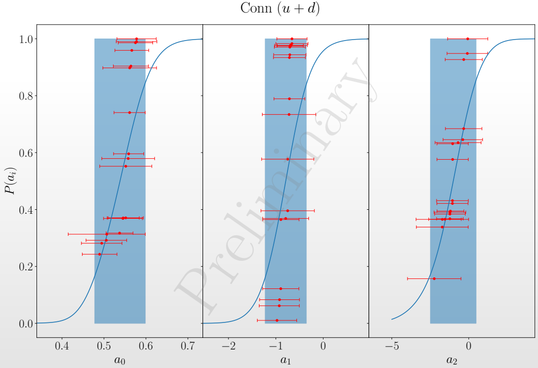
$w_k^{\mathrm{AIC}} \mathcal{N}(a_i; \mu_k, \sqrt{\lambda}\sigma_k)$ → weighted normal

$\sqrt{\lambda}$ → λ is used to disantangle statistical and systematic error



We need to preserve the correlations among a_i! We calculate also $P(a_i a_j)$ and its variance

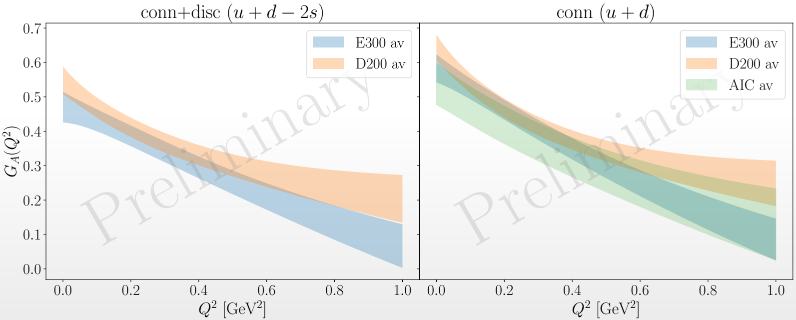
$$\mathrm{var}(a_i a_j) = \bar{a}_i^2 \sigma_j^2 + \bar{a}_j^2 \sigma_i^2 + 2\bar{a}_i \bar{a}_j \mathrm{cov}(a_i, a_j)$$

$$\bar{a}_i = a_i\big|_{50}$$

and extract the correlations manually

Preliminary results

- $u + d$: AIC average with cuts $M_\pi^{\rm cut}[\text{MeV}] = \{300, 285, 265\}$, and $a^{\rm cut}[\text{fm}] = \{0.08\}$ (exclude the coarsest)
- $u + d - 2s$: only window average of the coefficients (analysis ongoing)



Value of axial charge compatible with $g_A^{u+d-2s} = 0.46(5)$ obtained using the Cloudy Bag model [Bass and Thomas (2010)[16]]

Outlook and conclusions

So far:

- full analysis on the isoscalar connected case $(u+d)$ + preliminary results on the full case $u+d-2s$
- analysis with summation method + direct z-expansions at $n=2$
- various approaches to regulate the covariance matrix all consistent ($\rightarrow$ svd)
- z-expansion coefficients evaluated on each ensemble and extrapolated to the continuum limit

To do:

- include all the disconnected contributions (in progress)
- cross-check summation method with two-state fits
- explore more fit ansätze (e.g. dipole)
- refine the model average
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THANK YOU!

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