How much strangeness is needed for the axial-vector form factor?

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Overview

- we considered the axial-vector and induced pseudo-scalar form factor in SU(2) ChPT
- we obtained a simultaneous fit of both form factors to lattice data

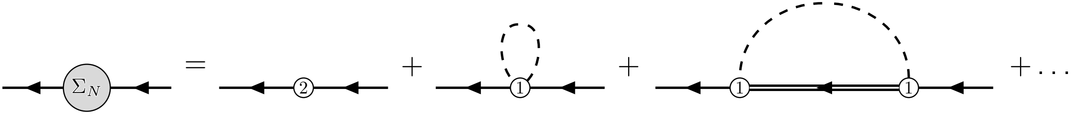
arXiv:2402.04905

Determination of nucleon and isobar mass

- using the flavor SU(2) chiral Lagrangian in the isospin limit with pion, nucleon and isobar as degrees of freedom
- masses are given by

$$M_N = M + \Sigma_N(M_N\,, M_\Delta)\,,$$
$$M_\Delta = M + \Delta + \Sigma_\Delta(M_N\,, M_\Delta)$$

- 1-loop calculation of the self-energy[1]



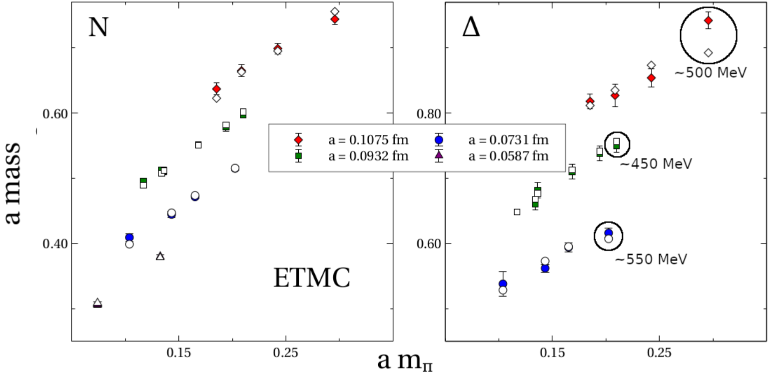
- **our approach**: insisting on on-shell masses inside the loop contributions

  → solve a system of two coupled equations

  → lattice data for M_N and M_Δ are important

[1] M.F.M. Lutz, Yonggoo Heo, and Xiao-Yu Guo. arXiv:1801.06417.

- inclusion of the isobar: consider lattice ensembles with $m_\pi > \Delta \approx 300$ MeV
- full consideration of finite volume effects (FVE)
- example[2]: fit to flavor-SU(2) data from ETMC[3]



[2] M. F. M. Lutz, U. Sauerwein, and R. G. E. Timmermans. arXiv:2003.10158.

[3] C. Alexandrou et al. arXiv:0803.3190.

Axial-vector form factors of the nucleon

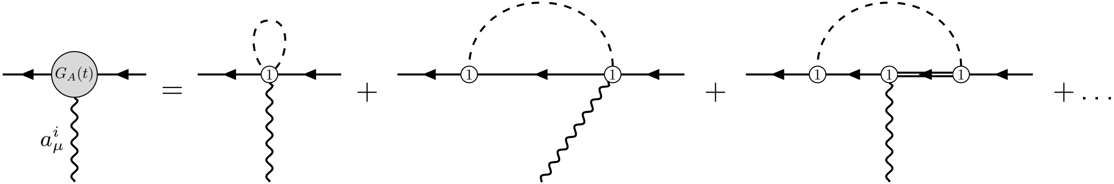
- definition ($q = \bar{p} - p$):

$$\langle N(\bar{p})|\, A_i^\mu(0)\, |N(p)\rangle = \bar{u}_N(\bar{p}) \left(\gamma^\mu\, G_A(q^2) + \frac{q^\mu}{2\, M_N}\, G_P(q^2)\right) \gamma_5\, \frac{\tau_i}{2}\, u_N(p)$$

- $G_A(0)$ through high precision measurement of β-decay

$$G_A(0) = 1.2732(23)$$

- Lutz et. al.[4] calculated G_A up to one-loop level:



[4] M. F. M. Lutz, U. Sauerwein, and R. G. E. Timmermans. arXiv:2003.10158.

Determination of the loop functions

- we insist on **on-shell** masses

  $\rightarrow$ no chiral expansion of M_N and M_Δ

  $\rightarrow$ result expressed in kinematical function times basis integrals
- full consideration of FVE in the hadron masses (*implicit*)
- no *explicit* FVE of the axial loop functions

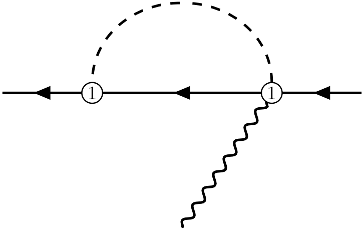
  $\rightarrow$ ongoing work
- chiral expansion only in the kinematic functions

  question: How to power count the mass difference $M_\Delta - M_N$?

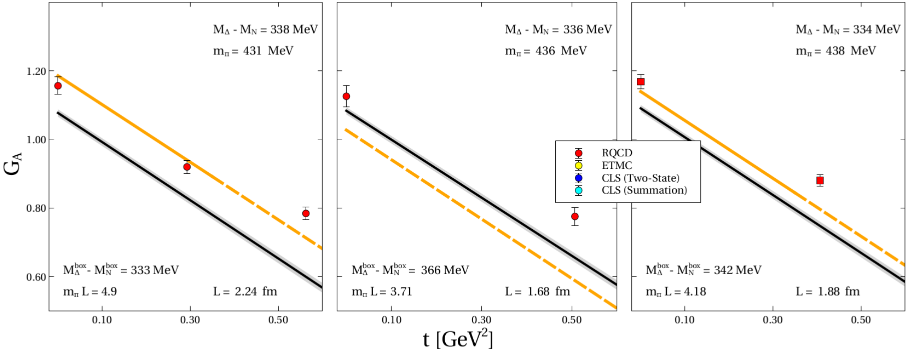
$$\delta = M_\Delta - M_N(1 + \frac{\Delta}{M}) \sim Q^2\,, \quad m_\pi^2 \sim t \sim Q^2$$

- no chiral expansion of the basis integrals

  $\rightarrow$ preserve analytic not polynomial structure



- flavor SU(2)-ensembles from ETMC[5], CLS[6] and RQCD[7]
- data points up to $m_\pi = 500$ MeV, $t \in [0, -0.36]$ GeV^2 and $m_\pi L \geq 4.0$
- lattice scales are fit parameters
- simultaneously fitting the masses and G_A with an evolutionary fit algorithm[8]



[5] C. Alexandrou et al. arXiv:1012.0857, [6] S. Capitani et al. arXiv:1705.06186, [7] Gunnar S. Bali et al. arXiv:1412.7336

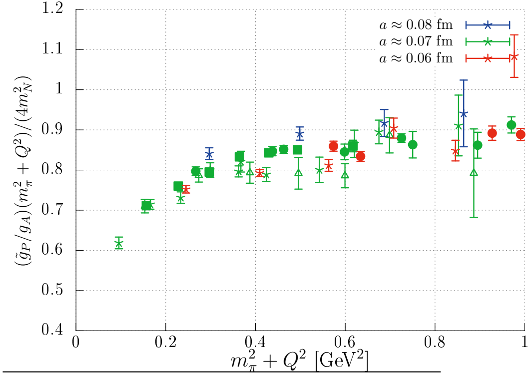
[8] Jonas Weßner et al. “Parametric Optimization on HPC Clusters with Geneva”. In: *Computing and Software for Big Science* (2023). DOI: 10.1007/s41781-023-00098-6.

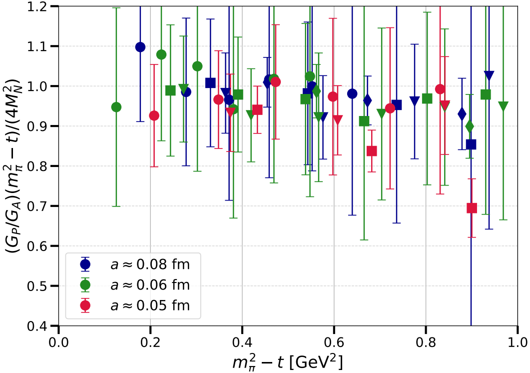
Motivation to determine G_P:

- lattice data on the same ensemble available
- only two additional LECs
- evidence if the data can be used[9]

} expect an improvement of the fit

$$\lim_{m \to 0} \frac{m_\pi^2 - t}{4\, M_N^2} \frac{G_P(t)}{G_A(t)} \to 1$$





[9] Left figure from Gunnar S. Bali et al. arXiv:1412.7336. Citation corrected (29.06.24, after talk)

Empirical knowledge of the induced pseudo-scalar form factor

- reminder:

$$\langle N(\bar{p})|\, A_i^{\mu}(0)\, |N(p)\rangle = \bar{u}_N(\bar{p}) \left(\gamma^{\mu}\, G_A(q^2) + \frac{q^{\mu}}{2\, M_N}\, G_P(q^2)\right) \gamma_5\, \frac{\tau_i}{2}\, u_N(p)$$

- empirical determination of induced pseudo-scalar coupling

$$g_P = \frac{m_{\mu}}{2\, M_N}\, G_P(-0.877\, m_{\mu}^2)$$

over ordinary muon capture process[10] $\mu^- + p \to n + \nu_{\mu}$

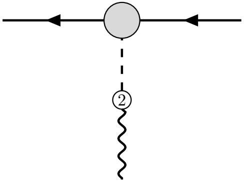
- empirical value

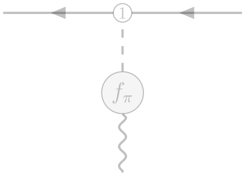
$$g_P = 10.6(2.7)$$

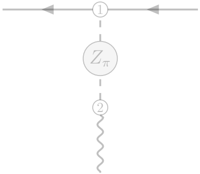
[10] Tim Gorringe and Harold W. Fearing. arXiv:0206039.

The induced pseudo-scalar form factor

- additional diagrams are needed







- Chiral Ward identity in the chiral limit:

$$\lim_{m\to 0}\left[G_A(t) + \frac{t}{4\,M_N^2}\,G_P(t)\right] = 0$$

Need to update expression of G_A in terms of an extended set of basis integrals[11]

[11] Tobias Isken et al. arXiv:2309.09695

The induced pseudo-scalar form factor

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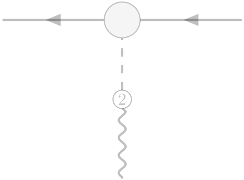

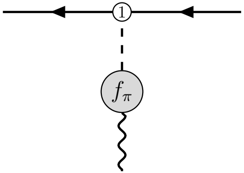

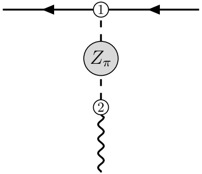

$$\frac{t - m_\pi^2}{4\,M_N^2}\,G_P(t) = -g_A\,\frac{f_\pi}{f}$$

$$f_\pi = f - \frac{1}{f}\,\bar{I}_\pi + \frac{m_\pi^2}{f}\,l_4$$

$$\frac{t - m_\pi^2}{4\,M_N^2}\,G_P(t) = -g_A\,Z_\pi$$

$$Z_\pi = 1 + \frac{2}{3\,f^2}\,\bar{I}_\pi - 2\,\frac{m_\pi^2}{f^2}\,l_4$$

Determination of the mesonic LEC l_4 and therefore the pion-decay constant f_π only through G_A and G_P possible

Fit details

- $G_P(t)$ data from CLS (Mainz) can be used, while RQCD can not be fitted
  $\rightarrow$ excited state contamination ?
- data points up to $m_\pi = 500$ MeV, $t \in [0, -0.36]$ GeV^2 and $m_\pi L \geq 4.0$
- using 124 lattice points
- 32 degrees of freedom (22 LEC and 10 lattice scales)
- simultaneous fit of the nucleon, isobar masses, G_A and G_P
  $\rightarrow$ to our knowledge: never been done successfully before

$$\chi^2_{\mathrm{min}}/N_{\mathrm{df}} = 99.12/(124-32) = 1.077$$

- **scale setting with the nucleon mass**

(1) Very good characterization of the $G_P(t)$ lattice data from Mainz
(2) Improved description of G_A compared to previous ChPT calculation due to updated basis functions

Fit results

- large subset of LEC is consistent with other works e.g.

LEC	f [MeV]	M [MeV]	$M+\Delta$
fit result	$83.43(^{+0.30}_{-0.81})$	$893.79(^{+0.55}_{-0.16})$	$1200.42(^{+0.72}_{-0.39})$

- axial radius, induced pseudo-scalar-coupling and the axial coupling

$$\langle r_A^2 \rangle = \frac{6\, G_A'(0)}{G_A(0)}\,, \qquad g_P = \frac{m_\mu}{2\, M_N}\, G_P(-0.877\, m_\mu^2)$$

are given by

Observable	$\langle r_A^2 \rangle$ [fm^2]	g_P	$G_A(0)$
Fit results	$0.20137(^{+0.0032}_{-0.0035})$	$8.2521(^{+0.039}_{-0.039})$	$1.2284(^{+0.0021}_{-0.0059})$
Empirical	0.46(24)	10.6(2.7)	1.2732(23)

- our axial coupling significantly below empirical value

$\rightarrow$ interpretation: due to neglect of strange quark effects

- pion-nucleon sigma term

$$\sigma_{\pi N} = m \frac{\partial}{m} M_N$$

- some values from the literature

	this work	Lutz et al. $SU(2)$	Lutz et al. $SU(3)$[12]	empirical[13]
$\sigma_{\pi N}$ [MeV]	$42.22(^{+0.02}_{-0.05})$	$49.31(^{+0.41}_{-0.12})$	$42.4(4)$	$58(6)$

$\rightarrow$ method of incorporation if isobar is relevant

- pion decay constant

	this work	PDG value
f_π [MeV]	$84.96(^{+0.29}_{-0.82})$	92.21 ± 0.14

- results largely stems from negative $l_4 = -0.0151(^{+0.0003}_{-0.0011})$

> Raises the question of the role of the strange quark in f_π but also in G_A and $\sigma_{\pi N}$.
> Reason of this discovery: scale setting with M_N and not f_π

[12] Matthias F. M. Lutz, Yonggoo Heo, and Renwick J. Hudspith. arXiv:2406.07442.

[13] e.g. Jacobo Ruiz de Elvira et al. arXiv:1706.01465

Summary

- calculation of G_A and G_P in a novel chiral framework for flavor $SU(2)$ ensembles
- simultaneous reproduction of **both** form factors on ensembles up to $m_\pi = 500$ MeV
- determination of f_π only through G_A and G_P
- results from observables indicate a crucial importance of strange quark effects

Outlook

- we plan a full calculation also of the explicit finite volume effects of the form factors
- our SU(2)-chiral approach should be used on ensembles with fixed physical strange quark mass

  $\rightarrow$ possibility to further scrutinize the importance of the strange quark

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Thank you for your attention

Backup slides

Framework

- as an effective field theory we use a flavor SU(2) chiral Lagrangian in the isospin limit with pion, nucleon and isobar as degrees of freedom
- the fields are defined as

$$\phi = \begin{pmatrix} \pi^0 & \sqrt{2}\,\pi^+ \\ \sqrt{2}\,\pi^- & -\pi^0 \end{pmatrix}, \qquad N = \begin{pmatrix} p \\ n \end{pmatrix},$$

$$\Delta_\mu^{111} = \Delta_\mu^{++}, \quad \Delta_\mu^{112} = \Delta_\mu^{+}/\sqrt{3}, \quad \Delta_\mu^{122} = \Delta_\mu^{0}/\sqrt{3}, \quad \Delta_\mu^{222} = \Delta_\mu^{-}$$

- example of building blocks

$$U_\mu = \tfrac{1}{2}\, u^\dagger \Big(\big(\partial_\mu e^{i\,\phi/f}\big) - \big\{ i\, a_\mu, e^{i\,\phi/f} \big\} \Big) u^\dagger ,$$

$$D_\mu N = \partial_\mu N + \Gamma_\mu N , \qquad u = e^{i\,\phi/f}$$

- construct terms of the Lagrangian up to chiral order Q^3
- reminder : $m_\pi \sim Q$

- example of the Lagrangian:

$$\mathcal{L} \supset \bar{N}\,(\mathrm{i}\,\not{D} - M)\,N + g_A\,\bar{N}\,\gamma^\mu\,\gamma_5 \mathrm{i}\,U_\mu N$$
$$+ f_S\,\big((\bar{\Delta}_\mu \cdot \mathrm{i}\,U^\mu)N + \text{h.c.}\big) + h_A\;\mathrm{tr}\left[(\bar{\Delta}_\mu \cdot \gamma_5\,\gamma_\nu\,\Delta^\mu)\,\mathrm{i}\,U^\mu\right] + \ldots$$

- interpretations:
  - M : chiral mass of the nucleon
  - g_A : axial coupling between two nucleons at tree level
  - f_S : axial coupling between an isobar and a nucleon at tree level
  - h_A : axial coupling between two isobars at tree level
- in sum we have 25 different **L**ow **E**nergies **C**onstants (LEC)
- subsets of LECs are related through Large-N_c, e.g.

$$h_A = 9\,g_A - 6\,f_S$$

strategy: fit simultaneously and consistently masses and form-factors to lattice-QCD data for varying pion masses to determine LECs

- reduction of the loop contributions into the Passarino-Veltman basis

$$I_{L\pi R} = \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{i}{((l-\bar{p})^2 - M_L^2)(l^2 - m_\pi^2)((l-p)^2 - M_R^2)} \,,$$

$$I_{\pi R} = \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{-i}{(l^2 - m_\pi^2)((l-p)^2 - M_R^2))} \,, \quad I_\pi = \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{i}{l^2 - m_\pi^2}$$

- to lift the singularities at $t = 0$ in the triangle contributions

$$\Delta I_{L\pi R}(t) = I_{L\pi R}(t) - I_{L\pi R}(t=0)$$

- to insure the right power counting subtractions are needed

$$\bar{I}_{L\pi R} = I_{L\pi R} - \gamma_{L\pi R} \sim Q^0 \,, \qquad \Delta\bar{I}_{L\pi R} = \Delta I_{L\pi R} - q^2\,\gamma'_{L\pi R} \sim Q^0 \,,$$
$$\bar{I}_{\pi R} = I_{\pi R} - \gamma_{\pi R} \sim Q^1$$

- chiral expansion only in the kinematic functions

- after the Passarino-Veltman reduction we find:

$$G_A(t) = g_A\, Z_N + 4\, g_\chi^+\, m_\pi^2 + g_R\, t + \frac{g_A}{f^2}\left\{J_\pi^A(t) + J_{\pi N}^A(t) + J_{N\pi}^A(t)\right\} + \frac{g_A^3}{4\, f^2}\, J_{N\pi N}^A(t) + \frac{5\, h_A\, f_S^2}{9\, f^2}\, J_{\Delta\pi\Delta}^A(t) + \frac{f_S}{3\, f^2}\left\{J_{\pi\Delta}^A(t) + J_{\Delta\pi}^A(t)\right\} + \frac{2\, g_A\, f_S^2}{3\, f^2}\left\{J_{N\pi\Delta}^A(t) + J_{\Delta\pi N}^A(t)\right\} + \mathcal{O}\left(Q^4\right),$$

$$\frac{t - m_\pi^2}{4\, M_N^2}\, G_P(t) = -g_A\left(Z_N + Z_\pi + f_\pi/f - 2\right) - m_\pi^2\,(4\, g_\chi^+ + g_\chi^-) - g_R\,(t - m_\pi^2) + \frac{g_A}{f^2}\left\{J_\pi^P(t) + J_{\pi N}^P(t) + J_{N\pi}^P(t)\right\} + \frac{g_A^3}{4\, f^2}\, J_{N\pi N}^P(t) + \frac{5\, h_A\, f_S^2}{9\, f^2}\, J_{\Delta\pi\Delta}^P(t) + \frac{f_S}{3\, f^2}\left\{J_{\pi\Delta}^P(t) + J_{\Delta\pi}^P(t)\right\} + \frac{2\, g_A\, f_S^2}{3\, f^2}\left\{J_{N\pi\Delta}^P(t) + J_{\Delta\pi N}^P(t)\right\} + \mathcal{O}\left(Q^4\right),$$

- Chiral Ward identity in the chiral limit:

$$\lim_{m\to 0}\left[G_A(t) + \frac{t}{4\, M_N^2}\, G_P(t)\right] = 0 \qquad \Rightarrow \qquad \lim_{m\to 0}\left[J_{\dots}^A + J_{\dots}^P\right] = 0$$

- triangle contribution in G_P pose a challenge
  - due to projection: $1/t^2$ singularity arise after reduction into the Passarino–Veltman basis
  - extended subtraction of $I_{L\pi R}$

$$\Delta\Delta I_{L\pi R}(t) = I_{L\pi R}(t) - I_{L\pi R}(t=0) - t\,\partial_t\, I_{L\pi R}(t=0)$$

technically possible → breaks Chiral Ward Identity

- **solution:** extend the Passarino–Veltman basis[14]

$$\bar{I}^{(m,n)}_{L\pi R}(t) = -\frac{\gamma^{(m,n)}_{L\pi R}}{16\,\pi^2\,M^2} + \int_0^1 \int_0^{1-u} \frac{dv\,du\,u^m\,v^n}{16\,\pi^2\,F_{L\pi R}(u,v)} \sim Q^0\,,$$

$$F_{L\pi R}(u,v) = m_\pi^2 + u\left(M_L^2 - m_\pi^2 - (1-u)\,M_N^2\right) + v\left(M_R^2 - m_\pi^2 - (1-v)\,M_N^2\right) + u\,v\left(2\,M_N^2 - t\right)$$

- due to recursion relations only $\bar{I}^{(0,n)}_{L\pi R}(t)$ and $\bar{I}^{(n,0)}_{L\pi R}(t)$ are needed
- no kinematical singularities and power counting violating terms
- Chiral Ward identity in the chiral limit is recovered

[14] Tobias Isken et al. arXiv:2309.09695.

- example

$$\bar{J}^P_{\Delta\pi\Delta}(t) = -\frac{2}{3}\left(2\,r\,t\,\alpha^P_{81} + \frac{5}{9}\,m_\pi^2\,\alpha^P_{82} - \frac{10}{3}\,\delta\,M_N\,\alpha^P_{83}\right)\bar{I}_{\pi\Delta}$$
$$-\frac{4}{3}\left[t\,\alpha^P_{91} + \frac{1}{3}\,m_\pi^2\,\alpha^P_{92}\right]M_N^2\left(\bar{I}^{(2,0)}_{\Delta\pi\Delta}(t) + \bar{I}^{(0,2)}_{\Delta\pi\Delta}(t)\right) + \mathcal{O}(Q^4)\,,$$
$$\bar{J}^A_{\Delta\pi\Delta}(t) = \frac{2}{3}\left(2\,r\,t\,\alpha^A_{81} + \frac{5}{9}\,m_\pi^2\,\alpha^A_{82} - \frac{10}{3}\,\delta\,M_N\,\alpha^A_{83}\right)\bar{I}_{\pi\Delta}$$
$$+\frac{4}{3}\,t\,\alpha^A_{91}\,M_N^2\left(\bar{I}^{(2,0)}_{\Delta\pi\Delta}(t) + \bar{I}^{(0,2)}_{\Delta\pi\Delta}(t)\right) + \mathcal{O}(Q^4)\,,$$

- $\alpha^{A/P}_{ab}$ are rational functions of $r = \Delta/M$ with $\alpha^{A/P}_{ab} \to 1$ if $r \to 0$
- explicit check that $\alpha^P_{n1} = \alpha^A_{n1}$
- typical values for $r = 0.343$ e.g.

$$\alpha^{A/P}_{81} = 1.554\,, \quad \alpha^P_{81}/\alpha^A_{81} = 2.783/3.481\,, \quad \alpha^{A/P}_{83} = 1.399$$

- partially large deviation from 1
- in small scale expansion: $\Delta \sim Q \to$ only leading term $\alpha^{A/P}_{ab} = 1$

Fit results

- large subset of LEC is consistent with other works e.g.

LEC	f [MeV]	M [MeV]	$M+\Delta$	g_A	l_3
fit result	$83.43(^{+0.30}_{-0.81})$	$893.79(^{+0.55}_{-0.16})$	$1200.42(^{+0.72}_{-0.39})$	$1.1449(^{+0.0019}_{-0.0049})$	$0.0193(^{+0.0003}_{-0.0003})$

- some LEC differ from other SU(2) results with no isobars as degree of freedom

LEC	g_S [GeV^{-1}]	g_V [GeV^{-2}]
fit result	$0.9163(^{+0.0060}_{-0.0072})$	$-0.8096(^{+0.0792}_{-0.1784})$

- two flipped signs to typically values

LEC	l_4	h_A^*
fit result	$-0.0151(^{+0.0003}_{-0.0011})$	$0.7893(^{+0.1123}_{-0.0229})$

- h_A^* fixed by large -N_c $h_A = 9\,g_A - 6\,f_S$ and sensitive how to incorporate the isobar
- negative sign was reported from a loop study of the pion-nucleon scattering[15]

[15] De-Liang Yao et al. In: *Journal of High Energy Physics* (May 2016). DOI: 10.1007/jhep05(2016)038.

No Contribution

Both diagrams do not contribute due the on-shell masses