Electromagnetic Form Factors of the Nucleon at Large Momentum

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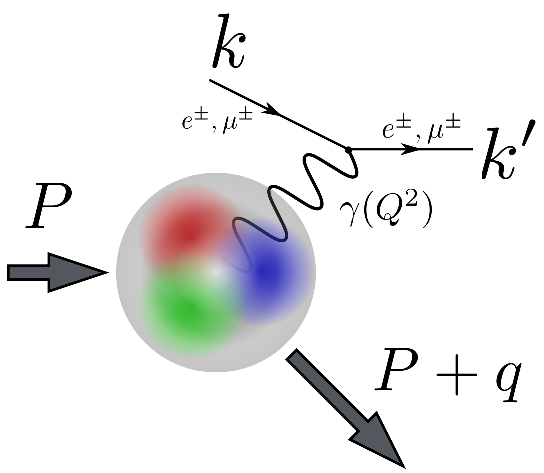
Outline

- Nucleon vector form factors at large momentum transfer Q^2
- Challenges for large-momentum hadron structure on lattice
- Connected contributions: examining excited states, discretization,
- Disconnected contributions to large-Q^2 form factors
- Summary

Nucleon Elastic E&M Form Factors



$$Q^2 = -(k - k')^2 > 0$$

Elastic e^-p amplitude

$$\langle P+q | \bar{q}\gamma^\mu q | P \rangle = \bar{U}_{P+q}\Big[\underbrace{F_1(Q^2)}_{\text{(Dirac)}} \gamma^\mu + \underbrace{F_2(Q^2)}_{\text{(Pauli)}} \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} \Big] U_P$$

Sachs Electric $G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$

Magnetic $G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$

Elastic e^-p cross-section

- $G_{E,M}$ from ϵ-dep. at fixed $\tau(Q^2)$ ("***Rosenbluth separation***")
- dominated by G_M at large Q^2
- 2γ corrections at $Q^2 \gtrsim 1$ GeV^2

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\text{Mott}}}{1+\tau}\Big[G_E^2 + \frac{\tau}{\epsilon} G_M^2 \Big]$$

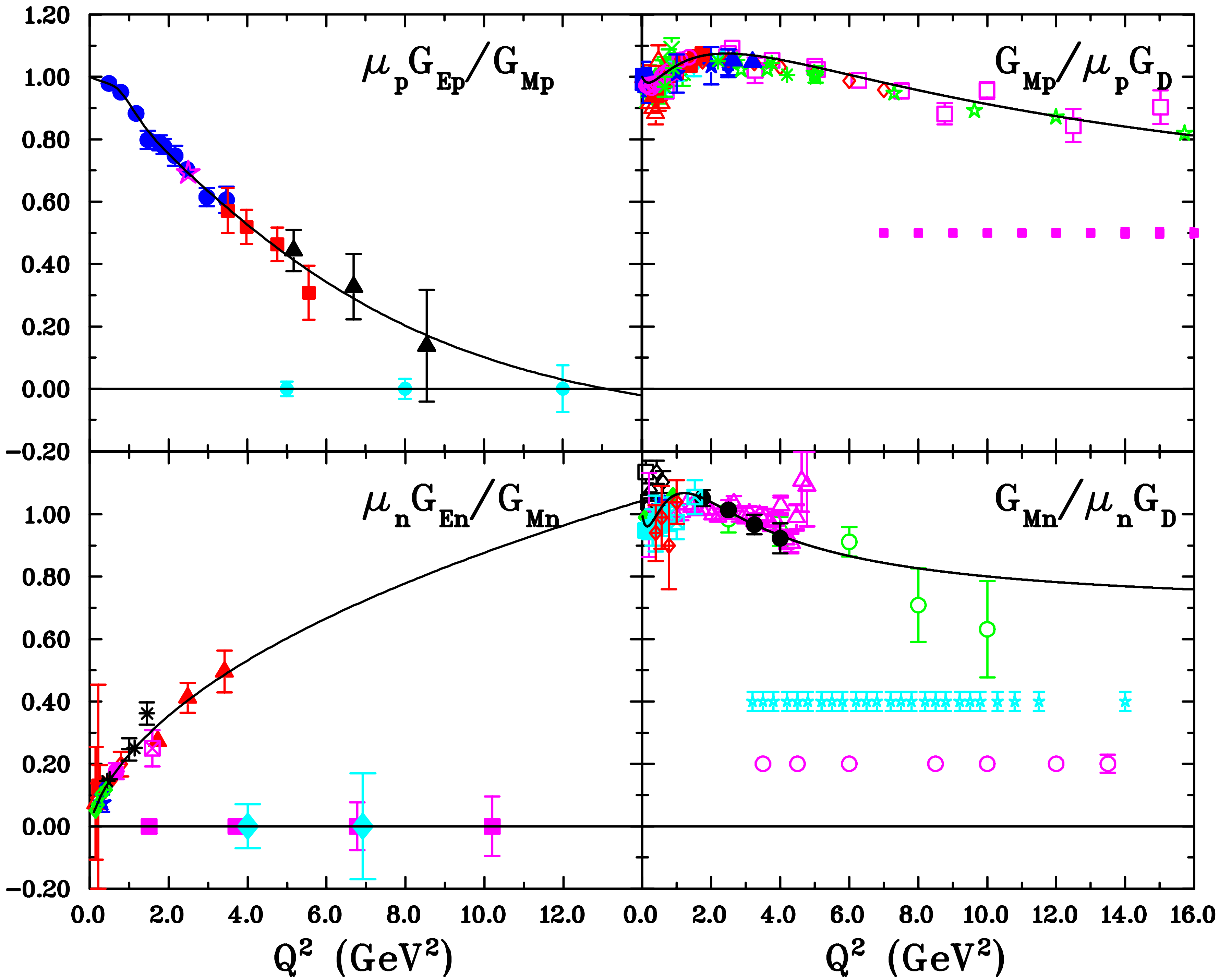
$$\tau = \frac{Q^2}{4M_N^2} \qquad \epsilon = \Big[1 + 2(1+\tau)\tan^2\frac{\theta}{2} \Big]^{-1}$$

Polarization transfer: polarized e^-beam
+ detect polarization of recoil nucleon
(alt.: transverse asymmetry on pol. target)

- *G_E/G_M ratio* (only small radiative corrections)

$$P_t / P_l \propto G_E / G_M$$

Recent/Ongoing Experiments

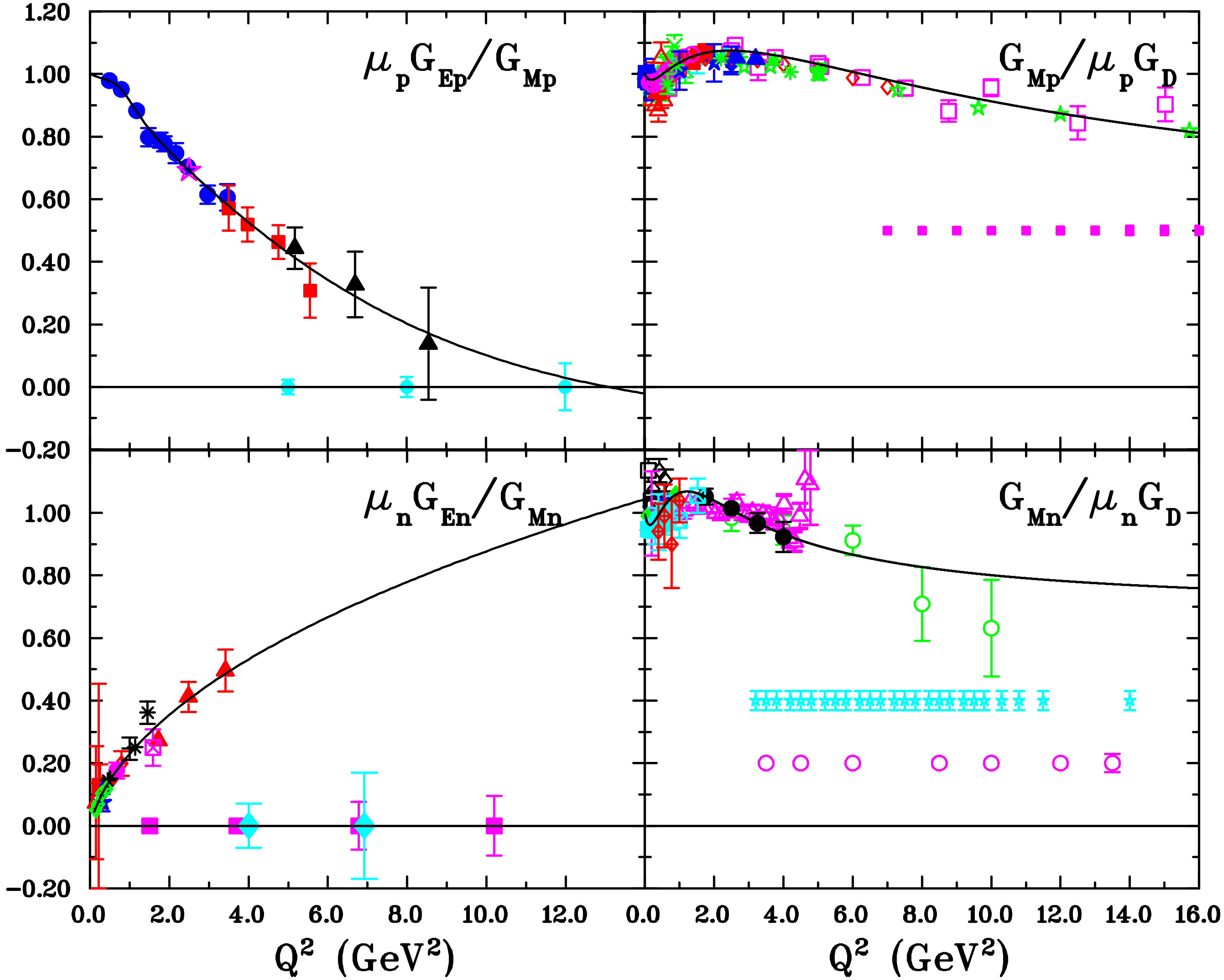


Projected new precision on proton & neutron form factors
[V. Punjabi et al, EPJ A51: 79 (2015); arXiv: 1503.01452]

Experiments at JLab@12GeV

- Hall A (HRS, SBS):
  - G_{Mp} @ $Q^2 \lesssim 17.5$ GeV^2
  - G_{Ep}/G_{Mp} @ $Q^2 \lesssim 15$ GeV^2 ;
  - G_{Mn} @ $Q^2 \lesssim 18$ GeV^2
  - G_{En}/G_{Mn} @ $Q^2 \lesssim 10.2$ GeV^2 ;
- Hall B (CLAS12):
  - G_{Mn} @ $Q^2 \lesssim 14$ GeV^2
- Hall C :
  - G_{En}/G_{Mn} @ $Q^2 \lesssim 6.9$ GeV^2

Recent/Ongoing Experiments



Projected new precision on proton & neutron form factors
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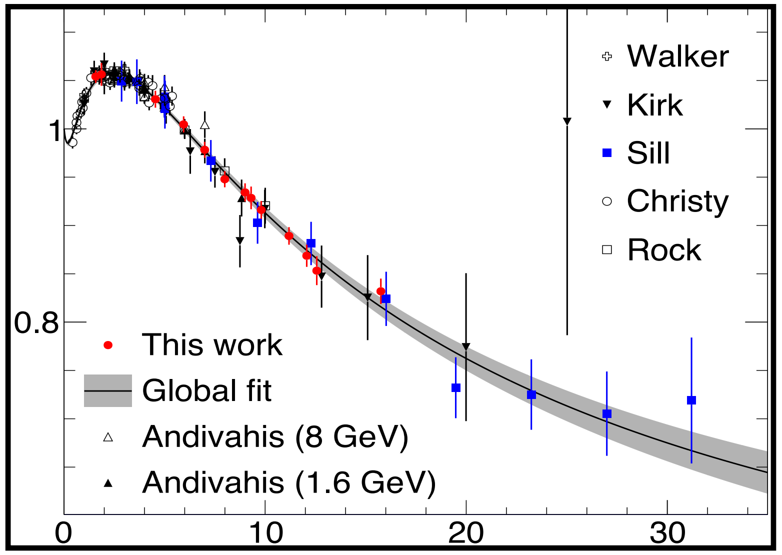


New G_{Mp} data from Hall A
[Christy et al, PRL'22]

Challenges at Large Q^2

- Discretization effects:
  O(a) Correction to current operator

$$(V_\mu)_I = [\bar{q}\gamma_\mu q] + c_V\, a\, \underbrace{\partial_\nu [\bar{q} i \sigma_{\mu\nu} q]}_{\propto Q}$$

- Stochastic noise grows faster with T [Lepage'89]:

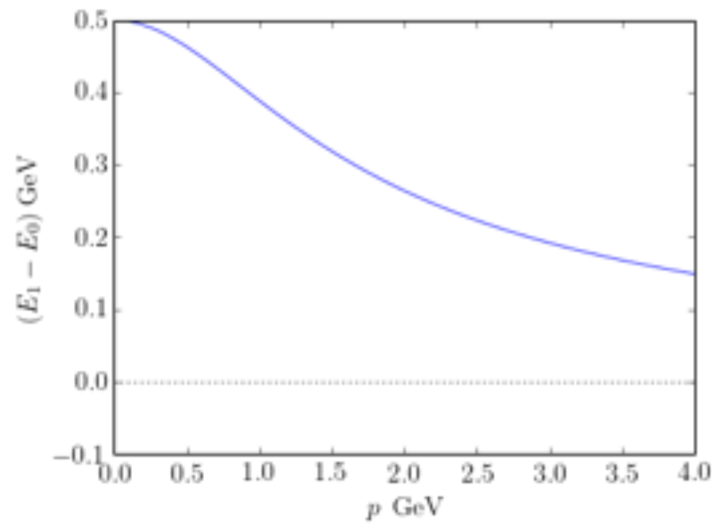
$$\text{Signal} \quad \langle N(T)\bar{N}(0)\rangle \sim e^{-E_N T}$$
$$\text{Noise} \quad \langle |N(T)\bar{N}(0)|^2\rangle - |\langle N(T)\bar{N}(0)\rangle|^2 \sim e^{-3m_\pi T}$$
$$\text{Signal/Noise} \sim e^{-(E_N - \frac{3}{2}m_\pi)T}$$

SNR reduction
*at 1 fm/c ~ **O(10^{-4})***
(phys.quarks, $Q^2 \approx 12$ GeV^2)

- Excited states: boosting "shrinks" the energy gap

$$E_1 - E_0 = \sqrt{M_1^2 + \vec{p}^2} - \sqrt{M_2^2 + \vec{p}^2} < M_1 - M_0$$

  - N(~1500): pN→1.5 GeV ⇒ ΔE = 500 → 300 MeV



- Quark-disconnected contributions:
  negligible (≲1%) at $Q^2 \leq 1$ GeV^2, unknown at large Q^2

- Large p_N: no reliable EFT/ChPT for m_π-, lattice size-extrapolation

Large statistics required to suppress MC noise in lattice correlators

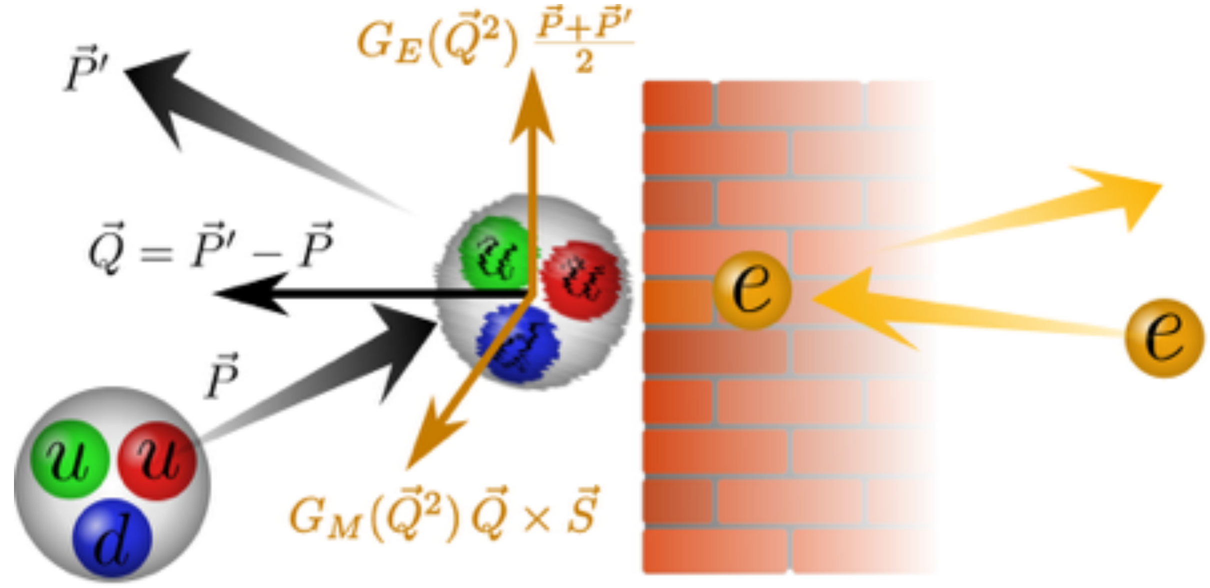
Accessing Large Q^2 : Breit Frame on a Lattice

Breit("Brick-Wall") frame: Minimize energies of in-/out-nucleon states for required Q^2

$$Q^2 = (\vec{p}_{in} - \vec{p}_{out})^2 - (E_{in} - E_{out})^2$$

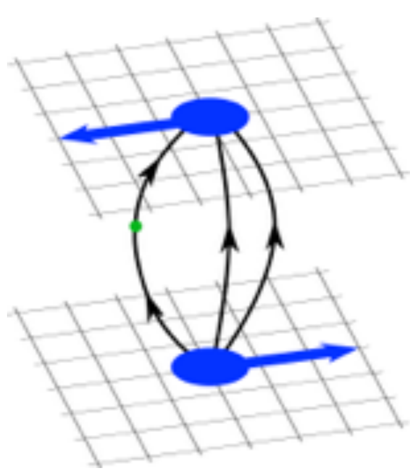
Back-to-back in/out momenta

$$Q^2 = 4\vec{p}^2$$

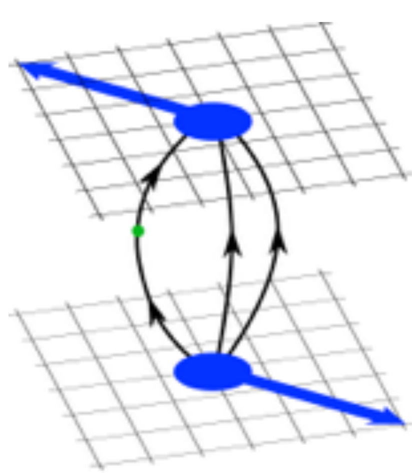


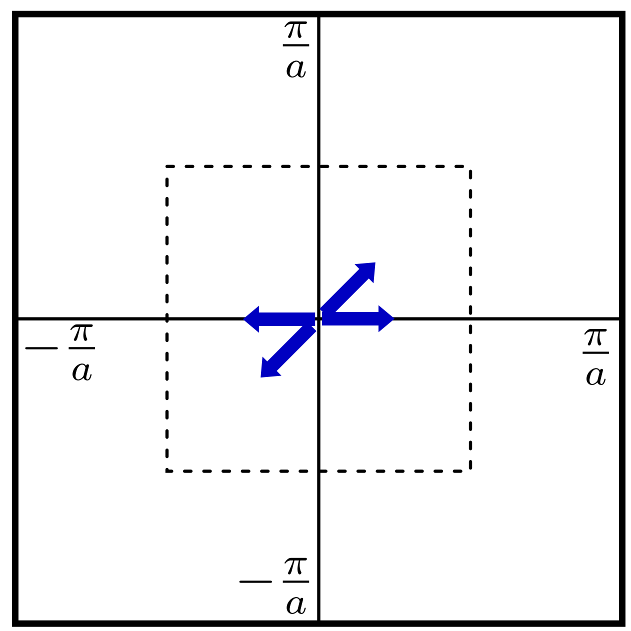
For $(Q^2)_{max}$ = *10 GeV²* $(E_N \approx 1.9\ GeV)$

$$|\vec{p}| = \frac{1}{2}\sqrt{Q^2_{\rm max}} \approx 1.6\ \mathrm{GeV}$$



examine O(a²) effects by changing momentum orientation?

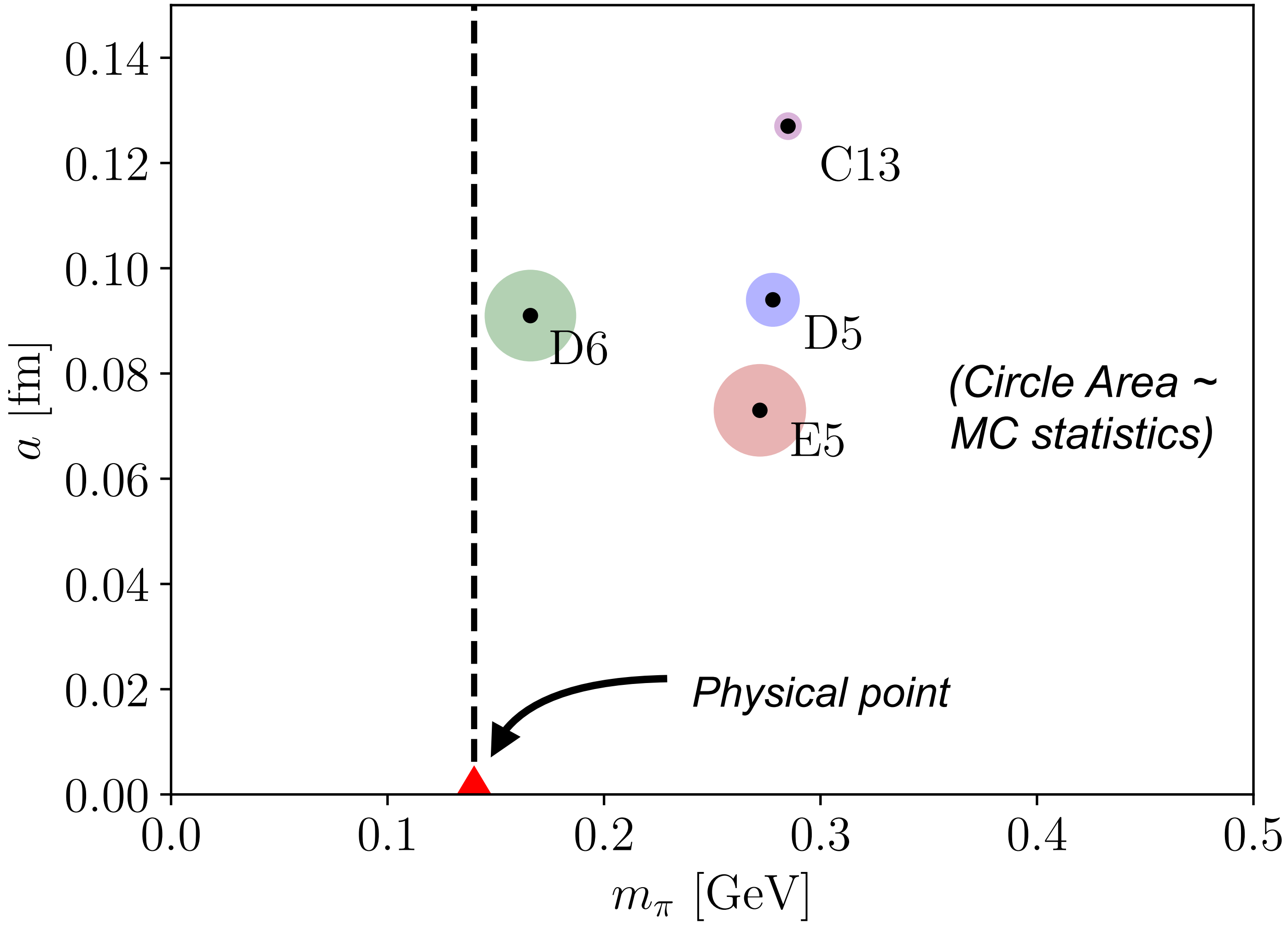




In/out momenta within Brillouin zone on a=0.09 fm lattice

Present QCD Calculation Parameters

- $N_F = 2{+}1$ clover-improved Wilson fermion ensembles (JLab / W&M / LANL / MIT)
- Lattice spacing $a \approx 0.073 \div 0.091$ fm
- Light quark masses approaching physical : $m\pi = 170 \div 280$ MeV
- Large physical volume $L \gtrsim 3.7\ (m\pi)^{-1}$
- Source-sink separation t_{sep}= 0.51 ÷ 1.09 fm
- Momentum smearing, AMA sampling

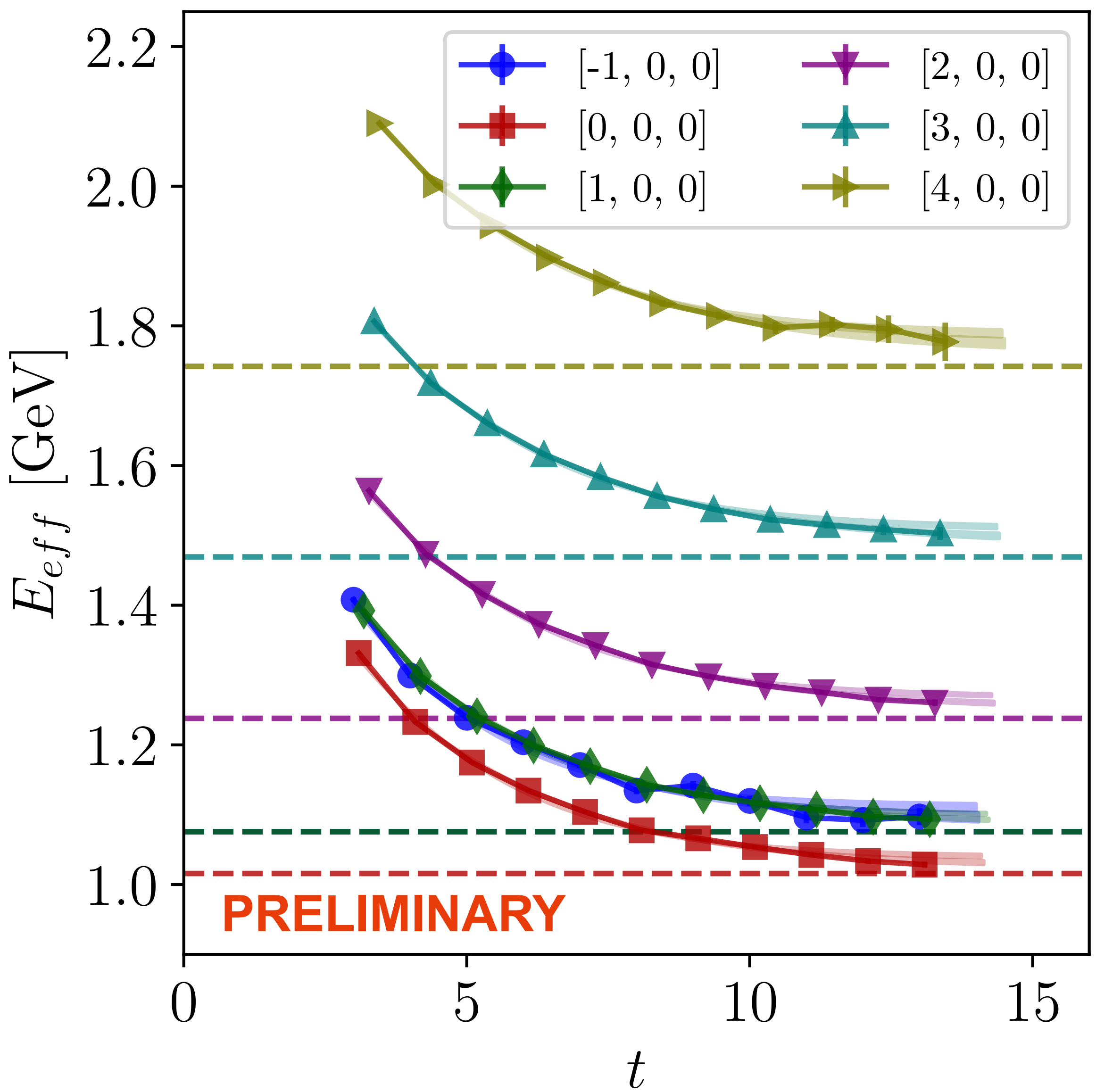


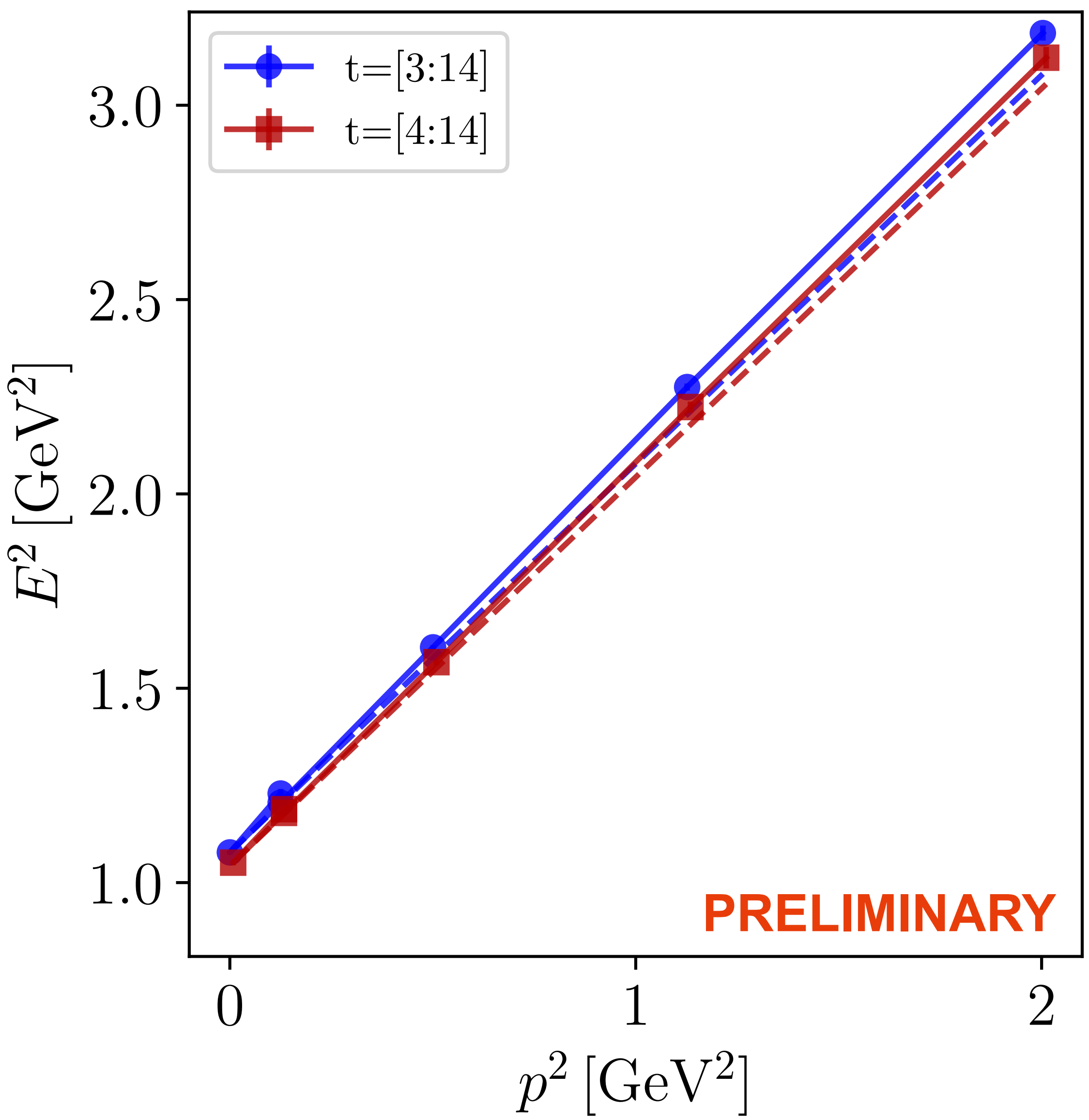
2022/24:

- MC Statistics ~250k on D6 (48^3 x 96), E5 (48^3 x 128)
- Disconnected contractions on D6 (1000+ configs)

Lattice Nucleon Energy & Dispersion Relation (E5)

- E5 : mπ = 272 MeV , spacing a = 0.073 fm , 266k MC samples





- Effective energy and 2-state fits

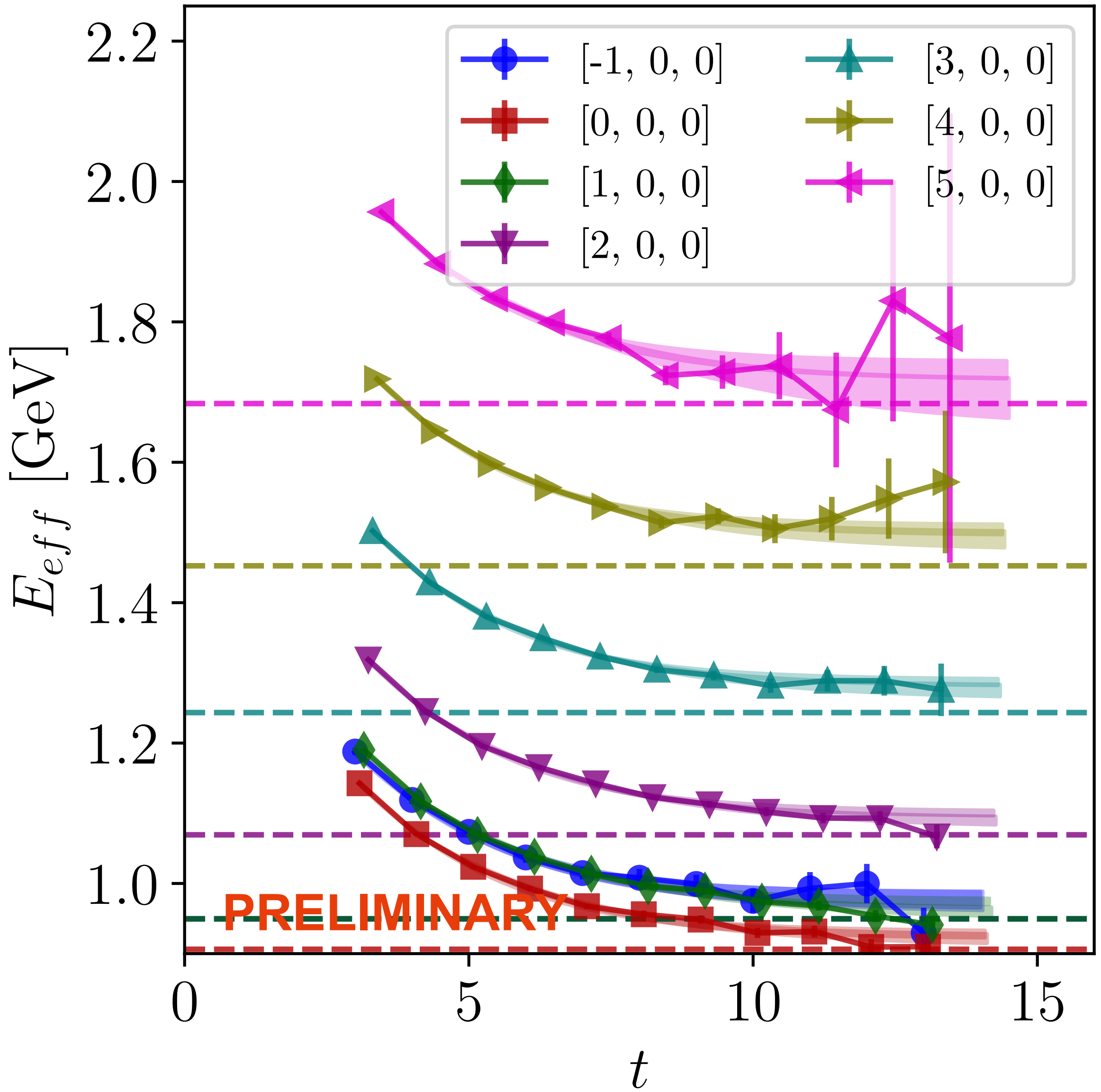
$$E_{eff} = \frac{1}{a} \log \frac{C_{N\bar{N}}(t)}{C_{N\bar{N}}(t+a)}$$

- Dispersion relation

  Dashed lines: cont. $E^2(p) = E^2(0) + p^2$

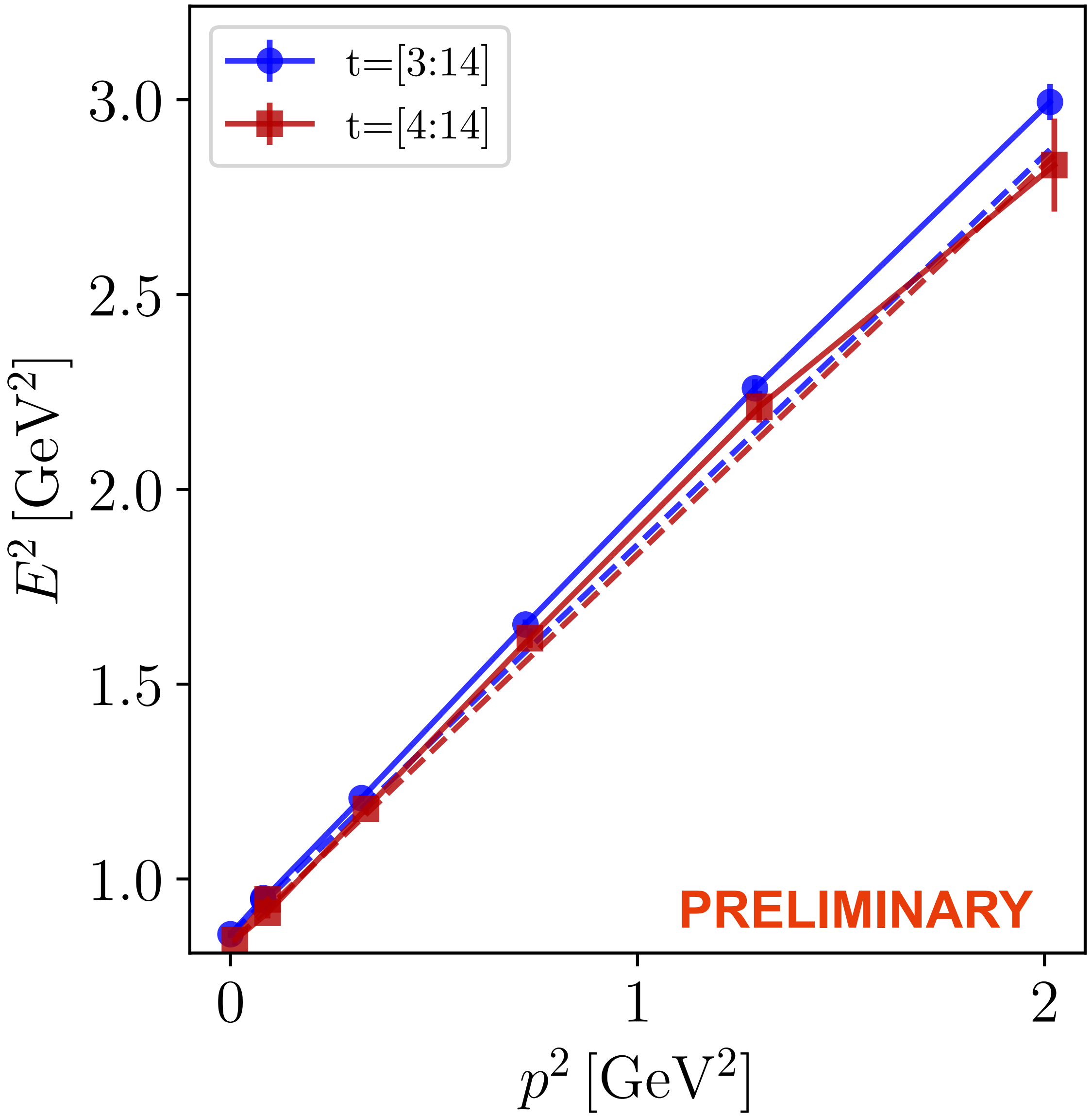
Lattice Nucleon Energy & Dispersion Relation (D6)

- D6 : $m\pi$ = 166 MeV , spacing a = 0.091 fm , 261k MC samples



- Effective energy and 2-state fits

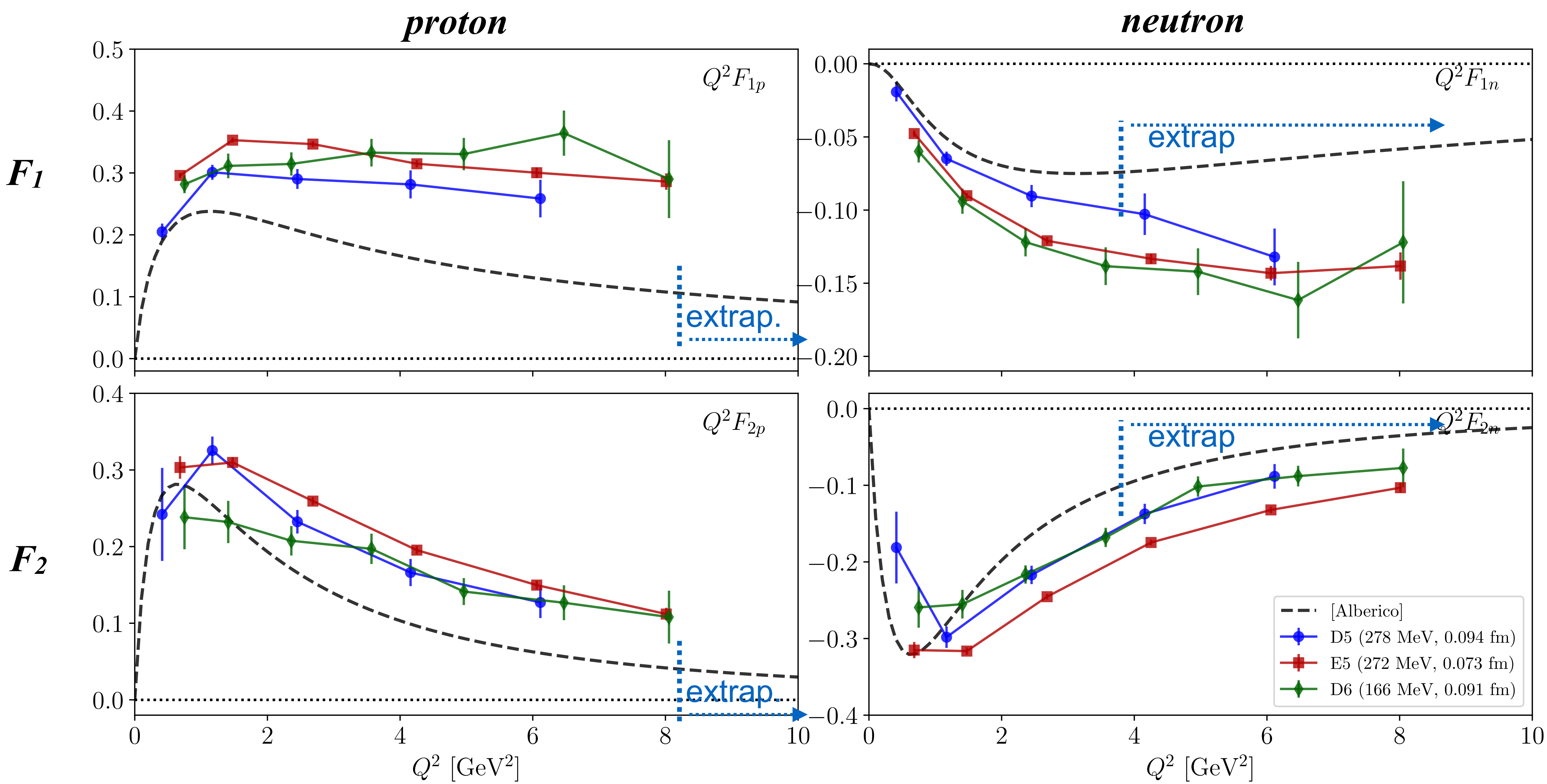
$$E_{eff} = \frac{1}{a}\log\frac{C_{N\bar{N}}(t)}{C_{N\bar{N}}(t+a)}$$



- Dispersion relation
  Dashed lines: cont. $E^2(p) = E^2(0) + p^2$

Nucleon Form Factors: Ensemble Comparison

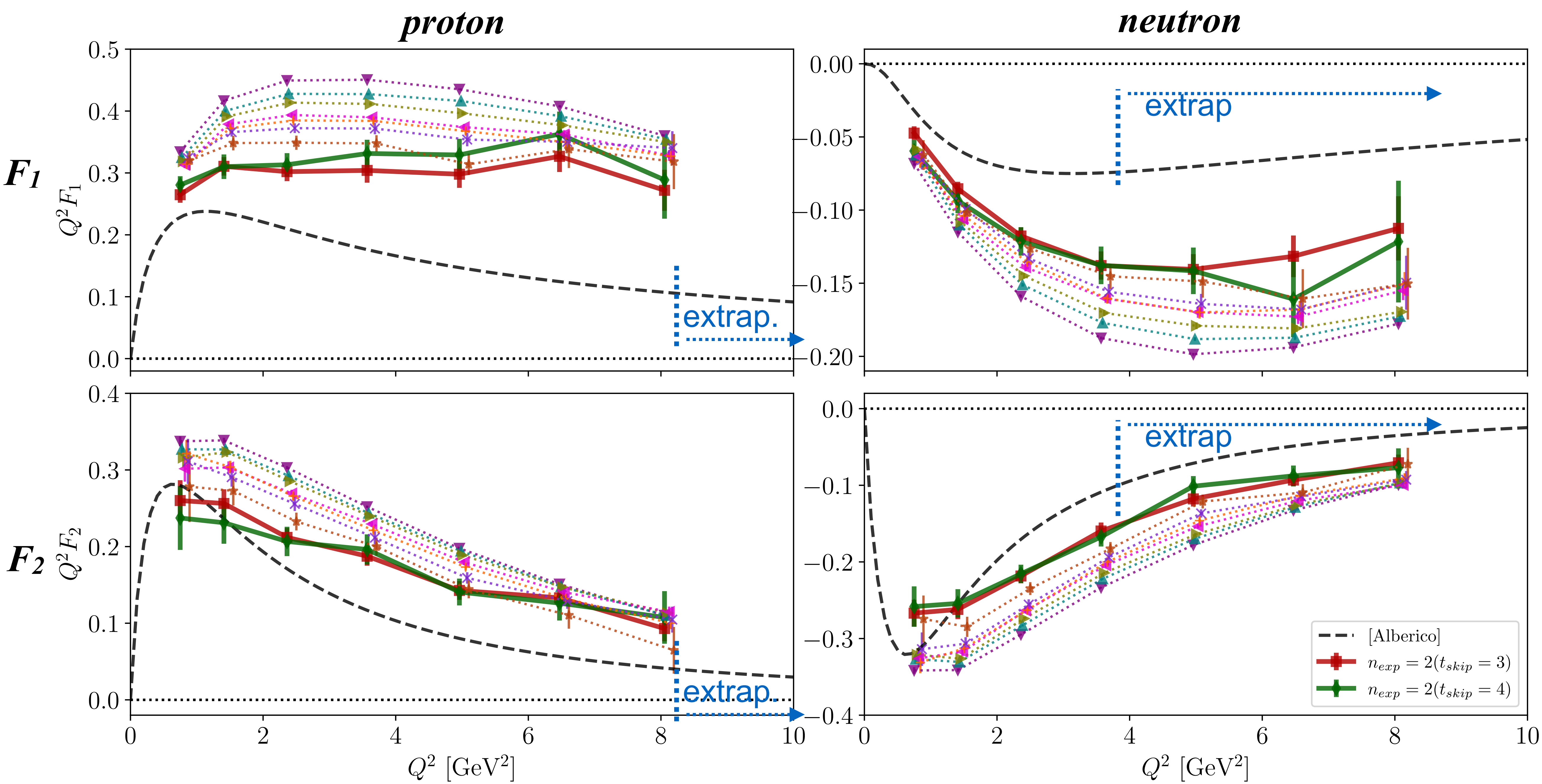
- Comparison of 3 ensembles (**D5** : 86k, **E5** : 266k, **D6** : 261k samples)
- "Ground" state from 2-state fits, **t_{sep}= 0.7÷1.1 fm**
- *Phenomenology (dashed)* : [Alberico et al, PRC79:065204 (2008)]



- *No disconnected diagrams*

Nucleon Form Factors: 2-state fit vs. fixed T_{sep}(D6)

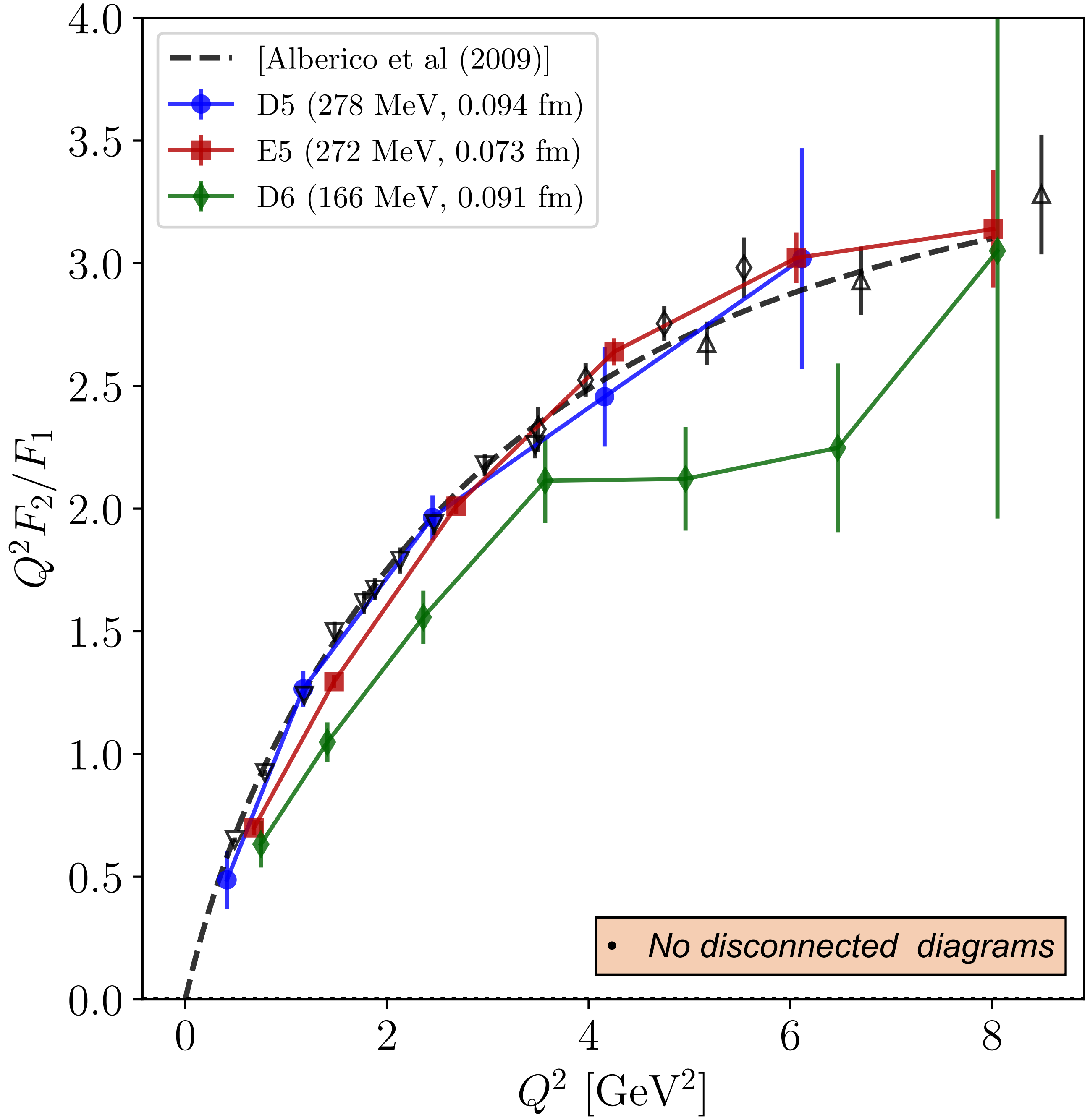
- D6 ensemble (260k samples) : Comparison of plateaus vs fits
- "Ground" state from 2-state fits, **t_{sep}= 0.7÷1.1 fm** and **t_{sep}=0.5÷1.1 fm**
- *Phenomenology (dashed)* : [Alberico et al, PRC79:065204 (2008)]



- *No disconnected diagrams*

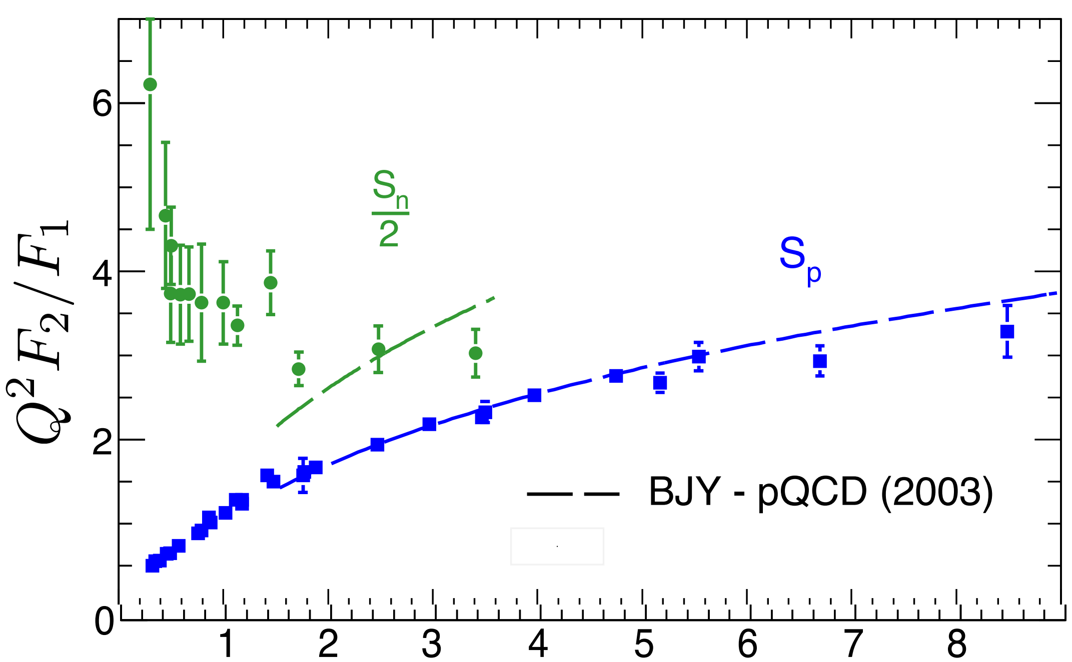
Proton F_2/F_1 Ratio

- Comparison of 3 ensembles (**D5** : 86k, **E5** : 266k, **D6** : 261k samples) ; fit **t_{sep}= 0.7÷1.1 fm**
- *Phenomenology (dashed)* : [Alberico et al, PRC79:065204 (2008)]
- Proton experimental data $Q^2 \lesssim 8.5$ GeV^2 (black points)



- Prediction from pQCD + quark OAM [Balitsky, Ji, Yuan (2003)]

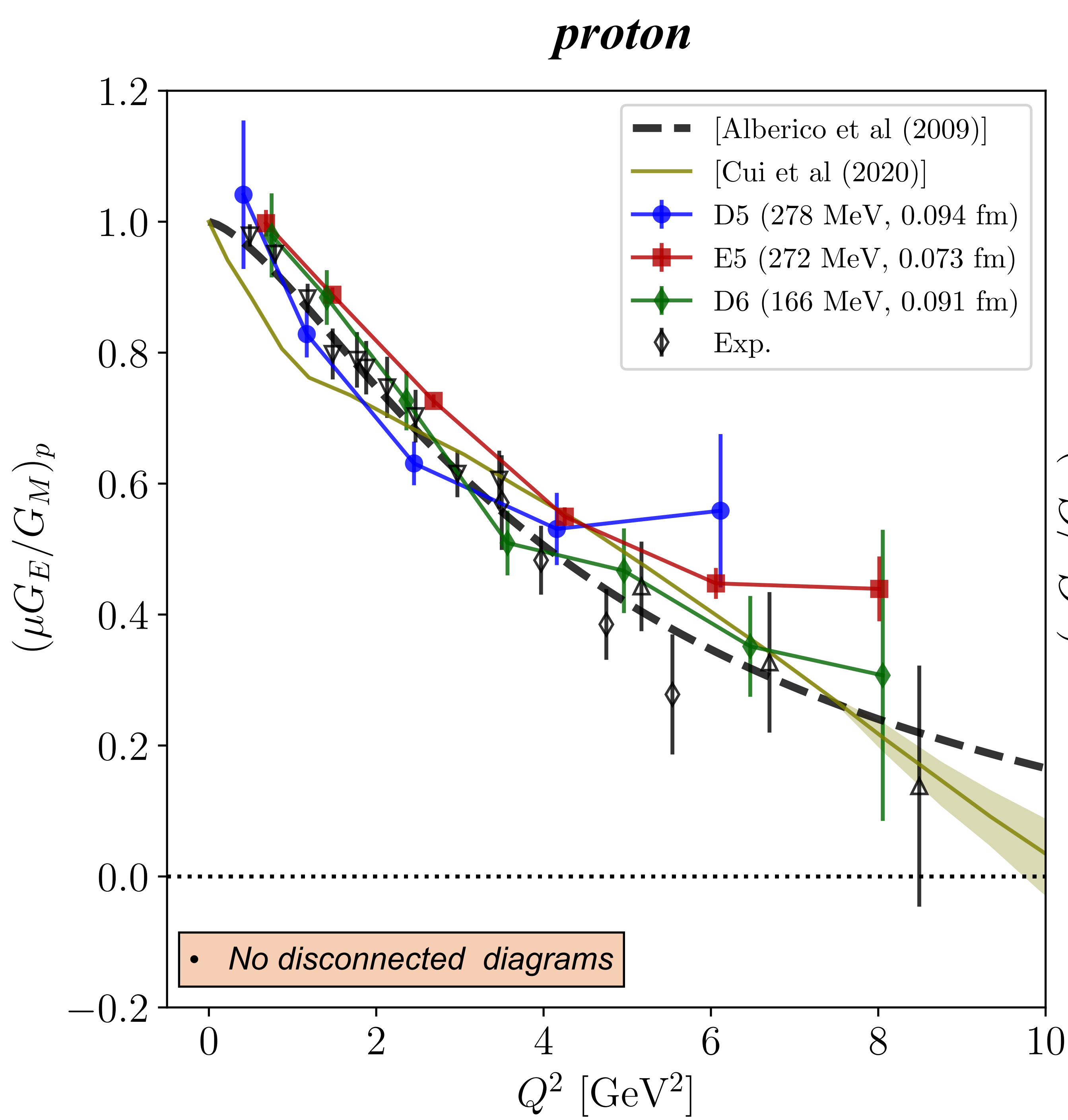
$$Q^2 F_{2p}/F_{1p} \overset{?}{\propto} \log^2(Q^2/\Lambda^2)$$

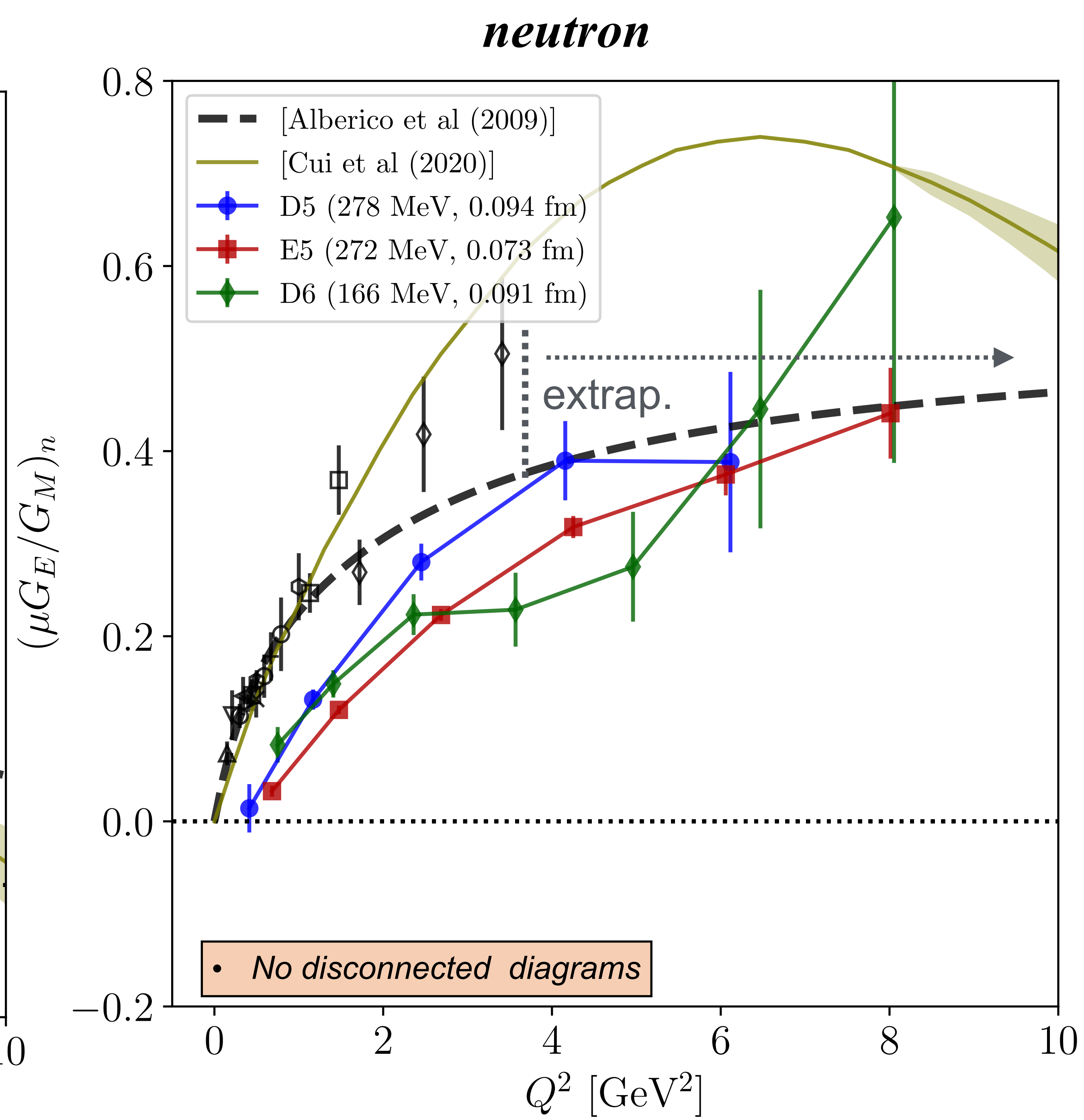


[G.D.Cates, et al, PRL106:252003 (2011)]

Proton & Neutron G_E/G_M Ratio (min. t_{sep}= 0.5 fm)

- Comparison of 3 ensembles (**D5** : 86k, **E5** : 266k, **D6** : 261k samples) ; fit **t_{sep}= 0.5÷1.1 fm**
- Phenomenology : [Alberico et al, PRC79:065204 (2008)] ;
- Experimental data (black points) $Q^2 \lesssim 8.5$ GeV^2 (proton) and $Q^2 \lesssim 3.4$ GeV^2 (neutron)

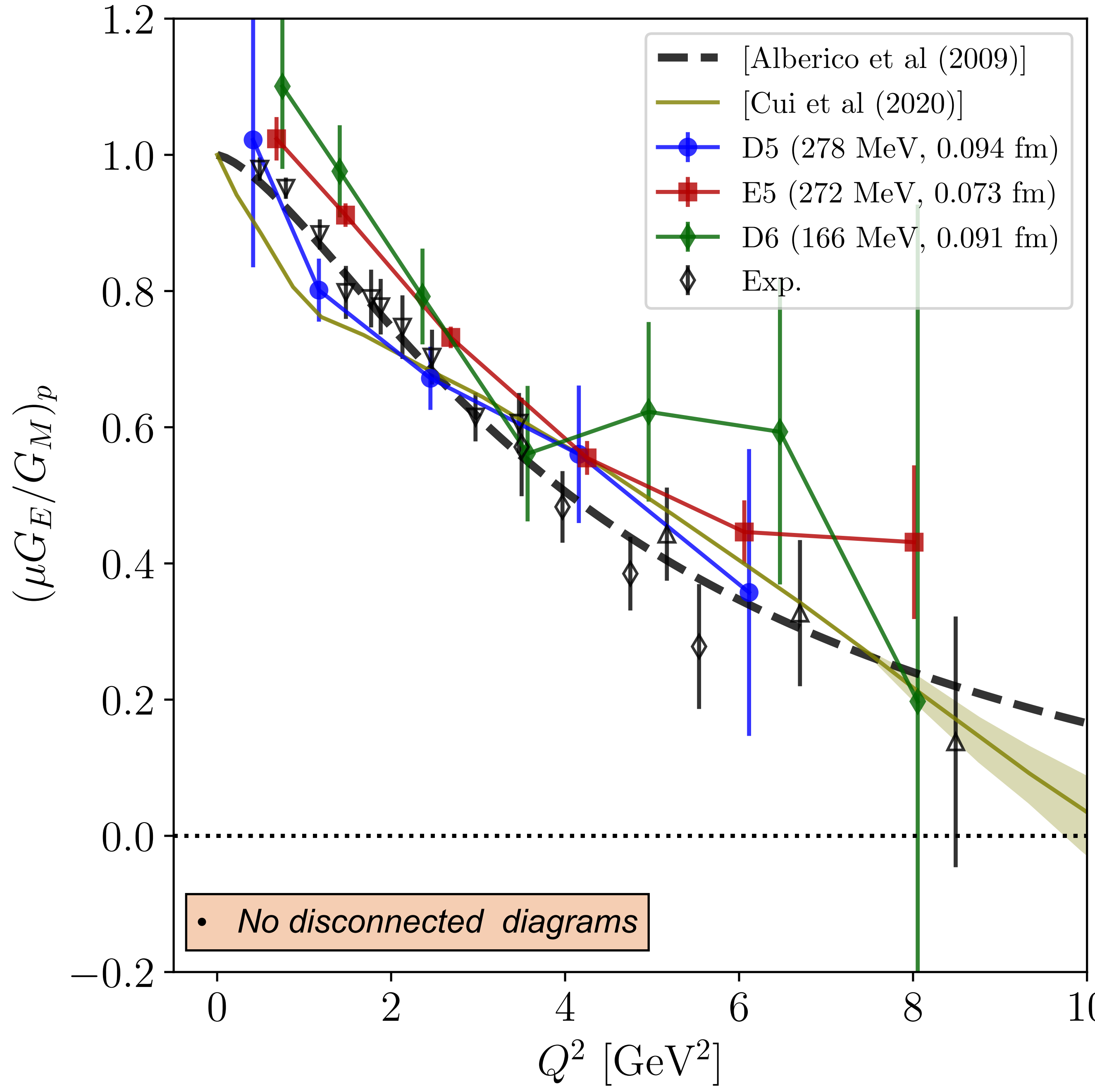




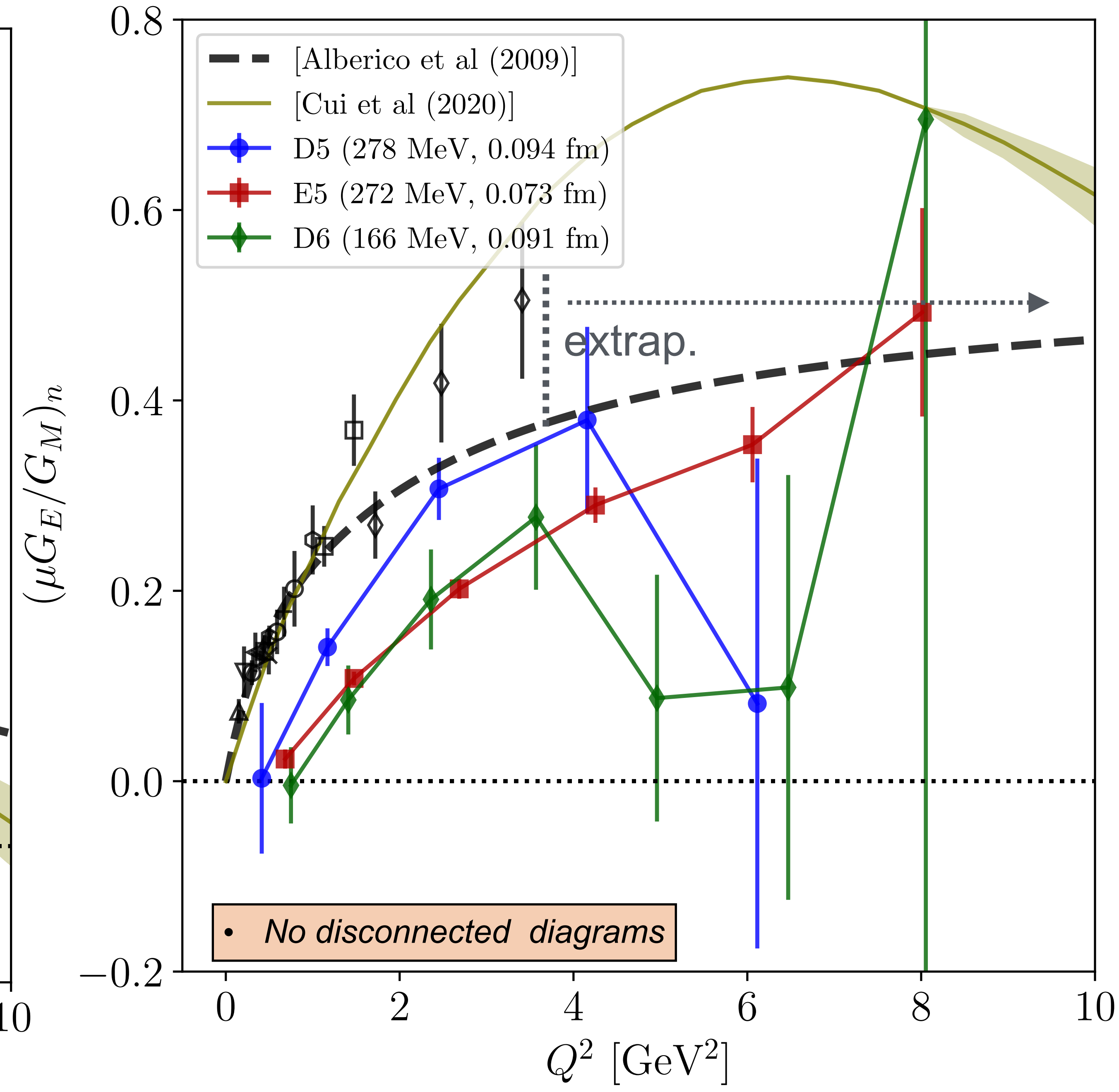
Proton & Neutron G_E/G_M Ratio (min. t_{sep}= 0.7 fm)

- Comparison of 3 ensembles (**D5** : 86k, **E5** : 266k, **D6** : 261k samples) ; fit **t_{sep}= 0.7÷1.1 fm**
- Phenomenology : [Alberico et al, PRC79:065204 (2008)] ;
- Experimental data (black points) $Q^2 \lesssim 8.5$ GeV^2 (proton) and $Q^2 \lesssim 3.4$ GeV^2 (neutron)

proton

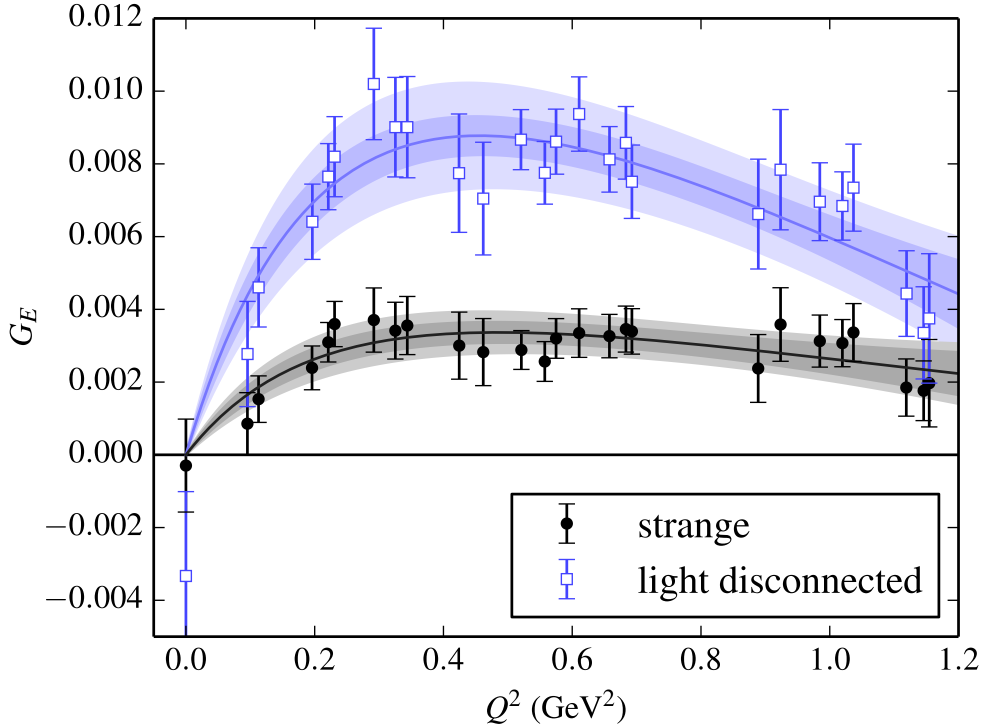


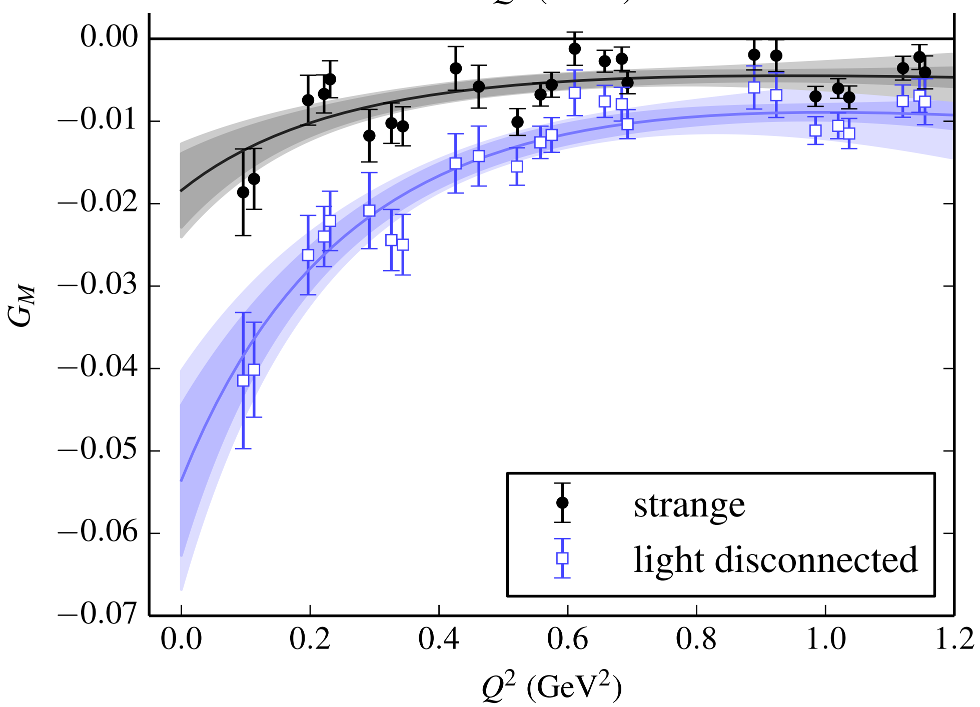
neutron



Disconnected Contributions to Vector FFs?

[J. Green, S. Meinel, S.S. et al; PRD92:031501 (2015)]

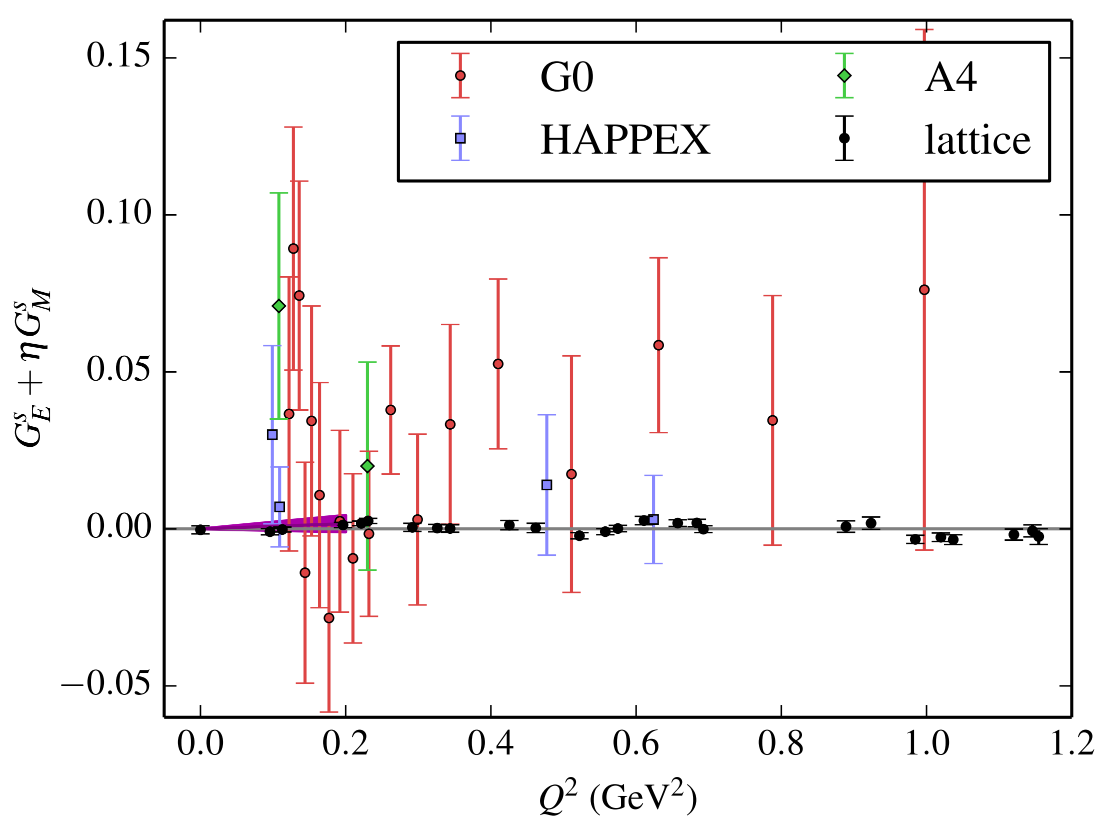




N_f=2+1 dynamical fermions, $m_\pi \approx$ 320 MeV (C13 ensemble)

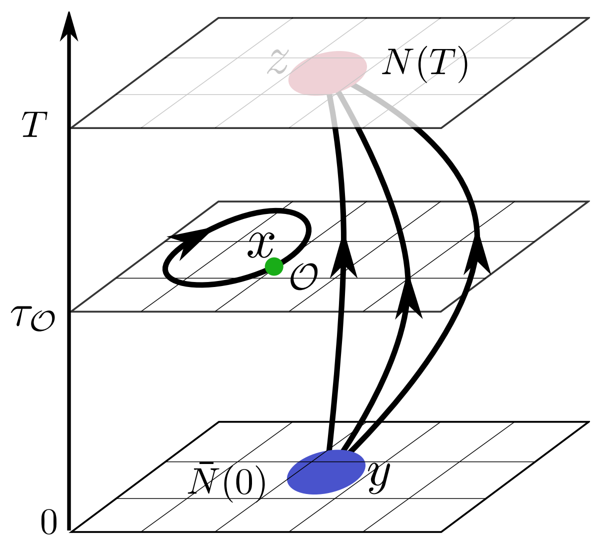
$$|(G_E^{u/d})_{\rm disc}| \lesssim 0.010 \text{ of } |(G_E^{u-d})_{\rm conn}|$$
$$|(G_E^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_E^{u-d})_{\rm conn}|$$

$$|(G_M^{u/d})_{\rm disc}| \lesssim 0.015 \text{ of } |(G_M^{u-d})_{\rm conn}|$$
$$|(G_M^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_M^{u-d})_{\rm conn}|$$



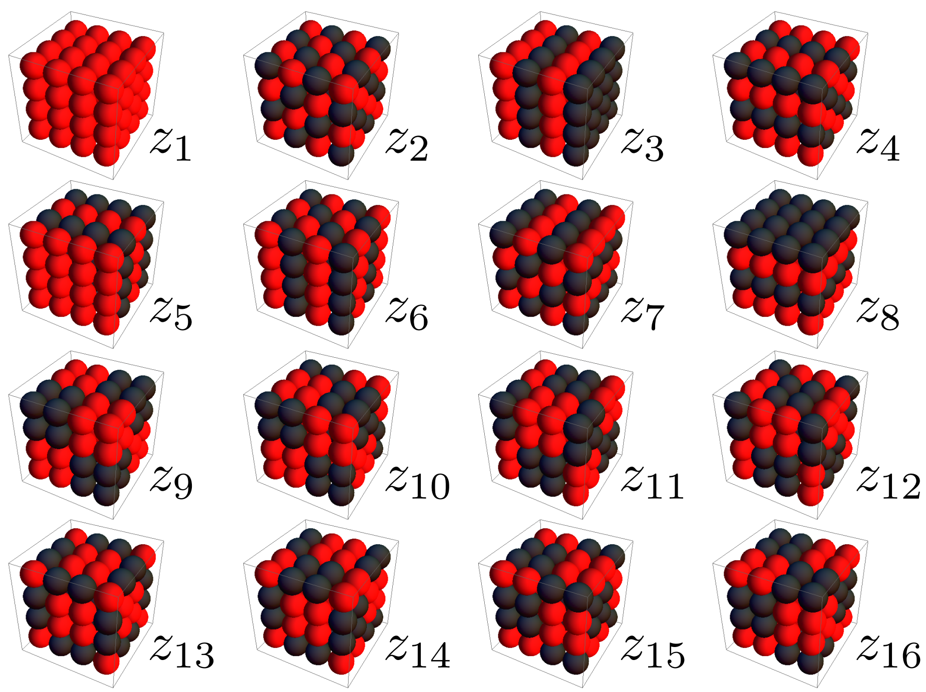
Disconnected Quark Loops

- Stochastic evaluation $\Longrightarrow$ large noise esp. at large Q^2
  Significant source of uncertainty in GE/GM



- Typically small relative to connected;
  $(U+D)^{disc}$ partially cancel with S in proton/neutron
  Important for individual-flavor FFs

- Efficient evaluation of quark loops:
  suppress noise from $\sum_{x \neq y} |\not{D}^{-1}(x,y)|^2$
  - Hierarchical probing with Hadamard vectors
    [K.Orginos, A.Stathopoulos, '13]
    eliminate noise from short-range (x,y)
  - combine with low-mode deflation
    [A.Gambhir, PhD thesis]
    eliminate noise from long-distance (x,y)

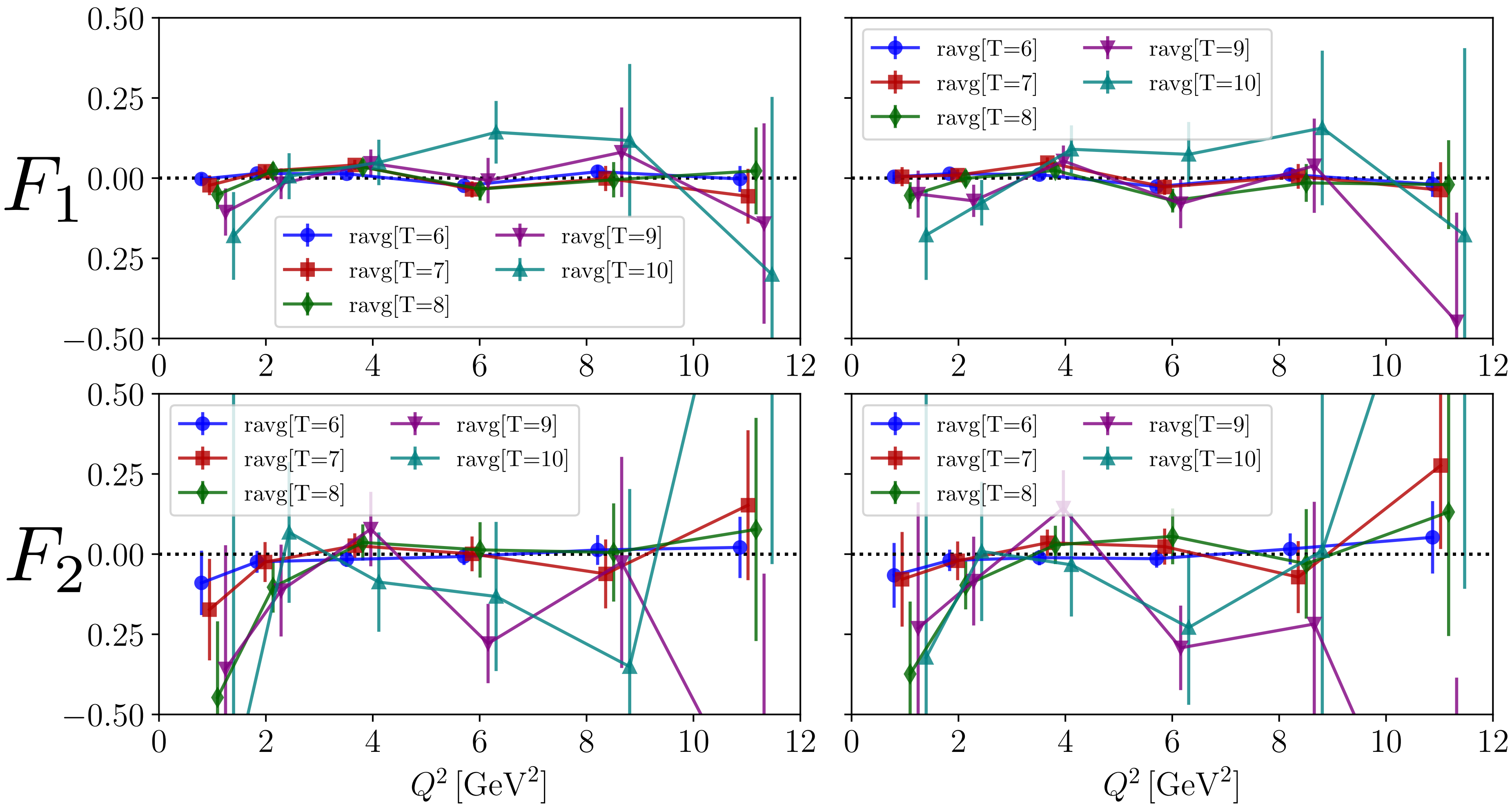


[figure: S. Meinel et al PRD92:031501 (2015)]

Disconnected Light & Strange vs. Connected (D5)

- *relative correction* $F_{1,2}^{disc}$ / $F_{1,2}^{conn}$ from plateau averages **t_{sep}= 0.5÷0.9 fm,** $Q^2 \lesssim 11$ GeV^2
- **D5 ensemble (m_π=280 MeV, a=0.094 fm)**, 1346 configs ⊗ 64 samples of ⟨N$\bar{N}$⟩

disconnected L=U or D ***disconnected S***

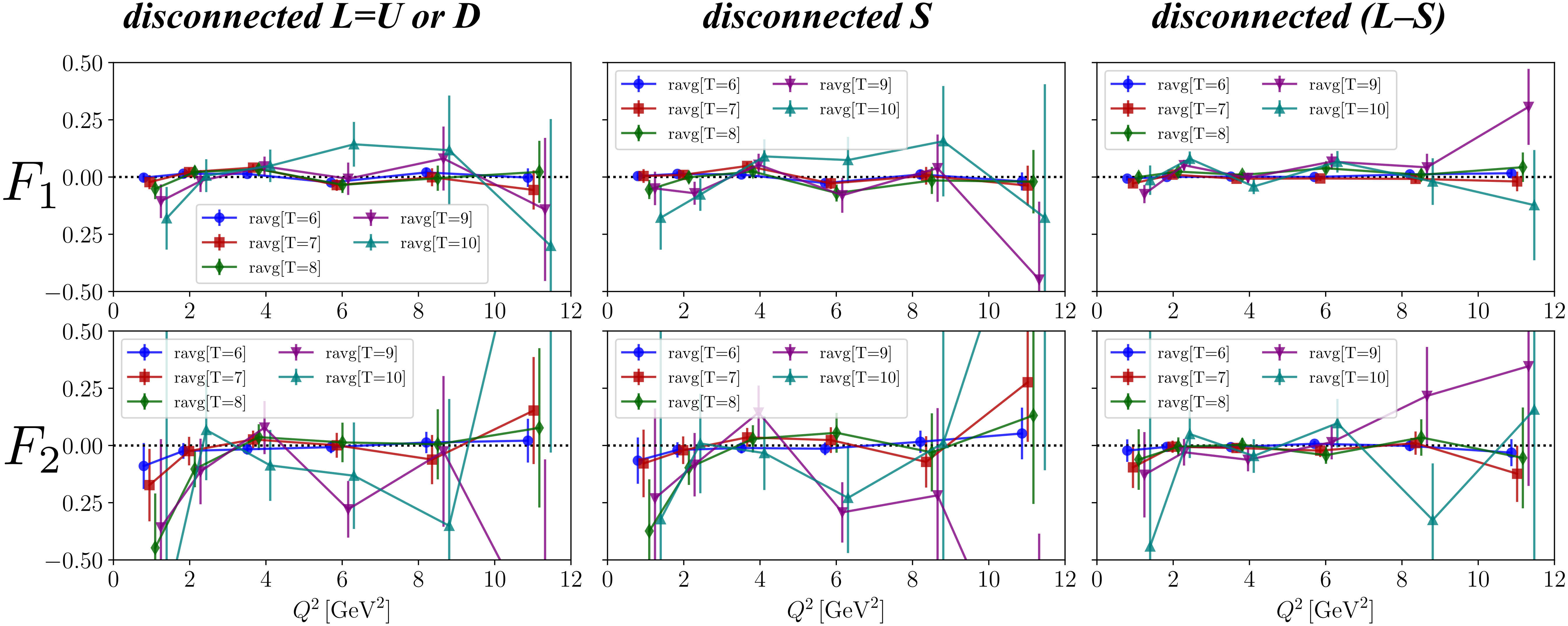


Disconnected Light & Strange vs. Connected (D5)

- *relative correction* $F_{1,2}^{disc} / F_{1,2}^{conn}$ from plateau averages **t_{sep}= 0.5÷0.9 fm**, $Q^2 \lesssim 11$ GeV^2
- **D5 ensemble (mπ=280 MeV, a=0.094 fm)**, 1346 configs ⊗ 64 samples of ⟨N$\bar{N}$⟩
- partial noise cancellation between L=U/D and S in proton & neutron

$$P = \frac{1}{3}\left[2U - D\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$

$$N = \frac{1}{3}\left[2D - U\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$

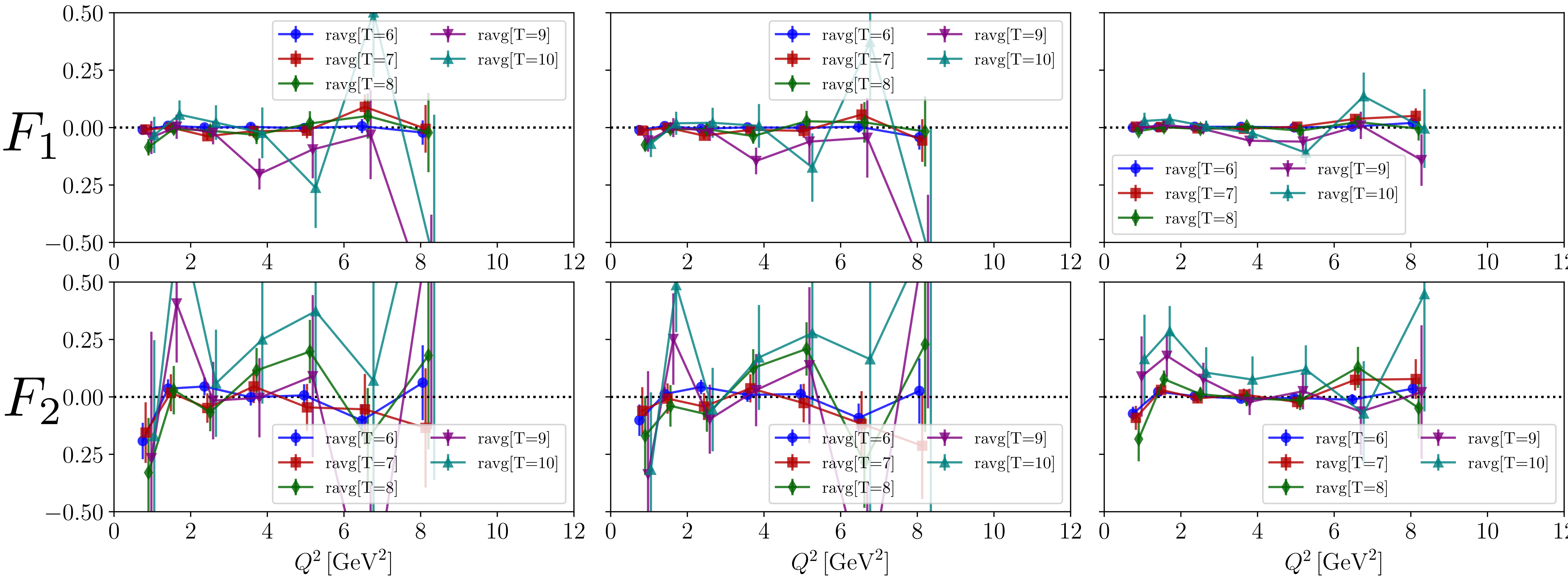


Disconnected Light & Strange vs. Connected (D6)

- *relative correction* $F_{1,2}^{disc} / F_{1,2}^{conn}$ from plateau averages **t_{sep}= 0.5÷0.74 fm,** $Q^2 \lesssim 8$ GeV^2
- **D6 ensemble (m_π=170 MeV, a=0.092 fm)**, 727 configs ⊗ 128 samples of ⟨N$\bar{\text{N}}$⟩
- partial noise cancellation between L=U/D and S in proton & neutron

$$P = \frac{1}{3}\left[2U - D\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$
$$N = \frac{1}{3}\left[2D - U\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$

disconnected L=U or D ***disconnected S*** ***disconnected (L–S)***



G_E/G_M from 3pt Correlator Ratio

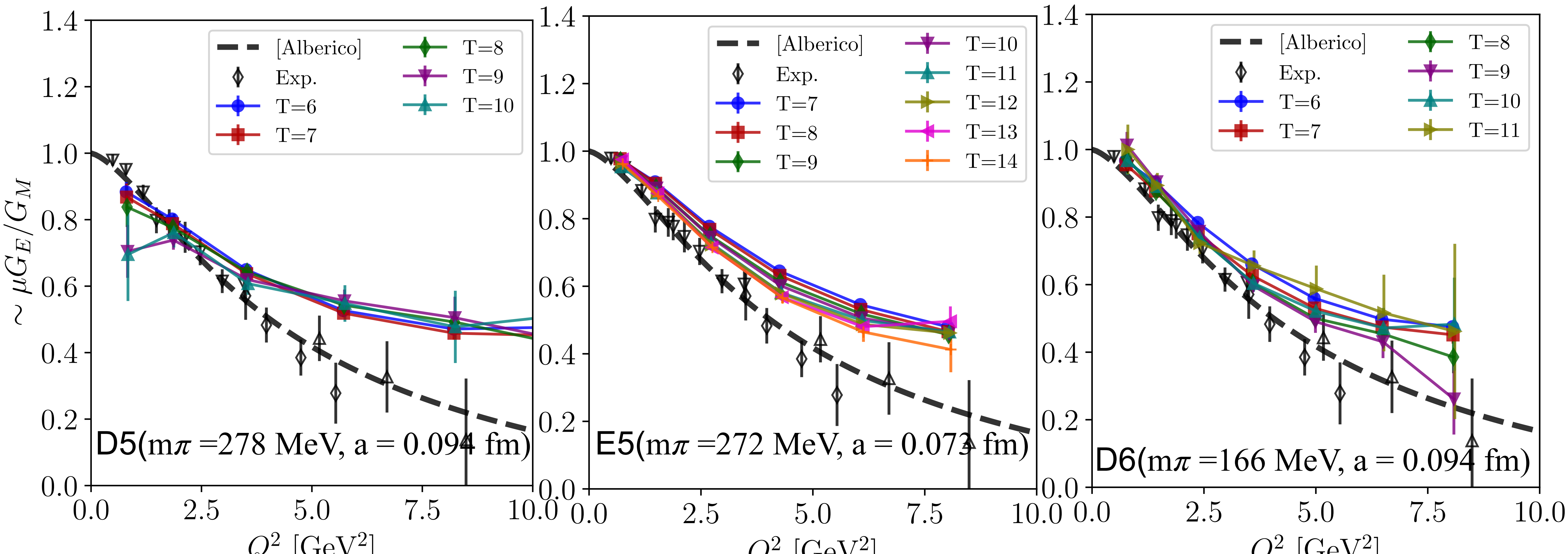
- Robust estimator from nucleon-current correlators: avoid fit-induced bias to examine other systematic effects (disconnected contractions, discretization, etc)

$$\mathrm{Re}\,\langle p'\hat{x}|J_t|p\hat{x}\rangle \;\propto\; \cosh\frac{\lambda'+\lambda}{2}G_E$$

$$\mathrm{Re}\,\langle p'\hat{x}|J_y|p\hat{x}\rangle \;\propto\; \sinh\frac{\lambda'-\lambda}{2}G_M$$

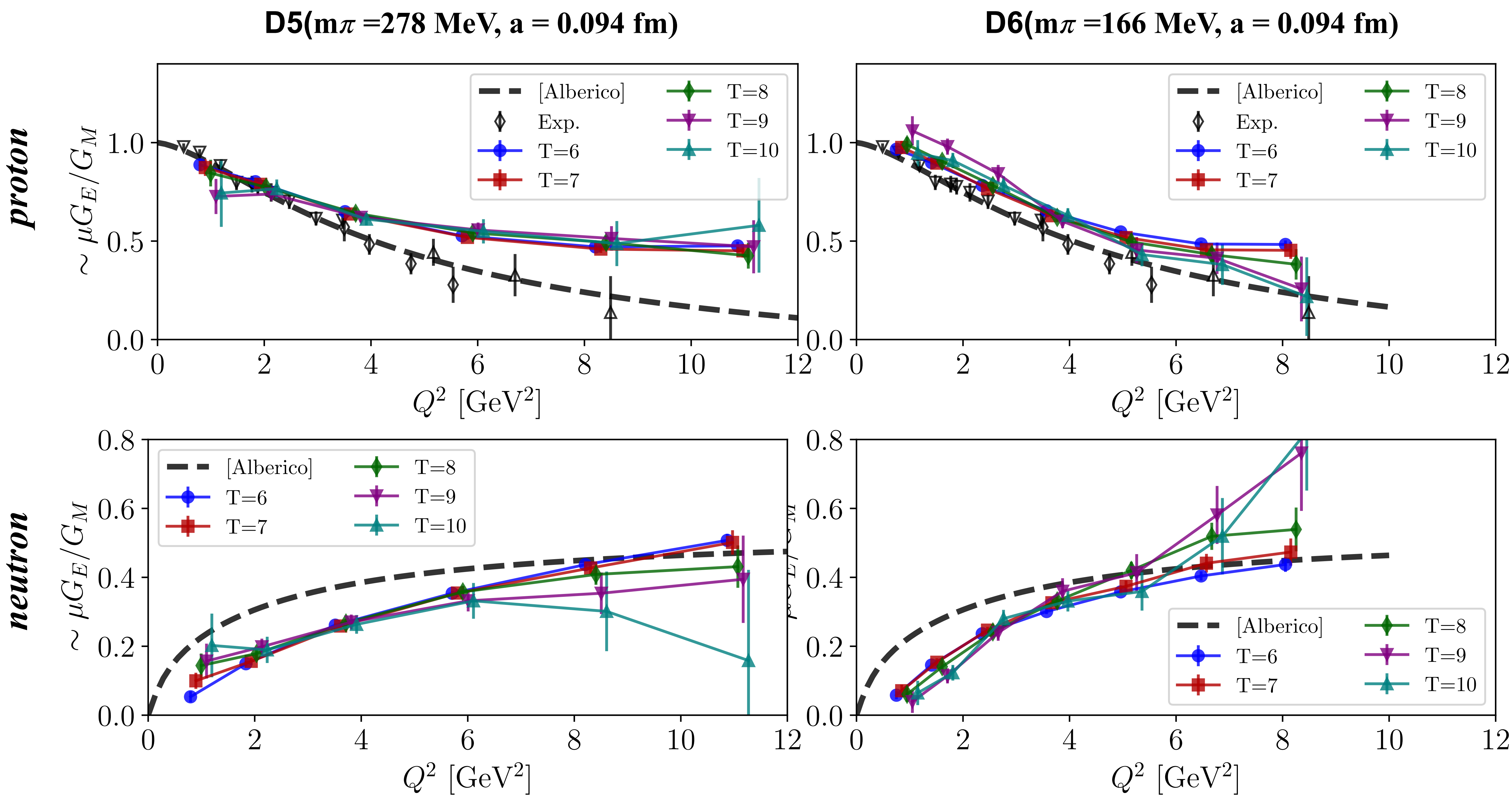
where $\begin{pmatrix} p^{(\prime)} & = m_N \sinh\lambda^{(\prime)} \\ E^{(\prime)} & = m_N \cosh\lambda^{(\prime)} \end{pmatrix}$

$$\left(\frac{\sinh\frac{\lambda'-\lambda}{2}}{\cosh\frac{\lambda'+\lambda}{2}}\right)\frac{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_t(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle}{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_y(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle} \overset{T\to\infty}{=} G_E/G_M$$



Proton&Neutron G_E/G_M : Connected-only

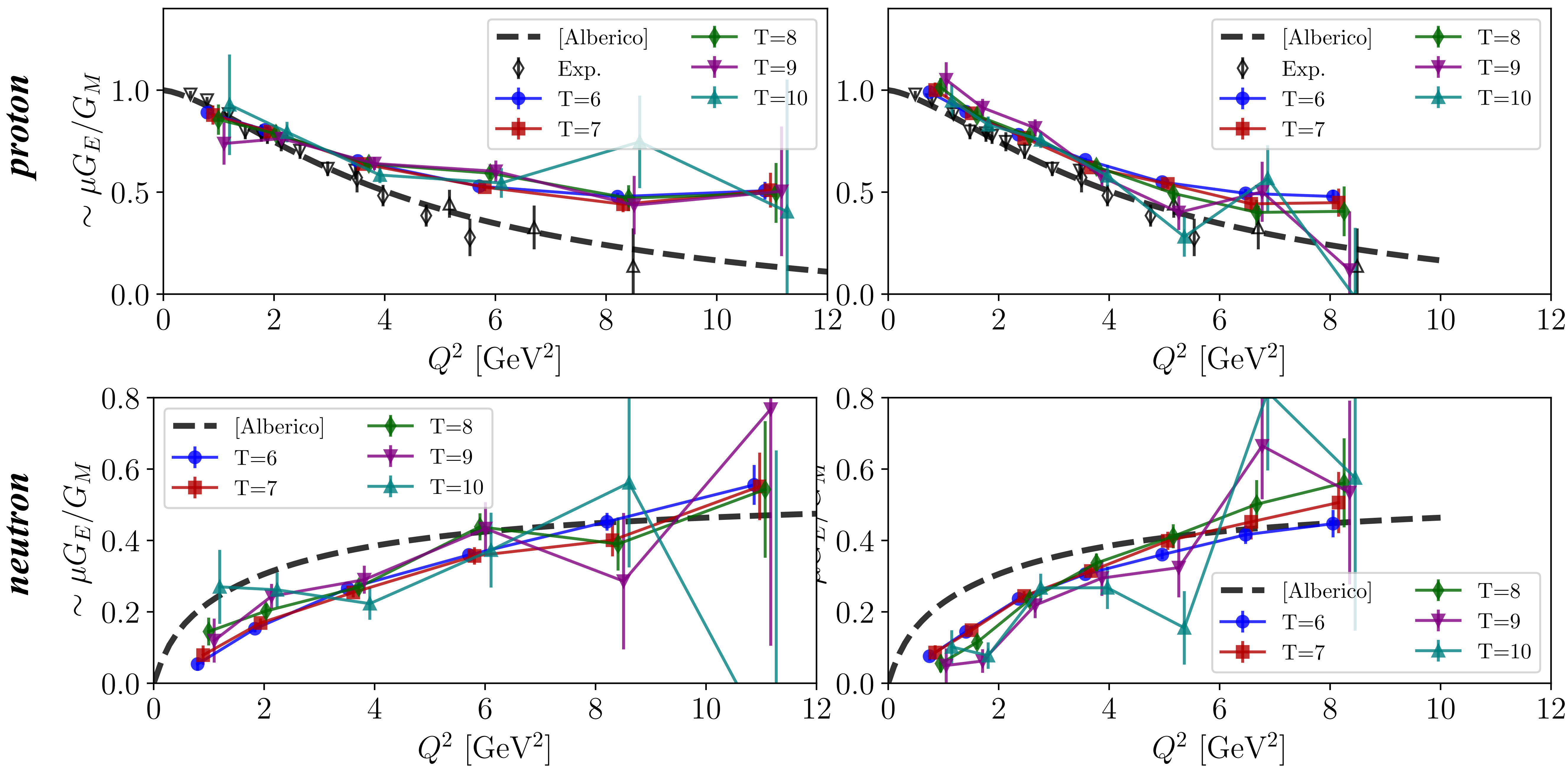
$$\left(\frac{\sinh\frac{\lambda'-\lambda}{2}}{\cosh\frac{\lambda'+\lambda}{2}}\right)\frac{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_t(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle}{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_y(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle} \overset{T\to\infty}{=} G_E/G_M$$



Proton&Neutron G_E/G_M : Connected+Disconnected

$$\left(\frac{\sinh\frac{\lambda'-\lambda}{2}}{\cosh\frac{\lambda'+\lambda}{2}}\right)\frac{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_t(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle}{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_y(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle} \overset{T\to\infty}{=} G_E/G_M$$

D5(mπ =278 MeV, a = 0.094 fm) **D6(mπ =166 MeV, a = 0.094 fm)**



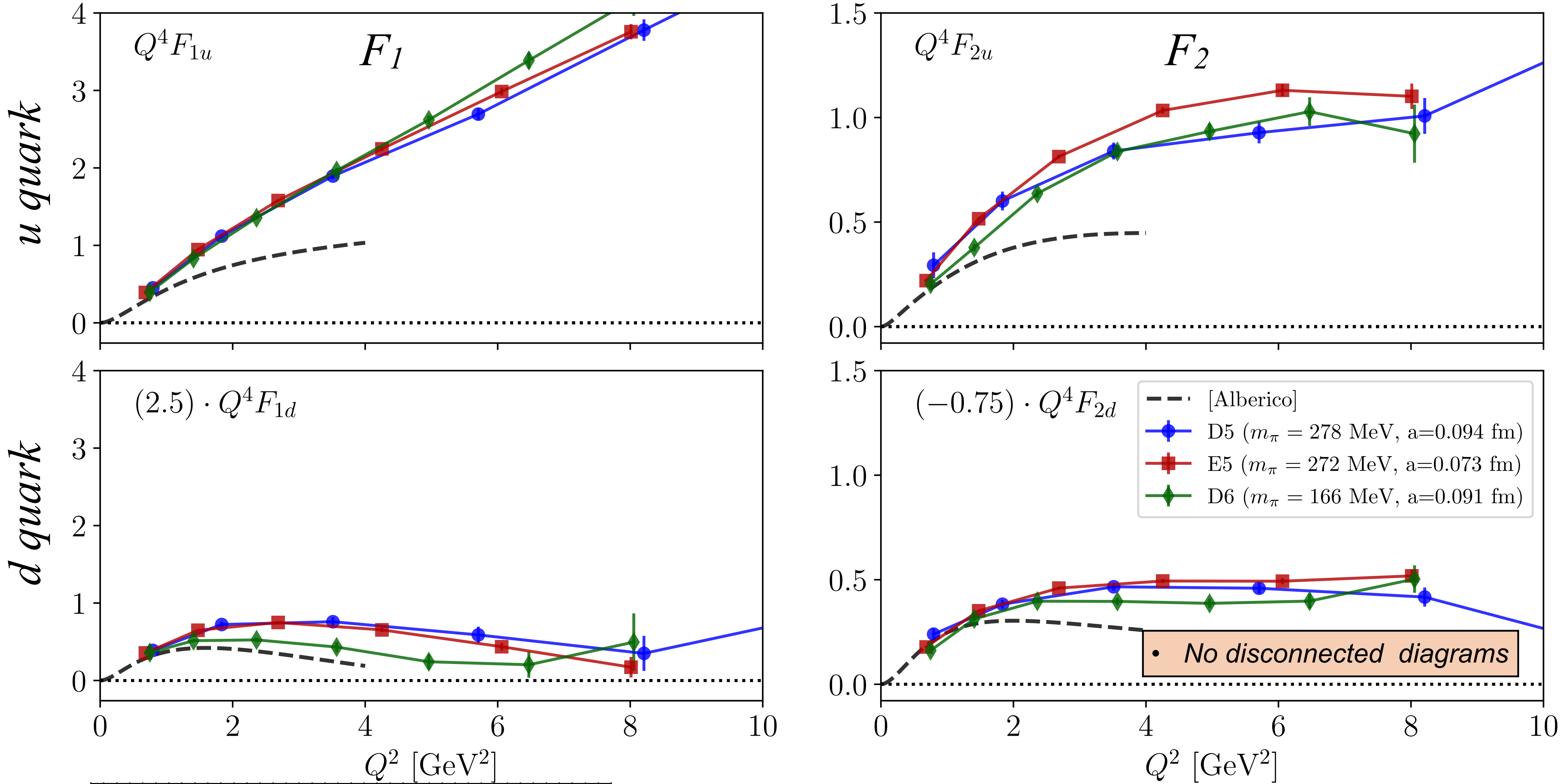
Summary

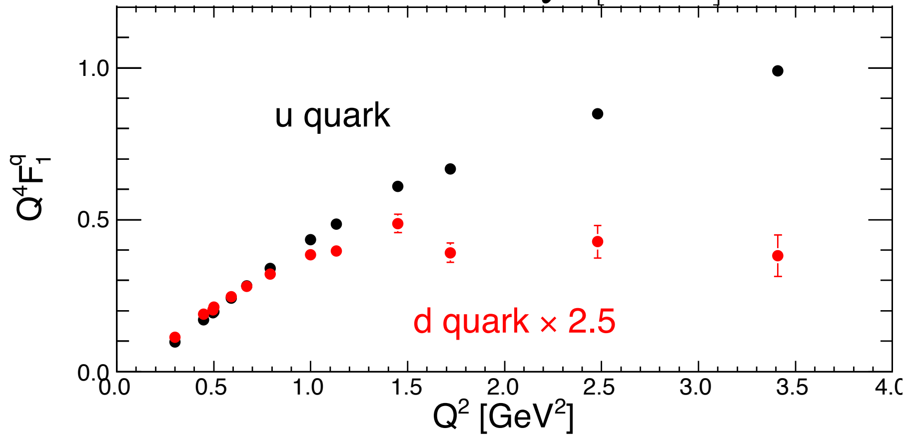
- Preliminary results for high MC-statistics high-momentum form factors
  - up to $Q^2 \lesssim 10\ \mathrm{GeV}^2$
  - two lattice spacings $a \gtrsim 0.07$ fm
  - two pion masses $m\pi \gtrsim 170\ \mathrm{MeV}$
- Quark-disconnected contributions evaluated at $a \approx 0.09$ fm, $m\pi$ down to $170\ \mathrm{MeV}$
  - little impact below $Q^2 \lesssim 6\ \mathrm{GeV}^2$ (except in G_{Ep}/G_{Mp} and G_{En})
  - large stoch. uncertainty above $Q^2 \gtrsim 8\ \mathrm{GeV}^2$
- Form factor results overshoot experimental data x(2 ... 2.5) ;
  G_E/G_M ratios in qualitative agreement
  - Excited states (most likely)
  - Non-physical quarks masses?
  - ~~Discretization?~~ (less likely)

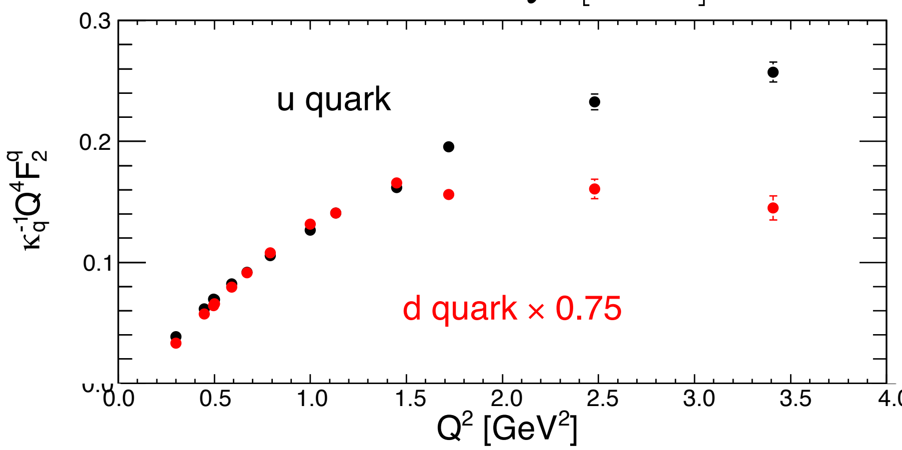
Important cross-check with experiments,
relevant for calculations of relativistic nucleon matrix elements
as well as TMDs, PDFs, DAs ...

BACKUP

Light-Flavor Decomposition (Proton)



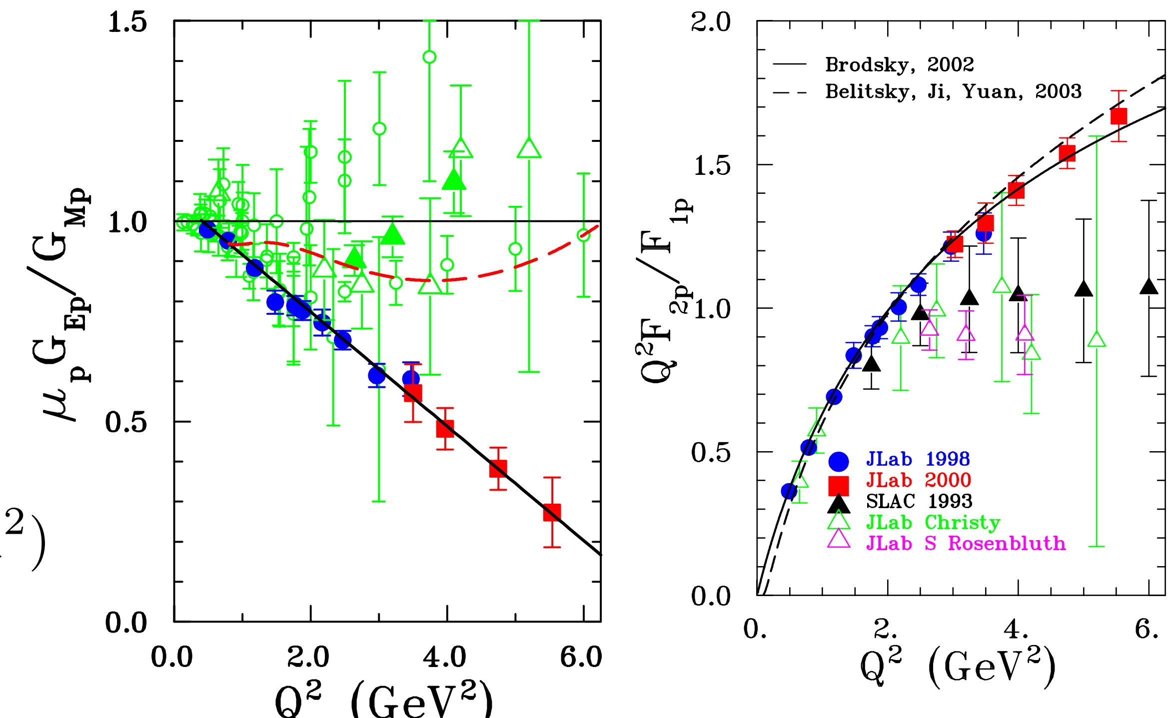




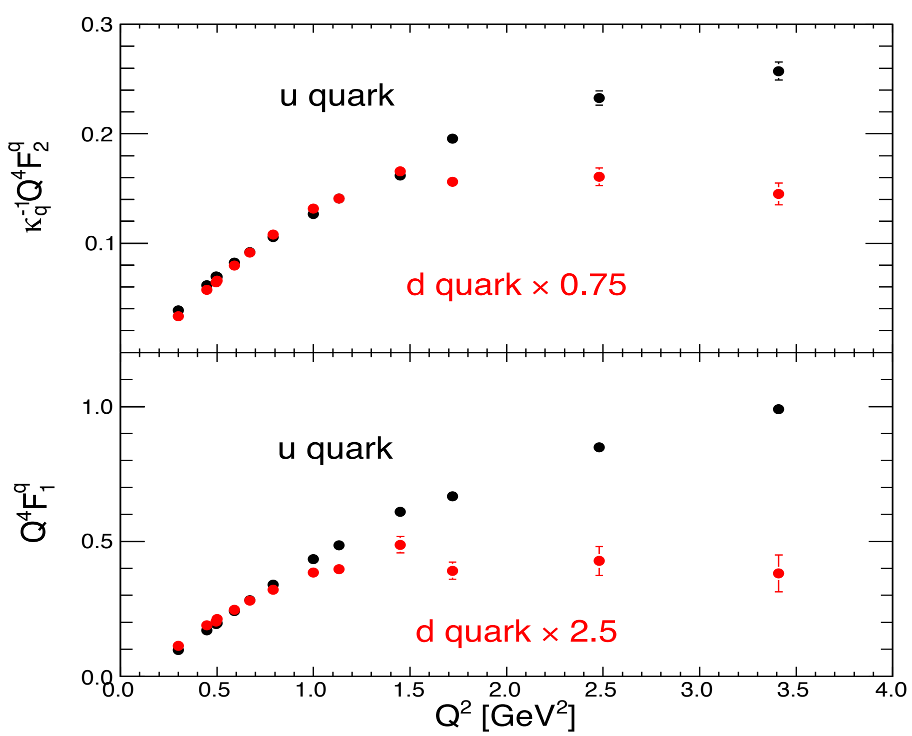
- Similar qual.features of flavor dependence [G.D.Cates, et al, PRL106:252003(2011)]

Nucleon Form Factors: Open Questions

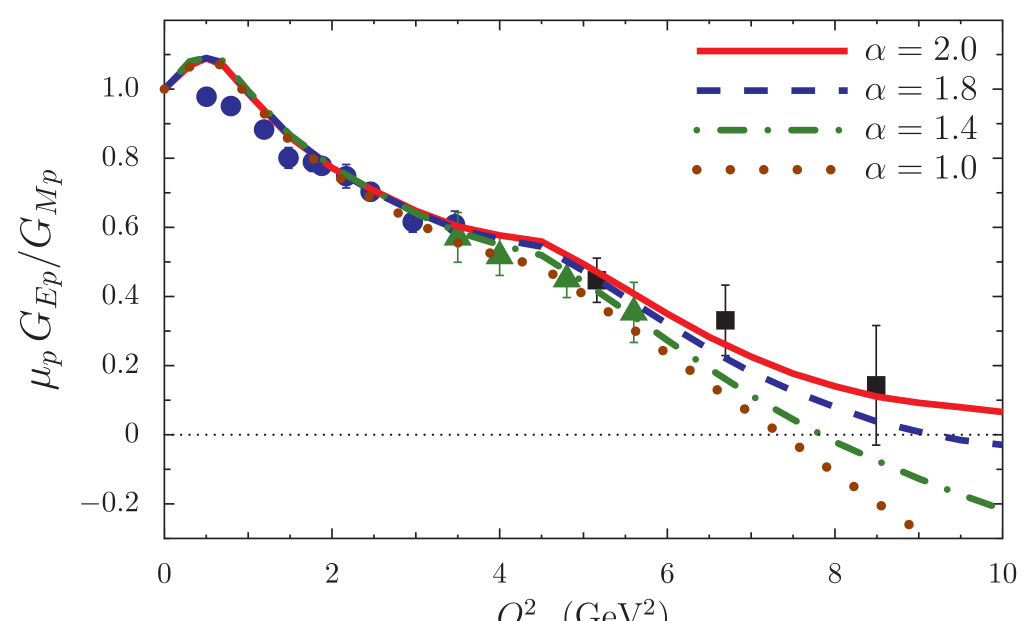
- *Are model descriptions of the nucleon viable ?*
  Nucleon models disagree beyond explored range
- *Role of diquark correlations in elastic scattering ?*
  Neutron & proton G_E/G_M at/above $Q^2 = 8$ GeV^2
- *Scale of transition to perturbative QCD ?*
  (F_2/F_1) scaling at large Q^2 : $Q^2 F_{2p}/F_{1p} \overset{?}{\propto} \log^2(Q^2/\Lambda^2)$
- *What are contributions from u and d flavors?*
  Proton <u>and</u> neutron data needed in wide Q^2 range



[Research Mgmt. Plan for SBS(JLab Hall A)]

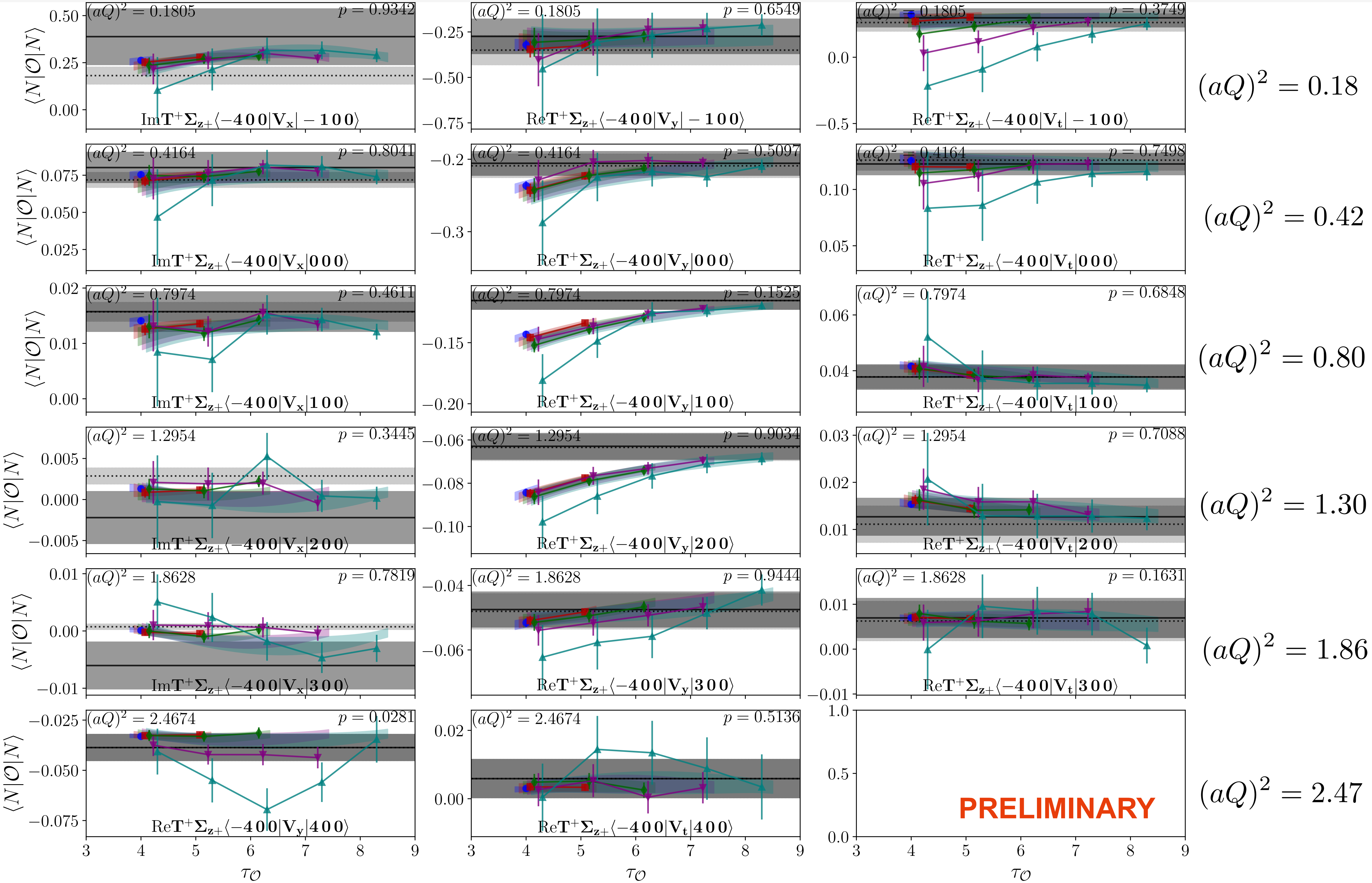


[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]



Dyson-Schwinger Eqns : quarks & 0^+, 1^+ diquarks
(α ≈ rate of transition const.quarks → pQCD with Q^2)
[Cloet, Roberts, Prog.Part.Nucl.Phys 77:1 (2014)]

Nucleon Matrix Element & Form Factor Fits (D5)



- 2-state fit t_{sep}=0.73 ÷1.09 fm (8a ÷12a); energies fixed from 2-state fits to $\langle N\bar{N}\rangle$

Neutron G_{En}/G_{Mn} Ratio

- Lattice data: 2-state fits
- Phenomenology curves : [Alberico et al, PRC79:065204 (2008)]
- Comparison to experimental data (black points)