Flavor diagonal charges of the nucleon

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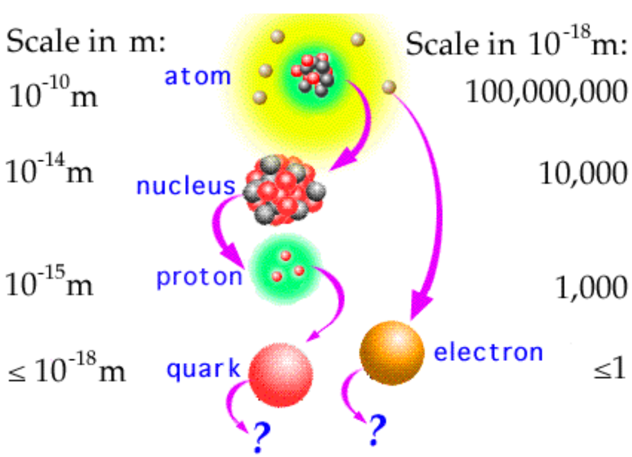
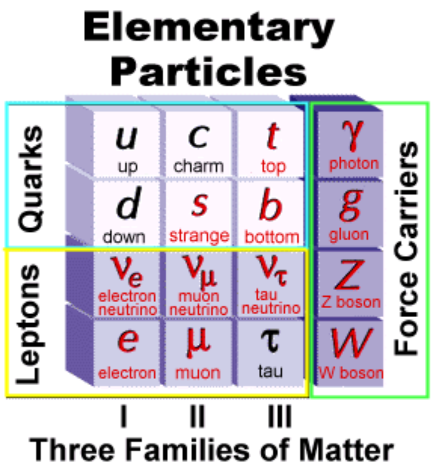
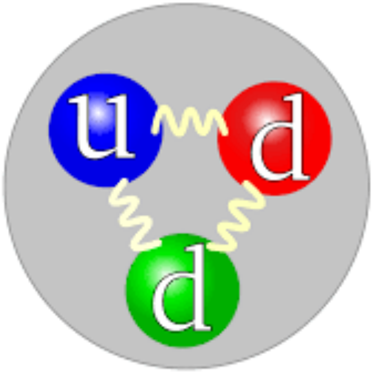






LA-UR: 23-27930

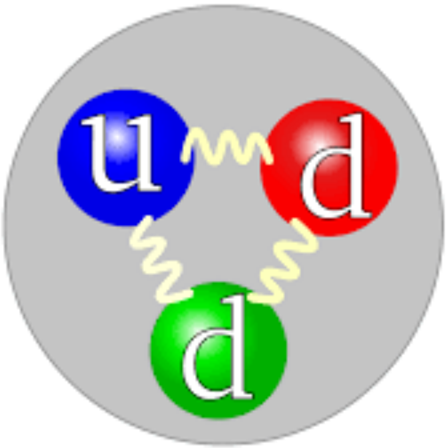
Lattice 2024, Liverpool, UK

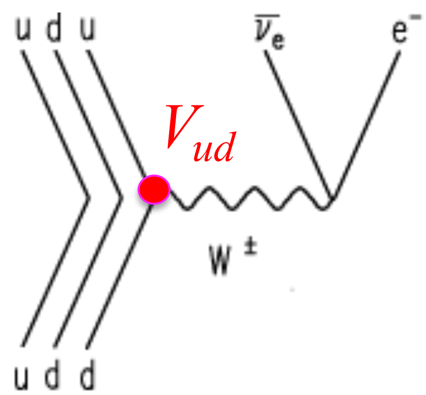
The neutron is a clean but challenging system

Decays weakly ⇒ a stable bound state of QCD

Properties:

- Charges g_A, g_P, g_S, g_T, g_V
- Spin content
  - Quark contribution
  - Gluon contribution
- nEDM
- Form factors
  - Electric, Magnetic
  - Axial
- Distribution functions, moments
  - PDF
  - GPD
- Radiative corrections to decay





PNDME Collaboration: **Eleven 2+1+1-flavor HISQ ensembles = clover-on-HISQ formulation**

NME Collaboration: **Thirteen 2+1-flavor clover ensembles = clover-on-clover formulation**

PNDME and NME members

- Tanmoy Bhattacharya (T-2)
- Vincenzo Cirigliano (T-2 → **INT, UW)**
- Rajan Gupta (T-2)
- Emanuele Mereghetti (T-2)
- Boram Yoon (CCS-7 → **NVIDIA**)
- Junsik Yoo (PD: 2022 May –)
- Yong-Chull Jang (PD: 2017-2018)
- Sungwoo Park (PD: 2018-2021)
- Santanu Mondal (PD: 2019-2021)
- Huey-Wen Lin (MSU)
- Balint Joo (NVIDIA)
- Frank Winter (Jlab)

References

- Charges: Gupta et al, PRD.98 (2018) 034503
- AFF: Gupta et al, PRD 96 (2017) 114503
- AFF: Jang et al, PRL 124 (2020) 072002
- AFF: Jang et al, PRD 109 (2024) 014503
- AFF: Tomalak et al, PRD 108 (2023) 074514
- VFF: Jang et al, PRD 100 (2020) 014507
- $\sigma_{\pi N}$ Gupta et al, PRL 127 (2021) 242002
- d_n from Θ-term Bhattacharya etal, PRD 103 (2021) 114507
- d_n from qEDM Gupta et al, PRD 98 (2018) 091501
- d_n from qcEDM Bhattacharya et al, PRD 98 (2018) 091501
- Moments of PDFs Mondal et al, PRD 102 (2020) 054512
- Proton spin: Lin et al, PRD 98 (2018) 094512

NME

- Charges, FF: Park et al, PRD 105 (2022) 054505
- Moments of PDFs Mondal et al, JHEP 04 (2021) 044

Acknowledgements for Computational Support:
MILC for HISQ ensembles.
DOE for computer allocations at NERSC and OLCF
USQCD
Institutional Computing at LANL

LDRD LABORATORY DIRECTED RESEARCH & DEVELOPMENT

U.S. DEPARTMENT OF ENERGY Office of Science

4-year odyssey

- Sungwoo Park et al., arXiv:2401.00721 @lattice 2023
- Sungwoo Park et al., arXiv:2301.07890 @lattice 2022
- Sungwoo Park et al., arXiv:2203.09584 @lattice 2021

Outline

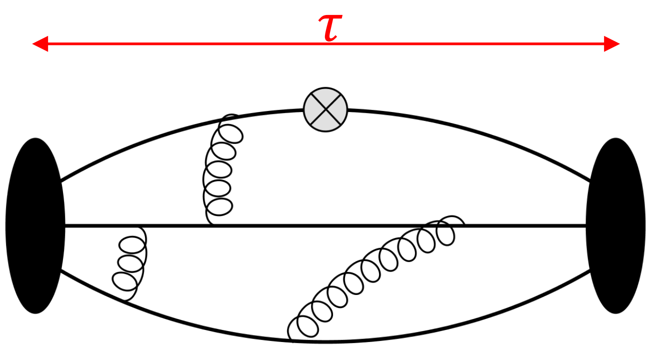
- Analysis of 8 HISQ Lattice Ensembles
- Clover-on-HISQ calculations
- Removing excited-state contributions
- Renormalization
- CCFV fits
- Results

Clover-on-HISQ calculations

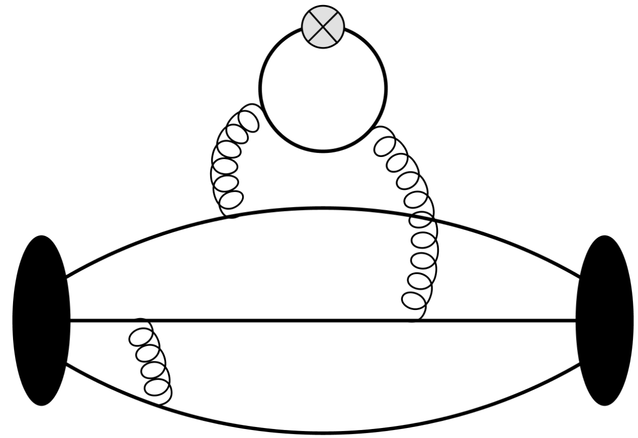
Ensemble ID	a [fm]	M_π [MeV]	$M_\pi L$	N_{cfg}^{conn}	$N_{cfg}^{disc,l}$	$N_{cfg}^{disc,s}$
a15m310	~0.15	320	3.93	1917	1917	1917
a12m310	~0.12	310	4.55	1013	1013	1013
a12m220	~0.12	228	4.38	744	958	870
a09m310	~0.09	313	4.51	2263	1017	1024
a09m220	~0.09	226	4.79	964	712	847
a09m130	~0.09	138	3.90	1290	1270	994
a06m310	~0.06	320	4.52	500	808	976
a06m220	~0.06	235	4.41	649	1001	1002

- HYP smeared $N_f = 2 + 1 + 1$ MILC HISQ lattices
- 8 ensembles including one with physical M_π^{phys}
- Correlation functions with Clover fermions with a tree-level tadpole improved c_{SW}
- Truncated solver method with bias correction and multigrid for high statistics
- Wuppertal smearing with $\langle r \rangle \approx 0.7 - 0.75$ fermi

Lattice Methodology well established for
"connected" and "disconnected" 3-point correlation functions



Connected



Disconnected

disconnected contributions (got via stochastic methods) are noisier for the same computational cost and smaller in value

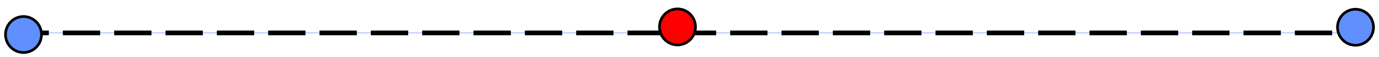
Isoscalar $$g_{A,S,T}^{u+d} = g_{A,S,T}^{u+d,\mathrm{conn}} + 2g_{A,S,T}^{l,disc}$$

Isovector $$g_{A,S,T}^{u-d} = g_{A,S,T}^{u-d,\mathrm{conn}}$$ In the isospin symmetric limit

Spectral decomposition of Γ^2 and Γ^3

Three-point function for matrix elements of axial current $\mathcal{A}_\mu$

$$\langle\Omega|N(\tau)\,\mathcal{A}_\mu(\mathrm{t})\overline{N}(0)|\Omega\rangle$$



Insert $T = e^{-H\Delta t}\sum_i |n_i\rangle\langle n_i|$ at each Δt with $T\,|n_i\rangle = e^{-H\Delta t}|n_i\rangle = e^{-E\Delta t}|n_i\rangle$

$$\langle\Omega|\overline{N}(\tau)\cdots e^{-\mathrm{H}\Delta t}\sum_j |n_j\rangle\langle n_j|\,\mathcal{A}_\mu e^{-\mathrm{H}\Delta t}\sum_i |n_i\rangle\langle n_i|\cdots \mathrm{N}(0)|\Omega\rangle$$

$$\sum_{i,j}\underbrace{\langle\Omega|\overline{N}|n_j\rangle}_{A_j^*}\,e^{-\mathrm{E_j}(\tau-t)}\,\underbrace{\langle n_j|A_\mu|n_i\rangle}_{\text{Matrix Elements}}\,e^{-\mathrm{E_i}t}\,\underbrace{\langle n_i|\mathrm{N}|\Omega\rangle}_{A_i}$$

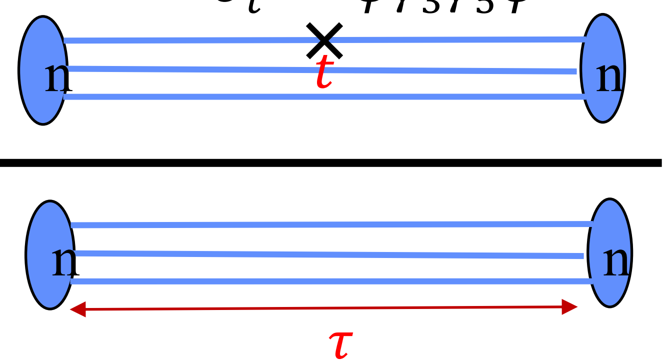
Calculating Nucleon Charges

$$\Gamma^2 = \sum_i A_i^* A_i \, e^{-E_i \tau} \qquad\qquad \Gamma^3 = \sum_{i,j} A_i^* A_j \langle N_i | O | N_j \rangle e^{-E_i t} \, e^{-E_j(\tau - t)}$$

$$\frac{\Gamma^3}{\Gamma^2} = \frac{\langle \Omega | \overline{N} \, \mathrm{A}_\mu \, \mathrm{N} | \Omega \rangle}{\langle \Omega | \overline{N} \, \mathrm{N} | \Omega \rangle} \rightarrow \langle N(p_f) | A_\mu \, (Q^2) | N(p_i) \rangle \rightarrow \boldsymbol{g_A}$$

$$O_t = \bar{\psi} \gamma_3 \gamma_5 \psi$$

$$\frac{\Gamma^3}{\Gamma^2} =$$



$$\xrightarrow[\boldsymbol{\tau \to \infty}]{} \boldsymbol{g_A}$$

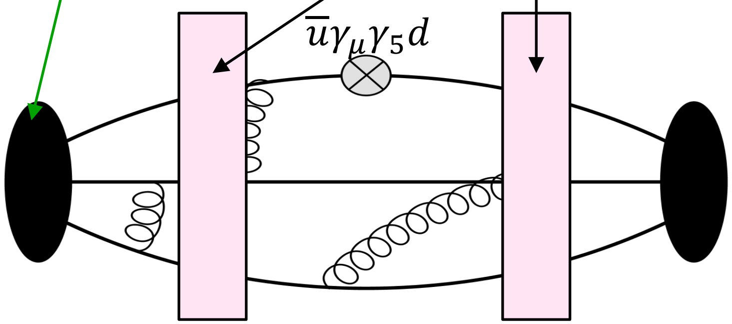
$N\pi$: Amplitudes A_i for local N fall as 1/V for each particle

Excited states in correlation functions

Challenge: To get the matrix elements within ground state of hadrons (nucleons), the contributions of all excited states must be removed.

Typical interpolating operators create (annihilate) all states with the same quantum numbers aa the Nucleon

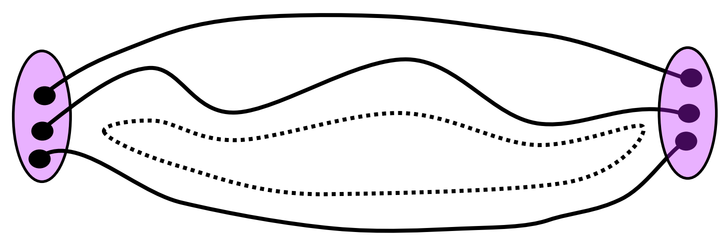
All intermediate states with nucleon quantum numbers are suppressed only by $A_i^2 e^{-(M_i - M_N)t}$



Towers of multihadron states

$$N(\vec{p})\pi(-\vec{p})$$
$$N(0)\pi(\vec{p})\pi(-\vec{p})$$
$$N(\vec{p})2\pi(-\vec{p})$$
$$\cdots$$

+ radial excitations

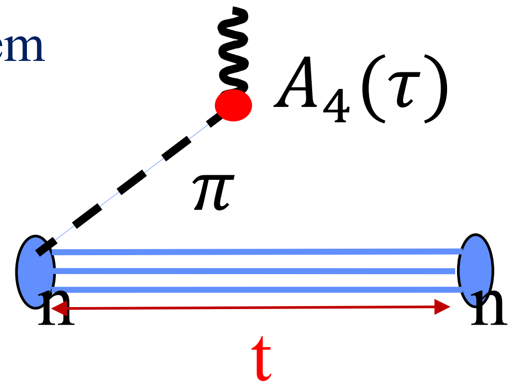


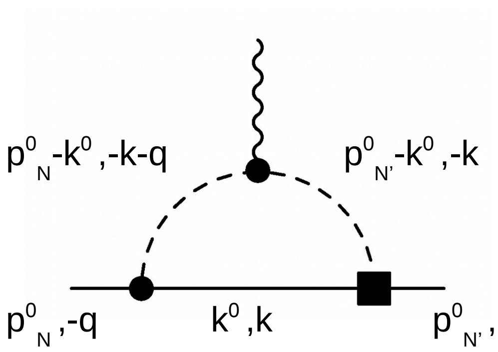
- Which excited states make significant contributions to a given matrix element?
- What are their energies in a finite box?

$N\pi$: Amplitudes A_i for local N fall as 1/V for each particle

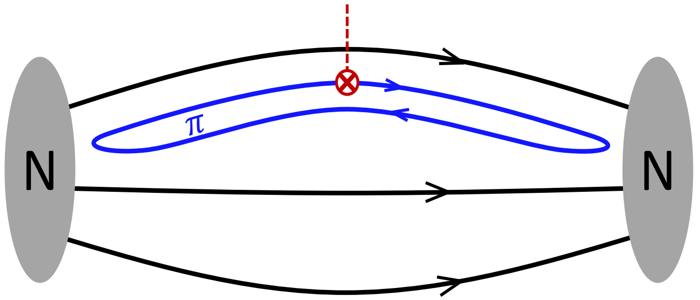
But excited-state matrix elements are enhanced

- Axial Form Factors must satisfy PCAC relation between them
  - Need to include $N(\vec{p})\pi(-\vec{p})$ states to satisfy PCAC
  - $\langle\Omega|N(\tau)\ \mathcal{A}_4(\mathrm{t})\overline{N}(0)|\Omega\rangle$ has very large ESC
  - Used $\langle\Omega|N\ A_4\overline{N}|\Omega\rangle$ to include $N\pi$ state. Data-driven method





- χ PT predicts large contributions from $N\pi$ state in
  - nEDM from Θ-term
  - The pion-nucleon sigma term $\sigma_{\pi N} = \mathrm{m_{ud}}\ g_S^{u+d}$



PCAC & 2 ways of extracting isovector g_A^{u-d}

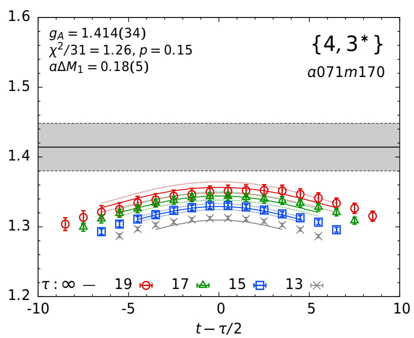
Spectrum from Γ^2

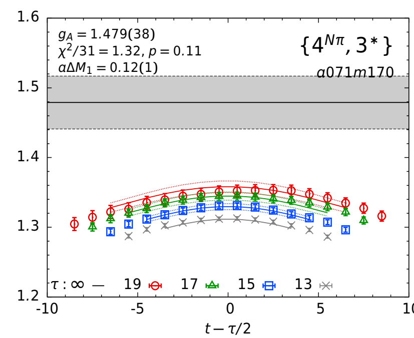
$N\pi$ included in fits (via A_4 or priors)

g_A (Forward ME)

1.22(5)

1.28(5)





$g_A = G_A(Q^2 \to 0)$

1.19(5)

1.32(6)

$G_A, \tilde{G}_P, G_P$ do not satisfy PCAC

$G_A, \tilde{G}_P, G_P$ satisfy PCAC

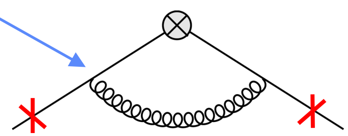
Operator mixing calculation in RI-sMOM

Calculated the 3×3 flavor (u, d, s) mixing matrices for 2+1-flavor theory in **RI-sMOM:** $g_\Gamma^f = \sum_{f'} Z_\Gamma^{ff'} g_\Gamma^{f'}|_{\text{bare}}$

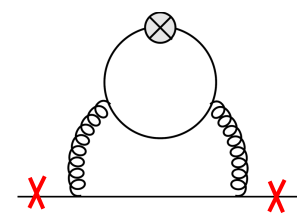
Landau gauge fixed quark propagators using momentum source with $p \propto (1,1,1,1)$

Projected amputated Green's function $\mathrm{Tr}[(..)\mathbb{P}] \equiv \Lambda_\Gamma^{\text{PA}}$

$$(Z_\Gamma^{-1})^{ff'} = \sum_{f'} \frac{1}{Z_\psi^f} \mathrm{Tr}\Big[\Big($$



$$\times\delta^{ff'} -$$



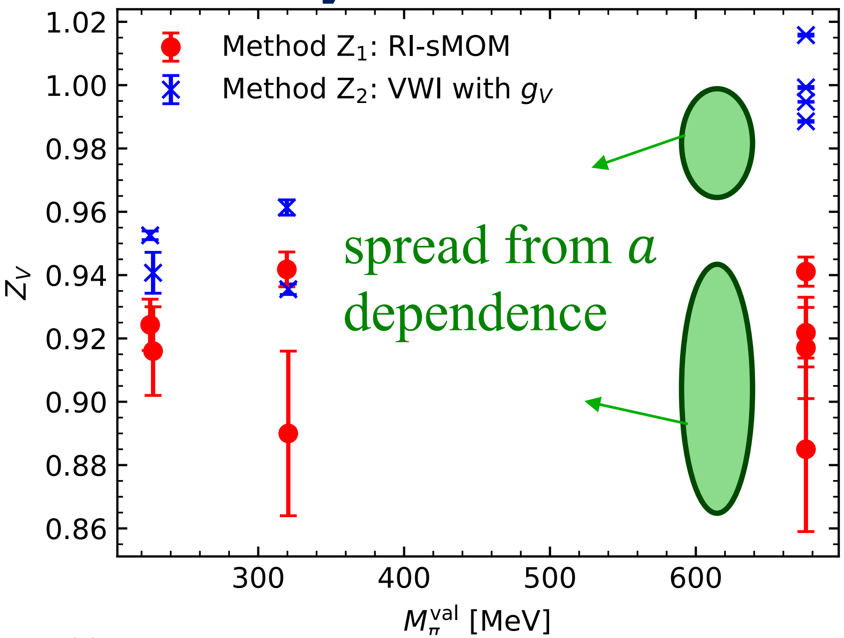
$$\Big)\mathbb{P}(p', p)\Big]$$

Z_1 method: $Z_\psi(p) \equiv \frac{i}{12p^2} \mathrm{Tr}[S^{-1}(p) p \cdot \gamma]$

Z_2 method: $Z_\psi^{\text{VWI}}(p) \equiv \Lambda_V^{\text{PA}}(p)/g_V$

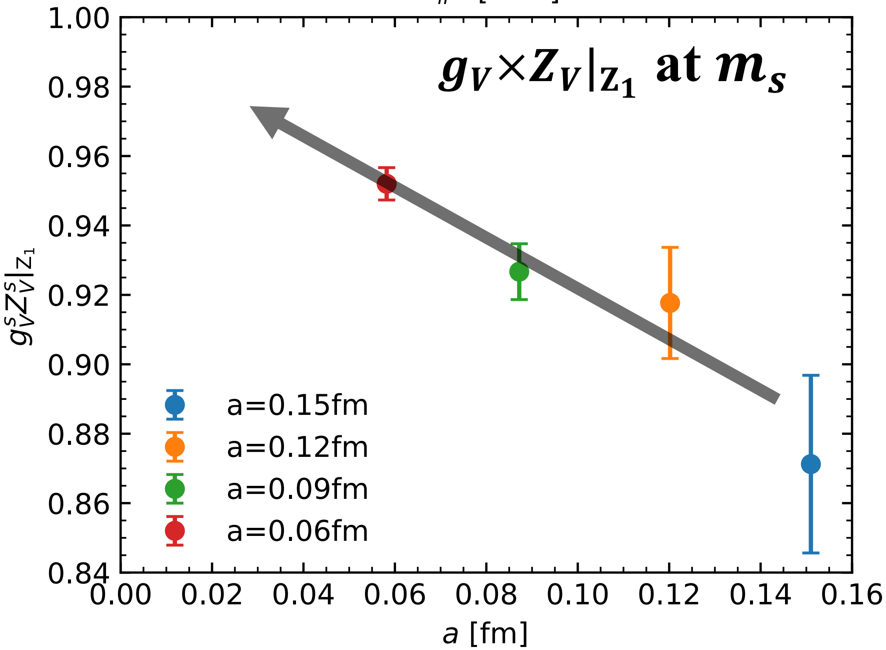
Using Vector Ward Identity (VWI): $g_V Z_V = 1$ with g_V from separate nucleon matrix element calculation

Z_V from methods Z_1, Z_2



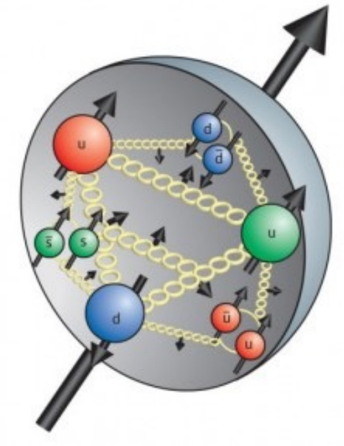
- $Z_V|_{Z_1}$ and $Z_V|_{Z_2}(= 1/g_V)$ have different M_π^{val} and a dependence

- $g_V \times Z_V|_{Z_1}$ deviates from VWI (=1) at large quark mass, but VWI restored in the continuum limit



- To study the systematic effect in two different methods, $\{Z_1, Z_2\}$ we chiral-continuum extrapolate $g_\Gamma|_{Z_1}$ and $g_\Gamma|_{Z_2}$, separately, and compare the results.

Intrinsic quark's spin contribution to proton spin



gauge invariant decomposition of the proton spin is given by

$$\frac{1}{2} = \sum_{\{u,d,s,c\}} \left(\frac{1}{2}\Delta q + L_q\right) + J_g$$

$$S_P^q = \sum_q S_q \equiv \sum_q \frac{\Delta q}{2} \equiv 0.5 \sum_q g_A^q \qquad g_A^q = \langle N(p_i) | Z_A A_\mu^q(0) | N(p_i) \rangle$$

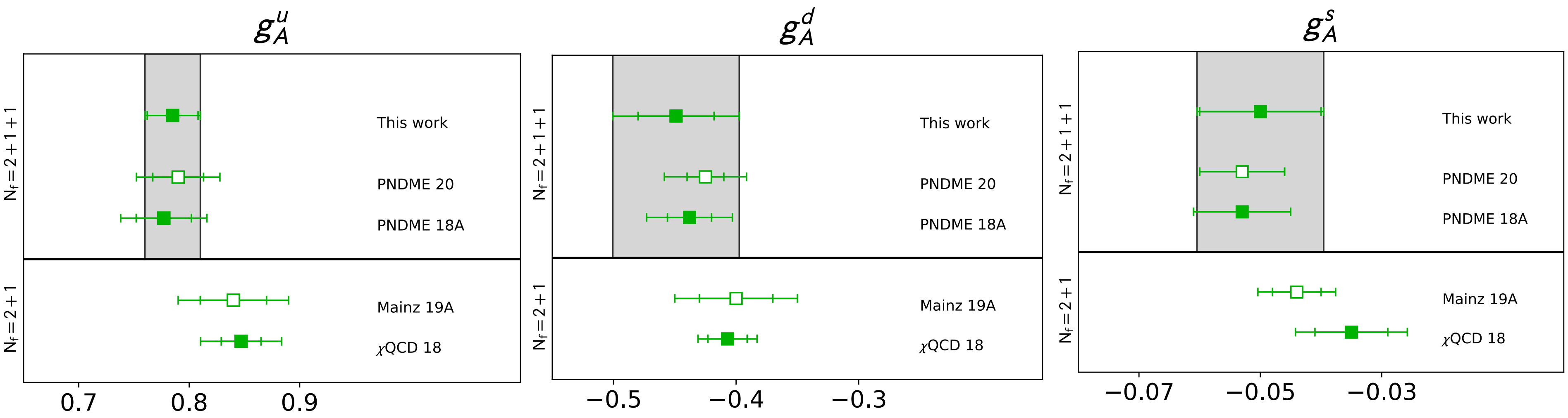
[X. Ji, PRL 78 (1997) 610]

LANL (PNDME) result (PRD 98 (2018) 094512):

$$0.5 \sum_q g_A^q = (0.777(39) - 0.438(35) - 0.053(8))/2 = 0.143(31)(36)$$

Compass result $0.13 \leq \sum_q S_q \equiv 0.5 \sum_q g_A^q \leq 0.18$

FD axial charges



LANL (PNDME) result (PRD 98 (2018) 094512):

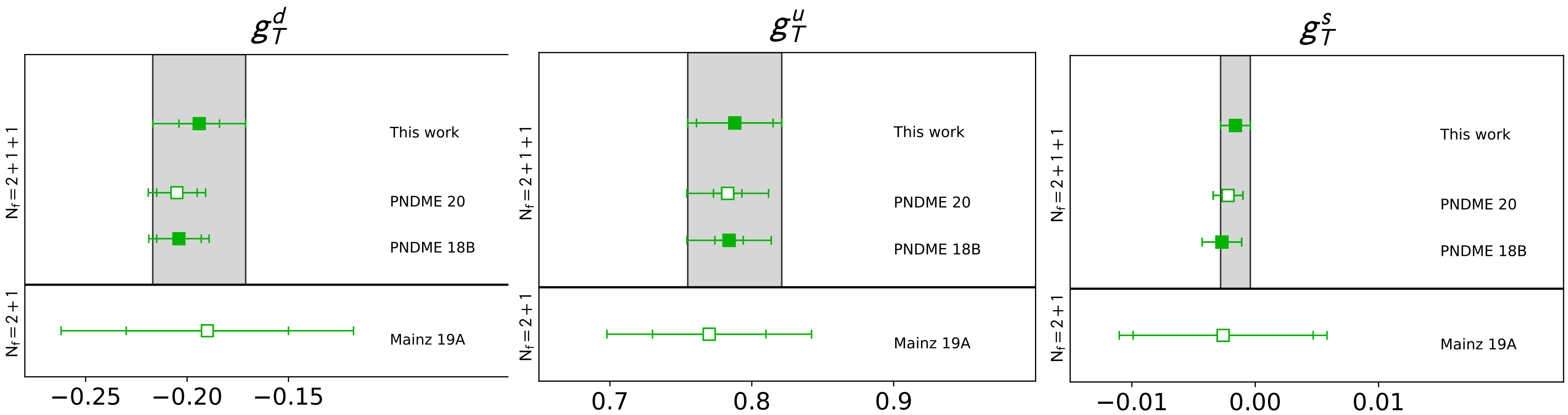
$$0.5 \sum_q g_A^q = (0.777(39) - 0.438(35) - 0.053(8))/2 = 0.143(31)(36)$$

New result

$$0.5 \sum_q g_A^q = (0.784(22) - [0.41 - 0.46] - [0.054 - 0.069])/2$$

Compass result $0.13 \leq \sum_q S_q \equiv 0.5 \sum_q g_A^q \leq 0.18$

$g_T^{u,d,s,c}$: Contribution of the quark EDM to neutron EDM

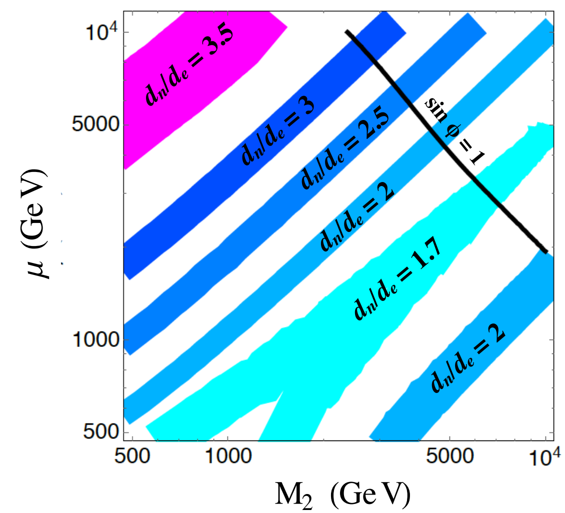


LANL (PNDME) results:

- g_T^u = 0.784(28)(10)
- g_T^d = -0.204(11)(10)
- g_T^s = -0.0027(16)

→

Constrains using nEDM on the parameter space of split SUSY model



PRD 98 (2018) 091501

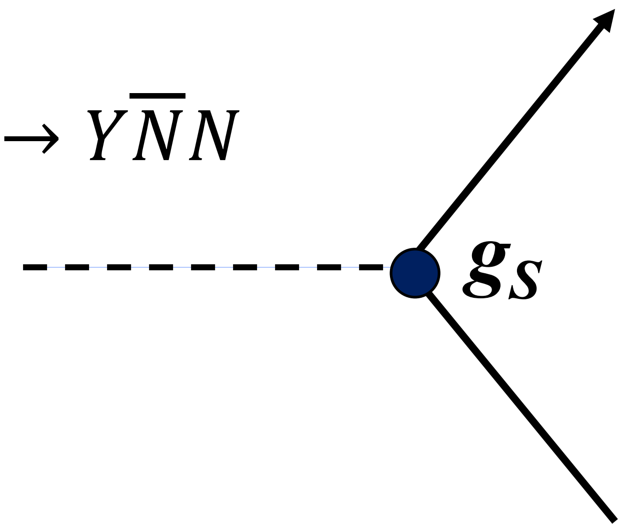
g_T^u = 0.781(27)(04)

g_T^d = -0.194(10)(20)

g_T^s = -0.0016(12)

Scalar charges $g_S^{u,d,s,c}$

Effective operator: $X\overline{q}q \rightarrow Y\overline{N}N$



g_S^{u-d}: novel scalar interaction measured in neutron decay

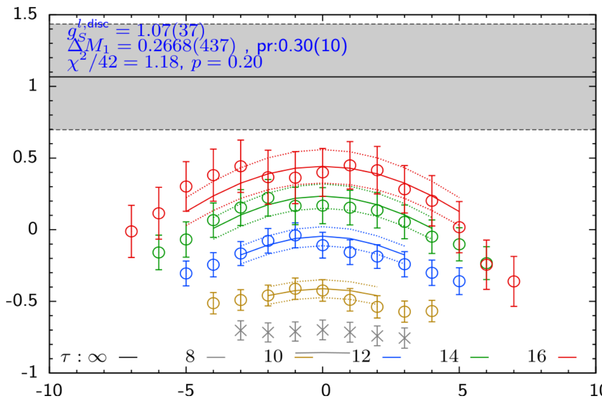
$g_S^{u,d,s,c}$: flavor independent interactions (dark matter)

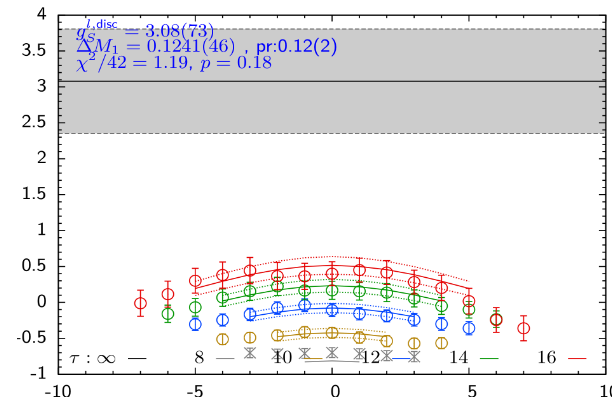
g_S^{u+d} : rate of change of nucleon mass with u,d quark mass

$g_S^{u,d}$: Excited-state effects are large and results very sensitive to $N\pi$ / $N\pi\pi$ states

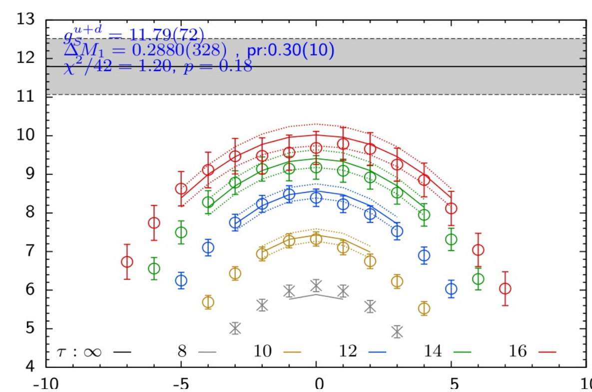
Fits without $N\pi/N\pi\pi$ $(M_1 \approx 1.6\ GeV)$ with $N\pi$ / $N\pi\pi$ $(M_1 \approx 1.2\ GeV)$

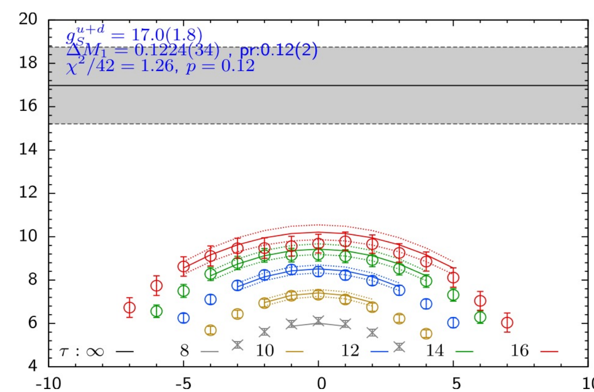
$g_S^{l,\text{disc}}$





$$g_S^{u+d} = g_S^{u+d,conn} + 2g_S^{l,disc}$$



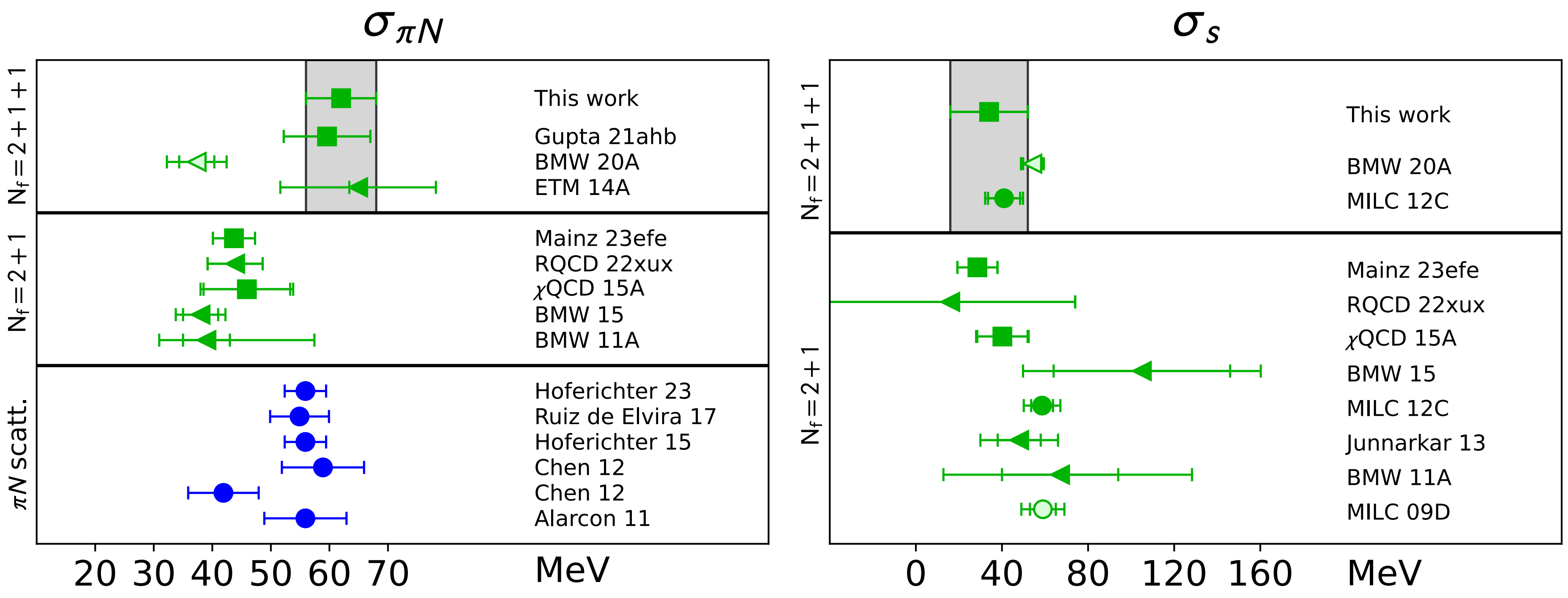


PRL 127 (2021) 242002

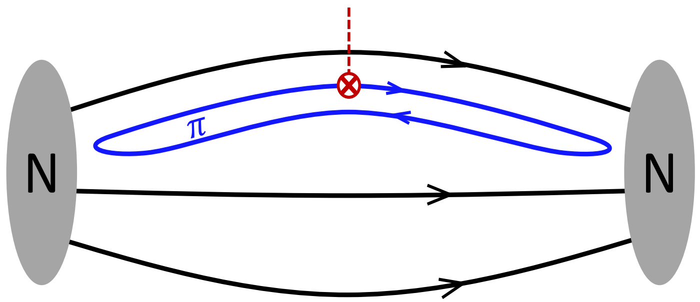
$\sigma_{\pi N} = m_l g_S^{u+d} \sim 40$ MeV

$\sigma_{\pi N} = m_l g_S^{u+d} \sim 60$ MeV

Sigma terms



$\sigma_{\pi N}$
$N_f = 2+1+1$
This work
Gupta 21ahb
BMW 20A
ETM 14A
$N_f = 2+1$
Mainz 23efe
RQCD 22xux
χQCD 15A
BMW 15
BMW 11A
πN scatt.
Hoferichter 23
Ruiz de Elvira 17
Hoferichter 15
Chen 12
Chen 12
Alarcon 11
20 30 40 50 60 70
MeV
σ_s
$N_f = 2+1+1$
This work
BMW 20A
MILC 12C
$N_f = 2+1$
Mainz 23efe
RQCD 22xux
χQCD 15A
BMW 15
MILC 12C
Junnarkar 13
BMW 11A
MILC 09D
0 40 80 120 160
MeV



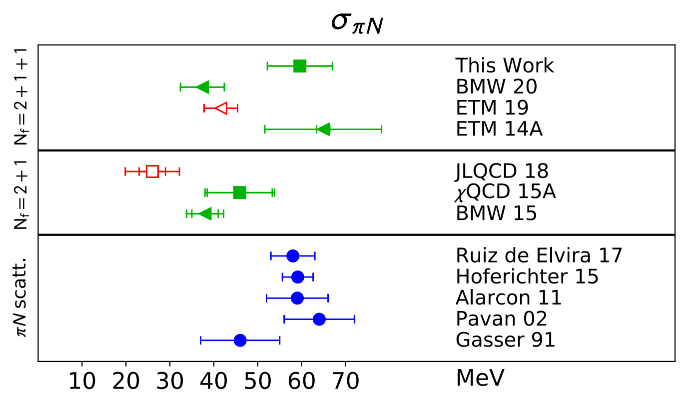
N
π
N

The pion-nucleon sigma term: Resolving tension between Lattice QCD and Phenomenology

$$\sigma_{\pi N} \equiv m_{ud} g_S^{u+d} \equiv m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle$$

FLAG Reports 2019, 2021:

- Lattice results ~40 MeV
- Phenomenology favors ~60 MeV



Post FLAG 2021 results

BMW (arXiv:2007.03319) $\sigma_{\pi N}$ = 37.4(5.1) MeV (FH)

RQCD (JHEP 05 (2023) 035) $\sigma_{\pi N}$ = 43.9(4.7) MeV (FH)

Mainz (PRL 131 (2023) 261902) $\sigma_{\pi N}$ = 43.7(3.6) MeV (FH)

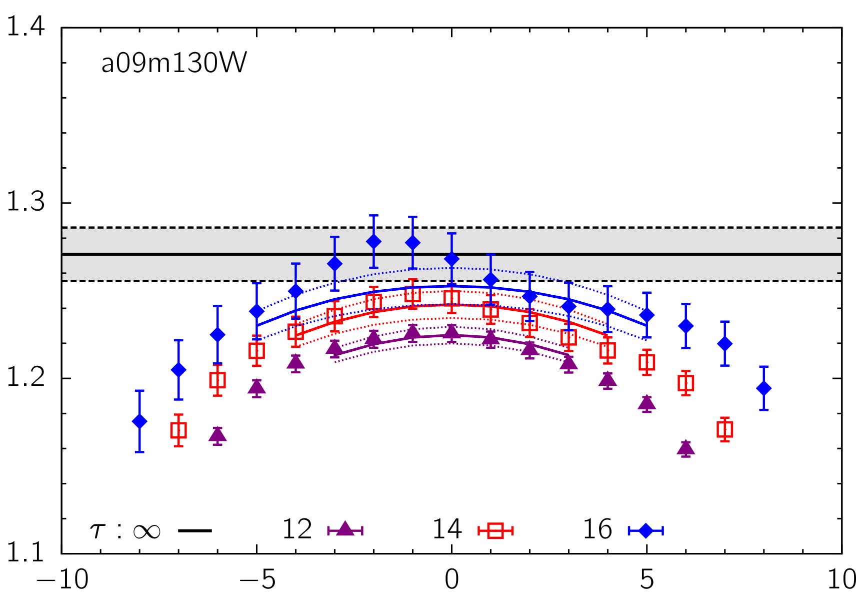
ETM (PRD **102**, 054517) $\sigma_{\pi N}$ = 41.6(3.8) MeV (Direct)

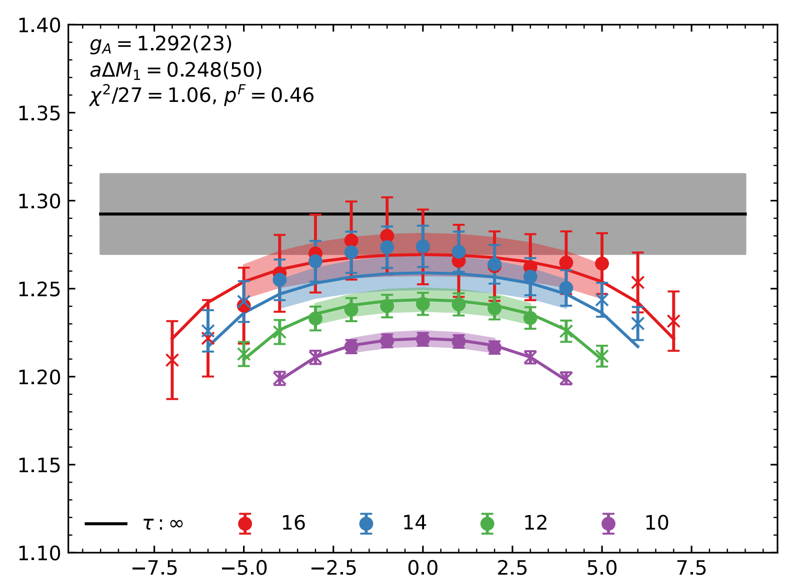
LANL Results: PRL 127 (2021) 242002; e-Print: 2105.12095

- Without including $\boldsymbol{N(\vec{k})\pi(-\vec{k})}$ and $\boldsymbol{N(0)\pi(\vec{k})\pi(-\vec{k})}$ states: = 41.9 (4.9) MeV
- Including $\boldsymbol{N(\vec{k})\pi(-\vec{k})}$ and $\boldsymbol{N(0)\pi(\vec{k})\pi(-\vec{k})}$ states: = 59.6 (7.4) MeV

Future

- Brute force: increase statistics to get to larger τ
  - Two $M_\pi = 135$ HISQ ensembles
- Variational basis of states including $N\pi$ to get results from smaller τ





Acknowledgements

- MILC collaboration for providing the 2+1+1-flavor HISQ lattices.
- The calculations used the CHROMA software suite.
- DOE HEP for support
- For computer allocations we thank
  - DOE NERSC through ERDCAP
  - DOE INCITE project hep133 @OLCF
  - USQCD collaboration
  - Institutional Computing at LANL