

Nucleon axial, tensor, and scalar charges and σ -terms from lattice QCD

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- 1 Methodology
- 2 Analysis
- 3 Results and Comparison
- 4 Summary and Outlook

- To extract the nucleon charges we use three- and two-point correlation functions:

$$C^\mu(x_s; x_{ins}) = \langle \mathcal{J}(x_s) \mathcal{O}^\mu(x_{ins}) \bar{\mathcal{J}}(0) \rangle$$

$$C(x_s) = \langle \mathcal{J}(x_s) \bar{\mathcal{J}}(0) \rangle$$

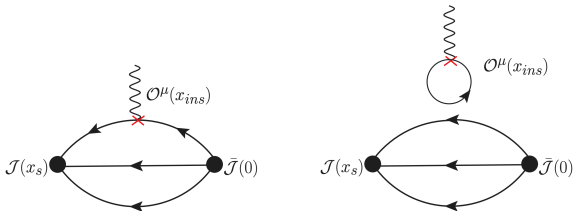
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Connected (left) and disconnected (right) contributions of a baryon three-point function.

- We can manipulate the three-point function and at the limit of large time, where $(t_s - t_{ins}) \rightarrow \infty$ and $t_{ins} \rightarrow \infty$ write it as:

$$C^\mu(\vec{p}', t_s; \vec{p}_1, t_{ins}) \rightarrow \langle \mathcal{J} | H(\vec{p}') \rangle \langle H(\vec{p}_1) | \bar{\mathcal{J}} \rangle \times \\ \langle H(\vec{p}') | \mathcal{O}^\mu | H(\vec{p}' - \vec{p}_1) \rangle e^{-E(\vec{p}')(t_s - t_{ins})} e^{-E(\vec{p}_1)t_{ins}}$$

- Similarly, the two-point function can be written as:

$$C(\vec{p}, t_s) \rightarrow \langle \mathcal{J} | H(\vec{p}) \rangle \langle H(\vec{p}) | \bar{\mathcal{J}} \rangle e^{-E(\vec{p})t_s}$$

- To isolate the matrix element from the overlap terms and the exponentials, we take the ratio of the three-point function with two-point functions:

$$R^\mu(\vec{p}', t_s; \vec{p}_1, t_{ins}) = \frac{C^\mu(\vec{p}', t_s; \vec{p}_1, t_{ins})}{C(\vec{p}', t_s)} \times \sqrt{\frac{C(\vec{p}_1, t_s - t_{ins})C(\vec{p}', t_{ins})C(\vec{p}', t_s)}{C(\vec{p}', t_s - t_{ins})C(\vec{p}_1, t_{ins})C(\vec{p}_1, t_s)}}$$

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- The spectral decomposition of the two- and three-point functions are given respectively by:

$$C(\Gamma_0, \vec{p}; t_s) = \sum_i^{\infty} c_i(\vec{p}) e^{-E_i(\vec{p})t_s} \quad \text{and}$$

$$C^\mu(\Gamma_k, \vec{q}; t_s, t_{ins}) = \sum_{i,j}^{\infty} A_{\mu}^{i,j}(\Gamma_k, \vec{q}) e^{-E_i(\vec{0})(t_s - t_{ins}) - E_j(\vec{q})t_{ins}}$$

- The nucleon charges are obtained at zero momentum transfer and the ratio at large times gives:

$$R_{A,S,T}^\mu(t_s, t_{ins}) = \frac{C^\mu(t_s, t_{ins})}{C(t_s)} \rightarrow g_{A,S,T}$$

- To extract the charge from the data we use:

$$g_{A,S,T} = \frac{A_{\mu}^{0,0}(\vec{0})_{A,S,T}}{c_0(\vec{0})}$$

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Model Average

- Since the optimal fit range in t_s and t_{ins} may vary for each case, we explore a wide parameter space in the fitting ranges.
- From each combination of the varied parameters, we obtain a different result. We average the results using the Akaike Information Criterion (AIC).
- For each fit i , we associate a weight w_i , which we define as:

$$\log(w_i) = -\frac{\chi_i^2}{2} + N_{\text{dof},i}$$

- We use the weights to define the probability:

$$p_i = \frac{w_i}{Z} \quad \text{with} \quad Z = \sum_i w_i$$

- We define the *Model Average (MA)* value of an observable \mathcal{O} as:

$$\langle \mathcal{O} \rangle_{MA} = \sum_i \bar{\mathcal{O}}_i p_i \quad \text{with} \quad \sigma_{MA}^2 = \sum_i (\sigma_i^2 + \bar{\mathcal{O}}_i^2) p_i - \langle \mathcal{O} \rangle_{MA}^2$$

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- Our data consists of two- and three-point functions from three $N_f = 2 + 1 + 1$ twisted mass fermion ensembles at the physical point, generated by the *Extended Twisted Mass Collaboration (ETMC)*.

cB211.072.64		
749 configurations		
t_s/a	t_s [fm]	n_{src}
8	0.64	1
10	0.80	2
12	0.96	5
14	1.12	10
16	1.28	32
18	1.44	112
20	1.60	128
Nucleon 2pt 477		

cC211.060.80		
400 configurations		
t_s/a	t_s [fm]	n_{src}
6	0.41	1
8	0.55	2
10	0.69	4
12	0.82	10
14	0.96	22
16	1.10	48
18	1.24	45
20	1.37	116
22	1.51	246
Nucleon 2pt 650		

cD211.054.96		
496 configurations		
t_s/a	t_s [fm]	n_{src}
8	0.46	1
10	0.57	2
12	0.68	4
14	0.80	8
16	0.91	16
18	1.03	32
20	1.14	64
22	1.25	16
24	1.37	32
26	1.48	64
Nucleon 2pt 480		

	cB211.072.64				cC211.060.80				cD211.054.96			
	749 configurations				400 configurations				496 configurations			
Flavour	N_{defl}	N_r	N_{Had}	N_{vect}	N_{defl}	N_r	N_{Had}	N_{vect}	N_{defl}	N_r	N_{Had}	N_{vect}
Light	200	1	512	6144	450	1	512	6144	0	8	512	49152
Strange	0	2	512	12288	0	4	512	24576	0	4	512	24576
Charm	0	12	32	4608	0	1	512	6144	0	1	512	6144

Statistics used for the connected two- and three-point functions (top) and disconnected loops (bottom)

- The nucleon isovector axial charge, g_A^{u-d} , governs the rate of weak decay of neutrons into protons.
- The flavor diagonal g_A^f , are related to the intrinsic spin carried by the quarks in the nucleon.
- The isovector axial-vector operator is given by:

$$O_A^{u-d} \equiv A_\mu = \bar{u}\gamma_\mu\gamma_5u - \bar{d}\gamma_\mu\gamma_5d$$

Axial Charge

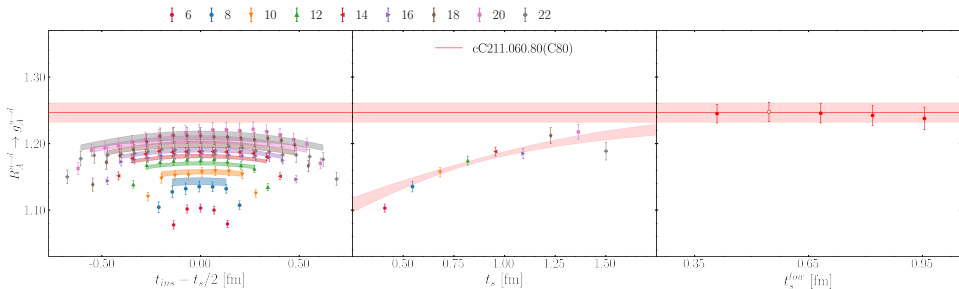
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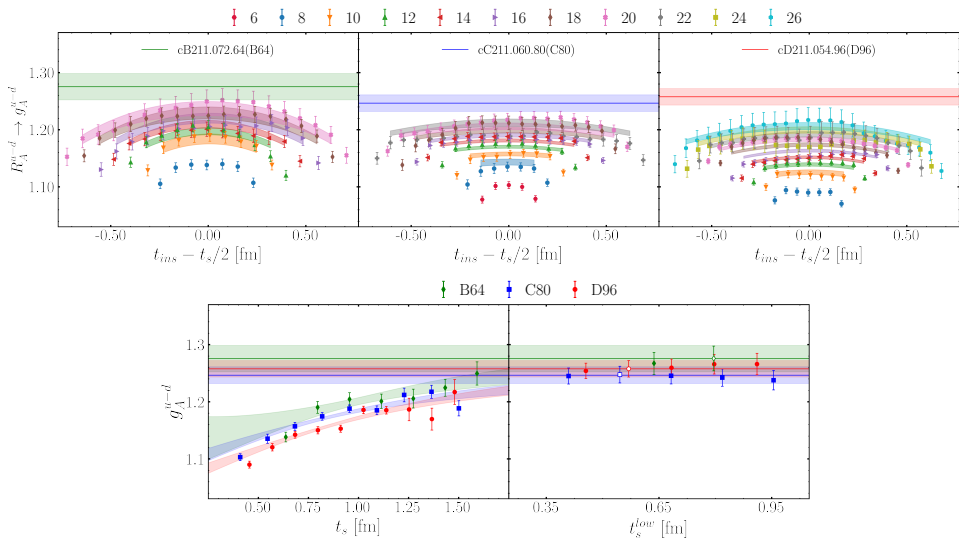
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Axial Charge



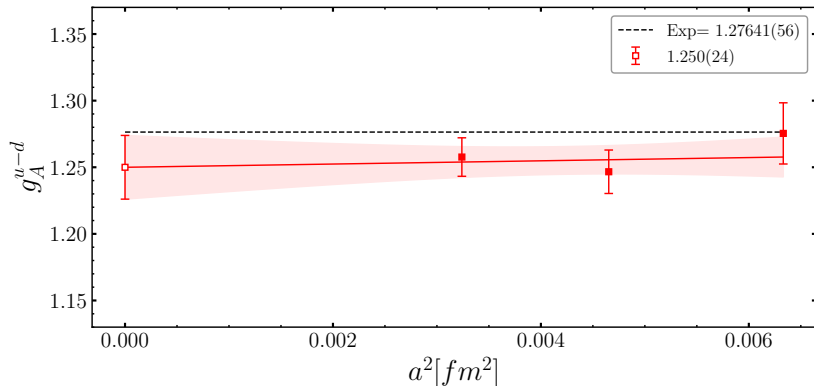
See G. Koutsou, after the lunch break for more details.

Axial Charge



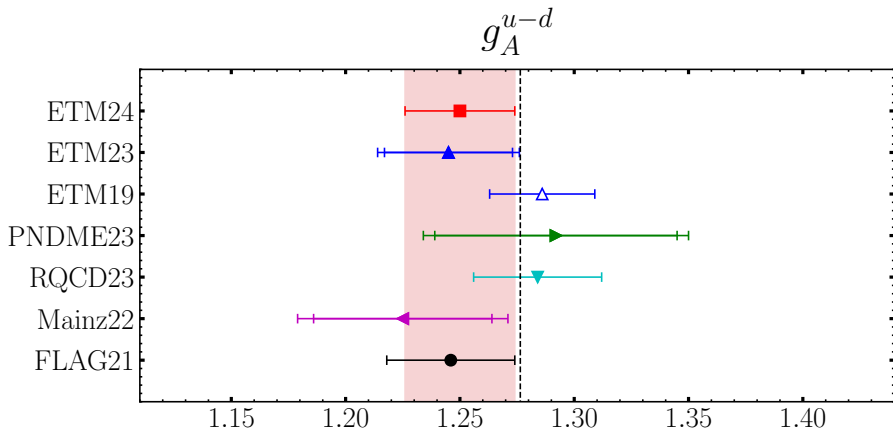
Analysis of the isovector axial charge g_A^{u-d} for all three ensembles.

Axial Charge - Preliminary Results



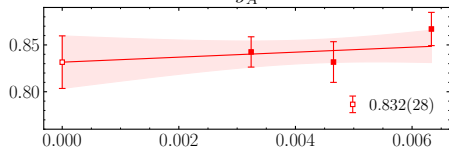
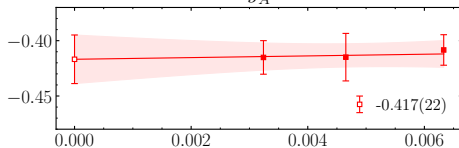
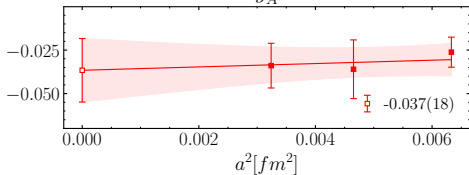
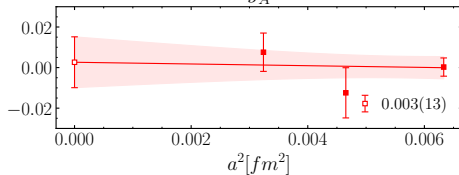
The continuum limit of the isovector axial charge g_A^{u-d} .

Axial Charge - Preliminary Results



Comparison of the results of this work (ETM24) with previous ETMC results and other lattice works.

Axial Charge - Preliminary Results

 g_A^u  g_A^d  g_A^s  g_A^c 

Continuum limit extrapolations for single flavor axial charges.

- Scalar charges are only known poorly from phenomenology.
- They can put limits on the existence of BSM interactions and are relevant in dark matter searches.
- The isovector scalar charge g_S^{u-d} measures the proportionality constant between the neutron-proton mass splitting and the up and down quark mass splitting.
- The nucleon matrix element of the single-flavor scalar operator g_S^f is directly connected to the quark content of the nucleon.
- We define the nucleon σ -terms as

$$\sigma^f = m_f \langle N | \bar{q}_f q_f | N \rangle, \quad \sigma^{u+d} = m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle$$

for a given quark q_f of flavor f , or for the isoscalar combination.

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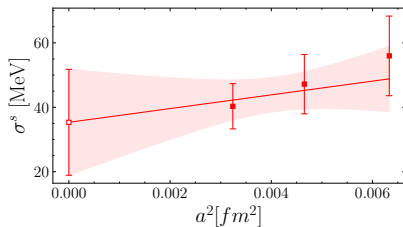
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σ -terms - Preliminary Results

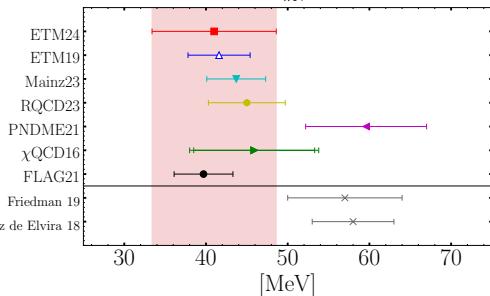
$$\sigma_{\pi N} = 41.0(7.6) \text{ MeV}$$

$$\sigma_s = 35(16) \text{ MeV}$$

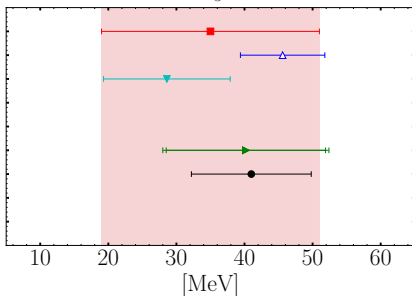
$$\sigma_c = 82(29) \text{ MeV}$$



$\sigma_{\pi N}$



σ_s



Comparison of the πN and strange sigma term results of this work (ETM24) with previous ETMC results and other lattice works.

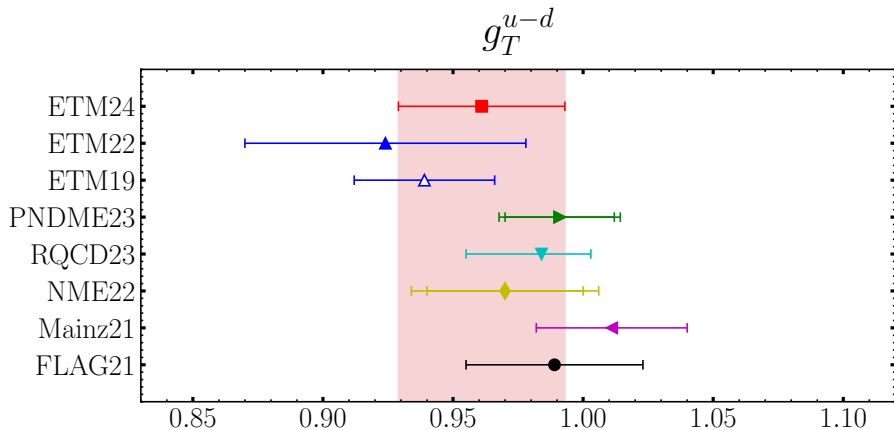
- The tensor charge is the first Mellin moment of the transversity parton distribution function (PDF).
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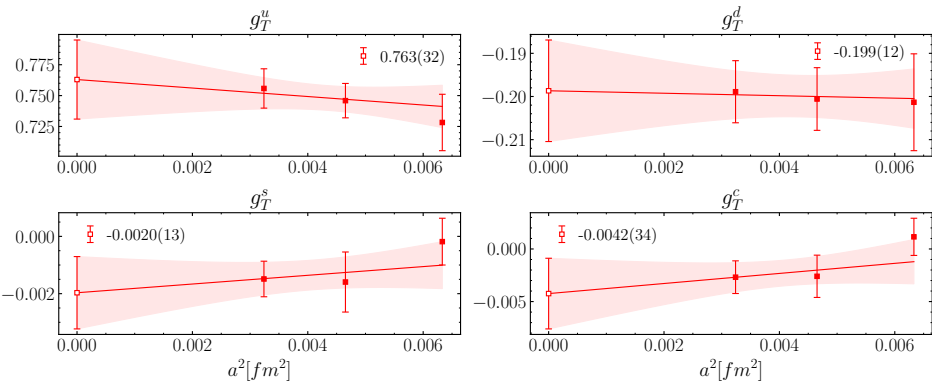
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Tensor Charge - Preliminary Results



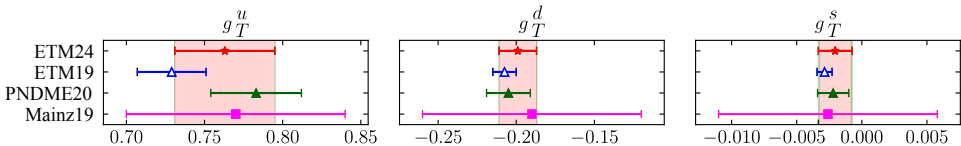
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Tensor Charge - Preliminary Results



Continuum limit extrapolations for single flavor tensor charges.

Tensor Charge - Preliminary Results



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Summary and Outlook

- In this work we used three $N_f = 2 + 1 + 1$ physical point ensembles to calculate the nucleon axial, scalar and tensor charges as well as the nucleon σ -terms.
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Summary and Outlook

- In this work we used three $N_f = 2 + 1 + 1$ physical point ensembles to calculate the nucleon axial, scalar and tensor charges as well as the nucleon σ -terms.
- Our results are in agreement with other lattice works and with the experimental value the case of g_A^{u-d} .
- Future analysis will include a fourth ensemble at smaller value of lattice spacing.

Nucleon axial, tensor, and scalar charges and σ -terms from lattice QCD

Thank you for your attention!



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