# Isovector Axial and Pseudoscalar Form Factors From Twisted Mass Lattice QCD at the Physical Point

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# Nucleon Structure with Physical Point Ensembles

**Outline**

- Ensembles
- Lattice setup and statistics
- Axial and pseudoscalar form factors
  - Excited state analysis
  - Continuum extrapolation
  - $Q^2$ – dependence
  - PCAC relation and pion pole dominance
- Conclusions

Extended Twisted Mass Collaboration

*ETM Collaboration*

*Cyprus* (Univ. of Cyprus, Cyprus Inst.), *Germany* (Berlin/Zeuthen, Bonn, Wuppertal), *Italy* (Rome I, II, III, Parma), *Poland* (Poznan), *Switzerland* (Bern), *US* (Temple, PA)

# Axial Form Factors

**Matrix element of the axial-vector current:**

$$\langle N(p', s')|A^3_\mu|N(p,s)\rangle = \bar{u}(p', s')\mathcal{O}^\mu u(p,s)$$

$$\mathcal{O}^\mu = \gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m_N}G_P(q^2), \quad q = p' - p$$

**Axial (**$G_A$**)** and **Induced Pseudoscalar (**$G_P$**)** form factors

- Axial charge $g_A = G_A(q^2{=}0)$
- Form factors known to less accuracy experimentally compared to, e.g. Electromagnetic
  - Via elastic scattering: $\nu_\mu + n \to \mu^- + p$
  - Via charged pion electroproduction
- Required in neutrino oscillation experiments. Traditionally modelled with dipole:

$$G_A(Q^2) = \frac{g_A}{(1 + Q^2/M_A^2)^2}$$

- Also of interest: $g_P^* = \frac{m_\mu}{2m_N}G_P(0.88m_\mu^2)$

# Axial Form Factors

**Matrix element of the pseudo scalar current:**

$$\langle N(p', s')|P|N(p, s)\rangle = \bar{u}(p', s')G_5(q^2)\gamma_5 u(p, s)$$

**Pseudoscalar (**$G_5$**)** form factor

– Goldberger Treiman relation

$$G_A(q^2) + \frac{q^2}{4m_N^2}G_P(q^2) = \frac{F_\pi m_\pi^2}{m_N(m_\pi^2 - q^2)}G_{\pi NN}(q^2)$$

where

$$m_q G_5(q^2) = \frac{F_\pi m_\pi^2}{m_\pi^2 - q^2}G_{\pi NN}(q^2)$$

– Requiring that close to the pole, the divergence cancels on both sides

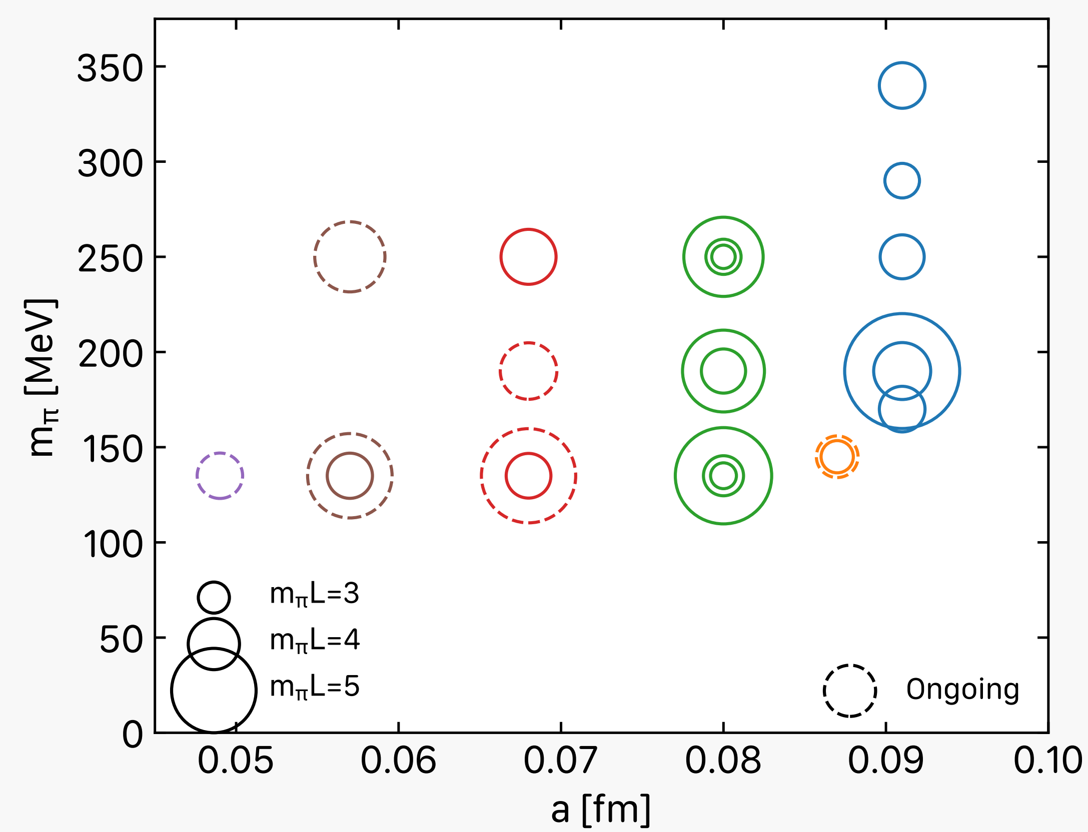
$$G_P(q^2) = \frac{4m_N^2}{m_\pi^2 - q^2}G_A(q^2)\Big|_{q^2 \to m_\pi^2}$$

Similarly we can relate $G_A$ to $G_{\pi NN}$ and $G_P$ to $G_5$

# Ensembles

**Three $N_f$=2+1+1 ensembles at physical pion mass**

| Ens. ID (abbrv.) | Vol. | $a$ [fm] |
|---|---|---|
| cB211.072.64 (cB64) | 64×128 | 0.07957(13) |
| cC211.060.80 (cC80) | 80×160 | 0.06821(13) |
| cD211.054.96 (cD96) | 96×192 | 0.05692(12) |




- Lattice spacings quoted as determined from meson sector
  - See paper on $a_\mu^{\mathrm{HVP}}$ for details – Phys. Rev. D107, 074506 • arXiv:2206.15084 [hep-lat].
- Axial form-factors for cB64 reported:
  - Isovector – Phys.Rev.D 103 (2021) 3, 034509 • arXiv:2011.13342 [hep-lat]
  - Flavor decomposition – Phys. Rev. D104 (2021) 074503 • arXiv:2106.13468 [hep-lat]
- **Here:** Isovector; continuum limit; more thorough excited state analysis;
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Poster on ETMC ensembles status by B. Kostrzewa, Tuesday @ 17:15

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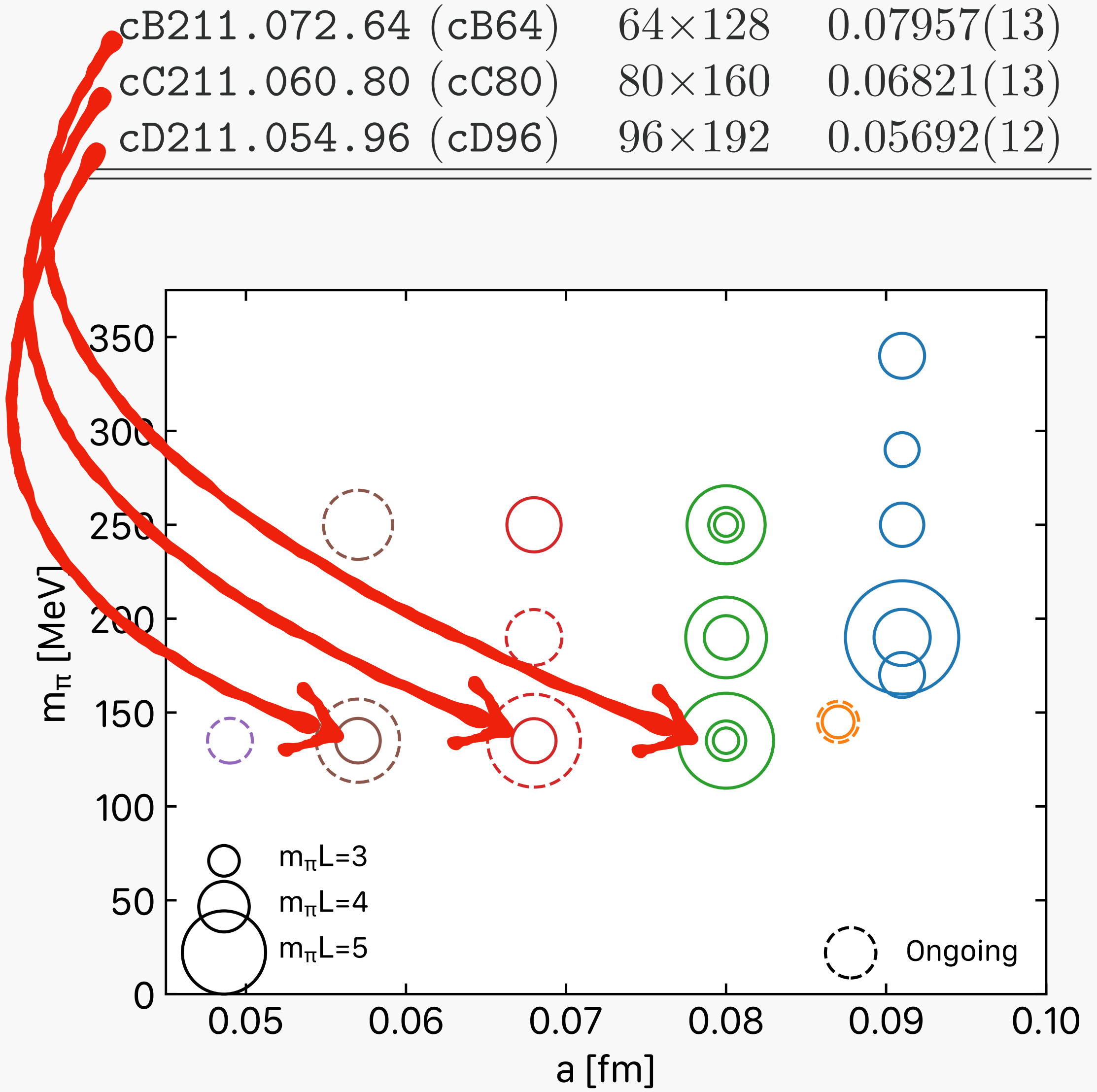


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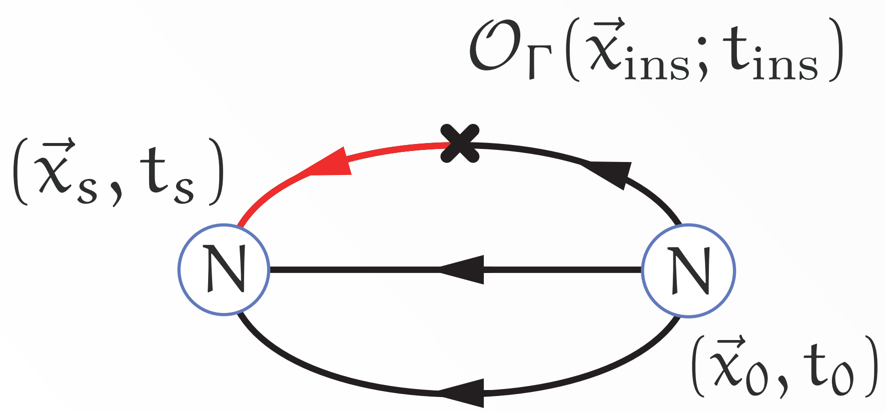
# Matrix elements on the Lattice

General three-point function:

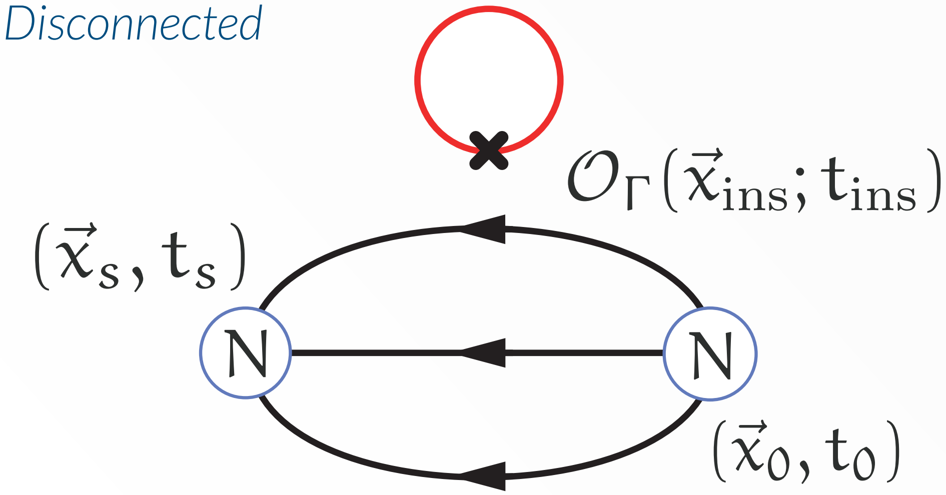
$$G_\Gamma(P;\vec{q};t_s,t_{ins}) = \sum_{\vec{x}_s\,\vec{x}_{ins}} e^{-i\vec{q}.\vec{x}_{ins}} P^{\alpha\beta} \langle \bar{\chi}_N^{\beta}(\vec{x}_s;t_s)|\mathcal{O}^{\Gamma}(\vec{x}_{ins};t_{ins})|\chi_N^{\alpha}(\vec{0};0)\rangle$$

Isovector case → *connected contributions* using *fixed-sink* sequential inversions

Connected




Disconnected




*Disconnected contributions*

– Vanish for isovector case as $a \to 0$

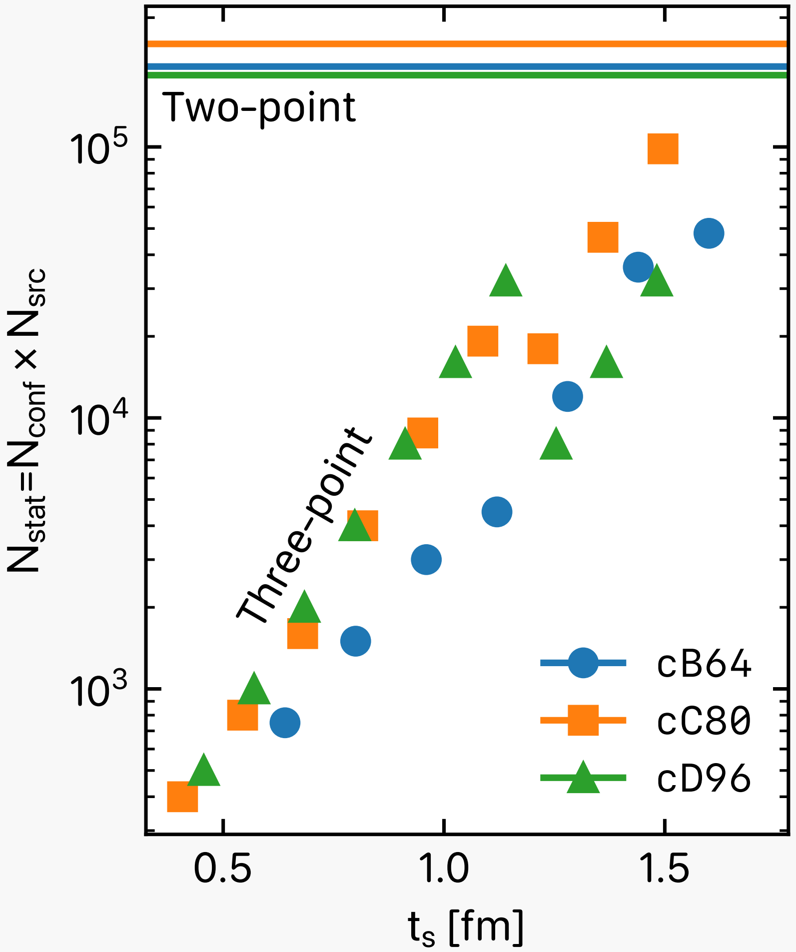
– Not included here. Inclusion may change approach to continuum limit

See also talk by Yan Li, Tuesday @ 11:55 – "*Investigation of πN contributions to nucleon matrix elements*"

# Statistics

$$R_\Gamma(P;\vec{q};t_s;t_{ins}) = \frac{G_\Gamma(P;\vec{q};t_s;t_{ins})}{G(\vec{0};t_s)}\sqrt{\frac{G(\vec{p};t_s-t_{ins})G(\vec{0};t_{ins})G(\vec{0};t_s)}{G(\vec{0};t_s-t_{ins})G(\vec{p};t_{ins})G(\vec{p};t_s)}}$$

*Connected:* Increasing $N_{src}$ with $t_s$



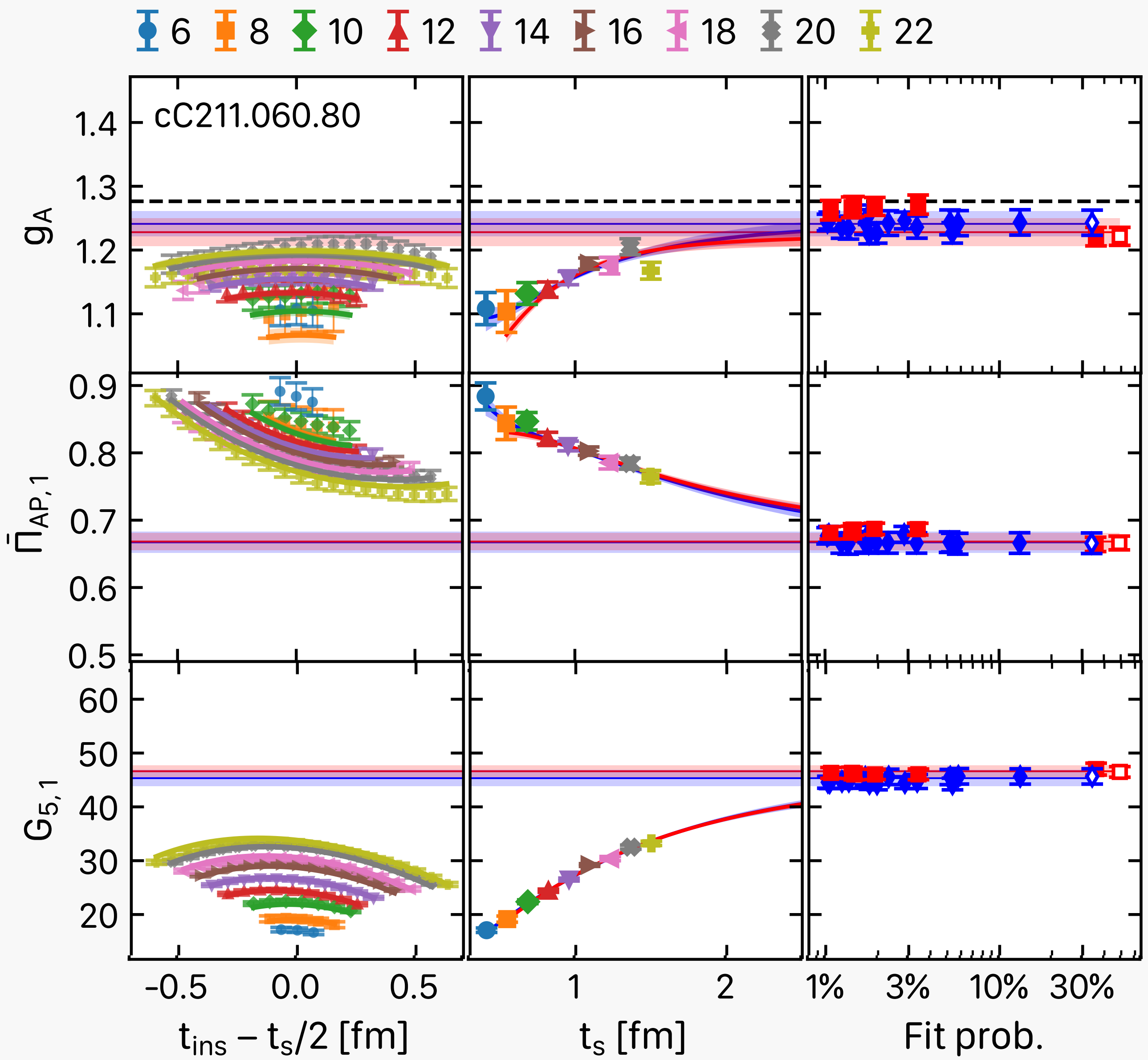

*Ideally:* Aim for *constant statistical errors* over all values of $t_s$ of a given ensemble

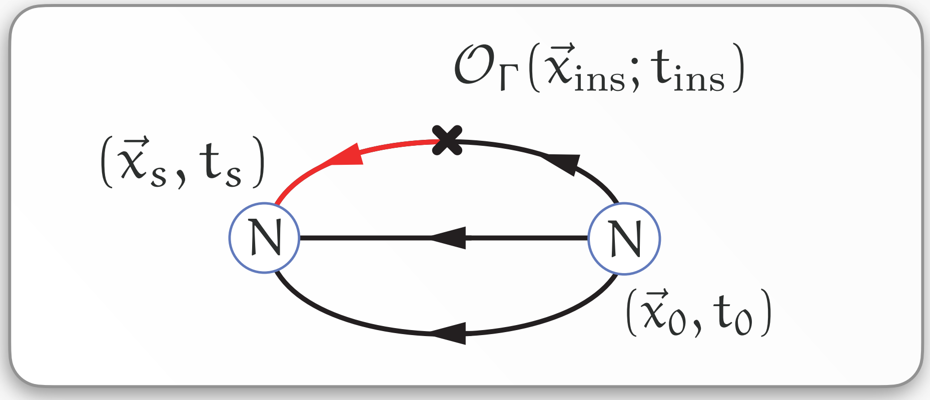
- Robust analysis of excited states: summation method, two- or three-state fits, etc.

*Two-point function:* High number of sources per config

- Improves signal for multi-state fits
- Improves signal of disconnected contributions

# Correlation function analysis

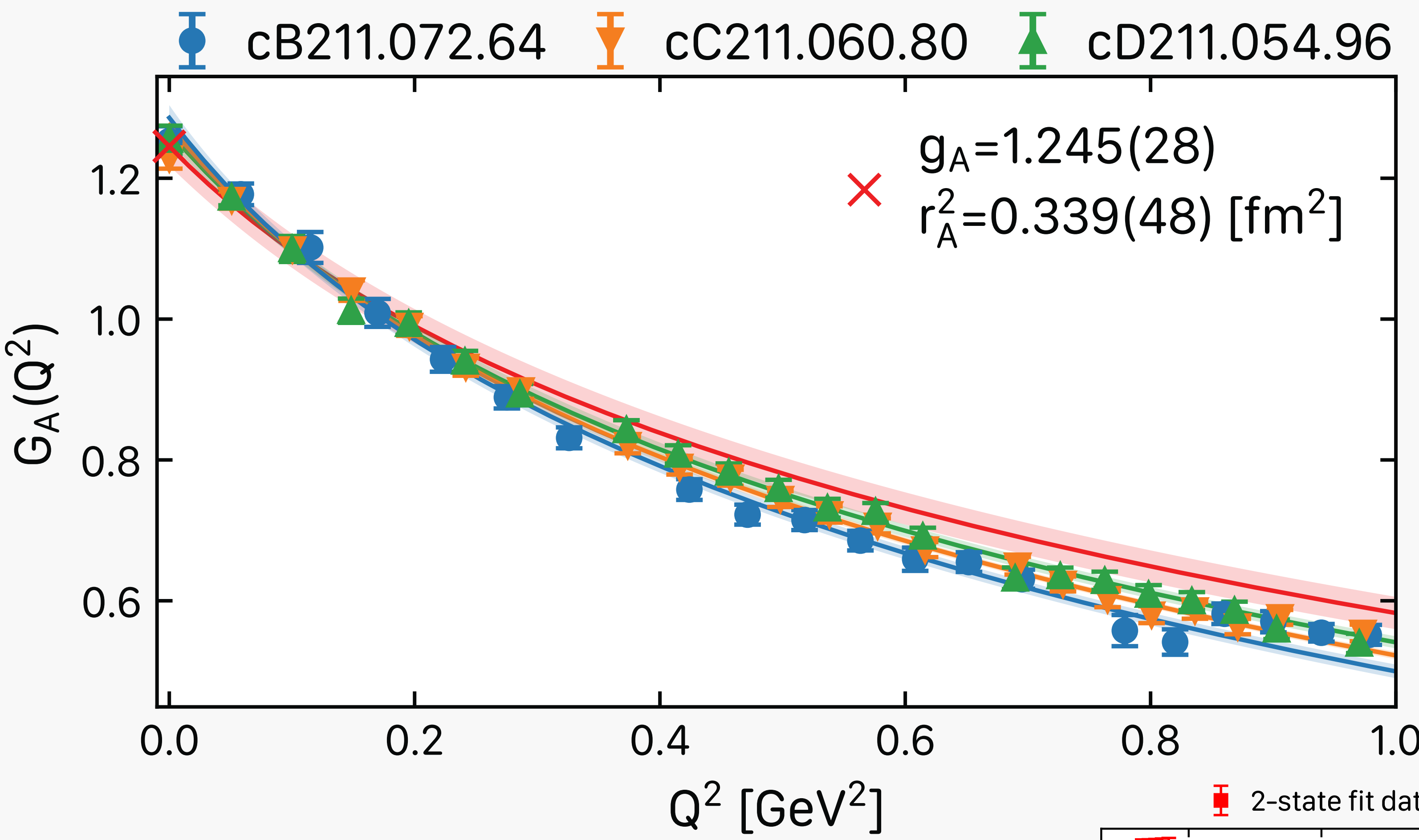



- 0 and 1 unit of momentum fit together first to extract $m_N$
- Temporal component of axial current included
- Larger momenta fitted with $m_N$ as prior
- Both two- and three-state fits

- Example from intermediate lattice spacing $a \simeq 0.07$ fm
- 9 separations, $t_s \simeq 0.4 - 1.5$ fm

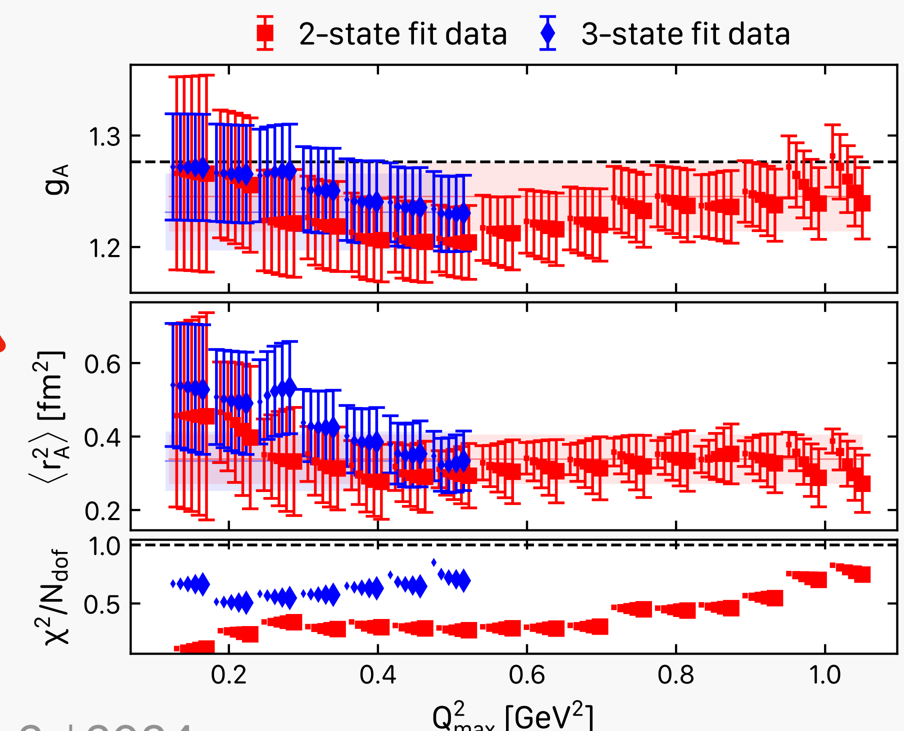
# Axial Form Factor



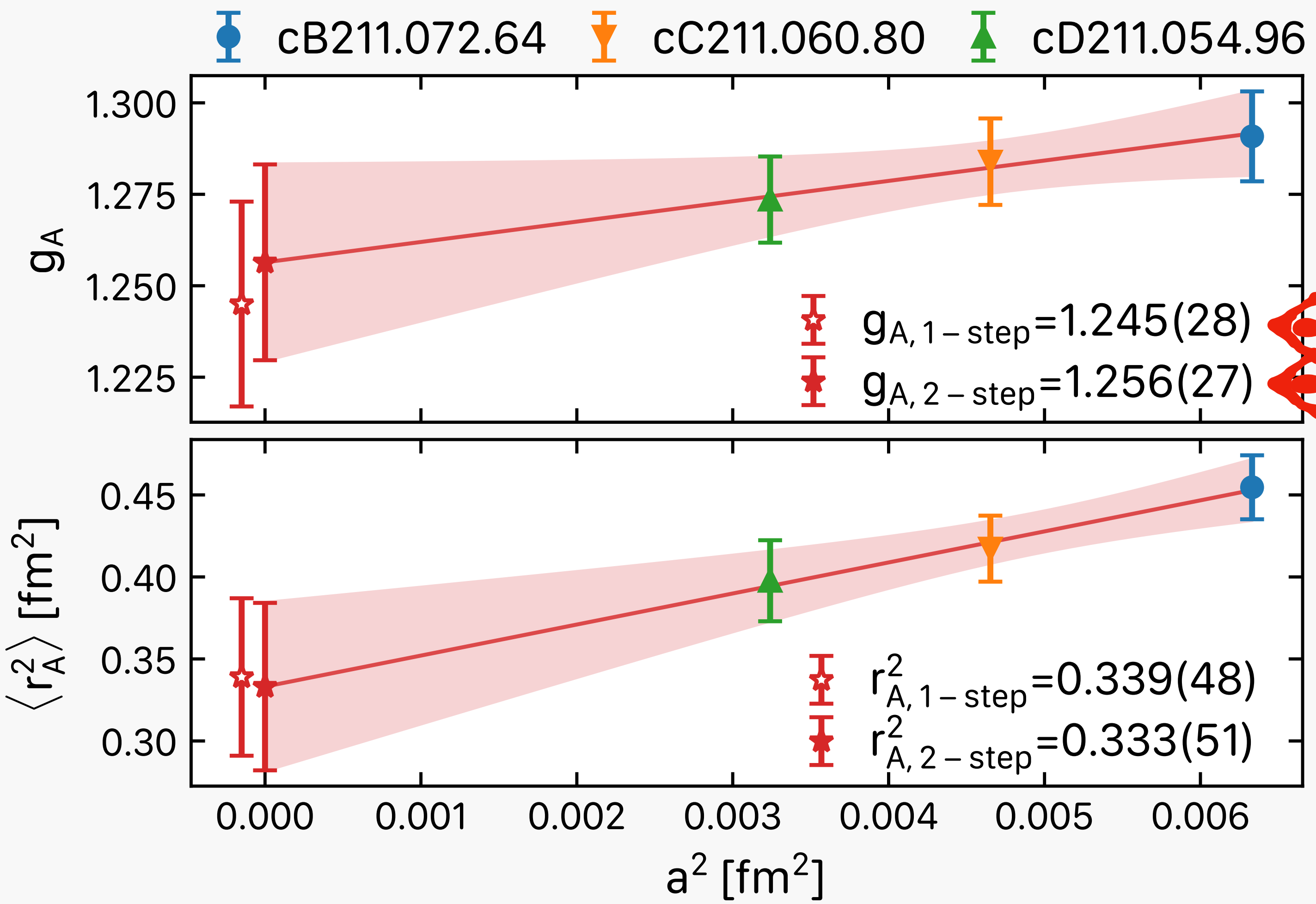

$z$-expansion; convergence after 3rd order

$$G(Q^2) = \sum_{k=0}^{k_{max}} a_k \, z^k(Q^2),$$

$$z(Q^2) = \frac{\sqrt{t_{cut} + Q^2} - \sqrt{t_{cut} + t_0}}{\sqrt{t_{cut} + Q^2} + \sqrt{t_{cut} + t_0}}$$



# Axial charge and radius

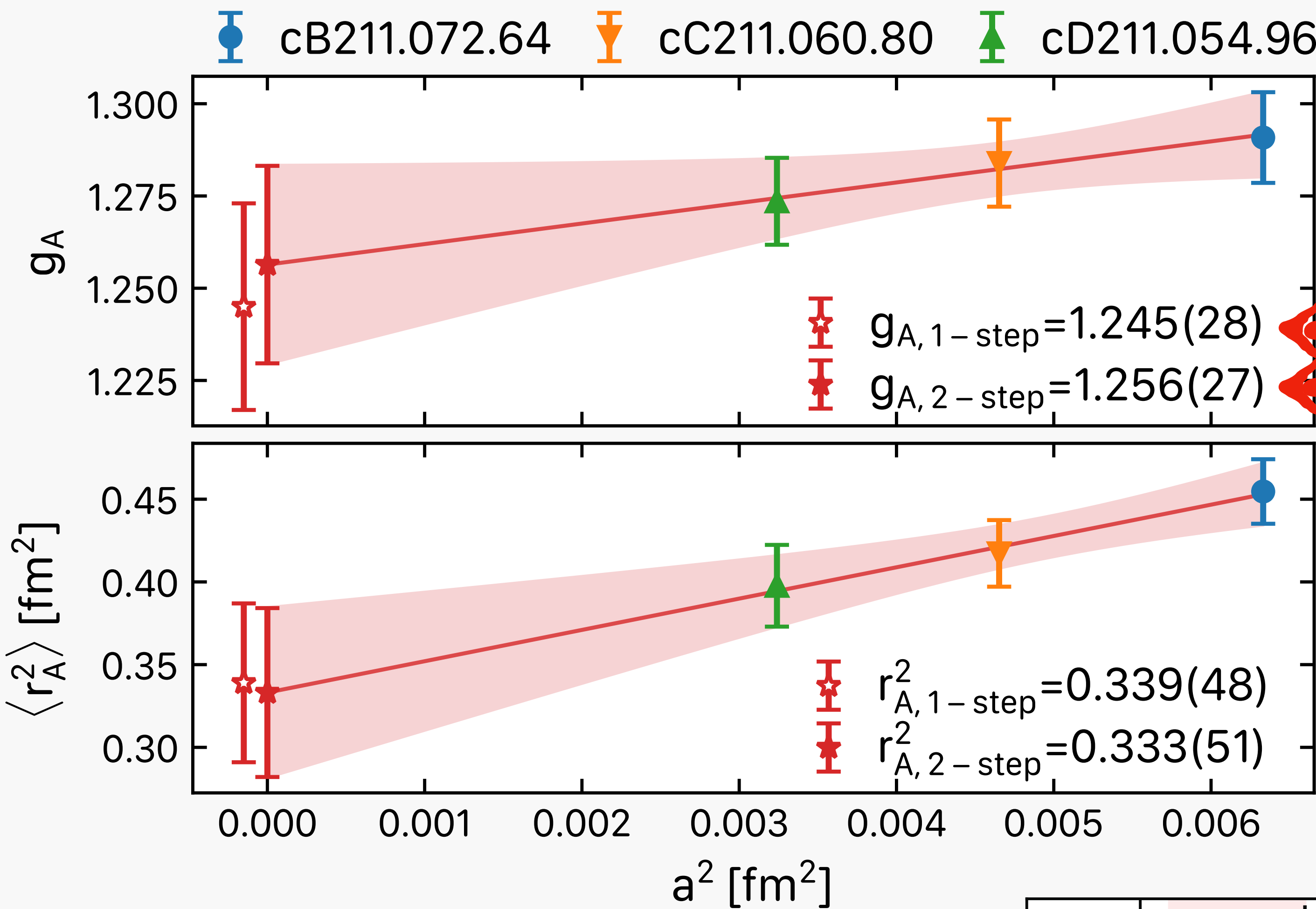



Fit $Q^2$- and $a$-dependence together

Fit $Q^2$-dependence for each $a$; then continuum limit

Christo Iona, Monday @ 11:15 – *"Nucleon axial, tensor, and scalar charges and σ-terms from lattice QCD"*

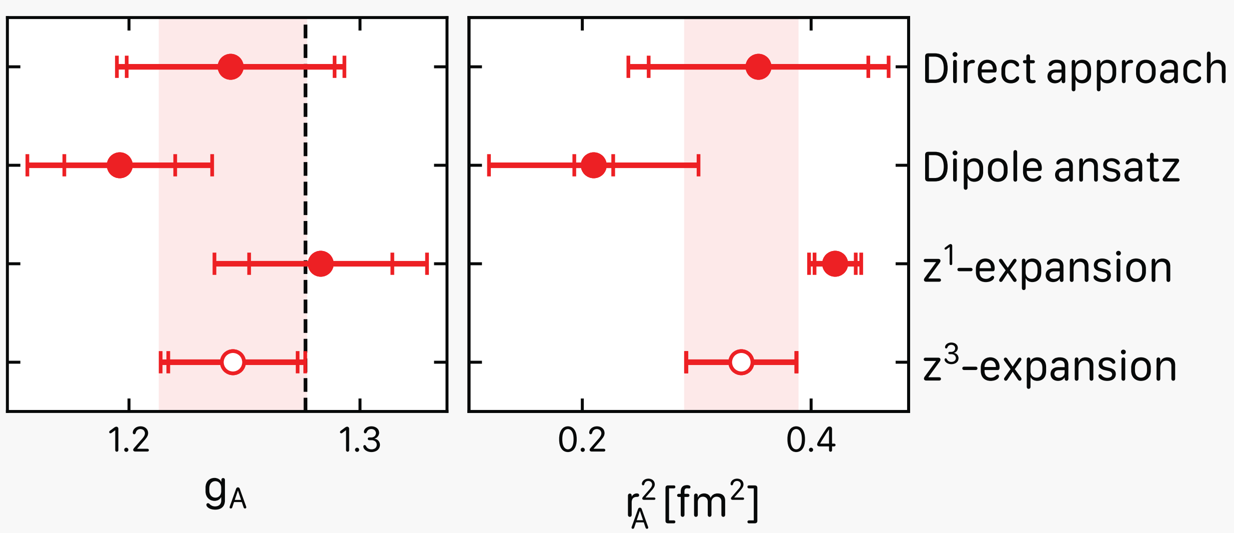
# Axial charge and radius




Fit $Q^2$- and $a$-dependence together

Fit $Q^2$-dependence for each $a$; then continuum limit

- $z^3$-expansion
- Two-state fits to correlators
- Agreement with dipole and "direct"

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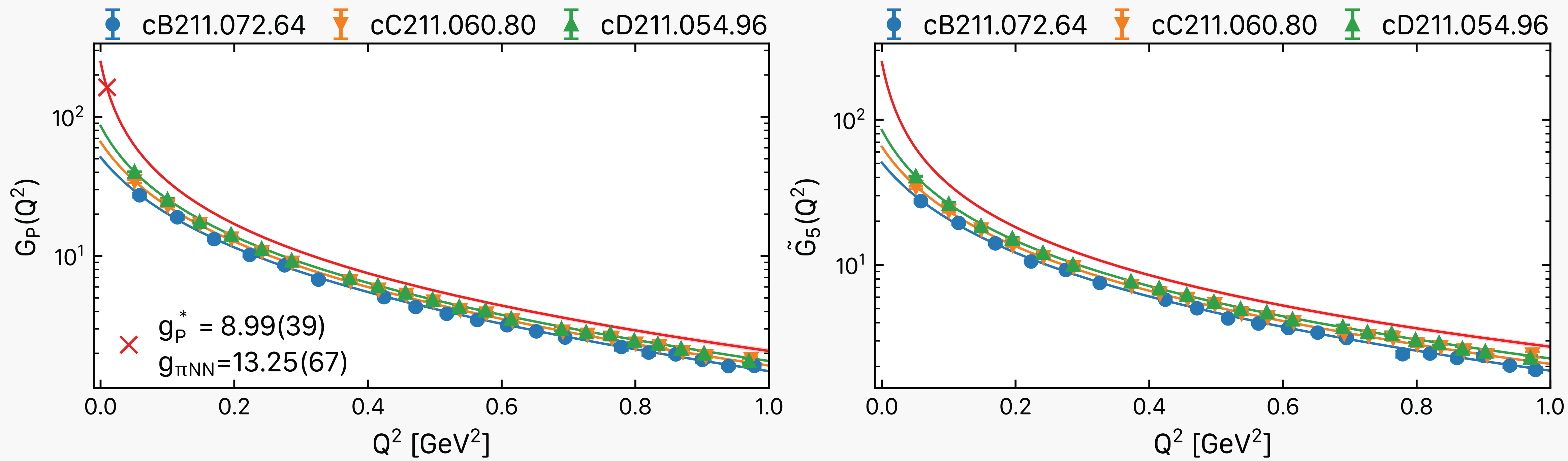
# Induced pseudo scalar & Pseudo scalar




- Note the logarithmic scale
- Relatively large cut-off effects, especially at low $Q^2$

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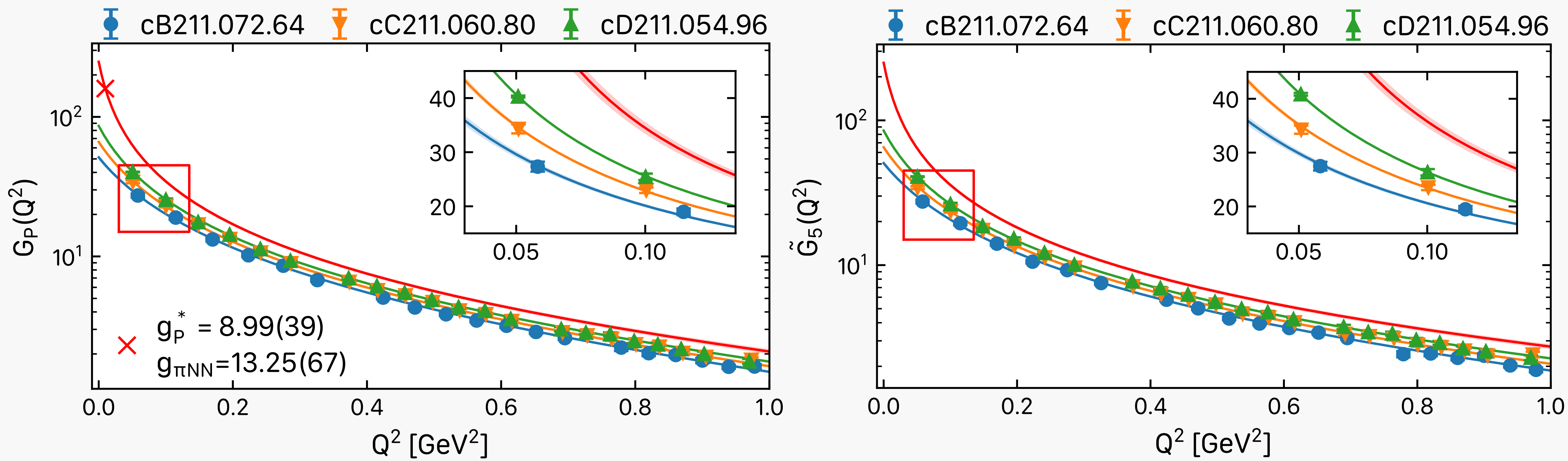


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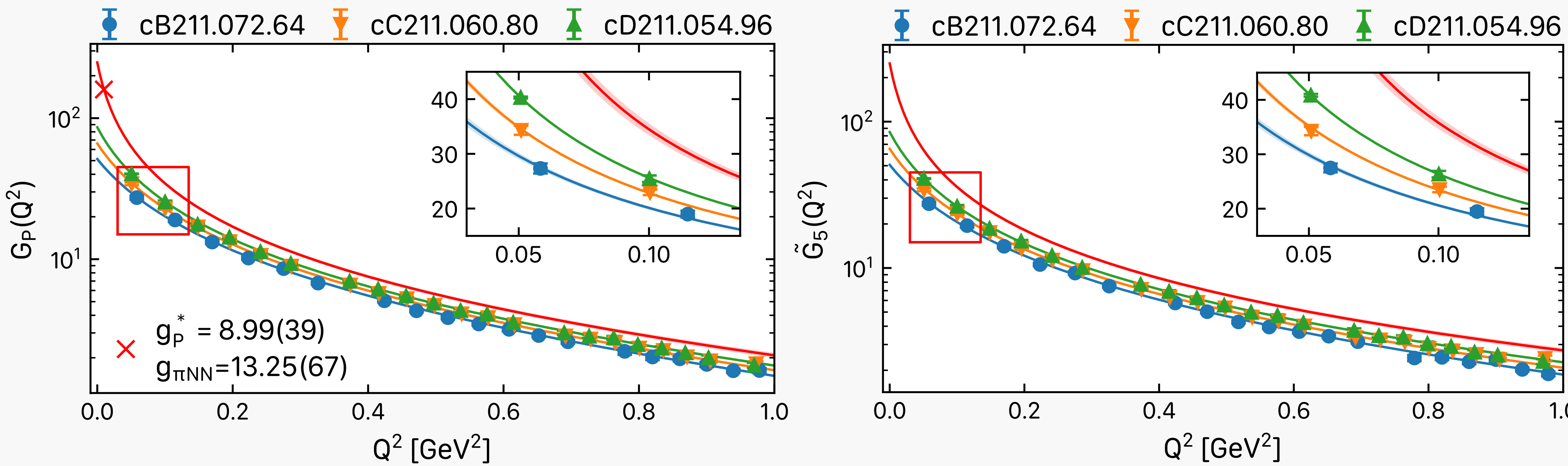



$$r_{PPD,2}(Q^2) = \frac{4m_N}{m_\pi^2}\frac{m_q G_5(Q^2)}{G_P(Q^2)} = \frac{4m_N^2}{m_\pi^2}\frac{G_A(Q^2)}{G_P(Q^2)} - \frac{Q^2}{m_\pi^2} = 1 + \left(\frac{\langle r_A^2\rangle m_\pi^2}{6} - \Delta_{GT}\right)\left(1 + \frac{Q^2}{m_\pi^2}\right)$$

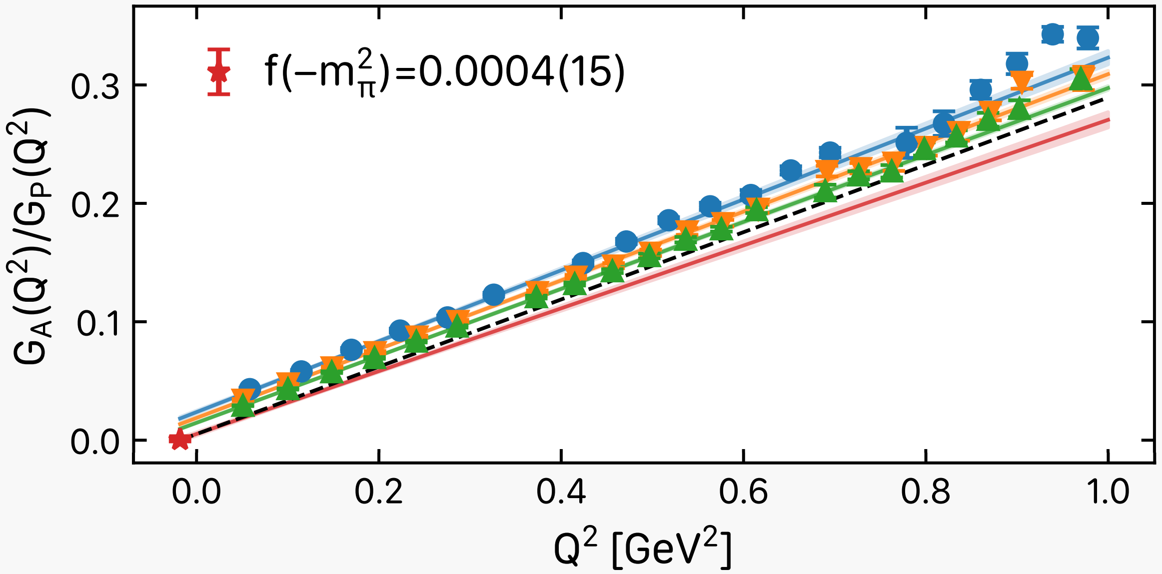
PCAC

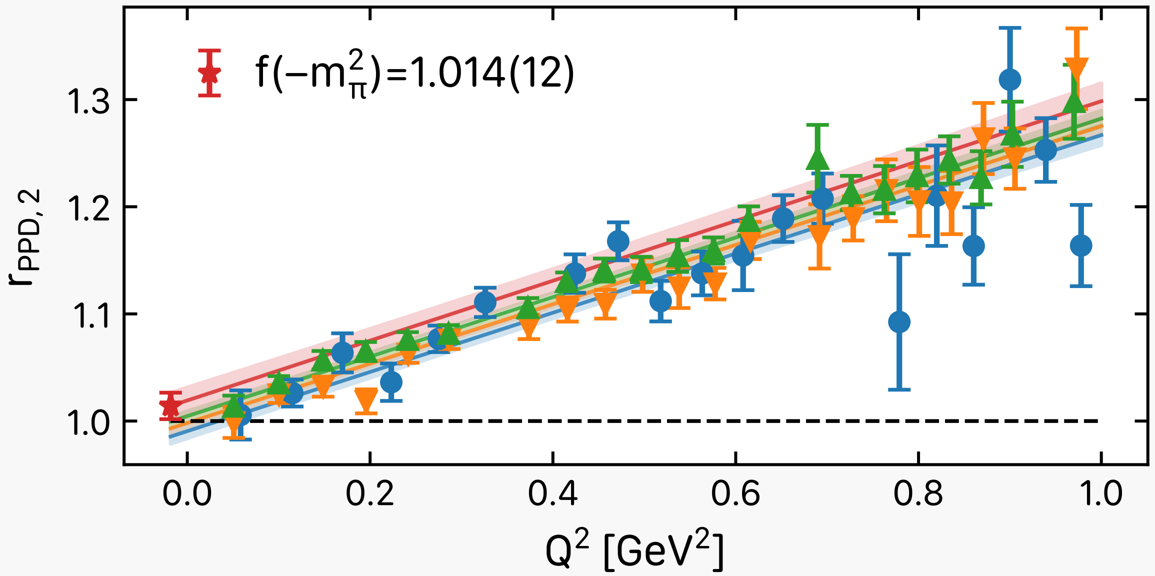
Expansion of $G_A(Q^2)$

$$\Delta_{GT} = 1 - \frac{g_A m_N}{g_{\pi NN} F_\pi}$$

$$\frac{G_A(Q^2)}{G_P(Q^2)} = \frac{Q^2 + m_\pi^2}{4m_N^2}\Bigg|_{Q^2 \to -m_\pi^2}$$

# Induced pseudo scalar & Pseudo scalar

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$$\frac{G_A(Q^2)}{G_P(Q^2)} = \frac{Q^2 + m_\pi^2}{4m_N^2}\bigg|_{Q^2\to -m_\pi^2}$$

$$\Delta_{GT} = -0.0213(38) \approx 2\%$$

# Axial & Pseudoscalar form factors

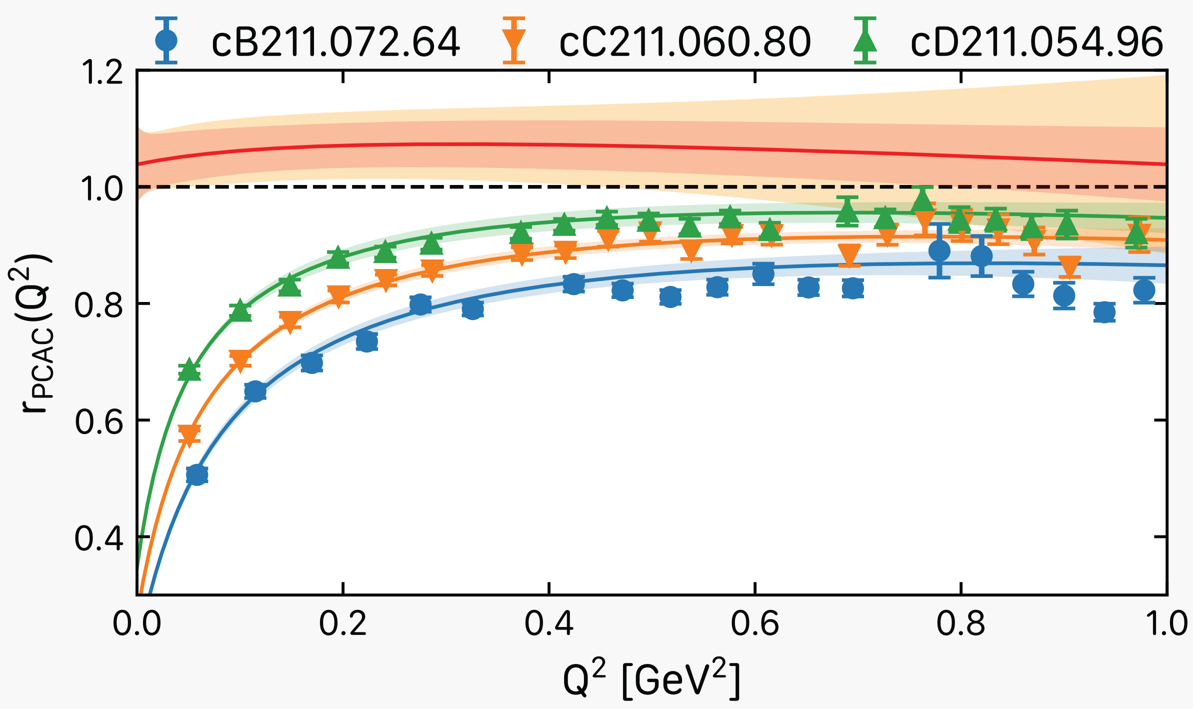
Check relation between axial and pseudoscalar form factors

– From PCAC relation

$$\partial^\mu A_\mu = 2m_q P$$

– Between nucleon states:

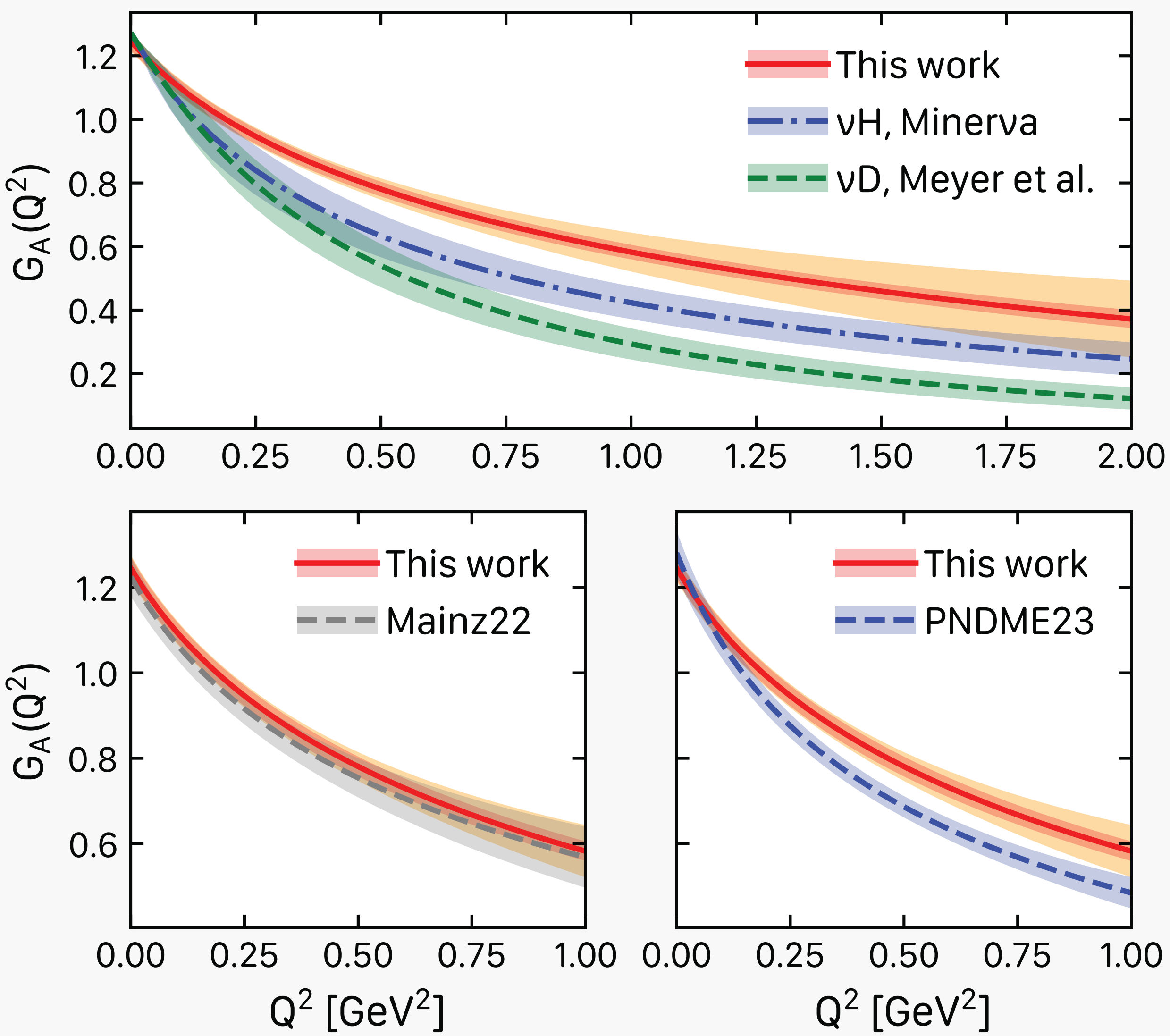
$$G_A(Q^2) - \frac{Q^2}{4m_N^2} G_P(Q^2) = \frac{m_q}{m_N} G_5(Q^2)$$

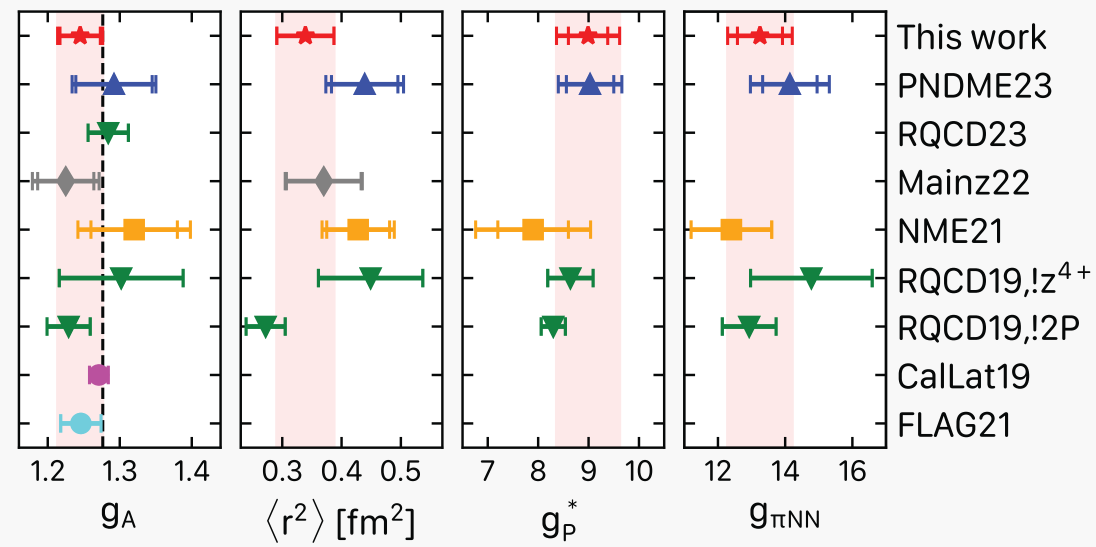



– Relation from PCAC restored at continuum limit ($a \to 0$)

# Axial & Pseudoscalar form factors

## Comparison with experiment and other lattice results




- Good agreement with other lattice results
- Note the MINERνA result (2023):
  - $r_A = 0.73(17)$ fm or
  - $(r_A)^2 = 0.53(25)$ fm$^2$

# Summary & Outlook

- Three twisted mass ensembles ➜ Continuum limit directly at physical poin
- Axial, induced pseudo scalar, and pseudo scalar form factors
- Multiple sink-source separations
  - high statistics with increasing separation
  - allow good two- and three-state fits to correlation functions
- PCAC relation
  - Not satisfied at finite $a$, but restored at continuum limit
- Relations arising from pion pole dominance assumption
  - Similarly to PCAC, restored at continuum limit
- Axial form factor, axial radius
  - Agreement with other lattice results
  - Predict slightly smaller axial radius compared to experiments

# Acknowledgements




Με τη συγχρηματοδότηση
της Ευρωπαϊκής Ένωσης

ΚΥΠΡΙΑΚΗ ΔΗΜΟΚΡΑΤΙΑ

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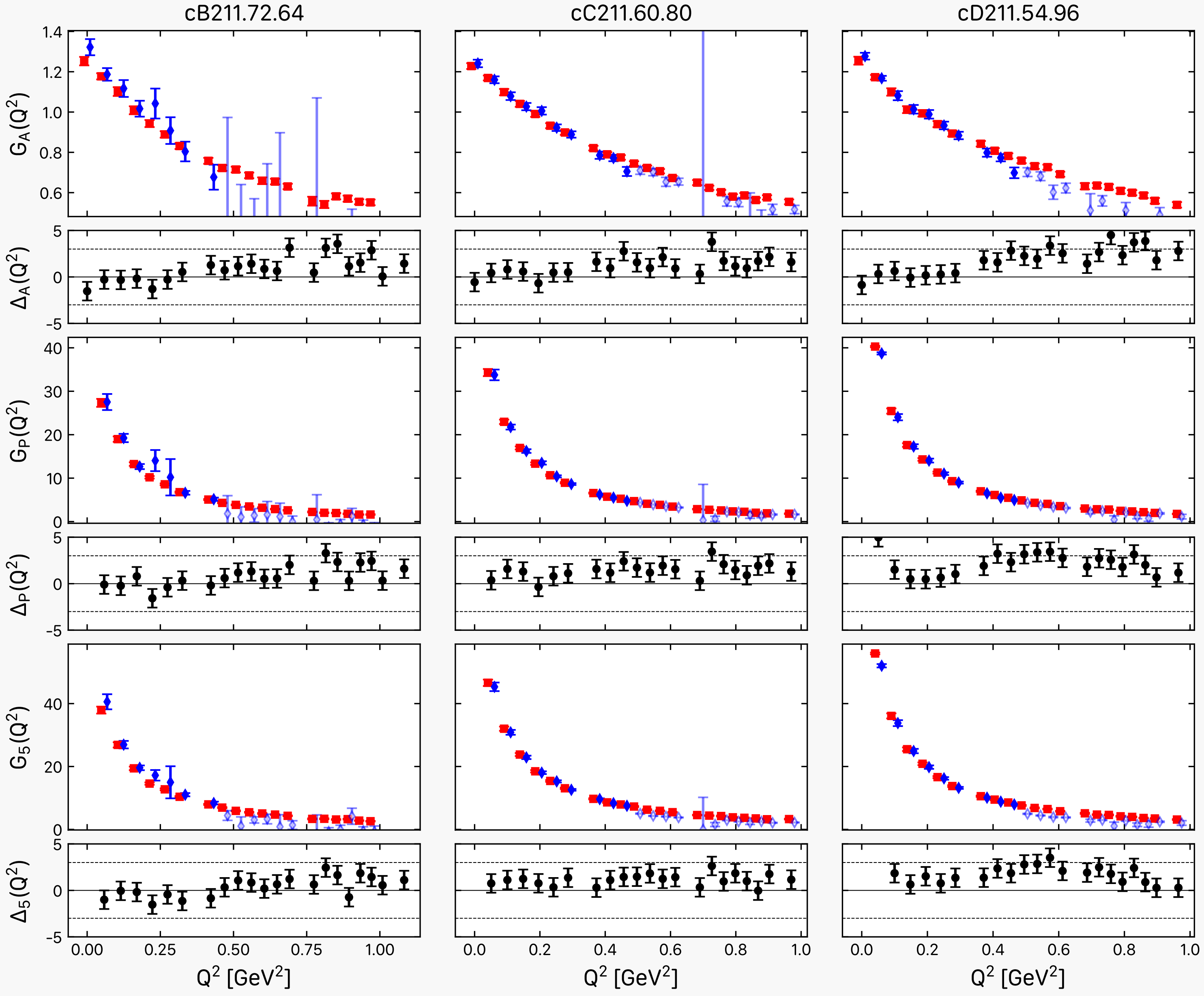
# Backup




- Two- vs Three-state fits
- Three-state fits stable for $Q^2 \lesssim 0.465\ \mathrm{GeV}^2$

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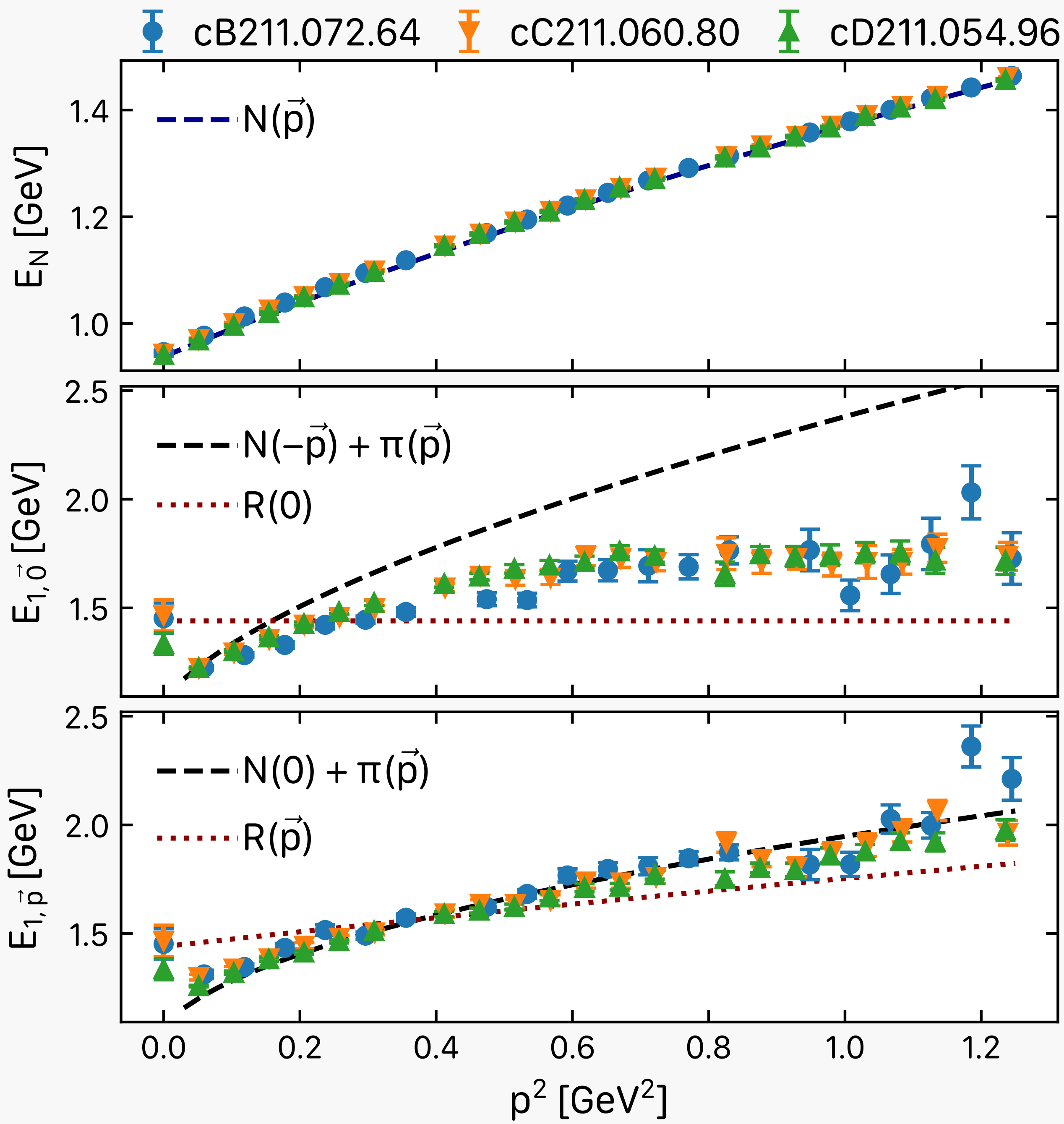
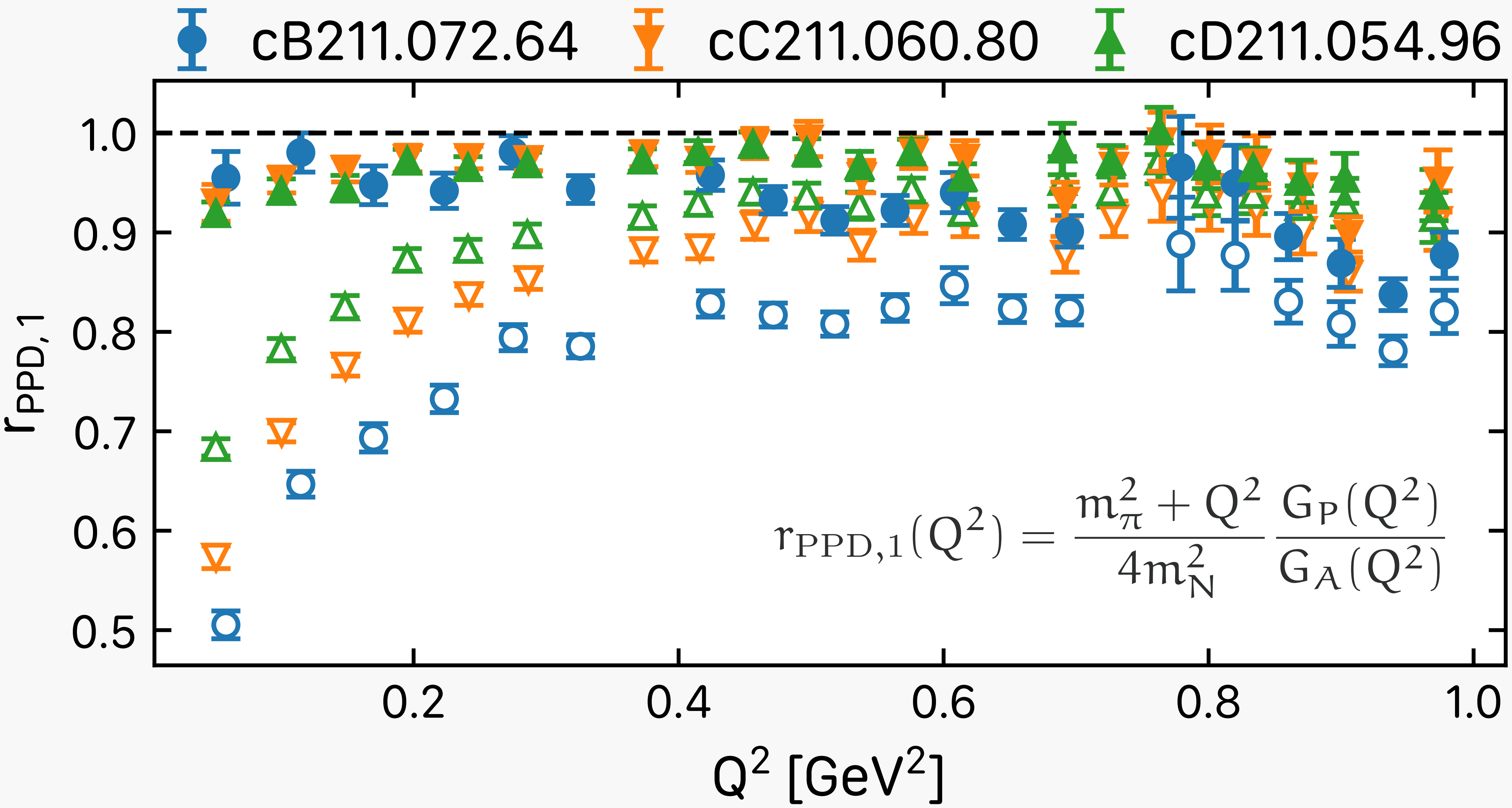



- From two-state fits to both two- and three-point functions

# Backup




- Open symbols: using unitary pion mass $m_\pi^{TM}$
- Filled symbols: using Osterwalder-Seiler pion mass $m_\pi^{OS}$