Update on pion scalar radii with $N_f = 2+1$ Clover-improved Wilson fermions

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Introduction

The pion scalar form factor and scalar radii with $N_f = 2+1$ quark flavors are given by

$$F_S^{\pi,f}(Q^2) = \langle \pi(p_f)|\mathcal{S}^f|\pi(p_i)\rangle\,, \qquad \langle r_S^2\rangle_\pi^f = \frac{-6}{F_S^{\pi,f}(0)} \cdot \left.\frac{dF_S^{\pi,f}(Q^2)}{dQ^2}\right|_{Q^2=0}\,,$$

where for $f = l, 0, 8$ we have

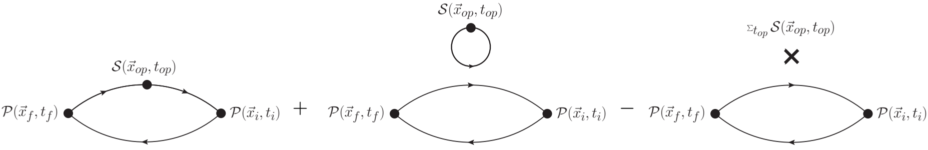
$$\begin{aligned} \mathcal{S}_l &= 2m_l\bar{l}l & \rightarrow \quad & F_S^{\pi,l}(Q^2) \quad \text{(light)} \\ \mathcal{S}_8 &= 2m_l\bar{l}l - 2m_s\bar{s}s & \rightarrow \quad & F_S^{\pi,8}(Q^2) \quad \text{(octet)} \\ \mathcal{S}_0 &= 2m_l\bar{l}l + m_s\bar{s}s & \rightarrow \quad & F_S^{\pi,0}(Q^2) \quad \text{(singlet)} \end{aligned}$$

They provide insight into the low-energy regime of QCD:

- At NLO in $SU(2)$ χPT $\langle r_S^2\rangle_\pi^l$ is parametrized by a single LEC $\bar{l}_4$. *Annals Phys. 158, 142 (1984)*
- At NLO in $SU(3)$ χPT $\langle r_S^2\rangle_\pi^{0,8,l}$ give access to f_0, L_4 and L_5. *Nucl. Phys. B250 (1985) 517-538*
- A few (mostly older) lattice calculations exist for $SU(2)$; very little is known for $SU(3)$.
  PRD 80 (2009) 034508 *PRD 89 (2014) 9, 094503* *PRD 93 (2016) 5, 054503* *PRD 105, 054502 (2022)*
- Arguably no calculation with controlled systematics; e.g. radii from linear slope between two momenta.
- $F_S^{\pi,f}(q^2)$ computationally demanding $\rightarrow$ significant **quark-disconnected contributions**.

Computational setup part I

Contributions from quark-connected and disconnected three-point functions:



- Sequential sink method for three-point functions: $1\,\mathrm{fm} \lesssim t_{\mathrm{sep}} \lesssim 3.5\,\mathrm{fm}$ and $\mathbf{p}_f \in \{(0,0,0), (1,0,0)\}$.
- Truncated solver method $\rightarrow$ speedup of 2-5. Phys.Rev. D91 (2015) 11, 114511
- Point-to-all forward propagators re-used for two-point functions.
- $\mathcal{O}(500)$ additional two-point functions for 2+1 disconnected diagrams per config.
- **On periodic BC boxes:**
  - Sources randomly distributed; 8 sources for quark-connected three-point functions per config.
- **On open BC boxes:**
  - Source setup symmetric around $T/2$ $\Rightarrow$ 4+4 sources for three-point functions at each t_{sep}.
  - Additional two-point function sources in bulk, i.e. $t_i \in \left[t_{\mathrm{ex}}^{\mathrm{oBC}}, T - t_{\mathrm{ex}}^{\mathrm{oBC}}\right]$, $t_{\mathrm{ex}}^{\mathrm{oBC}} = 1.75 \ldots 2.5\,\mathrm{fm}$.

Computational setup part II

Quark-disconnected loops (here: $\mathcal{O}_f(\mathbf{x},t) = \mathcal{S}_{l,s}(\mathbf{x},t)$)

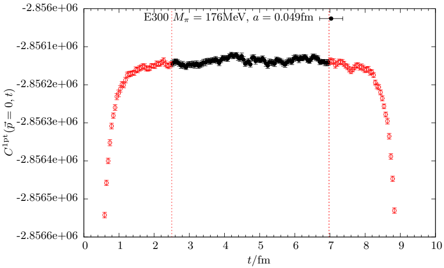
$$L_{\mathcal{O}_f}(\mathbf{p},t) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \langle \mathcal{O}_f(\mathbf{x},t)\rangle_F \,,$$

are computed using **stochastic all-to-all propagators** combining:

1. the one-end trick / frequency splitting to compute $L_1 - L_2, \ldots, L_{n-1} - L_n$, for $m_1 < m_2 < \ldots < m_n$
   EPJ C58, 261 (2008) *EPJ C79, 586 (2019)*
2. with the hopping parameter expansion + hierarchical probing for the heaviest quark n.
   PRD 89, 094503 (2014) *arXiv:1302.4018 [hep-lat]*

- Loops at gauge noise for any $\mathcal{O}_f(\mathbf{x},t)$
- Statistics for $2+1$ diagrams remains limited by two-point functions measurements.
- For periodic BC: foward + backward averaging for all sources used for two-point functions.
- For open BC: Stay away from boundary!

$\Rightarrow$ **2+1 statistics may get severely reduced at larger values of $t_{\rm sep}$, depending on T and $t_{\rm ex}^{\rm oBC}$.**



Ensembles

ID^{BC}	Traj.	a/fm	T/a	L/a	M_π / MeV	$M_\pi L$	N_{conf}	$N^{3\mathrm{pt}}_{\mathrm{meas}}$	$N^{(2+1)\mathrm{pt}}_{\mathrm{meas}}$
$\mathrm{C101}^o$	tr[M]	0.086	96	48	225	4.68	2000	16000	800000
$\mathrm{C102}^o$	m_s^{phys}		96	48	228	4.75	1500	12000	600000
$\mathrm{N101}^o$	tr[M]		128	48	283	5.89	1596	12768	893760
$\mathrm{H102}^o$	tr[M]		96	32	358	4.97	2037	16296	1140720
$\mathrm{D450}^p$	tr[M]	0.076	128	64	219	5.36	1028	12000	526336
$\mathrm{D451}^p$	m_s^{phys}		128	64	218	5.35	1500	8224	768000
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$\mathrm{S400}^o$	tr[M]		128	32	356	4.37	1001	8008	448448
$\mathrm{E250}^p$	**both**	0.064	192	96	132	4.07	1000	8000	512000
$\mathrm{D200}^o$	tr[M]		128	64	204	4.22	2000	16000	1120000
$\mathrm{D201}^o$	m_s^{phys}		128	64	204	4.21	1078	8624	603680
$\mathrm{N200}^o$	tr[M]		128	48	287	4.45	1712	13696	958720
$\mathrm{N203}^o$	tr[M]		128	48	349	5.40	1543	12344	864080
$\mathrm{E300}^o$	tr[M]	0.049	192	96	176	4.22	1139	9112	485973
$\mathrm{J303}^o$	tr[M]		192	64	260	4.17	1073	8584	600880

- $N_f = 2+1$ flavors of non-perturbatively improved Wilson clover fermions provided by CLS.

  JHEP 1502 (2015) 043 Commun.Math.Phys. 97 (1985) PoS LATTICE2008 (2008) 049

- **Production almost complete**; ensembles only listed if they have reached target statistics.
- 13 Ensembles on tr[M] $=$ const trajectory, 3 more on $m_s = m_s^{\mathrm{phys}}$ trajcetory.

  $\rightarrow$ J304 almost done, a few more $m_s = \mathrm{phys}$ ensembles available.

Ensembles

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- **Ensembles cover four values of the lattice spacing a**
- Pion masses range from $\sim 130\,\mathrm{MeV}$ to $\sim 350\,\mathrm{MeV}$
- Many different physical volumes with $L \approx 2.4, \ldots, 6.1\,\mathrm{fm}$, $M_\pi L > 4$.
- Two very large and fine boxes at (near) physical quark mass and high momentum resolution.

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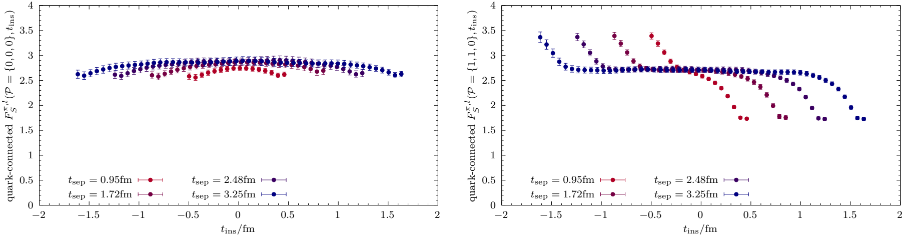
Ratio method and effective form factor



Light quark-connected form factors on E250 ($M_\pi = 132\,\mathrm{MeV}$, $a = 0.064\,\mathrm{fm}$); momentum labels: $\mathcal{P} \equiv (p_f^2, q^2, p_i^2)$

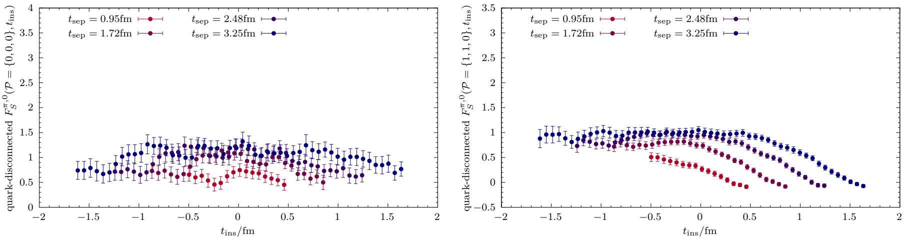
Effective form factors from **ratio method**:

$$R(p_f^2, q^2, p_i^2, t_f - t_i, t_{op} - t_i) = \frac{C_3(p_f^2, q^2, p_i^2, t_f - t_i, t_{op} - t_i)}{C_2(p_f^2, t_f - t_i)} \sqrt{\frac{C_2(p_i^2, t_f - t_{op}) C_2(p_f^2, t_{op} - t_i) C_2(p_f^2, t_f - t_i)}{C_2(p_f^2, t_f - t_{op}) C_2(p_i^2, t_{op} - t_i) C_2(p_i^2, t_f - t_i)}}.$$

$\Rightarrow$ ground state matrix elements $\langle \pi(p_f^2) | S_f(q^2) | \pi(p_i^2) \rangle \sim F_S^{\pi,f}(Q^2)$ for $t_{op} - t_i \to \infty$ and $t_f - t_{op} \to \infty$.

- Quark-connected data very precise at zero- and non-zero Q^2.

Ratio method and effective form factor



Singlet quark-disconnected form factors on E250 ($M_\pi = 132\,\mathrm{MeV}$, $a = 0.064\,\mathrm{fm}$); momentum labels: $\mathcal{P} \equiv (p_f^2, q^2, p_i^2)$

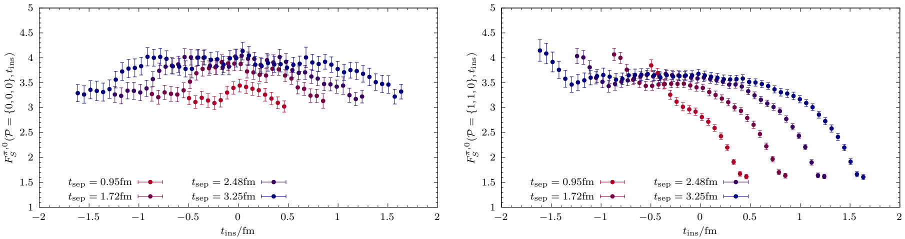
Effective form factors from **ratio method**:

$$R(p_f^2, q^2, p_i^2, t_f - t_i, t_{op} - t_i) = \frac{C_3(p_f^2, q^2, p_i^2, t_f - t_i, t_{op} - t_i)}{C_2(p_f^2, t_f - t_i)} \sqrt{\frac{C_2(p_i^2, t_f - t_{op}) C_2(p_f^2, t_{op} - t_i) C_2(p_f^2, t_f - t_i)}{C_2(p_f^2, t_f - t_{op}) C_2(p_i^2, t_{op} - t_i) C_2(p_i^2, t_f - t_i)}}.$$

$\Rightarrow$ ground state matrix elements $\langle \pi(p_f^2) | S_f(q^2) | \pi(p_i^2) \rangle \sim F_S^{\pi,f}(Q^2)$ for $t_{op} - t_i \to \infty$ and $t_f - t_{op} \to \infty$.

- Quark-connected data very precise at zero- and non-zero Q^2.
- Quark-disconnected contribution large at small Q^2 $\to$ up to $\sim 100\%$ correction on $a = 0.086\,\mathrm{fm}$.

Ratio method and effective form factor



Full singlet form factors on E250 ($M_\pi = 132\,\mathrm{MeV}$, $a = 0.064\,\mathrm{fm}$); momentum labels: $\mathcal{P} \equiv (p_f^2, q^2, p_i^2)$

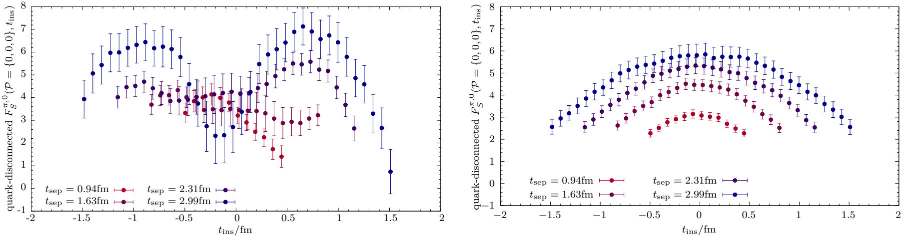
Effective form factors from **ratio method**:

$$R(p_f^2, q^2, p_i^2, t_f - t_i, t_{op} - t_i) = \frac{C_3(p_f^2, q^2, p_i^2, t_f - t_i, t_{op} - t_i)}{C_2(p_f^2, t_f - t_i)} \sqrt{\frac{C_2(p_i^2, t_f - t_{op}) C_2(p_f^2, t_{op} - t_i) C_2(p_f^2, t_f - t_i)}{C_2(p_f^2, t_f - t_{op}) C_2(p_i^2, t_{op} - t_i) C_2(p_i^2, t_f - t_i)}}.$$

$\Rightarrow$ ground state matrix elements $\langle \pi(p_f^2)|S_f(q^2)|\pi(p_i^2)\rangle \sim F_S^{\pi,f}(Q^2)$ for $t_{op} - t_i \to \infty$ and $t_f - t_{op} \to \infty$.

- Quark-connected data very precise at zero- and non-zero Q^2.
- Quark-disconnected contribution large at small Q^2 $\to$ up to $\sim 100\%$ correction on $a = 0.086\,\mathrm{fm}$.
- Error of full form factor dominated by disconnected piece.

Technical aside: Vacuum expectation value (VEV) subtraction



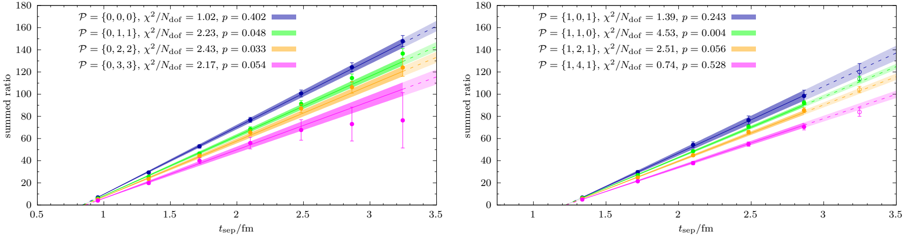
Singlet quark-disconnected contribution at Q^2 on C101 ($M_\pi = 225\,\mathrm{MeV}$, $a = 0.086\,\mathrm{fm}$). Left: Standard VEV subtraction. Right: Improved method.

At $Q^2 = 0$ we initially observed large fluctuations in the quark-disconnected signal on **open boundary ensembles**:

- Correlated fluctuations already visible in the gauge average of the loops (one-point functions).
- Two-point function source-positions t_i often restricted to rather small strip.
- Usual method of (global) VEV-subtraction fails.
- Instead subtract VEV **per-timeslice:**

$$\left\langle C^{3pt}_{\rm disc}(t_{\rm sep}, t_{\rm ins}) \right\rangle = \frac{1}{N_{t_i}} \sum_{t_i} \left[\left\langle C^{2\rm pt}(t_f - t_i) C^{1pt}(t_{op} - t_i) \right\rangle - \left\langle C^{2\rm pt}(t_f - t_i) \right\rangle \cdot \left\langle C^{1pt}(t_{op} - t_i) \right\rangle \right] .$$

Summation method



Summation method for $F_S^{\pi,0}(Q^2)$ on E250 ($M_\pi = 132\,\mathrm{MeV}$, $a = 0.064\,\mathrm{fm}$); momentum labels: $\mathcal{P} \equiv (p_f^2, q^2, p_i^2)$

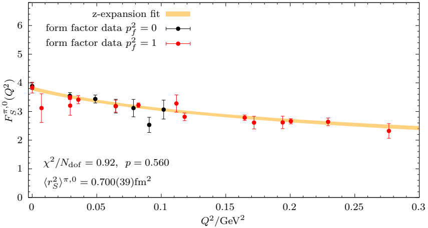
Summation method to extract groundstate matrix elements

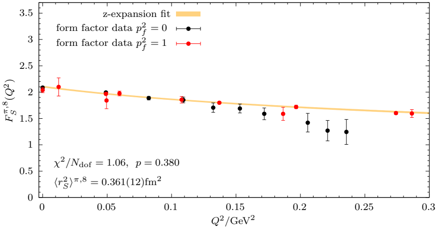
$$\sum_{t_{\rm ins}=t_{\rm ex}}^{t_{\rm sep}-t_{\rm ex}} R(\mathbf{p}_f^2, \mathbf{q}^2, \mathbf{p}_i^2, t_{\rm sep}, t_{\rm ins}) = \mathrm{const} + \langle \pi(p_f^2) | S(Q^2) | \pi(p_i^2) \rangle (t_{\rm sep} - t_0) + \mathcal{O}(e^{-\Delta t_{\rm sep}})$$

- Choose $t_{\rm ex} \simeq t_{\rm sep}^{\rm min}/2$ to improve signal quality for $Q^2 > 0$.
- Use values of $t_{\rm sep}^{\rm min} \in [\sim 1.0, \ldots, \sim 1.5]\,\mathrm{fm}$.
- Combined with any $t_{\rm sep}^{\rm max} \in [\sim 2.25, \ldots, \sim 3.25]\,\mathrm{fm}$ s.t. $t_{\rm sep}^{\rm max} - t_{\rm sep}^{\rm min} \geq 1\,\mathrm{fm}$.

$\rightarrow$ **Carry out remaining analysis for all variations and include them in final model averages**

z-expansion fits





Fits for $Q^2 \leq 0.3\,\mathrm{GeV}^2$, $1.25 \lesssim t_{\rm sep} \lesssim 3.25\,\mathrm{fm}$. Left: $F_S^{\pi,0}(Q^2)$ on **E250** ($M_\pi = 132\,\mathrm{MeV}$, $a = 0.064\,\mathrm{fm}$). Right: $F_S^{\pi,8}(Q^2)$ on **E300** ($M_\pi = 172\,\mathrm{MeV}$, $a = 0.049\,\mathrm{fm}$)

Use z-expansion to extract radii from **unrenormalized** form factor:

$$F_S^{\pi,f}(Q^2) = \sum_{n=0}^{N_z} a_n z^n, \qquad z = \frac{\sqrt{t_{\rm cut} + Q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} + Q^2} + \sqrt{t_{\rm cut} - t_0}}, \qquad a_1 \sim \langle r_S^2 \rangle_\pi^f = -\frac{6}{F_S^{\pi,f}(0)} \cdot \frac{dF_S^{\pi,f}(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

We use $N_z = 1$, $t_{\rm cut} = 4M_\pi^2$ and $t_0 = t_0^{\rm opt} = t_{\rm cut}(1 - \sqrt{1 + Q^2_{\rm max}/t_{\rm cut}})$. *PRD 92, 013013 (2015)*

- **Unprecedented Q^2-resolution at (near) physical quark mass.**
- $\mathbf{p}_f = (1, 0, 0)$ data greatly improves signal quality and Q^2-resolution.
- No renormalization needed for radii.
- Again we apply data cuts, i.e. $Q^2_{\rm cut} \in \{0.20, 0.25, 0.30, 0.35, 0.40\}\,\mathrm{GeV}^2$.

Physical extrapolation

NLO χPT fit ansatz with quark mass proxies $m_l \sim M_\pi^2$ and $m_s \sim 2M_K^2 - M_\pi^2$,

$$\langle r_S^2\rangle_\pi^0 = \frac{1}{(4\pi f_0)^2}\left[768\pi^2\left(3L_4^r + L_5^r\right) - 19 + \frac{m_l}{m_l+2m_s} - 12\log(m_l) - 6\log\left(\frac{m_l+m_s}{2}\right)\right] + c_0 a^2,$$

$$\langle r_S^2\rangle_\pi^l = \frac{1}{(4\pi f_0)^2}\left[768\pi^2\left(2L_4^r + L_5^r\right) - 16 + \frac{m_l}{3(m_l+2m_s)} - 12\log(m_l) - 3\log\left(\frac{m_l+m_s}{2}\right)\right] + c_l a^2,$$

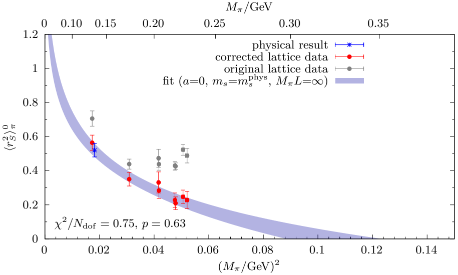
$$\langle r_S^2\rangle_\pi^8 = \frac{1}{(4\pi f_0)^2}\left[768\pi^2 L_5^r - 10 - \frac{m_l}{m_l+2m_s} - 12\log(m_l) + 3\log\left(\frac{m_l+m_s}{2}\right)\right] + c_8 a^2.$$

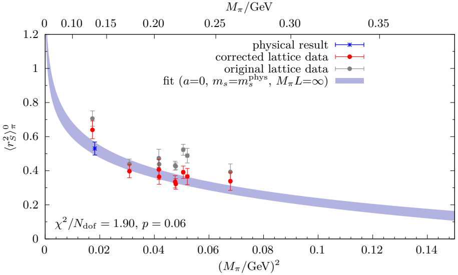
- Correlated fits are carried out in units of t_0 with $N_b = 1000$ bootstrap samples.
- Scale setting: $\sqrt{t_0} = 0.14464(87)\,\mathrm{fm}$. *Eur. Phys. J. C 82 (2022) 10, 869 (FLAG Review 2021)*
- Physical point (isospin limit): $M_\pi^{\mathrm{phys}} = 134.8(3)\,\mathrm{MeV}$, $M_K^{\mathrm{phys}} = 494.2(3)\,\mathrm{MeV}$. *Eur. Phys. J. C 77 (2017) 2, 112*
- Radii are fitted individually to compute $\langle r_S^2\rangle_{\pi,\mathrm{phys}}^{0,l,8}$.
- LECs f_0, L_4^r, L_5^r are obtained from fitting the following expressions:

$$\begin{aligned}
f_0 &: \quad \langle r_S^2\rangle_\pi^0,\ \langle r_S^2\rangle_\pi^l,\ \langle r_S^2\rangle_\pi^8,\ \langle r_S^2\rangle_\pi^0 - \langle r_S^2\rangle_\pi^l,\ \langle r_S^2\rangle_\pi^0 - \langle r_S^2\rangle_\pi^8,\ \langle r_S^2\rangle_\pi^l - \langle r_S^2\rangle_\pi^8,\\
L_4^r &: \quad \langle r_S^2\rangle_\pi^0 - \langle r_S^2\rangle_\pi^l,\ \langle r_S^2\rangle_\pi^0 - \langle r_S^2\rangle_\pi^8,\ \langle r_S^2\rangle_\pi^l - \langle r_S^2\rangle_\pi^8,\\
L_5^r &: \quad \langle r_S^2\rangle_\pi^8,\ 3\langle r_S^2\rangle_\pi^l - 2\langle r_S^2\rangle_\pi^0
\end{aligned}$$

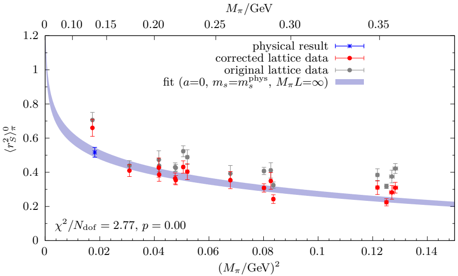
- Systematics from (combinations of) data cuts $M_\pi < \{230, 265, 290\}\,\mathrm{MeV}$, $a < 0.08\,\mathrm{fm}$ and $L > 3.5\,\mathrm{fm}$.

Example: Fit to $\langle r_S^2\rangle_\pi^0$ data (from $1.5\,\mathrm{fm} \lesssim t_{\mathrm{sep}} \lesssim 3.25\,\mathrm{fm}$ and $Q^2 \leq 0.3\,\mathrm{fm}$)

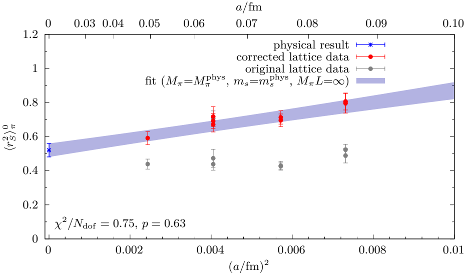


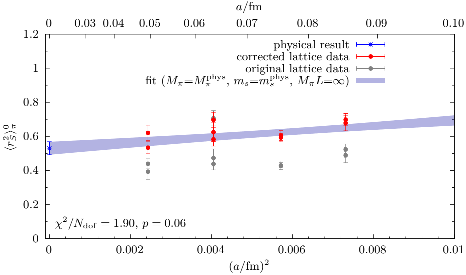


- Data set for $\langle r_S^2\rangle_\pi^0$ with $1.5\,\mathrm{fm} \lesssim t_{\mathrm{sep}} \lesssim 3.25\,\mathrm{fm}$ and $Q^2 \leq 0.3\,\mathrm{fm}$.
- Steep slope towards chiral limit (chiral log).
- **Pion mass cuts greatly improve fits.**
- E250 falls on fit curve for $M_\pi^{\mathrm{cut}} = 230\,\mathrm{MeV}$.
- Data receive significant corrections from fit...

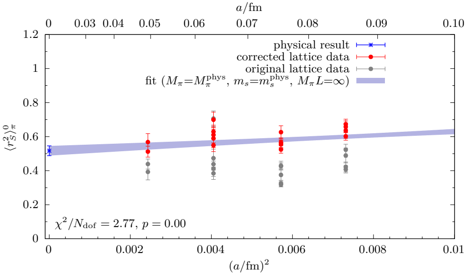


Example: Continuum extrapolation

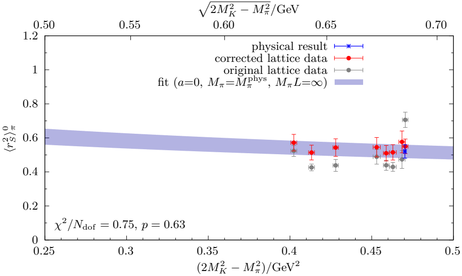


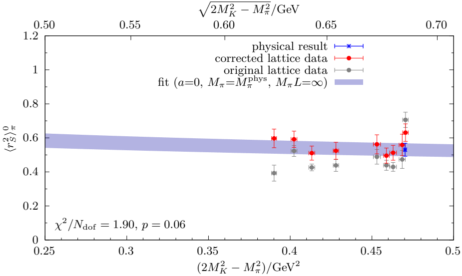


- **Corrections dominated by continuum extrapolation.**
- (Much) more pronounced for stricter M_π cuts.
  $\rightarrow$ Higher order corrections?
- Effect from cut $a < 0.08\,\mathrm{fm}$ less severe (not shown).

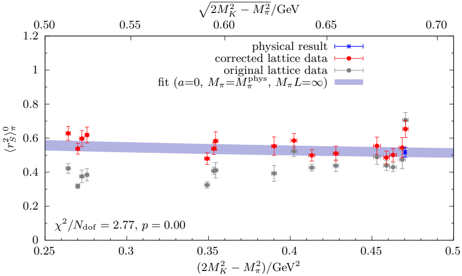


m_s–dependence





- m_s-depence very mild.
- Cuts in M_π also remove ensembles further away from m_s^{phys} on $\text{tr}[M] = const$ trajectory
- Physical point in m_s well determined by E250, E300 and ensembles with $m_s \approx \text{phys}$.
- Quite stable under various data cuts.



Model averages

Assign a weight to each model (fit) Phys. Rev. D 103,114502 (2021)

$$w_i \sim \exp\left(-\frac{1}{2}\left[\chi^2 + 2(N_{\rm para} - N_{\rm prio}) - 2N_{\rm data}\right]\right).$$

Central value and total err for an observable y are given by median and 16% and 84% percentiles of the CDF

$$CDF(y, \lambda) = \int_{-\infty}^{y} d\tilde{y} \sum_i w_i N(\tilde{y}, m_i, \sigma_i\sqrt{\lambda}).$$

Separate statistical ($\sigma_{\rm stat}$) and systematic ($\sigma_{\rm sys}$) errors from solving

$$\lambda\sigma^2_{\rm stat} + \sigma^2_{\rm sys} = \left(\frac{y_{\rm hi} - y_{\rm lo}}{2}\right)^2 \quad \text{where} \quad CDF(y_{\rm hi}, \lambda) = 0.84, \quad \text{and} \quad CDF(y_{\rm lo}, \lambda) = 0.16,$$

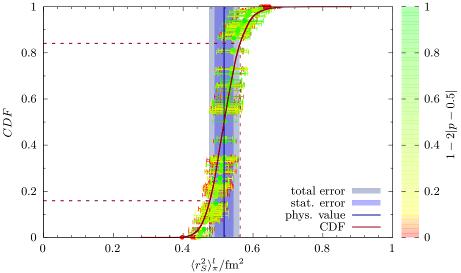
where $\lambda = 2.0$ rescales the statistical errors Nature 593 (2021) 7857, 51-55

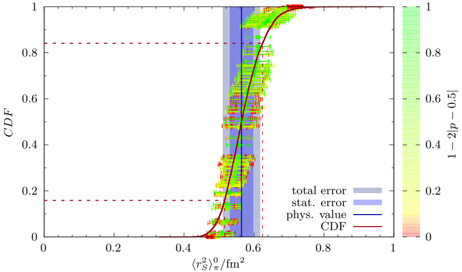
Final set of models: $\underbrace{\left\{(t_{\rm sep}^{\rm min}, t_{\rm sep}^{\rm max})\text{-pairs}\right\}}_{\text{summation method}} \bigotimes \underbrace{\left\{Q^2\text{-cuts}\right\}}_{\text{z-expansion}} \bigotimes \underbrace{\left\{M_\pi\text{-, } a\text{- and } L\text{-cuts}\right\}}_{\text{physical extrapolation}}$

$\Rightarrow$

observable	$\langle r_S^2\rangle_{\pi,\rm phys}^{0,l,8}$	f_0	L_4^r	L_5^r
#models	910	5460	2730	1820

Results for radii

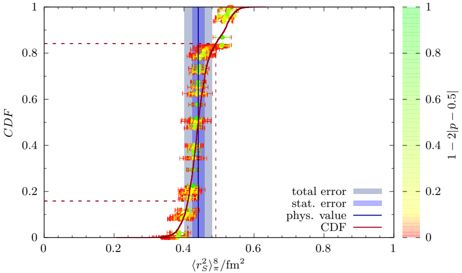




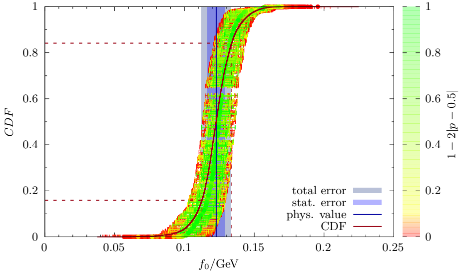
- **Results for radii (preliminary):**

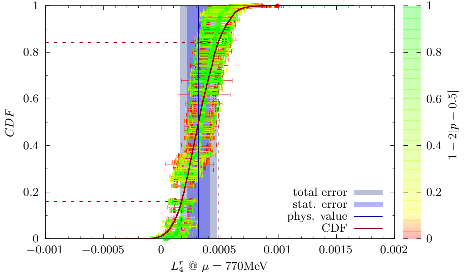
$$\langle r_S^2\rangle_\pi^0 = 0.564(34)_{\mathrm{stat}}(42)_{\mathrm{sys}}\ \mathrm{fm}^2\,,$$
$$\langle r_S^2\rangle_\pi^l = 0.517(28)_{\mathrm{stat}}(33)_{\mathrm{sys}}\ \mathrm{fm}^2\,,$$
$$\langle r_S^2\rangle_\pi^8 = 0.441(18)_{\mathrm{stat}}(36)_{\mathrm{sys}}\ \mathrm{fm}^2\,.$$

- Error for $\langle r_S^2\rangle_\pi^8$ dominated by systematics.
- Expected hierarchy: $\langle r_S^2\rangle_\pi^8 < \langle r_S^2\rangle_\pi^l < \langle r_S^2\rangle_\pi^0$.
- Compatible with only other lattice calculation by HPQCD. *Phys. Rev. D93, 054503 (2016)*



Results for LECs



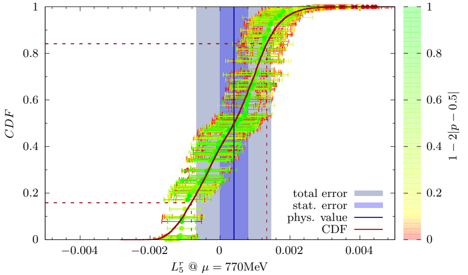


- **Results for LECs @ $\mu = 770\,\mathrm{MeV}$ (preliminary):**

$$f_0 = 122.7(6.5)_{\mathrm{stat}}(8.5)_{\mathrm{sys}}\ \mathrm{MeV}$$
$$L_4^r(\mu) = +0.32(10)_{\mathrm{stat}}(12)_{\mathrm{sys}} \times 10^{-3}$$
$$L_5^r(\mu) = +0.40(41)_{\mathrm{stat}}(1.00)_{\mathrm{sys}} \times 10^{-3}$$

- FLAG 2021 estimates ($N_f = 2+1$):

$$f_0 = 114.0(8.5)\ \mathrm{MeV}$$
$$L_4^r(\mu) = -0.02(56) \times 10^{-3}$$
$$L_5^r(\mu) = +0.95(41) \times 10^{-3}$$



Summary and Outlook

- **Study of the pion scalar radii on 16 CLS $N_f = 2+1$ ensembles:**
  - Results for radii at the physical point and results for $SU(3)$ χPT LECs.
  - **First calculation with fully controlled systematics and corresponding error budget, i.e. excited states, momentum dependence and physical extrapolation.**
  - Most precise existing determination of L_4^r.
- **Future plans:**
  - Finish production and carry out final analysis.
  - Dedicated determination of $\bar{l}_4$ from $SU(2)$ χPT fit.

    $\rightarrow$ possibly add a few more ensembles on m_s^{phys}-trajectory.
  - Analyze further form factors, e.g. $F_V^{\pi,K}(Q^2)$.

    $\rightarrow$ data for all 16 local and one-link displaced operator insertions