$B^*\pi$ excited-state contamination in B-physics observables

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41st Lattice Conference
Liverpool - July 30, 2024

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CPT

- Excited states are present in (almost) any lattice calculation
  - $\rightarrow$ smearing (gauss, Wuppertal, distillation, ...)
  - $\rightarrow$ use several operators (GEVP, global fits, ...)
- Multi-hadron excited states (e.g. $B^{(*)}\pi$ or $N\pi$ for the nucleon)
  - $\rightarrow$ lowest excited states ($m_\pi \rightarrow m_\pi^{\mathrm{phys}}$, $L \rightarrow \infty$)
  - $\rightarrow$ dominate at long distances (where we extract relevant information)
  - $\rightarrow$ spectrum becomes more dense in large volume
  - $\rightarrow$ effective field theories can do predictions (needs low-energy constants, LECs)
- **Question** : Do standard techniques handle these states correctly ?
  - $\rightarrow$ statistical errors might hide a systematic bias due to multi-hadronic states
- Focus on the $B \rightarrow \pi$ form factors (but the same LECs enter other relevant observales)

  1) compute the relevant LECs of HMChiPT

  2) test HMChiPT against lattice data

- Semileptonic decay $B \to \pi \ell \bar{\nu}_\ell$: form factors decomposition

$$m_B^{-1/2} \langle \pi(p_\pi) | V^\mu | B(p_B) \rangle = (p_\pi^\mu - (v \cdot p_\pi) v^\mu) \; h_\perp(v \cdot p_\pi) + v^\mu \; h_\parallel(v \cdot p_\pi)$$

- Rest frame of the B meson ($\vec{v} = \vec{0}$) and using the HQET normalization of states :

$$\langle \pi(p_\pi) | V^k | B(p_B) \rangle_{\mathrm{HQET}} = p_\pi^k \, h_\perp(E_\pi) \, ,$$
$$\langle \pi(p_\pi) | V^0 | B(p_B) \rangle_{\mathrm{HQET}} = h_\parallel(E_\pi)$$

- CKM matrix element $|V_{ub}|$: form factor $f_+(q^2)$

$$f_+(q^2) = \frac{\sqrt{m_B}}{2} \left[\left(1 - \frac{E_\pi}{m_B} \right) h_\perp(E_\pi) + \frac{1}{m_B} h_\parallel(E_\pi) \right]$$

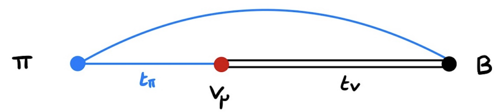
$\rightarrow$ at small pion energies, the form factor f_+ is dominated by $h_\perp$
$\rightarrow$ but $h_\perp$ potentially strongly affected by $B^*\pi$ excited states contamination

- Extraction from standard ratio of three- and two-point functions

$$h_\perp^{\mathrm{eff}}(t_v, t_\pi, E_\pi) = h_\perp(E_\pi) \times (1 \; + \; \Delta h_\perp(t_v, E_\pi) \; + \; \cdots)$$

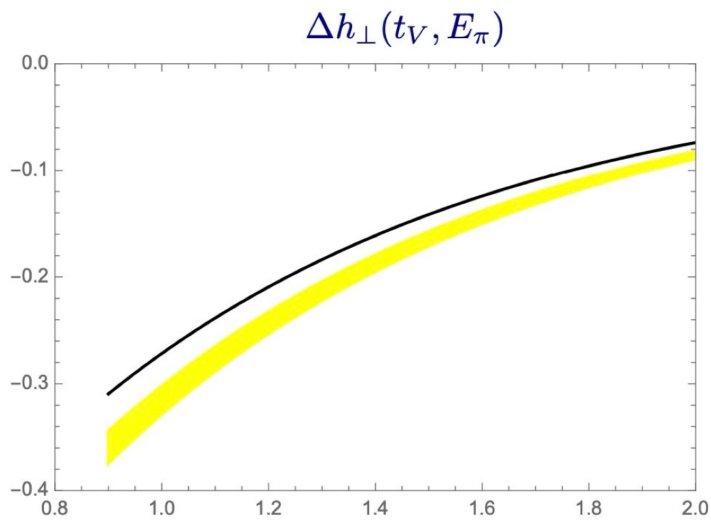
$B^*\pi$ excited states — other excited states

- Observable dependent : predicted to be large for $h_\perp$
- Prediction from HMChiPT : [O. Bär, A. Broll, R. Sommer '23]



$$\Delta h_\perp(t_v;\mathbf{p}) = -\frac{1+\widetilde{\beta}_1 E_\pi(\mathbf{p})/g}{1-\beta_1 E_\pi(\mathbf{p})/g}\; e^{-E_\pi(\mathbf{p})t_v}$$

→ prediction depends on LECs (β_1, $\widetilde{\beta}_1$, g)
→ there is a tree level contribution !



$$|\vec{p}| = \frac{2\pi}{L} \qquad M_\pi L = 4$$

$$\Rightarrow \quad E_{\pi,\vec{p}} \approx 260\,\mathrm{MeV}$$

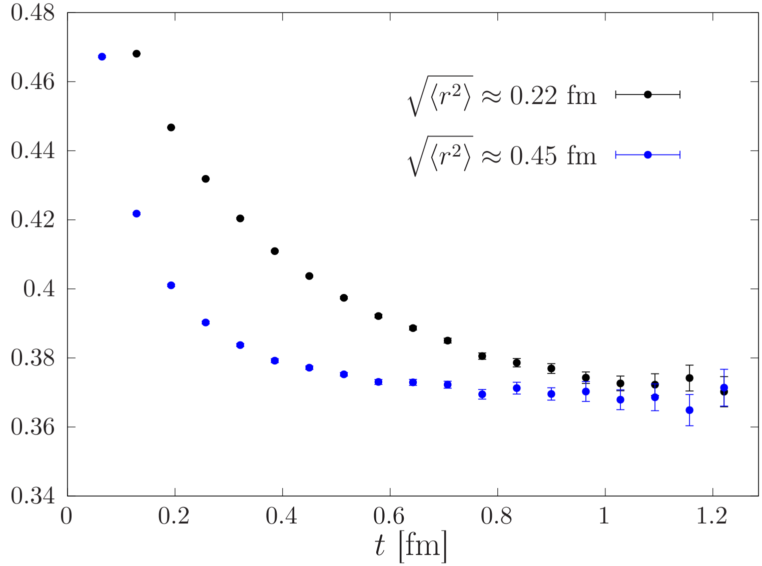
figure taken from Oliver Bär

- **Does smearing help to suppress $B^*\pi$ excited states ?**
  → compute β_1, $\widetilde{\beta}_1$ on the lattice

- Standard approach to suppress excited states : smearing (e.g. Gauss smearing)

$$\mathcal{O}_B(t) = \sum_{\mathbf{x}} \bar{b}(x)\gamma_5 d_{\rm smr}(x) \qquad \text{with} \qquad d_{\rm smr}(x) = \int_{L^3} \mathrm{d}^3y\; K(x,y) d(y)$$

- Example : Effective mass from 2-point heavy-light correlators $\langle \overline{\mathcal{O}}_B(t)\mathcal{O}_B(0)\rangle$



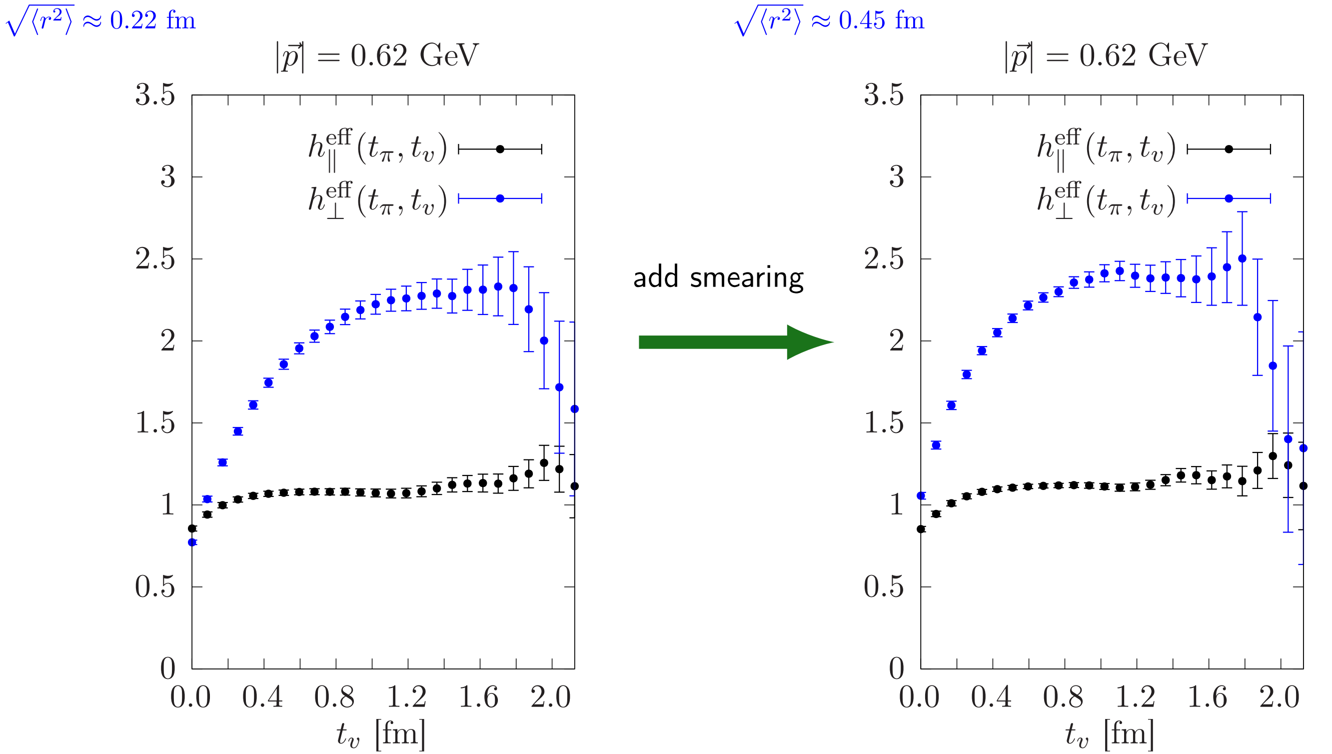
→ Smearing helps to reduce excited state contamination, plateau at earlier time

→ HMChiPT predicts a "small" $B^*\pi$ contribution (→ loop contribution, only few % for $t > 1$ fm)

→ Systematic procedure : use N operators and solve the Generalized Eigenvalue Problem (GEVP)

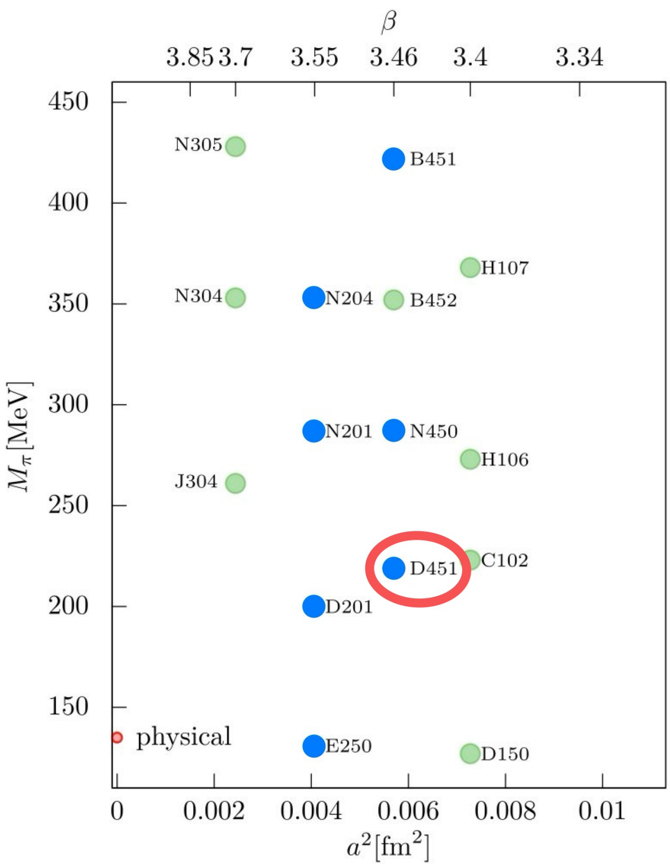
[Blossier et al. '09]

- Gaussian smearing and $B \to \pi$ form factors (static limit of HQET)



$\to$ Smearing does not help that much $\Rightarrow$ ineffective in removing $B^*\pi$ excited states ?

$\to$ HMChiPT prediction needs (unknown) LECs as input : β_1 and $\widetilde{\beta}_1$ (smearing dependent)



- CLS trajectory with $m_s \approx m_s^{\rm phys}$

  $\rightarrow$ SU(2) ChiPT

  $\rightarrow$ LECs (β_1 and $\widetilde{\beta}_1$) available on blue ensembles

- HMχPT prediction (at NLO) for the form factor $h_\perp$ [A. F. Falk et al '94] [D. Becirevic et al. '23]

$$\frac{\langle \pi(\mathbf{p})|V_k|B\rangle}{\langle \pi(\mathbf{p}^\star)|V_k|B\rangle} = \frac{1-\beta_1/g\,E_\pi(\mathbf{p})}{1-\beta_1/g\,E_\pi(\mathbf{p}^\star)} \times \frac{E_\pi(\mathbf{p}^\star)}{E_\pi(\mathbf{p})} \times \frac{p_k}{(p^\star)_k}$$

$\mathbf{p}^\star$: reference momentum

$\to$ extract β_1 from the pion energy dependence

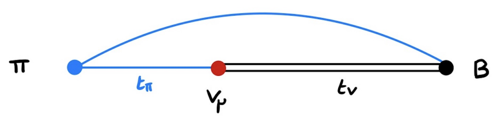
$\to$ smearing of the vector current ($V_\mu \to \widetilde{V}_\mu$) : gives access to $\widetilde{\beta}_1$ (LECs for smeared B operators)

[O. Bär, A. Broll, R. Sommer '23]

- Matrix element obtained from 3-point functions in the static limit

$$C^{(3)}_\mu(t_\pi, t_v; \mathbf{p}) = \frac{a^9}{V^2} \sum_{\mathbf{x_f},\mathbf{y},\mathbf{x_i}} \langle \overline{\mathcal{O}}_\pi(\mathbf{x_f}, t_v+t_\pi)\, V_\mu(\mathbf{y}, t_v)\, \mathcal{O}_B(\mathbf{x_i}, 0)\rangle\; e^{-i\mathbf{p}(\mathbf{x_f}-\mathbf{y})}$$

Replace local by smeared vector current : $\widetilde{V}_\mu \longrightarrow \widetilde{C}^{(3)}_\mu$



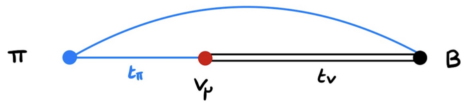
- Lattice estimator :

$$R^{\mathrm{eff}}(t, t_v; \mathbf{p}) \equiv \frac{E_\pi(\mathbf{p})}{E_\pi(\mathbf{p}^\star)} \frac{(p^\star)_k}{p_k} \times \frac{\widetilde{C}^{(3)}_k(t_\pi, t_v; \mathbf{p})}{\widetilde{C}^{(3)}_k(t_\pi, t_v; \mathbf{p}^\star)} \frac{C^{(2)}_\pi(t_\pi, \mathbf{p}^\star)}{C^{(2)}_\pi(t_\pi, \mathbf{p})}$$

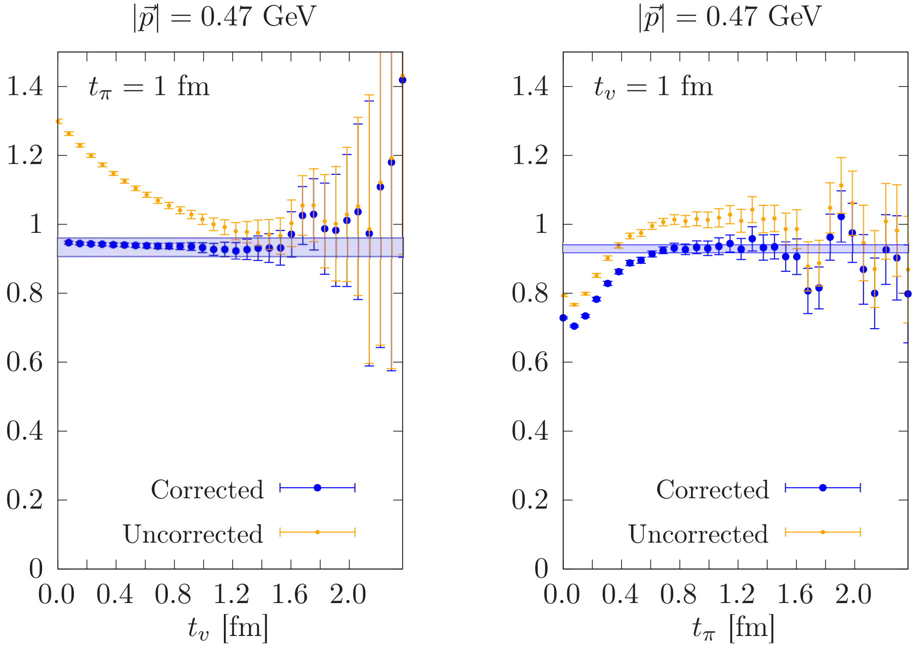
$\to$ this estimator is itself affected by excited states : can be used to correct our data

$$1+\delta_{B\pi}(t_v;\mathbf{p}) = \frac{1+\Delta h_\perp(t_v;\mathbf{p})}{1+\Delta h_\perp(t_v;\mathbf{p}^\star)} \approx 1 + e^{-(E_\pi(\mathbf{p})-E_\pi(\mathbf{p}^\star))t_v} + \frac{\beta_1+\tilde{\beta}_1}{g}\left(E_\pi(\mathbf{p})e^{-E_\pi(\mathbf{p})t} - E_\pi(\mathbf{p}^\star)e^{-E_\pi(\mathbf{p}^\star)t}\right) + \cdots$$

$$R^{\text{eff}}(t,t_v;\mathbf{p}) \equiv \frac{E_\pi(\mathbf{p})}{E_\pi(\mathbf{p}^\star)} \frac{(p^\star)_k}{p_k} \times \frac{\widetilde{C}_k^{(3)}(t,t_v;\mathbf{p})}{\widetilde{C}_k^{(3)}(t,t_v;\mathbf{p}^\star)} \frac{C_\pi^{(2)}(t-t_v,\mathbf{p}^\star)}{C_\pi^{(2)}(t-t_v,\mathbf{p})}$$



- Plateaus at fixed $t_\pi = 1$ fm (left) or at fixed $t_v = 1$ fm (right). We have $t = t_v + t_\pi$.

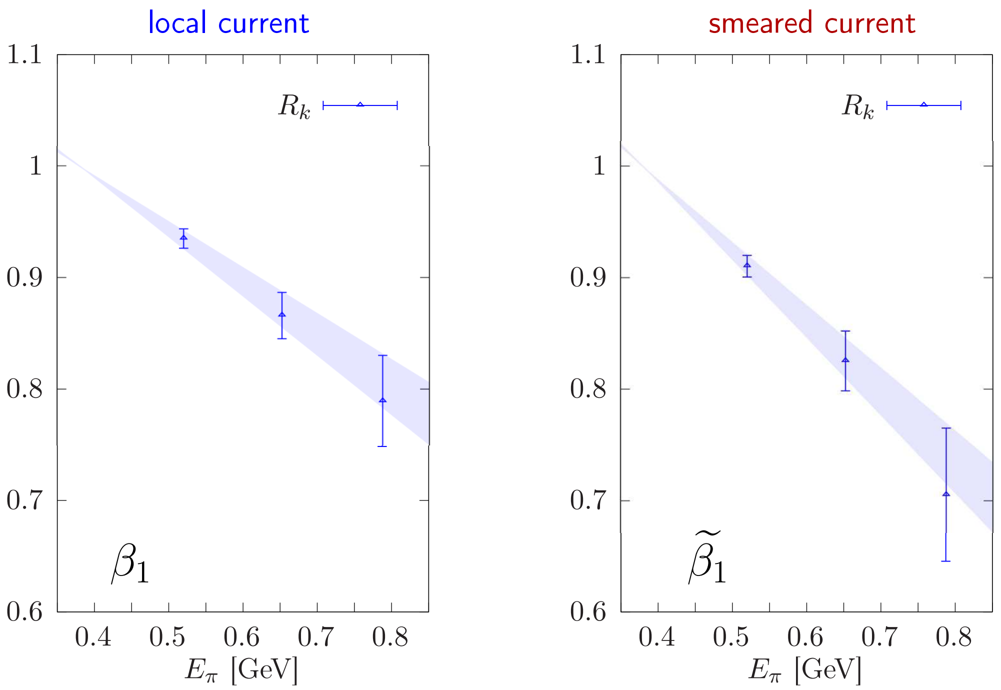


$\approx 10\%$ correction $(B^*\pi)$

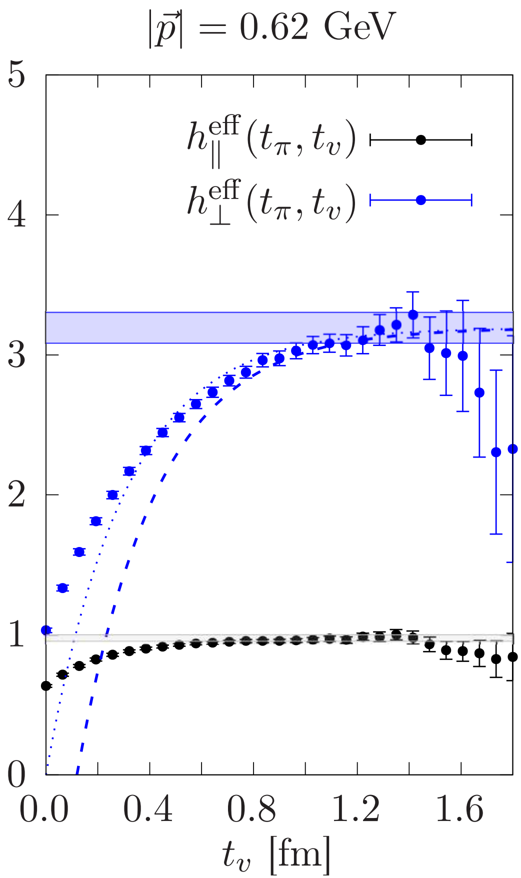
$(t_v = 1$ fm$)$

- Repeat the analysis for different values of E_π in the range [0.29 , 0.85] GeV

- HMχPT prediction : $$R(\mathbf{p}) = \frac{1 - \beta_1/g\, E_\pi(\mathbf{p})}{1 - \beta_1/g\, E_\pi(\mathbf{p}^\star)}$$ $\mathbf{p}^\star$: reference momentum



- **Preliminary result** : $\beta_1 = 0.20(2)$ GeV^{-1} and $\widetilde{\beta}_1 = 0.23(3)$ GeV^{-1} (@ our largest smearing radius)
- **Conclusion** : $\widetilde{\beta}_1 \approx \beta_1 \;\Rightarrow$ small impact of smearing on $B^*\pi$ excited states !



Dominant excited states contamination :

$$\Delta h_\perp(t_v; \mathbf{p}) = -\frac{1 + \widetilde{\beta}_1 E_\pi(\mathbf{p})/g}{1 - \beta_1 E_\pi(\mathbf{p})/g}\, e^{-E_\pi(\mathbf{p}) t_v}$$

- HMChitPT predicts the size of excited states contribution $\rightarrow$ depends on a few LECs
- Can we further test HMChiPT ?

  $\rightarrow$ start with a simpler case : (static) heavy-light 2pt function

$$C_B(t) = |\langle 0|\mathcal{O}_B|B\rangle|^2 e^{-E_B t} \ \times \ (1 + \delta C_B(t) + \cdots) \ ,$$

The correction term is now a sum over a tower of $B^*(\mathbf{p})\pi(-\mathbf{p})$ states with all allowed lattice momenta [O. Bär, A. Broll, R. Sommer '23]

$$\delta C_B(t) = \sum_{\mathbf{p}} \underbrace{\frac{C_N}{2} \frac{1}{(fL)^2 (E_\pi(\mathbf{p})L)} \frac{|\mathbf{p}|^2}{E_\pi(\mathbf{p})^2} \left(g + \widetilde{\beta}_1 E_\pi(\mathbf{p})\right)^2} e^{-E_\pi(\mathbf{p})t}$$

$\rightarrow \left| \frac{\langle 0|\mathcal{O}_B|B^*\pi\rangle}{\langle 0|\mathcal{O}_B|B\rangle} \right|^2$ in spectral decomposition

- Comment : preliminary results obtained with $m_\pi \approx 400$ MeV $\gg m_\pi^{\rm phys}$

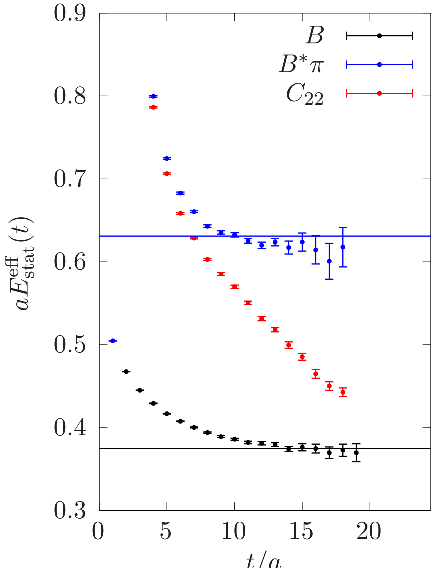
- Include explicitly a two-particle interpolator and solve a GEVP : $C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0)$

$$C_{ij}(t) = \begin{pmatrix} \langle \overline{O}_B(t) O_B(0) \rangle & \langle \overline{O}_{B\pi}(t) O_B(0) \rangle \\ \langle \overline{O}_B(t) O_{B\pi}(0) \rangle & \langle \overline{O}_{B\pi}(t) O_{B\pi}(0) \rangle \end{pmatrix} = \sum_{n=1}^{\infty} \psi_{ni}^* \psi_{nj} e^{-E_n t}$$

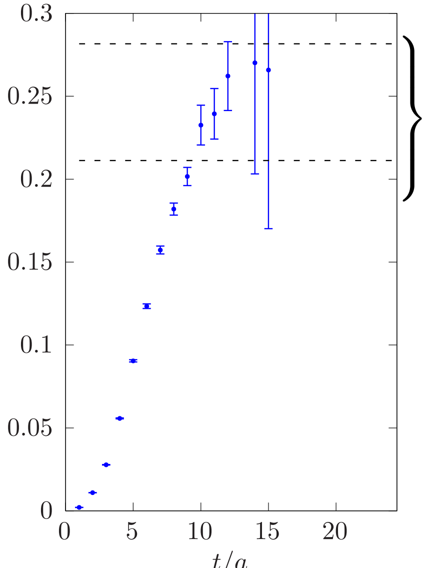
$\rightarrow$ we consider the simplest case : 2×2 GEVP (single $B^*(\mathbf{p})\pi(-\mathbf{p})$ with smallest $|\mathbf{p}|$)

$$\psi_{ni}^{\mathrm{eff}}(t) = C_{ij}(t) v_{nj}(t,t_0) \times \left(\frac{\lambda_n(t,t_0)}{\lambda_n(t+1,t_0)} \right)^{t-t_0/2} \qquad (n=1 : B, \quad n=2 : B^*\pi)$$

Spectrum



Ratio of overlap factors

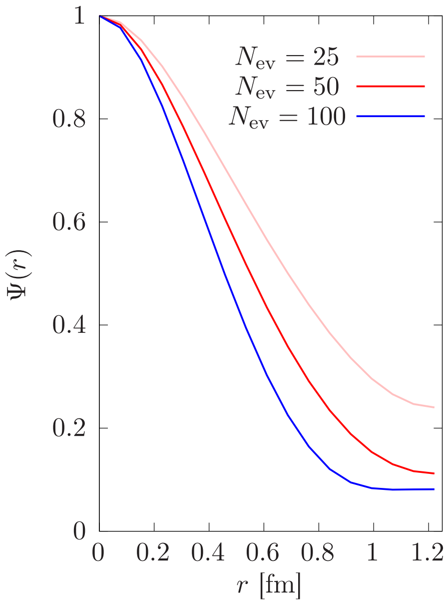


$$\left| \frac{\langle 0 | \mathcal{O}_B | B^*\pi \rangle}{\langle 0 | \mathcal{O}_B | B \rangle} \right|$$

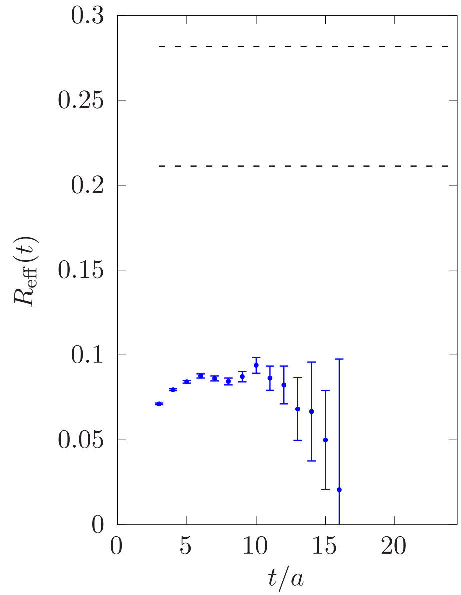
$\rightarrow$ right ballpark

- $\widetilde{\beta}_1$ depends on the detail of the smearing operator
  $\rightarrow$ other smearings may yield better results (?)
- Preliminary results obtained with distillation [M Peardon et al. '09]

Profiles



Ratio of overlap factors



$\rightarrow$ preliminary results suggest smaller overlap with $B^*\pi$ states as compared to gauss smearing

$\rightarrow$ Applicability of HLChPT is unclear. Non-local smearing.)

- HMChiPT provides predictions for excited states
  - $\rightarrow$ depends on a few LECs : accessible on the lattice
  - $\rightarrow$ observable dependent but can be (very) large : $h_\perp$
- Calculation of the LECs relevant for $B^*\pi$ is almost complete
  - $\rightarrow$ better understanding of smearing on multi-hadron states
- We can test these predictions
- Perspective
  - $\rightarrow$ apply the method to extract $B \rightarrow \pi$ form factors