Gradient Flow Renormalisation for Meson Mixing and Lifetimes

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In collaboration with:
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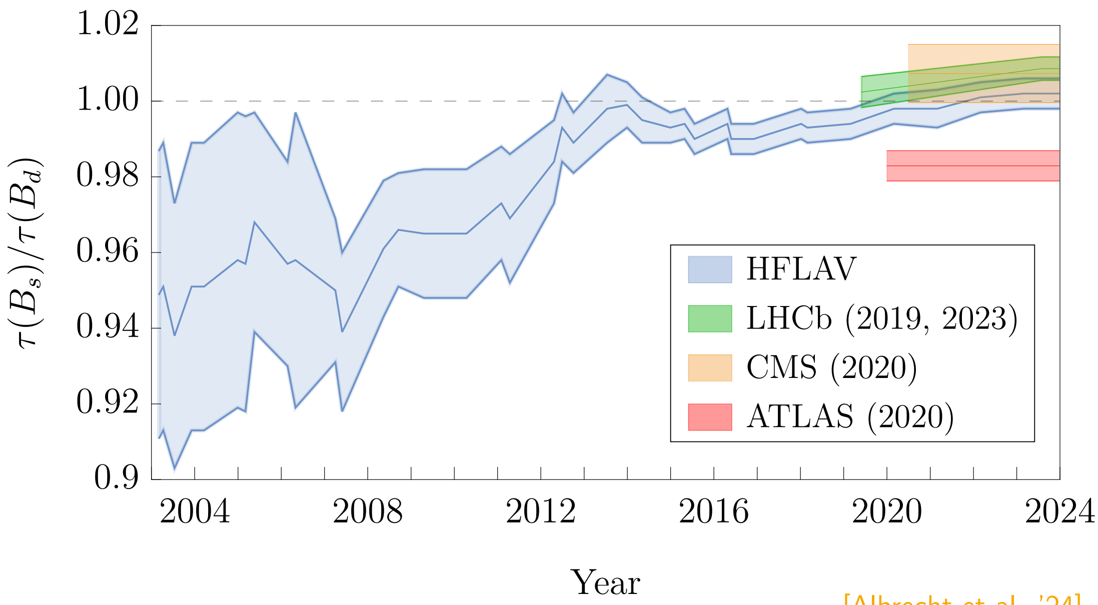
July 30, 2024

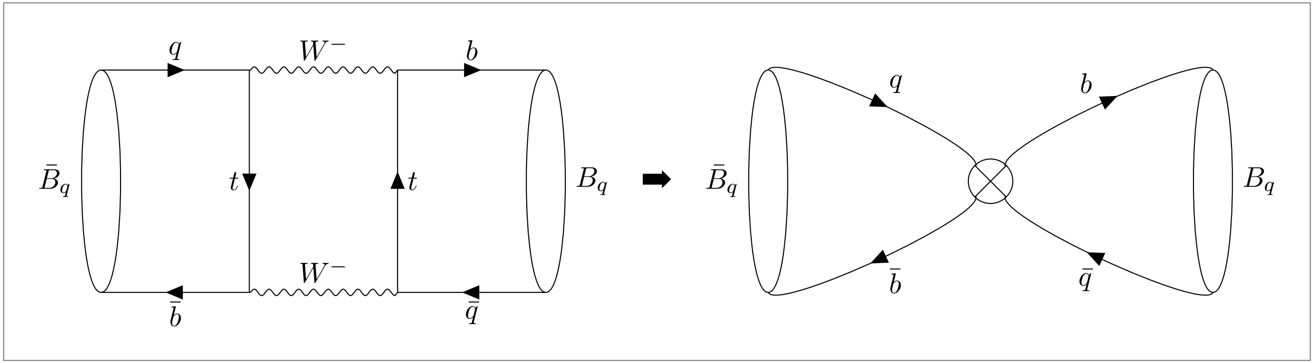
- B-meson mixing and lifetimes are measured experimentally to high precision
  - Key observables for probing New Physics ➡ **high precision in theory needed!**



[Albrecht et al. '24]

- B-meson mixing and lifetimes are measured experimentally to high precision
  - ➥ Key observables for probing New Physics ➡ **high precision in theory needed!**
- For B lifetimes and mixing, we use the **Heavy Quark Expansion**

$$\Gamma_{B_q} = \Gamma_3 \langle \mathcal{O}_{D=3} \rangle + \Gamma_5 \frac{\langle \mathcal{O}_{D=5} \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_{D=6} \rangle}{m_b^3} + ... + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_{D=6} \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_{D=7} \rangle}{m_b^4} + ... \right]$$



- Factorise observables into ➡ perturbative QCD contributions
  ➡ **Non-Perturbative Matrix Elements**

- Four-quark $\Delta B = 0$ and $\Delta B = 2$ matrix elements can be determined from lattice QCD simulations
- $\Delta B = 2$ well-studied by several groups ➡ precision increasing
  - ➥ preliminary $\Delta K = 2$ for Kaon mixing study with gradient flow [Suzuki et al. '20], [Taniguchi, Lattice '19]
- $\Delta B = 0$ ➡ exploratory studies from ∼20 years ago
  - ➥ contributions from gluon disconnected diagrams
  - ➥ mixing with lower dimension operators in renormalisation

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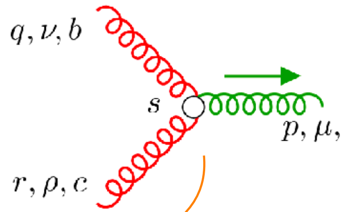
Recent Developments:

- [Lin, Detmold, Meinel '22] ➡ spectator effects in b hadrons **➜ Lin, Talk @ 11:50 Thursday**
  - ➥ focus on lifetime ratios for both B mesons and Λ_b baryon
  - ➥ isospin breaking, $\langle B|\mathcal{O}^d - \mathcal{O}^u|B\rangle$
  - ➥ position-space renormalisation + perturbative matching to $\overline{\text{MS}}$
- this work, [Black et al. '23]
  - ➥ goal is individual $\Delta B = 0$ matrix elements for B mesons
  - ➥ non-perturbative gradient flow renormalisation
  - ➥ perturbative matching to $\overline{\text{MS}}$ in short-flow-time expansion

- Use 6 RBC/UKQCD's 2+1 flavour DWF + Iwasaki gauge action ensembles
- For pilot study, simplified setup without additional extrapolations
  ➥ physical charm and strange quarks ➡ simulating a **charm-strange meson**
- Stout-smeared Möbius DWF for charm [Cho et. al '15]
- Neutral charm-strange meson mixing ➡ proxy to short-distance D^0 mixing up to spectator effects
- Charm-strange meson $\Delta Q = 0$ operators ➡ D_s meson lifetimes
- Non-perturbatively renormalise four-quark operators via gradient flow evolution
  ➥ Match to $\overline{\mathrm{MS}}$ with perturbative coefficients in the short-flow-time expansion

Gradient Flow – Short-Flow-Time Expansion

- Well-studied for e.g. ➡ energy-momentum tensor [Makino, Suzuki '14] [Harlander, Kluth, Lange '18]
  ➡ neutron EDM [Rizik, Monahan, Shindler '20]
- Re-express effective Hamiltonian in terms of 'flowed' operators:



$$\mathcal{H}_{\text{eff}} = \sum_n C_n \mathcal{O}_n = \sum_n \widetilde{C}_n(\tau) \widetilde{\mathcal{O}}_n(\tau).$$

- Relate to regular operators in 'short-flow-time expansion':

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau) \mathcal{O}_m + O(\tau)$$

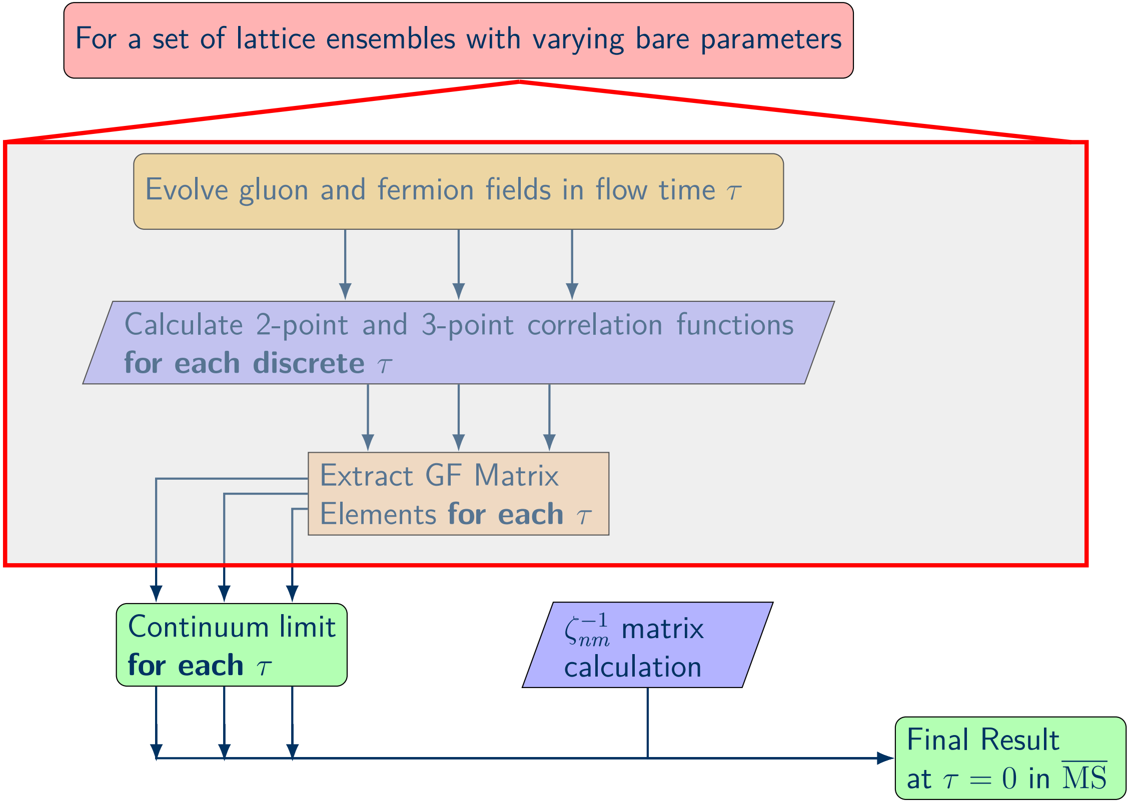
new Feynman diagrams

'flowed' MEs calculated on lattice replacing $A_\mu, q \to B_\mu, \chi$

matching matrix calculated perturbatively

$$\sum_n \zeta^{-1}_{nm}(\mu, \tau) \langle \widetilde{\mathcal{O}}_n \rangle(\tau) = \langle \mathcal{O}_m \rangle(\mu)$$

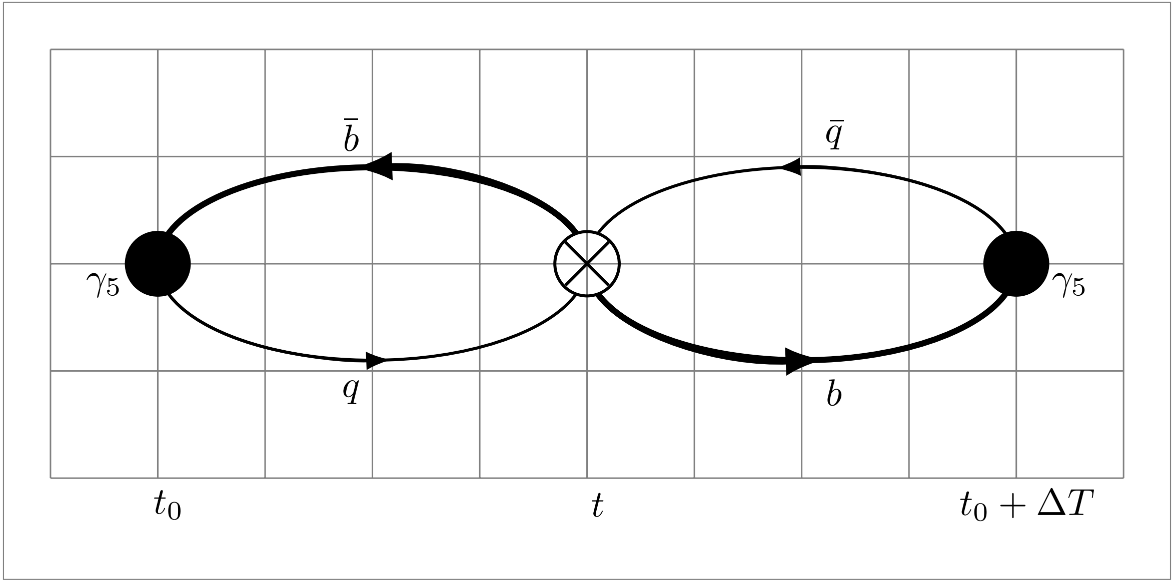
- Matrix element $\langle \mathcal{O}_m \rangle(\mu)$ in $\overline{\text{MS}}$ found in $\tau \to 0$ limit ➡ 'window' problem
  - ➥ large systematic effects at very small flow times
  - ➥ large flow time dominated by operators $\propto O(\tau)$

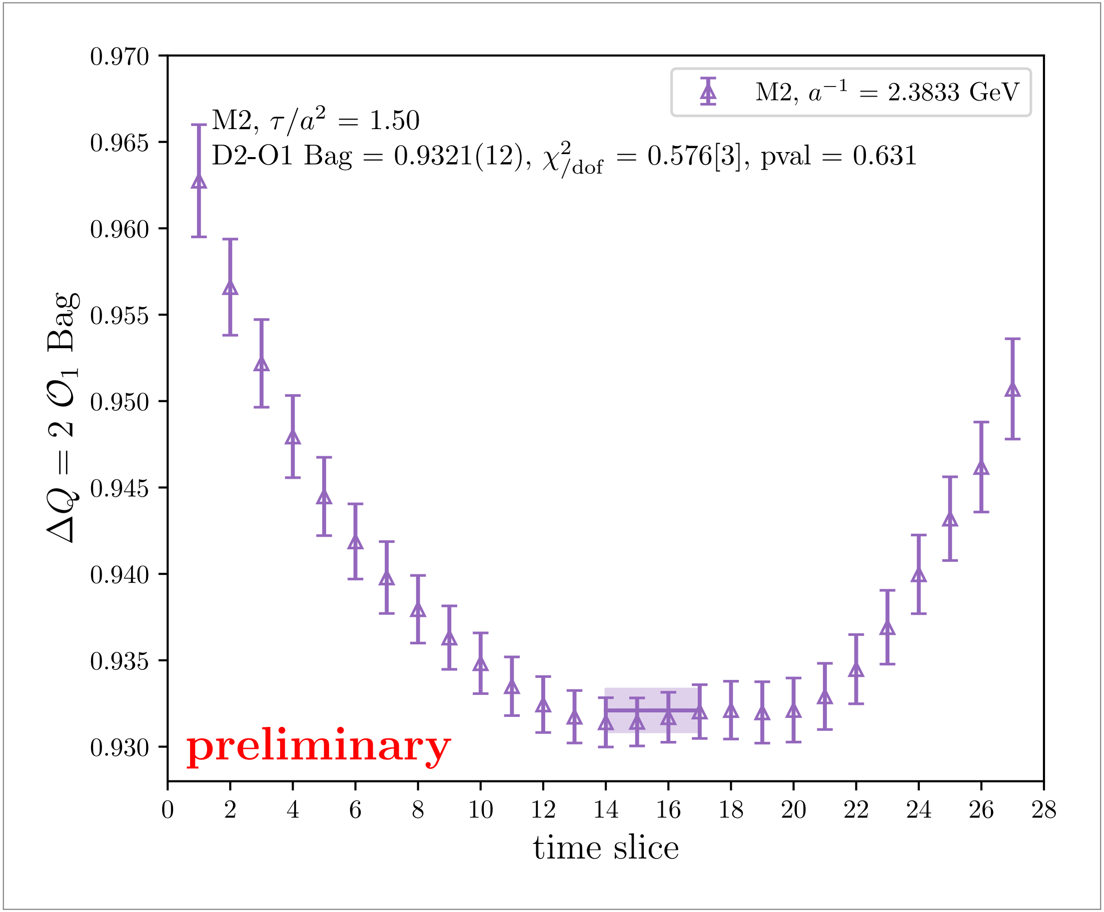

For a set of lattice ensembles with varying bare parameters
Evolve gluon and fermion fields in flow time τ
Calculate 2-point and 3-point correlation functions **for each discrete** τ
Extract GF Matrix Elements **for each** τ
Continuum limit **for each** τ
ζ_{nm}^{-1} matrix calculation
Final Result at $\tau = 0$ in $\overline{\mathrm{MS}}$

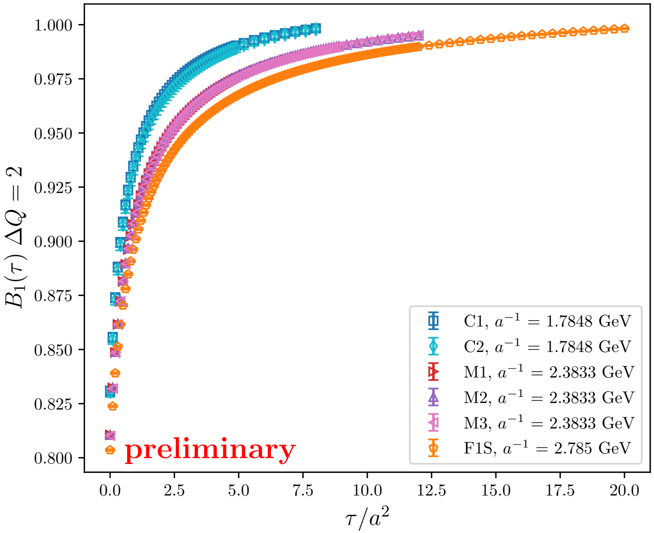
- Three-point correlation function:

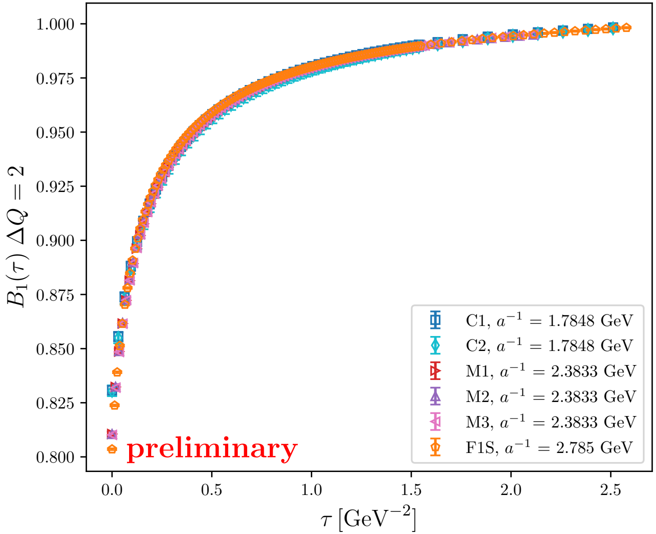
$$C^{\rm 3pt}_{\mathcal{Q}_i}(t, \Delta T, \tau) = \sum_{n,n'} \frac{\langle P_n|\mathcal{Q}_i|P_{n'}\rangle(\tau)}{4M_nM_{n'}} e^{-(\Delta T - t)M_n} e^{-tM_{n'}} \underset{t_0 \ll t \ll t_0 + \Delta T}{\Longrightarrow} \frac{\langle P\rangle^2}{4M^2} \langle \mathcal{Q}_i\rangle(\tau) e^{-\Delta T M}$$

- Measure along positive flow time τ



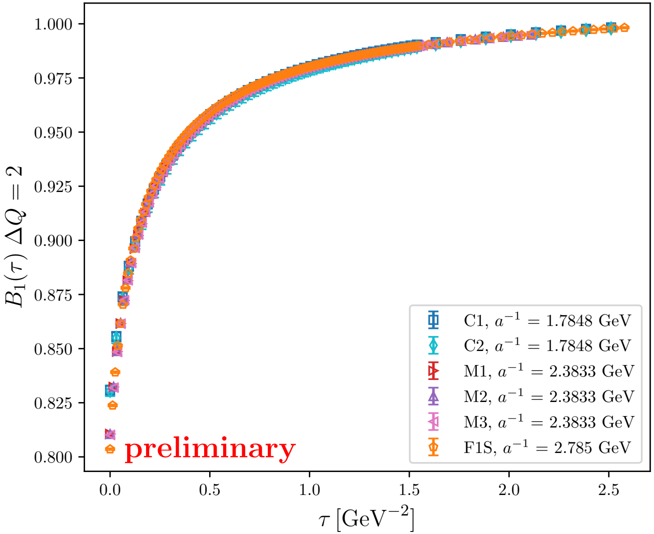

M2, a^{-1} = 2.3833 GeV
M2, $\tau/a^2 = 1.50$
D2-O1 Bag = 0.9321(12), $\chi^2_{/\mathrm{dof}} = 0.576[3]$, pval = 0.631
preliminary
$\Delta Q = 2\ \mathcal{O}_1$ Bag
time slice
0.970
0.965
0.960
0.955
0.950
0.945
0.940
0.935
0.930
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28

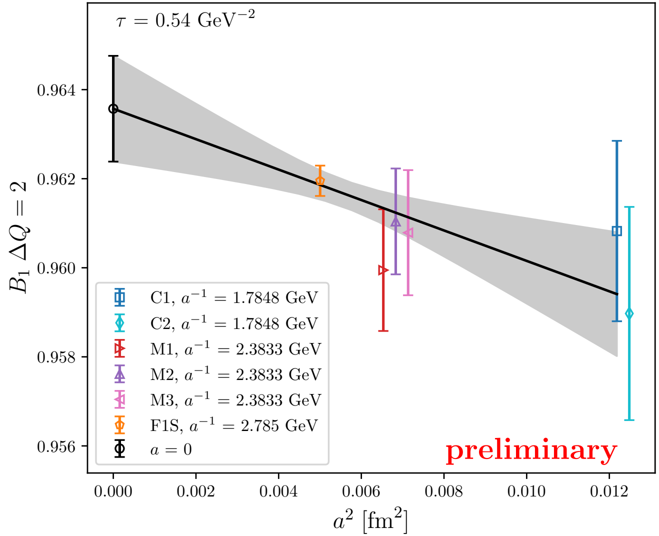




- operator is renormalised in ‘GF’ scheme as it is evolved along flow time
- different lattice spacings overlap in physical flow time

Mixing $\mathcal{O}_1$ vs GF time and Continuum Limit





- ➤ operator is renormalised in 'GF' scheme as it is evolved along flow time
- ➤ different lattice spacings overlap in physical flow time
- ➤ continuum limit well-controlled at positive flow time ✔

- Relate to regular operators in 'short-flow-time expansion':

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

'flowed' MEs calculated on lattice

matching matrix
calculated perturbatively

- Relate to regular operators in 'short-flow-time expansion':

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

'flowed' MEs calculated on lattice

matching matrix
calculated perturbatively

$$\sum_n \zeta^{-1}_{nm}(\mu,\tau)\langle\widetilde{\mathcal{O}}_n\rangle(\tau) = \langle\mathcal{O}_m\rangle(\mu)$$

- Relate to regular operators in 'short-flow-time expansion':

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

'flowed' MEs calculated on lattice

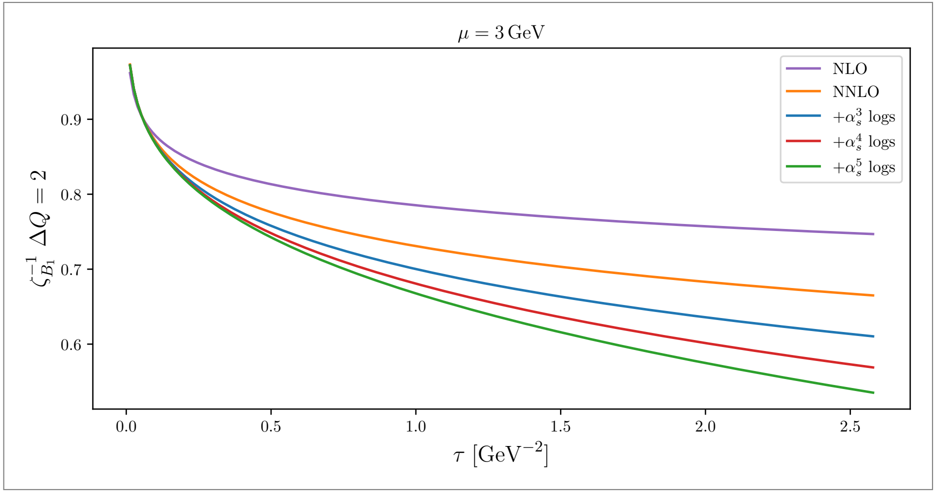
matching matrix calculated perturbatively

$$\sum_n \zeta^{-1}_{nm}(\mu,\tau)\langle\widetilde{\mathcal{O}}_n\rangle(\tau) = \langle\mathcal{O}_m\rangle(\mu)$$

- Calculated at two-loop for $\mathcal{B}_1$ based on [Harlander, Lange '22] [Borgulat et al. '23]:

$$\zeta^{-1}_{\mathcal{B}_1}(\mu,\tau) = 1 + \frac{a_s}{4}\left(-\frac{11}{3} - 2L_{\mu\tau}\right) + a_s^2\left(\dots\right)$$

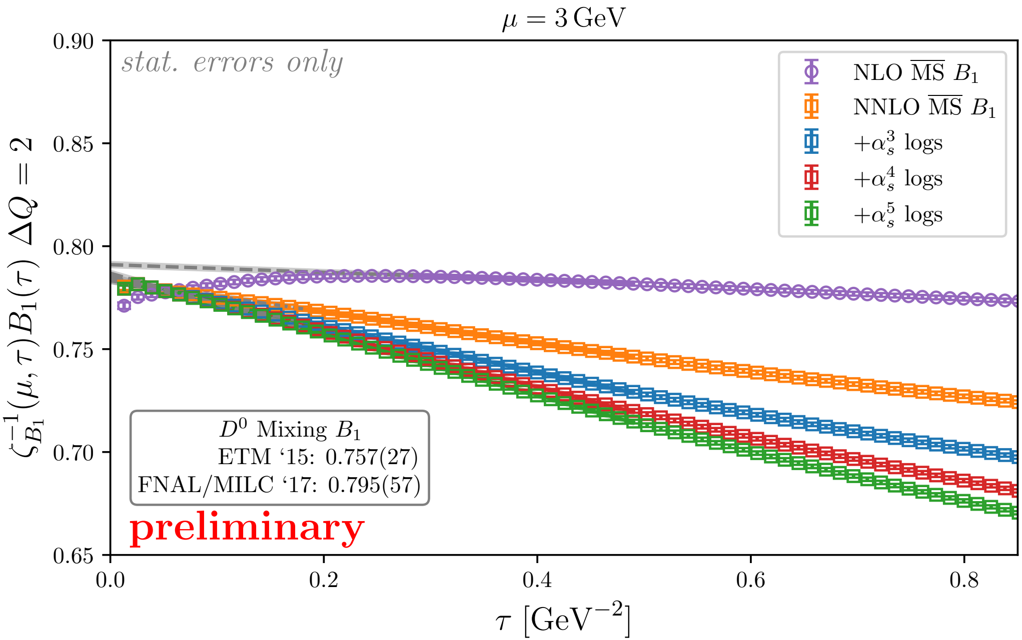
$$L_{\mu\tau} = \log(2\mu^2\tau) + \gamma_E, \quad \mu = 3\,\text{GeV}$$

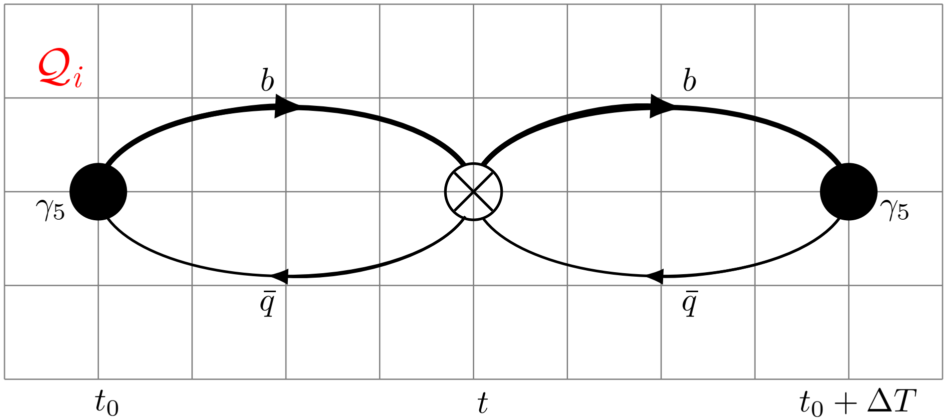


- ➤ Promising first signs of agreement
  - ➥ statistical errors only
- ➤ Different perturbative orders "in same ball park"
  - ➥ systematic errors needed for meaningful comparison
- ➤ Consider existing short-distance D^0 mixing results

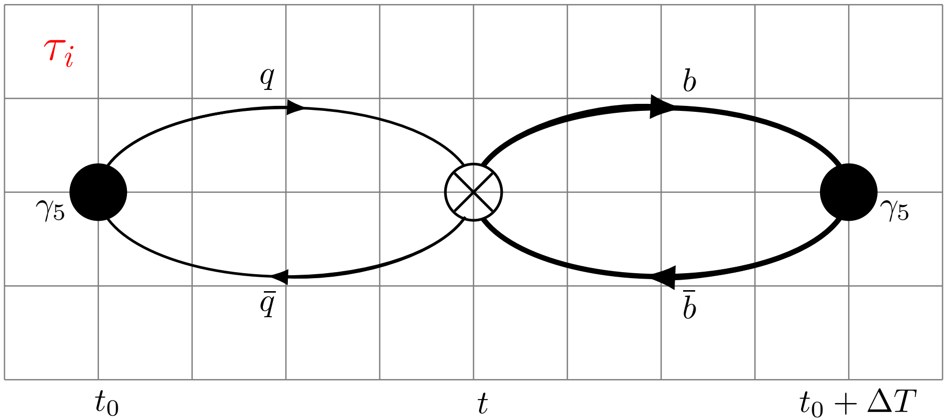
[ETM '15] $B_1^{\overline{\rm MS}} = 0.757(27)$

[FNAL/MILC '17] $B_1^{\overline{\rm MS}} = 0.795(56)$



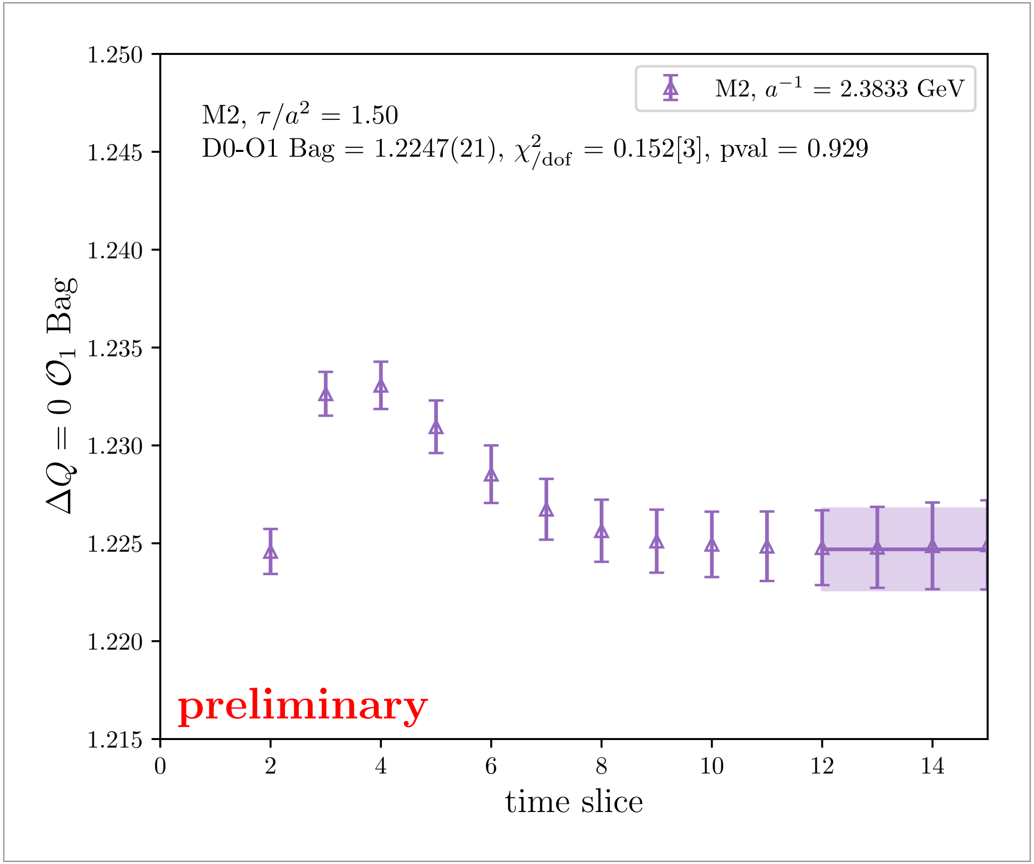


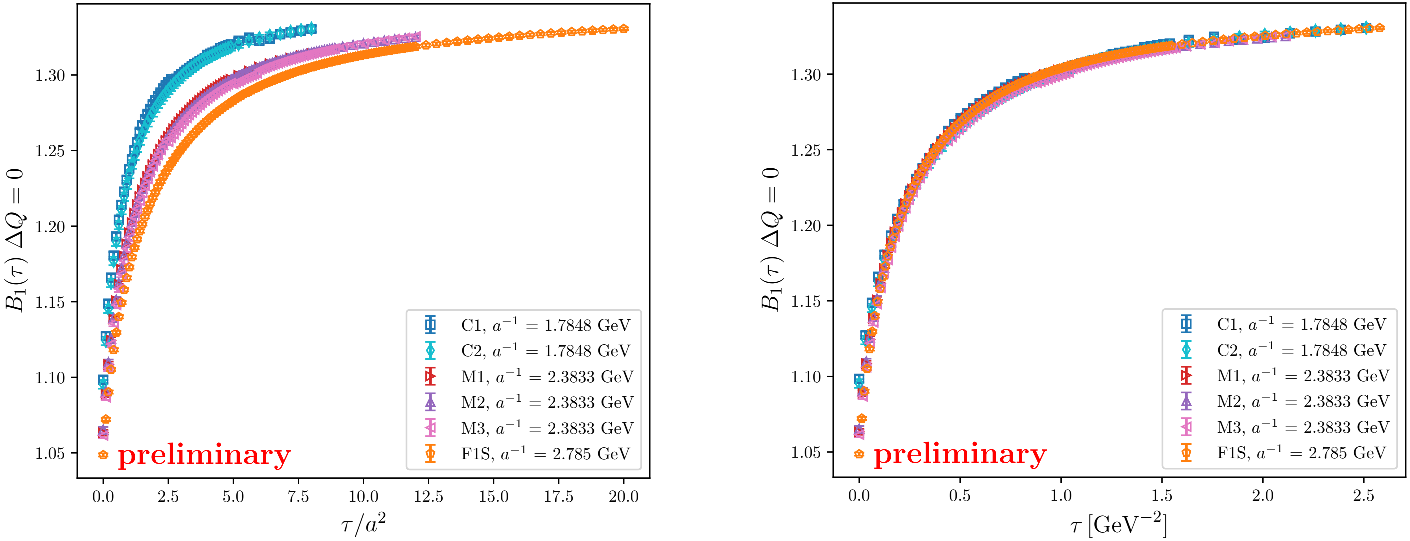
$\mathcal{Q}_i$
b
b
γ_5
γ_5
$\bar{q}$
$\bar{q}$
t_0
t
$t_0 + \Delta T$



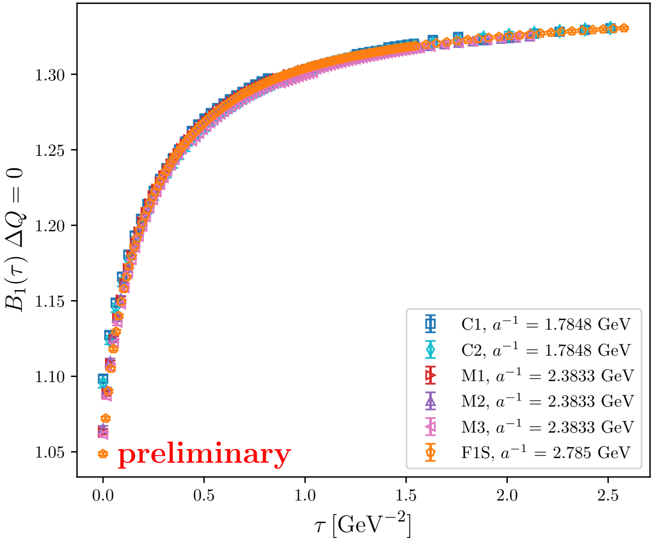
$\mathcal{T}_i$
q
b
γ_5
γ_5
$\bar{q}$
$\bar{b}$
t_0
t
$t_0 + \Delta T$

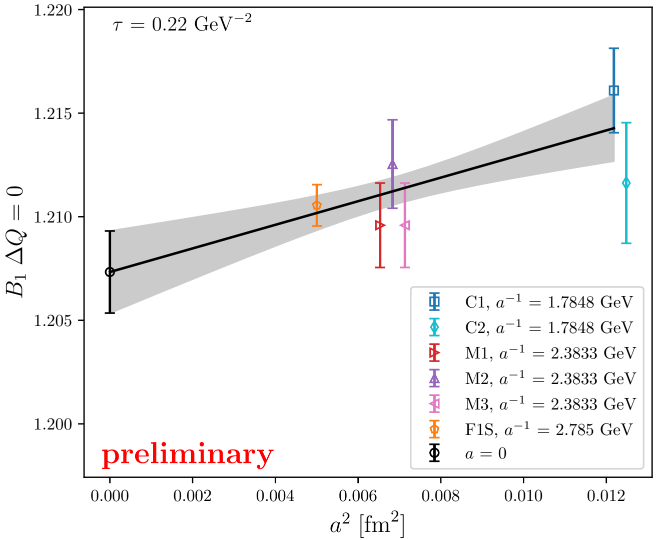
- Bag parameters for $\mathcal{Q}_i$ extracted as for $\Delta B = 2$ operators





- ➤ operator is renormalised in 'GF' scheme as it is evolved along flow time
- ➤ different lattice spacings overlap in physical flow time ➡ mild continuum limit

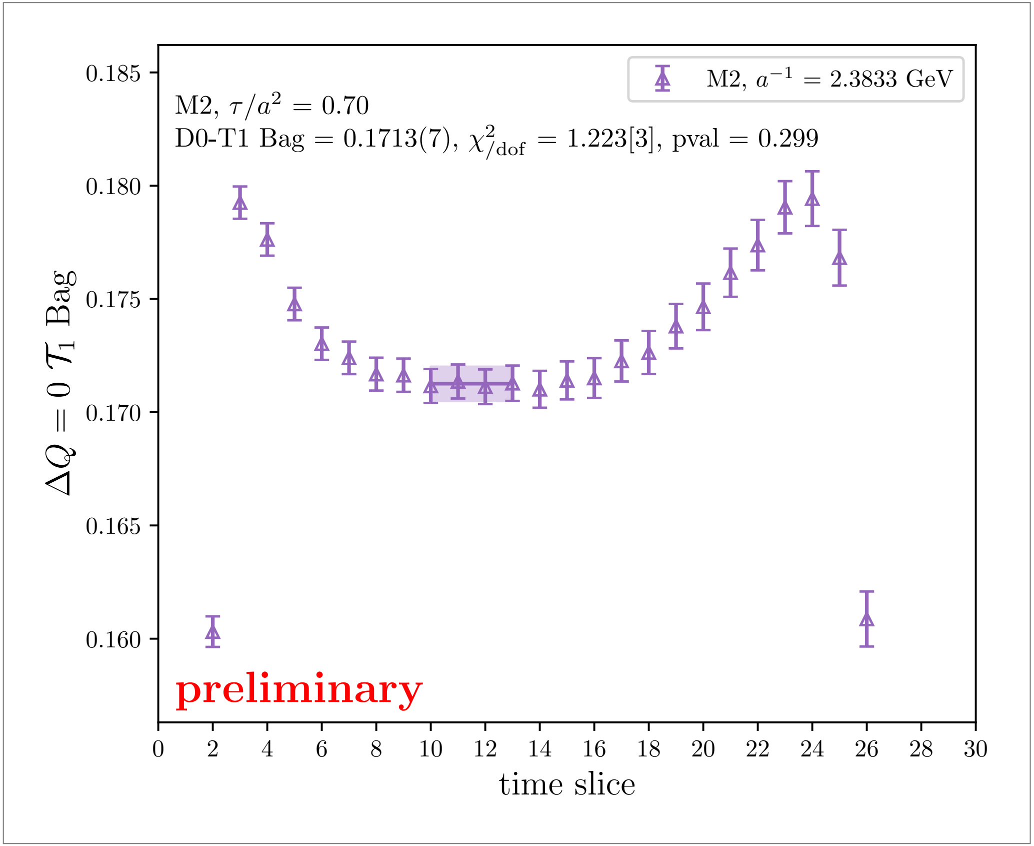




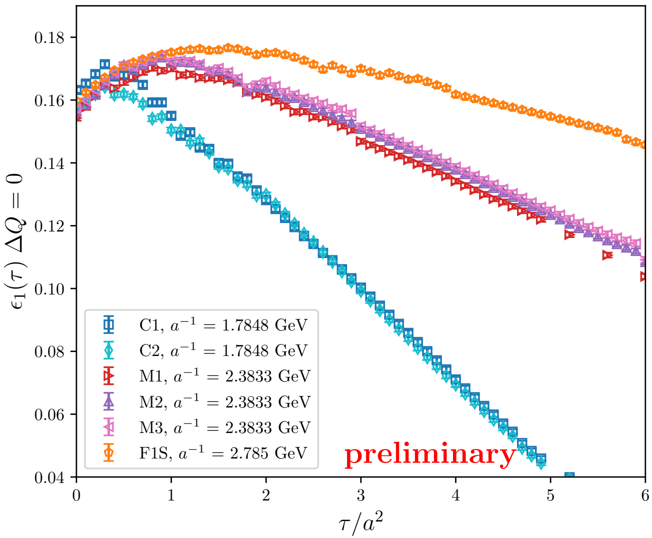
- different lattice spacings overlap in physical flow time ➡ mild continuum limit

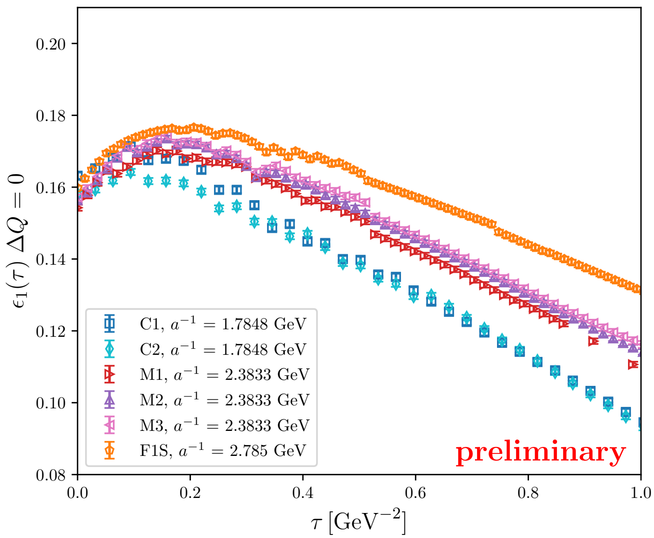
- Three-point functions for τ_i have different functional form

- asymmetric signal: $(b\bar{b}) \to (s\bar{s})$
- O_1 and T_1 mix in renormalisation
  - ➥ need both for preliminary results



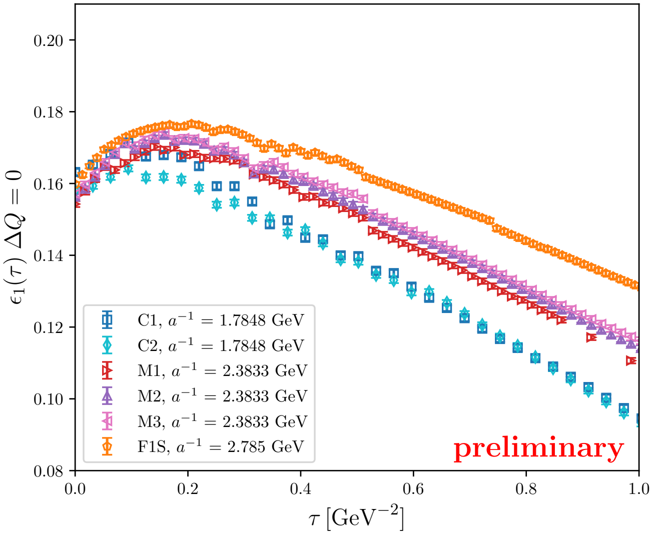
- first attempt at T_1

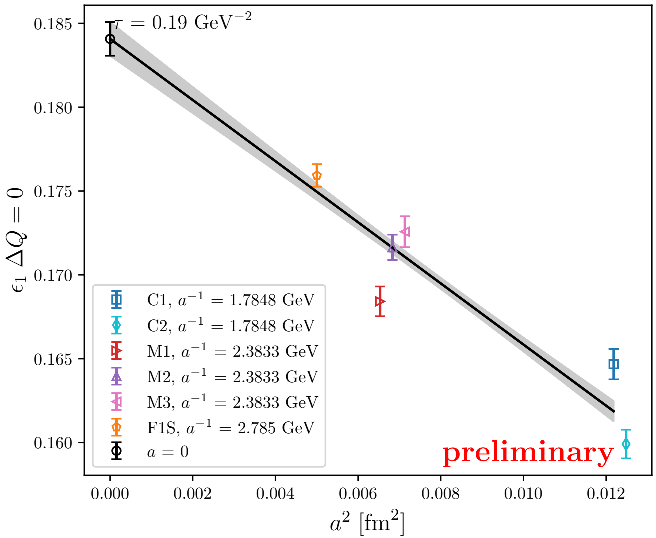




- larger systematic effects in correlator fitting

- first attempt at T_1



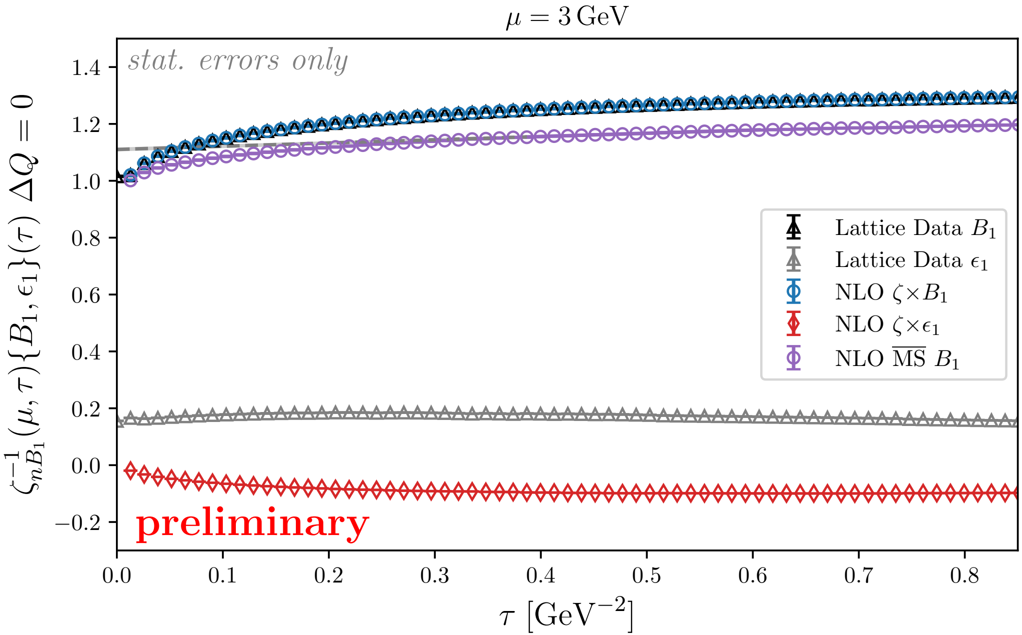


- larger systematic effects in correlator fitting
- steeper continuum limit

- Result for $B_1^{\overline{\text{MS}}}$ mixes B_1^{GF} and ϵ_1^{GF}
- Simplifications:
  - perturbative matching taken for lifetime ratios
  - missing 'eye' diagrams
- Compare existing D^0 lifetime result (HQET Sum Rules):

[Kirk, Lenz, Rauh '17]

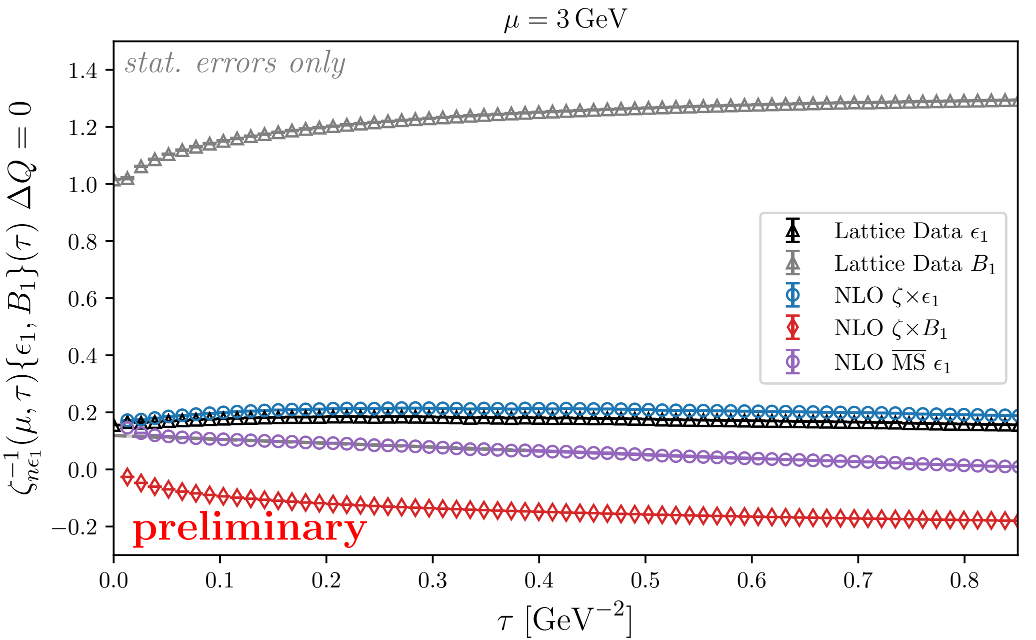
$$B_1^{\overline{\text{MS}}} = 0.902^{+0.077}_{-0.051}$$



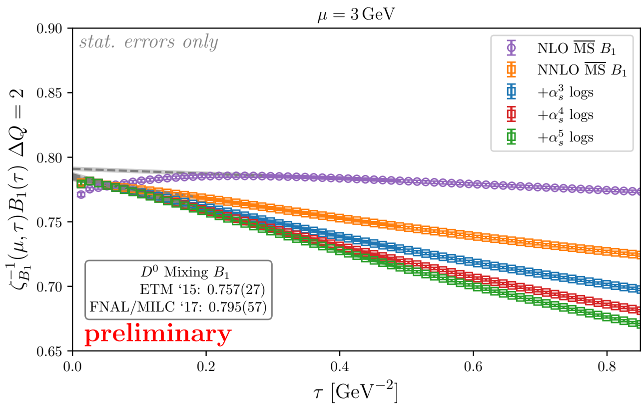
- Result for $\epsilon_1^{\overline{\text{MS}}}$ mixes ϵ_1^{GF} and B_1^{GF}
- Simplifications:
  - perturbative matching taken for lifetime ratios
  - missing 'eye' diagrams
- Compare existing D^0 lifetime result (HQET Sum Rules):

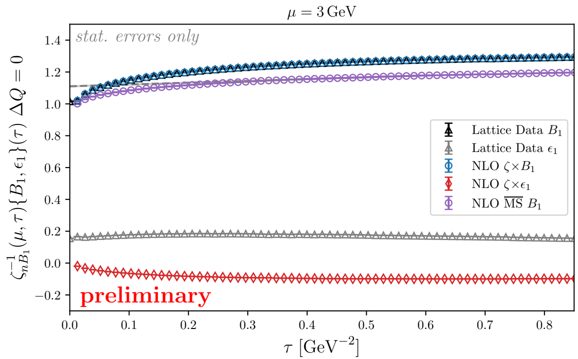
[Kirk, Lenz, Rauh '17]

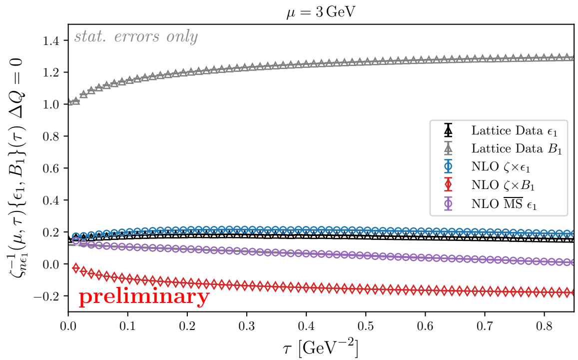
$$\epsilon_1^{\overline{\text{MS}}} = -0.132^{+0.041}_{-0.046}$$



- ➤ $\Delta B = 0$ four-quark matrix elements are strongly-desired quantities
  - ➥ We aim to use the fermionic gradient flow as a non-perturbative renormalisation procedure
- ➤ Shown first analysis for short-distance charm-strange mixing and charm-strange lifetimes







Lattice meets Continuum 3rd edition
Seminarzentrum Unteres Schloss, Universität Siegen
September 30 – October 3, 2024
https://indico.physik.uni-siegen.de/event/158/
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Alessandro Barone (U Mainz)
Vadim Baru (U Bochum)
Alessandro De Santis (U Roma Tor Vergata)
Felix Erben (CERN)
Martin Gorbahn (U Liverpool)
Christoph Hanhart (FZ Jülich)
Robert Harlander (RWTH Aachen)
Florian Herren (U Zürich)
Martin Jung (U Torino)
Takashi Kaneko (KEK)
Alexander Khodjamirian (U Siegen)
Daniel Mohler (TU Darmstadt)
Maria Laura Piscopo (U Siegen)
Fernando Romero-Lopez (MIT)
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Backup Slides

- Full BSM basis:

$$\mathcal{O}_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta, \qquad \langle \mathcal{O}_1^q \rangle = \langle \bar{B}_q | \mathcal{O}_1^q | B_q \rangle = \frac{8}{3} f_{B_q}^2 M_{B_q}^2 B_1^q$$

$$\mathcal{O}_2^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1-\gamma_5) q^\beta, \qquad \langle \mathcal{O}_2^q \rangle = \langle \bar{B}_q | \mathcal{O}_2^q | B_q \rangle = \frac{-5 M_{B_q}^2}{3(m_b + m_q)^2} f_{B_q}^2 M_{B_q}^2 B_2^q,$$

$$\mathcal{O}_3^q = \bar{b}^\alpha (1-\gamma_5) q^\beta \; \bar{b}^\beta (1-\gamma_5) q^\alpha, \qquad \langle \mathcal{O}_3^q \rangle = \langle \bar{B}_q | \mathcal{O}_3^q | B_q \rangle = \frac{M_{B_q}^2}{3(m_b + m_q)^2} f_{B_q}^2 M_{B_q}^2 B_3^q,$$

$$\mathcal{O}_4^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1+\gamma_5) q^\beta, \qquad \langle \mathcal{O}_4^q \rangle = \langle \bar{B}_q | \mathcal{O}_4^q | B_q \rangle = \left[\frac{2 M_{B_q}^2}{(m_b + m_q)^2} + \frac{1}{3} \right] f_{B_q}^2 M_{B_q}^2 B_4^q,$$

$$\mathcal{O}_5^q = \bar{b}^\alpha (1-\gamma_5) q^\beta \; \bar{b}^\beta (1+\gamma_5) q^\alpha, \qquad \langle \mathcal{O}_5^q \rangle = \langle \bar{B}_q | \mathcal{O}_5^q | B_q \rangle = \left[\frac{2 M_{B_q}^2}{3(m_b + m_q)^2} + 1 \right] f_{B_q}^2 M_{B_q}^2 B_5^q.$$

- Transformed basis (colour singlets only)

$$
\begin{aligned}
\mathcal{Q}_1^q &= \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta, \\
\mathcal{Q}_2^q &= \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1+\gamma_5) q^\beta, \\
\mathcal{Q}_3^q &= \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1+\gamma_5) q^\beta, \\
\mathcal{Q}_4^q &= \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1-\gamma_5) q^\beta, \\
\mathcal{Q}_5^q &= \frac{1}{4} \bar{b}^\alpha \sigma_{\mu\nu} (1-\gamma_5) q^\alpha \; \bar{b}^\beta \sigma_{\mu\nu} (1-\gamma_5) q^\beta
\end{aligned}
$$

$$
\begin{pmatrix} \mathcal{O}_1^+ \\ \mathcal{O}_2^+ \\ \mathcal{O}_3^+ \\ \mathcal{O}_4^+ \\ \mathcal{O}_5^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathcal{Q}_1^+ \\ \mathcal{Q}_2^+ \\ \mathcal{Q}_3^+ \\ \mathcal{Q}_4^+ \\ \mathcal{Q}_5^+ \end{pmatrix}
$$

- Advantages for both lattice calculation and the NPR procedure
- We are only concerned with parity-even components which then can be transformed back to SUSY basis

- For lifetimes, the dimension-6 $\Delta B = 0$ operators are:

$$Q_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \, \bar{q}^\beta \gamma_\mu (1-\gamma_5) b^\beta, \qquad \langle Q_1^q \rangle = \langle B_q | Q_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 \mathcal{B}_1^q,$$

$$Q_2^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \, \bar{q}^\beta (1-\gamma_5) b^\beta, \qquad \langle Q_2^q \rangle = \langle B_q | Q_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b + m_q)^2} f_{B_q}^2 M_{B_q}^2 \mathcal{B}_2^q,$$

$$T_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) (T^a)^{\alpha\beta} q^\beta \, \bar{q}^\gamma \gamma_\mu (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta, \qquad \langle T_1^q \rangle = \langle B_q | T_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 \epsilon_1^q,$$

$$T_2^q = \bar{b}^\alpha (1-\gamma_5) (T^a)^{\alpha\beta} q^\beta \, \bar{q}^\gamma (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta, \qquad \langle T_2^q \rangle = \langle B_q | T_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b + m_q)^2} f_{B_q}^2 M_{B_q}^2 \epsilon_2^q.$$

- For simplicity of computation, we transform to a colour-singlet operator basis:

$$\begin{aligned}
\mathcal{Q}_1 &= \bar{b}^\alpha \gamma_\mu (1-\gamma_5) q^\alpha \, \bar{q}^\beta \gamma_\mu (1-\gamma_5) b^\beta \\
\mathcal{Q}_2 &= \bar{b}^\alpha (1-\gamma_5) q^\alpha \, \bar{q}^\beta (1+\gamma_5) b^\beta) \\
\tau_1 &= \bar{b}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha \, \bar{q}^\beta \gamma_\mu (1-\gamma_5) q^\beta \\
\tau_2 &= \bar{b}^\alpha \gamma_\mu (1+\gamma_5) b^\alpha \, \bar{q}^\beta \gamma_\mu (1-\gamma_5) q^\beta
\end{aligned}
\qquad
\begin{pmatrix} Q_1^+ \\ Q_2^+ \\ T_1^+ \\ T_2^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{2N_c} & 0 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2N_c} & 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} \mathcal{Q}_1^+ \\ \mathcal{Q}_2^+ \\ \tau_1^+ \\ \tau_2^+ \end{pmatrix}$$

➤ We use RBC/UKQCD's 2+1 flavour DWF + Iwasaki gauge action ensembles
[Shamir '93] [Iwasaki, Yoshie '84] [Iwasaki '85]

	L	T	a^{-1}/GeV	am_l^{sea}	am_s^{sea}	M_π/MeV	srcs × N_{conf}
C1	24	64	1.7848	0.005	0.040	340	32 × 101
C2	24	64	1.7848	0.010	0.040	433	32 × 101
M1	32	64	2.3833	0.004	0.030	302	32 × 79
M2	32	64	2.3833	0.006	0.030	362	32 × 89
M3	32	64	2.3833	0.008	0.030	411	32 × 68
F1S	48	96	2.785	0.002144	0.02144	267	24 × 98

[Allton et al. '08]
[Aoki et al. '10]
[Blum et al. '14]
[Boyle et al. '17]

➤ For strange quarks tuned to physical value, $am_q \ll 1$ ✔ ➡ Shamir DWF

➤ For heavy b quarks, $am_q > 1$ ➡ large discretisation effects ✗

➥ manageable for physical c quarks instead

➥ stout-smeared Möbius DWF [Morningstar, Peardon '03] [Brower, Neff, Orginos '12] [Cho et. al '15]